CHAPTER 4

ALGEBRAIC IDENTITIES

REVISION OF KEY CONCEPTS AND FORMULAE

Following are some useful identities:

(i) $(a+b)^2 = a^2 + b^2 + 2ab$ (ii) $(a-b)^2 = a^2 + b^2 - 2ab$

(iii) $(a+b)(a-b) = a^2 - b^2$

(iv) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(v) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(vi) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(vii) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(viii) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(ix) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(x) $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\}$

(xi) If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *The square root of* $a^2 + \frac{1}{a^2} + 2$ *is*

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$ (c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$

Ans. (a)

SOLUTION We find that

$$a^2 + \frac{1}{a^2} + 2 = a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} = \left(a + \frac{1}{a}\right)^2$$

So, the square root of $a^2 + \frac{1}{a^2} + 2$ is $a + \frac{1}{a}$.

EXAMPLE 2 *The square root of* $a + \frac{1}{a} - 2$ *is*

(a) $a - \frac{1}{a}$ (b) $\sqrt{a} + \frac{1}{\sqrt{a}}$ (c) $\pm\left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)$ (d) $a + \frac{1}{a}$

Ans. (c)

SOLUTION We find that

$$a + \frac{1}{a} - 2 = (\sqrt{a})^2 + \left(\frac{1}{\sqrt{a}}\right)^2 - 2 \times \sqrt{a} \times \frac{1}{\sqrt{a}} = \left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)^2$$

$\therefore \qquad \sqrt{a+\frac{1}{a}-2} = \pm\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)$

EXAMPLE 3 *The value of* $\frac{(a+b)^2}{(b-c)(c-a)}+\frac{(b+c)^2}{(a-b)(c-a)}+\frac{(c+a)^2}{(a-b)(b-c)}$ *is*

(a) -1 (b) 0 (c) 1 (d) 2

Ans. (a)

SOLUTION We find that

$$\frac{(a+b)^2}{(b-c)(c-a)}+\frac{(b+c)^2}{(a-b)(c-a)}+\frac{(c+a)^2}{(a-b)(b-c)} = \frac{(a-b)(a+b)^2+(b-c)(b+c)^2+(c-a)(c+a)^2}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a+b)(a^2-b^2)+(b+c)(b^2-c^2)+(c+a)(c^2-a^2)}{(a-b)(b-c)(c-a)} = \frac{a^2b-ab^2+b^2c-bc^2+c^2a-ca^2}{(a-b)(b-c)(c-a)}$$

$$= \frac{(b^2c-b^2a)+(a^2b-bc^2)+(c^2a-ca^2)}{(a-b)(b-c)(c-a)} = \frac{b^2(c-a)-b(c^2-a^2)+ca(c-a)}{(a-b)(b-c)(c-a)} = \frac{(c-a)(b^2-b(c+a)+ca)}{(a-b)(b-c)(c-a)}$$

$$= \frac{(c-a)\{(b^2-bc)+(ca-ba)\}}{(a-b)(b-c)(c-a)} = \frac{(c-a)\{b(b-c)-a(b-c)\}}{(a-b)(b-c)(c-a)} = \frac{(c-a)(b-c)(b-a)}{(a-b)(b-c)(c-a)} = -1.$$

EXAMPLE 4 *The square root of the expression* $\frac{1}{abc}(a^2+b^2+c^2)+2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ *is*

(a) $\frac{a+b+c}{abc}$ (b) $\sqrt{a}+\sqrt{b}+\sqrt{c}$ (c) $\sqrt{\frac{bc}{a}}+\sqrt{\frac{ca}{b}}+\sqrt{\frac{ab}{c}}$ (d) $\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ca}}+\sqrt{\frac{c}{ab}}$

Ans. (d)

SOLUTION We have,

$$\frac{1}{abc}(a^2+b^2+c^2)+2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = \frac{1}{abc}(a^2+b^2+c^2)+2\left(\frac{ab+bc+ca}{abc}\right)$$

$$= \frac{1}{abc}(a^2+b^2+c^2+2ab+2bc+2ca) = \frac{1}{abc}(a+b+c)^2 = \left(\frac{a+b+c}{\sqrt{abc}}\right)^2 = \left(\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ca}}+\sqrt{\frac{c}{ab}}\right)^2$$

So, the square root of the given expression is $\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ca}}+\sqrt{\frac{c}{ab}}$.

