CHAPTER 6

FACTORIZATION OF POLYNOMIALS

REVISION OF KEY CONCEPTS AND FORMULAE

1. An algebraic expression of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0.$$

where $a_0, a_1, a_2, \ldots, a_n$ are constants, is known as a polynomial in variable x.

2. In the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0$ each of $a_n x^n$, $a_{n-1} x^{n-1}, \ldots, a_1 x, a_0$ is called its term and $a_n x^n, a_n \neq 0$ called the leading term. a_0 is known as the constant term.

3. A polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0$ is of degree n, if $a_n \neq 0$.

4. A polynomial of degree 1 is called a linear polynomial. For example, $f(x) = ax + b,\ a \neq 0$ is a linear polynomial.

5. A polynomial of degree 2 is called a quadratic polynomial. Thus, $f(x) = ax^2 + bx + c,\ a \neq 0$ is the general form of a quadratic polynomial.

6. A polynomial of degree 3 is called a cubic polynomial. Thus, $f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$ is the general form of a cubic polynomial.

7. A real number α is a zero (or root) of a polynomial $f(x)$, if $f(\alpha) = 0$.

8. A polynomial of degree n has n roots.

9. A linear polynomial $f(x) = ax + b, a \neq 0$ has a unique root given by $x = -\frac{b}{a}$.

10. A non-zero constant polynomial has no root.

11. Every real number is a root of the zero polynomial.

12. If $f(x)$ is a polynomial with integral coefficients and the leading coefficient is 1, then any integer root of $f(x)$ is a factor of the constant term.

13. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0, a_n \neq 0$ be a polynomial. Then, $\frac{b}{c}$ (a rational fraction in lowest terms) is a root of $f(x)$, if b is a factor of constant term a_0 and c is a factor of the leading term a_n.

14. *Remainder Theorem*: Let $f(x)$ be a polynomial of degree greater than or equal to one and a be any real number. If $f(x)$ is divisible by $(x - a)$, then the remainder is equal to $f(a)$.

15. *Factor Theorem*: Let $f(x)$ be a polynomial of degree greater than or equal to one and a be real number such that $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$. Conversely, if $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

16. (i) If a polynomial $p(x)$ is divided by $(x + a)$, the remainder is the value of $p(x)$ at $x = -a$ i.e. $p(-a)$.

(ii) If a polynomial $p(x)$ is divided by $(ax-b)$, the remainder is the value of $p(x)$ at $x = b/a$ i.e. $p(b/a)$.

(iii) If a polynomial $p(x)$ is divided by $(ax+b)$, the remainder is the value of $p(x)$ at $x = -b/a$ i.e. $p(-b/a)$.

(iv) If a polynomial $p(x)$ is divided by $(b-ax)$, the remainder is the value of $p(x)$ at $x = b/a$ i.e. $p(b/a)$.

17. (i) $(x-a)$ is a factor of a polynomial $f(x)$ if and only if $f(a) = 0$.

(ii) If $f(-a) = 0$, then $(x+a)$ is a factor of a polynomial $f(x)$.

(iii) If $f\left(\frac{b}{a}\right) = 0$, then $(ax-b)$ is a factor of a polynomial $f(x)$.

(iv) If $f\left(-\frac{b}{a}\right) = 0$, then $(ax+b)$ is a factor of a polynomial $f(x)$.

18. (i) If sum of all the coefficients of a polynomial is zero, then $(x-1)$ is one of its factors.

(ii) If sum of the coefficients of odd powers of x is equal to the sum of the coefficients of even powers of x in a polynomial $f(x)$, then $(x+1)$ is one of the factors of $f(x)$.

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *Which one of the following is a polynomial?*

(a) $f(x) = x + \frac{1}{x}$ (b) $g(x) = \frac{(x-1)(x-3)}{x}$

(c) $h(x) = \frac{x+2}{x+1}$ (d) $p(x) = 2x^2 + \frac{5x^{3/2} + 4\sqrt{x}}{\sqrt{x}}$

Ans. (d)

SOLUTION $f(x) = x + \frac{1}{x}$ may be written as $f(x) = x + x^{-1}$. We find that in one term the exponent of x is -1, which is a negative integer. So, $f(x)$ is not a polynomial.

In $g(x) = \frac{(x-1)(x-3)}{x} = \frac{x^2 - 4x + 3}{x} = x - 4 + 3x^{-1}$, there is one term in which the exponent of x is -1. So, it is not a polynomial.

In $h(x) = \frac{x+2}{x+1} = \frac{(x+1)+1}{x+1} = 1 + (x+1)^{-1}$, one term contains $(x+1)^{-1}$. So, it is not a polynomial.

$p(x) = 2x^2 + \frac{5x^{3/2} + 4\sqrt{x}}{\sqrt{x}} = 2x^2 + 5x + 4$, clearly, it is a polynomial.

EXAMPLE 2 $\sqrt{3}$ *is a polynomial of degree*

(a) 2 (b) 0 (c) 1 (d) 1/2

Ans. (b)

SOLUTION $f(x) = \sqrt{3}$ is a constant polynomial. The degree of a non-zero constant polynomial is zero.

EXAMPLE 3 *Which of the following is a polynomial?*

(a) $x^{-2}+2x^{-1}+3$ (b) $x+x^{-1}+5$ (c) $2x^{-1}$ (d) 0

Ans. (d)

SOLUTION Expressions in first three options contain negative powers of variable x. So, none of