EXAMPLE 5 *The square root of* $\frac{x^2}{9}+\frac{9}{4x^2}-\frac{x}{3}-\frac{3}{2x}+\frac{5}{4}$ *is*

(a) $\frac{2x}{3}+\frac{3}{2x}-\frac{1}{2}$ (b) $\frac{x}{3}-\frac{3}{2x}+1$ (c) $\frac{3}{x}+\frac{2}{3x}-\frac{1}{2}$ (d) $\frac{x}{3}+\frac{3}{2x}-\frac{1}{2}$

Ans. (d)

SOLUTION $\frac{x^2}{9}+\frac{9}{4x^2}-\frac{x}{3}-\frac{3}{2x}+\frac{5}{4}$

$$= \left(\frac{x}{3}\right)^2+\left(\frac{3}{2x}\right)^2+\left(-\frac{1}{2}\right)^2+2\left(\frac{x}{3}\right)\left(-\frac{1}{2}\right)+2\left(\frac{3}{2x}\right)\left(-\frac{1}{2}\right)+2\left(\frac{x}{3}\right)\left(\frac{3}{2x}\right) = \left(\frac{x}{3}+\frac{3}{2x}-\frac{1}{2}\right)^2$$

Hence, the required square root is $\frac{x}{3}+\frac{3}{2x}-\frac{1}{2}$.

EXAMPLE 6 *The square root of the expression* $(xy + xz - yz)^2 - 4xyz(x - y)$ *is*

(a) $xy + yz - 2xyz$ (b) $x + y - 2xyz$ (c) $xy + z - y$ (d) $xy + yz - zx$

Ans. (d)

SOLUTION We have, $(xy + xz - yz)^2 - 4xyz(x - y)$

$$= (xy + xz - yz)^2 - 4(xy)(xz) + 4(xy)(yz)$$

$$= (xy)^2 + (xz)^2 + (yz)^2 - 2(xy)(yz) - 2(xz)(yz) - 4(xy)(xz) + 4(xy)(yz) + 2(xy)(xz)$$

$$= (xy)^2 + (xz)^2 + (yz)^2 + 2(xy)(yz) - 2(xy)(xz) - 2(xz)(yz) = (xy - xz + yz)^2$$

So, required square root is $(xy - xz + yz)$

EXAMPLE 7 *The square root of* $\frac{a^2}{4} + \frac{1}{a^2} - \frac{1}{a} + \frac{a}{2} - \frac{3}{4}$ *is*

(a) $\frac{a}{2} - \frac{1}{a} + \frac{1}{2}$ (b) $\frac{a}{2} + \frac{2}{a} - 1$ (c) $\frac{a}{2} + \frac{1}{a} - \frac{1}{2}$ (d) $\frac{a}{2} - \frac{2}{a} - \frac{1}{2}$

Ans. (a)

SOLUTION $\frac{a^2}{4} + \frac{1}{a^2} - \frac{1}{a} + \frac{a}{2} - \frac{3}{4}$

$$= \left(\frac{a}{2}\right)^2 + \left(-\frac{1}{a}\right)^2 + \left(\frac{1}{2}\right)^2 + 2\left(\frac{a}{2}\right)\left(\frac{1}{2}\right) + 2\left(-\frac{1}{a}\right)\left(\frac{1}{2}\right) + 2\left(\frac{a}{2}\right)\left(-\frac{1}{a}\right) = \left(\frac{a}{2} - \frac{1}{a} + \frac{1}{2}\right)^2$$

Hence, required square root is $\left(\frac{a}{2} - \frac{1}{a} + \frac{1}{2}\right)$

EXAMPLE 8 *The expression* $(4a + 5b + 5c)^2 - (5a + 4b + 4c)^2 + 9a^2$ *is a perfect square of the expression*

(a) $\sqrt{3}(b + c)$ (b) $3(b + c - a)$ (c) $3(b + c)$ (d) $3(-b + c - a)$

Ans. (c)

SOLUTION $(4a + 5b + 5c)^2 - (5a + 4b + 4c)^2 + 9a^2$

$$= (4a + 5b + 5c + 5a + 4b + 4c)(4a + 5b + 5c - 5a - 4b - 4c) + 9a^2$$

$$= 9(a + b + c)(-a + b + c) + 9a^2$$

$$= 9\{(b + c) + a\}\{(b + c) - a\} + 9a^2 = 9\left\{(b + c)^2 - a^2\right\} + 9a^2 = 9(b + c)^2 = \{3(b + c)\}^2$$

EXAMPLE 9 *The expression* $(3a + 2b + 3c)^2 - (2a + 3b + 2c)^2 + 5b^2$ *is perfect square of the expression*

(a) $\sqrt{5}(a + b + c)$ (b) $\sqrt{5}(a + b)$ (c) $\sqrt{5}(a + c)$ (d) $\sqrt{5}(a + c - b)$

Ans. (c)

SOLUTION $(3a + 2b + 3c)^2 - (2a + 3b + 2c)^2 + 5b^2$

$$= (3a + 2b + 3c + 2a + 3b + 2c)(3a + 2b + 3c - 2a - 3b - 2c) + 5b^2$$

$$= (5a + 5b + 5c)(a - b + c) + 5b^2 = 5\{(a + c) + b\}\{(a + c) - b\} + 5b^2$$

$$= 5\{(a + c)^2 - b^2\} + 5b^2 = 5(a + c)^2 = \left\{\sqrt{5}(a + c)\right\}^2$$

EXAMPLE 10 *If* $\frac{a}{b} + \frac{b}{a} = 2$, *then* $\left(\frac{a}{b}\right)^{10} - \left(\frac{b}{a}\right)^{10}$ *is equal to*

(a) $\frac{2^{10}-1}{2^{10}}$ (b) 2 (c) 0 (d) $\frac{2^{20}+1}{2^{10}}$

Ans. (c)

SOLUTION We have, $\frac{a}{b}+\frac{b}{a}=2$

$$\Rightarrow \left(\sqrt{\frac{a}{b}}\right)^2+\left(\sqrt{\frac{b}{a}}\right)^2-2\sqrt{\frac{a}{b}}\times\sqrt{\frac{b}{a}}=0$$

$$\Rightarrow \left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2=0 \Rightarrow \sqrt{\frac{a}{b}}=\sqrt{\frac{b}{a}} \Rightarrow \frac{a}{b}=\frac{b}{a} \Rightarrow \left(\frac{a}{b}\right)^{10}=\left(\frac{b}{a}\right)^{10} \Rightarrow \left(\frac{a}{b}\right)^{10}-\left(\frac{b}{a}\right)^{10}=0.$$

EXAMPLE 11 *If* $abc=6$ *and* $a+b+c=6$, *then* $\frac{1}{ac}+\frac{1}{ab}+\frac{1}{bc}=$

(a) 2 (b) 1 (c) 3 (d) 0

Ans. (b)

SOLUTION We have, $a+b+c=6$ and $abc=6$

$$\Rightarrow \frac{a+b+c}{abc}=\frac{6}{6} \Rightarrow \frac{1}{bc}+\frac{1}{ca}+\frac{1}{ab}=1$$

EXAMPLE 12 $\sqrt{(a+b+c)^2+(a+b-c)^2+2(c^2-a^2-b^2-2ab)}$ *is equal to*

(a) $2c$ (b) $2a$ (c) $2b$ (d) $a+b+c$

Ans. (a)

SOLUTION We find that

$$(a+b+c)^2+(a+b-c)^2+2(c^2-a^2-b^2-2ab)$$

$$=2(a^2+b^2+c^2+2ab)+2(c^2-a^2-b^2-2ab)=2(2c^2)=4c^2$$

$$\therefore \sqrt{(a+b+c)^2+(a+b-c)^2+2(c^2-a^2-b^2-2ab)}=2c$$

EXAMPLE 13 *If* $\frac{a}{b}+\frac{b}{a}=-1$, *then* $a^3-b^3=$

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 0

Ans. (d)

SOLUTION We have $\frac{a}{b}+\frac{b}{a}=-1 \Rightarrow a^2+b^2+ab=0$

$$\therefore a^3-b^3=(a-b)(a^2+ab+b^2)=(a-b)\times 0=0$$

EXAMPLE 14 *If* $a+b=8$ *and* $ab=12$, *then* $a^3+b^3=$

(a) 244 (b) 288 (c) 144 (d) 284

Ans. (b)

SOLUTION We have, $a+b=8$ and $ab=12$

Now, $a+b=8 \Rightarrow (a+b)^2=64 \Rightarrow a^2+b^2+2ab=64 \Rightarrow a^2+b^2+2\times 8=64 \Rightarrow a^2+b^2=48$

$$\therefore a^3+b^3=(a+b)(a^2-ab+b^2)=8(48-12)=8\times 36=288$$

EXAMPLE 15 *If* $\left(a+\frac{1}{a}+2\right)^2=4$, *then* $a^2+\frac{1}{a^2}=$

(a) 12 (b) 13 (c) 14 (d) – 14

Ans. (c)

SOLUTION We have, $\left(a+\frac{1}{a}+2\right)^2=4$

$$\Rightarrow \quad a+\frac{1}{a}+2=\pm 2 \Rightarrow a+\frac{1}{a}=0 \text{ or, } a+\frac{1}{a}=-4$$

Now, $a+\frac{1}{a}=0 \Rightarrow a^2+1=0$, which is impossible. Therefore, $a+\frac{1}{a}\neq 0$

$$\therefore \quad a+\frac{1}{a}=-4 \Rightarrow \left(a+\frac{1}{a}\right)^2=16 \Rightarrow a^2+\frac{1}{a^2}+2=16 \Rightarrow a^2+\frac{1}{a^2}=14$$

EXAMPLE 16 *If* $x+\frac{1}{x}=7$, *then* $x^3-\frac{1}{x^3}=$

(a) $9\sqrt{5}$ (b) $144\sqrt{5}$ (c) $135\sqrt{5}$ (d) $\sqrt{5}$

Ans. (b)

SOLUTION We have, $x+\frac{1}{x}=7$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2=7^2 \Rightarrow x^2+\frac{1}{x^2}+2=49 \Rightarrow x^2+\frac{1}{x^2}=47$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=45 \Rightarrow \left(x-\frac{1}{x}\right)^2=(3\sqrt{5})^2 \Rightarrow x-\frac{1}{x}=3\sqrt{5}$$

$$\therefore \quad x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)=3\sqrt{5}\,(47+1)=144\sqrt{5}$$

EXAMPLE 17 $\frac{(a-b)^3-(a+b)^3}{2}+a\,(a^2+3b^2)=$

(a) a^3-b^3 (b) $(a+b)^3$ (c) a^3+b^3 (d) $(a-b)^3$

Ans. (d)

SOLUTION $\frac{(a-b)^3-(a+b)^3}{2}+a\,(a^2+3\,b^2)$

$$=\frac{(a^3-b^3-3a^2b+3ab^2)-(a^3+b^3+3a^2b+3ab^2)}{2}+a^3+3ab^2$$

$$=-b^3-3a^2b+a^3+3ab^2=(a-b)^3$$

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 18 *Statement-1 (Assertion):* $\sqrt{(a+b+c)^2 + (a-b+c)^2 + 2(b^2 - a^2 - c^2 - 2ac)} = 2b$

Statement-2 (Reason): $(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

Ans. (a)

SOLUTION Statement-2 is true, being a standard formula, using statement-2, we obtain

$$(a+b+c)^2 + (a-b+c)^2 + 2(b^2 - a^2 - c^2 - 2ac) = 2(a^2 + b^2 + c^2 + 2ac) + 2(b^2 - a^2 - c^2 - 2ac) = 4b^2$$

$$\therefore \quad \sqrt{(a+b+c)^2 + (a-b+c)^2 + 2(b^2 - a^2 - c^2 - 2ac)} = 2b$$

So, statement-1 is also true. Also, statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 19 *Statement-1 (Assertion):* $a^3 + b^3 + 3ab - 1 = (a+b-1)(a^2 + b^2 + a + b - ab + 1)$

Statement-2 (Reason): $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 + ab + bc + ca)$

Ans. (c)

SOLUTION Statement-2 is not true, because

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Using this formula, we obtain

$$a^3 + b^3 + (-1)^3 - 3ab(-1) = (a+b+(-1))\left(a^2 + b^2 + (-1)^2 - ab - a(-1) - b(-1)\right)$$

$$\Rightarrow \quad a^3 + b^3 + 3ab - 1 = (a+b-1)(a^2 + b^2 + a + b - ab + 1)$$

So, statement-1 is true. Hence, option (c) is correct.

EXAMPLE 20 *Statement-1 (Assertion):* $(a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$

Statement-2 (Reason): If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$

Ans. (a)

SOLUTION Statement-2, being a standard result, is ture. We find that

$$(a-b) + (b-c) + (c-a) = 0$$

Therefore, using statement-2, we obtain

$$(a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$$

So, statement-1 is true. Also, statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 21 *Statement-1 (Assertion):* $a^2 + b^2 + c^2 - ab - bc - ca = 0$ *if and only if* $a = b = c$.

Statement-2 (Reason): $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Ans. (b)

SOLUTION Statement-2, being a standard result, is ture.

Now, $\quad a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Leftrightarrow \quad 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$ [Multiplying both sides by 2]

$\Leftrightarrow \quad (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) = 0$

$\Leftrightarrow \quad (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Leftrightarrow \quad a - b = 0$ and $b - c = 0$ and $c - a = 0 \Leftrightarrow a = b = c$.

So, statement-1 is also true. Hence, option (b) is correct.

EXAMPLE 22 Statement-1 (Assertion): $a+b+c=6$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{3}{2}$, then

$$\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}=6$$

Statement-2 (Reason): $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Ans. (b)

SOLUTION Statement-2 is true.

We have, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{3}{2} \Rightarrow 2(ab+bc+ca)=3abc \Rightarrow ab+bc+ca=\frac{3}{2}abc$

Thus, we have,

$$a+b+c=6 \text{ and } ab+bc+ca=\frac{3}{2}abc$$

$$\Rightarrow (a+b+c)(ab+bc+ca)=6\times\frac{3}{2}abc$$

$$\Rightarrow a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)+3abc=9abc \Rightarrow a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)=6abc.$$

$$\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}=\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{a}{c}+\frac{c}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)=\frac{a^2+b^2}{ab}+\frac{a^2+c^2}{ca}+\frac{b^2+c^2}{bc}$$

$$=\frac{a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)}{abc}=\frac{6abc}{abc}=6$$

So, statement-1 is true. But, statement-2 is not a correct explanation for statement-1.

Hence, option (b) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. If $x+\frac{1}{x}=5$, then $x^2+\frac{1}{x^2}=$
   (a) 25 (b) 10 (c) 23 (d) 27

2. If $x+\frac{1}{x}=2$, then $x^3+\frac{1}{x^3}=$
   (a) 64 (b) 14 (c) 8 (d) 2

3. If $x+\frac{1}{x}=4$, then $x^4+\frac{1}{x^4}=$
   (a) 196 (b) 194 (c) 192 (d) 190

4. If $x+\frac{1}{x}=3$, then $x^6+\frac{1}{x^6}=$
   (a) 927 (b) 414 (c) 364 (d) 322

5. If $x^2+\frac{1}{x^2}=102$, then $x-\frac{1}{x}=$
   (a) 8 (b) 10 (c) 12 (d) 13

6. If $x^3 + \frac{1}{x^3} = 110$, then $x + \frac{1}{x} =$
(a) 5 (b) 10 (c) 15 (d) none of these

7. If $x^3 - \frac{1}{x^3} = 14$, then $x - \frac{1}{x} =$
(a) 5 (b) 4 (c) 3 (d) 2

8. If $a + b + c = 9$ and $ab + bc + ca = 23$, then $a^2 + b^2 + c^2 =$
(a) 35 (b) 58 (c) 127 (d) none of these

9. $(a-b)^3 + (b-c)^3 + (c-a)^3 =$
(a) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ (b) $(a-b)(b-c)(c-a)$
(c) $3(a-b)(b-c)(c-a)$ (d) none of these

10. If $a + b = 3$ and $ab = 2$, then $a^3 + b^3 =$
(a) 6 (b) 4 (c) 9 (d) 12

11. If $a - b = -8$ and $ab = -12$, then $a^3 - b^3 =$
(a) -244 (b) -240 (c) -224 (d) -260

12. If the volume of a cuboid is $3x^2 - 27$, then its possible dimensions are
(a) $3, x^2, -27x$ (b) $3, x-3, x+3$ (c) $3, x^2, 27x$ (d) $3, 3, 3$

13. $75 \times 75 + 2 \times 75 \times 25 + 25 \times 25$ is equal to
(a) 10000 (b) 6250 (c) 7500 (d) 3750

14. $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$ is equal to
(a) $x^{16} - y^{16}$ (b) $x^8 - y^8$ (c) $x^8 + y^8$ (d) $x^{16} + y^{16}$

15. If $x^4 + \frac{1}{x^4} = 623$, then $x + \frac{1}{x} =$
(a) 27 (b) 25 (c) $3\sqrt{3}$ (d) $-3\sqrt{3}$

16. If $x^4 + \frac{1}{x^4} = 194$, then $x^3 + \frac{1}{x^3} =$
(a) 76 (b) 52 (c) 64 (d) none of these

17. If $x - \frac{1}{x} = \frac{15}{4}$, then $x + \frac{1}{x} =$
(a) 4 (b) $\frac{17}{4}$ (c) $\frac{13}{4}$ (d) $\frac{1}{4}$

18. If $3x + \frac{2}{x} = 7$, then $\left(9x^2 - \frac{4}{x^2}\right) =$
(a) 25 (b) 35 (c) 49 (d) 30

19. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then
(a) $a + b = c$ (b) $b + c = a$ (c) $c + a = b$ (d) $a = b = c$

20. If $a+b+c=0$, then $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=$

(a) 0 (b) 1 (c) -1 (d) 3

21. If $a^{1/3}+b^{1/3}+c^{1/3}=0$, then

(a) $a+b+c=0$ (b) $(a+b+c)^3=27abc$

(c) $a+b+c=3abc$ (d) $a^3+b^3+c^3=0$

22. If $a+b+c=9$ and $ab+bc+ca=23$, then $a^3+b^3+c^3-3abc=$

(a) 108 (b) 207 (c) 669 (d) 729

23. $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}=$

(a) $3(a+b)(b+c)(c+a)$ (b) $3(a-b)(b-c)(c-a)$

(c) $(a-b)(b-c)(c-a)$ (d) $(a+b)(b+c)(c+a)$

24. The product $(a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$ is equal to

(a) a^6+b^6 (b) a^6-b^6 (c) a^3-b^3 (d) a^3+b^3

25. The product $(x^2-1)(x^4+x^2+1)$ is equal to

(a) x^8-1 (b) x^8+1 (c) x^6-1 (d) x^6+1

26. If $\frac{a}{b}+\frac{b}{a}=1$, then $a^3+b^3=$

(a) 1 (b) -1 (c) 1/2 (d) 0

27. If $49a^2-b=\left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)$, then the value of b is

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

28. One of the factors of $(5x+1)^2-(5x-1)^2$ is

(a) $5+x$ (b) $5-x$ (c) $5x-1$ (d) $20x$

29. If $9x^2-b=\left(3x+\frac{1}{2}\right)\left(3x-\frac{1}{2}\right)$, then the value of b is

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

30. The coefficient of x in $(x+3)^3$ is

(a) 1 (b) 9 (c) 18 (d) 27

31. The value of 249^2-248^2 is

(a) 1 (b) 477 (c) 487 (d) 497

32. Which of the following is a factor of $(x+y)^3-(x^3+y^3)$?

(a) $x^2+2xy+y^2$ (b) x^2-xy+y^2 (c) xy^2 (d) $3xy$

33. If $\frac{x}{y}+\frac{y}{x}=-1\,(x,y\neq 0)$, the value of x^3-y^3 is

(a) 1 (b) –1 (c) 0 (d) $\frac{1}{2}$

34. If $x+y=2$ and $xy=1$, then $x^4+y^4=$

(a) 6 (b) 4 (c) 8 (d) 2

35. If $x^2+y^2+xy=1$ and $x+y=2$, then $xy=$

(a) –3 (b) 3 (c) $-\frac{3}{2}$ (d) 0

36. If a, b, c are natural numbers such that $a^2+b^2+c^2=29$ and $ab+bc+ca=26$, and $a+b+c=$ ____.

(a) 9 (b) 6 (c) 7 (d) 10

37. If $2x+\frac{y}{3}=12$ and $xy=30$, then $8x^3+\frac{y^3}{27}=$ ______

(a) 1008 (b) 168 (c) 106 (d) none of these

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

38. Statement-1 (Assertion): If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Statement-2 (Reason): $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

39. Statement-1 (Assertion): $(a+b+c)^2=a^2+b^2+c^2-2(ab+bc+ca)$

Statement-2 (Reason): $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

40. Statement-1 (Assertion): $a^3+\frac{3}{8}ax+\frac{1}{64}x^3-\frac{1}{8}=\left(a+\frac{x}{4}-\frac{1}{2}\right)\left(a^2+\frac{x^2}{16}+\frac{1}{4}-\frac{ax}{4}+\frac{x}{8}+\frac{a}{2}\right)$

Statement-2 (Reason): $a^3+b^3+c^3+3abc=(a+b+c)(a^2+b^2+c^2+ab+bc+ca)$

41. Statement-1 (Assertion): If $a+b+c=6$, $ab+bc+ca=11$, then $a^2+b^2+c^2=14$

Statement-2 (Reason): $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

42. Statement-1 (Assertion): $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}=(x+y)(y+z)(z+x)$

Statement-2 (Reason): If $a+b+c=0$, then $a^3+b^3+c^3=3abc$.

43. Statement-1 (Assertion): The square root of $\frac{1}{abc}(a^2+b^2+c^2)+2\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is $\sqrt{\frac{a}{bc}}+\sqrt{\frac{b}{ca}}+\sqrt{\frac{c}{ab}}$.

Statement-2 (Reason): $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

ANSWERS

1. (c)	2. (d)	3. (b)	4. (d)	5. (b)	6. (a)	7. (d)
8. (a)	9. (c)	10. (c)	11. (c)	12. (b)	13. (a)	14. (b)
15. (c)	16. (b)	17. (b)	18. (b)	19. (d)	20. (d)	21. (b)
22. (a)	23. (d)	24. (b)	25. (c)	26. (d)	27. (b)	28. (d)
29. (c)	30. (d)	31. (d)	32. (d)	33. (c)	34. (d)	35. (b)
36. (a)	37. (a)	38. (a)	39. (d)	40. (c)	41. (a)	42. (a)
43. (b)						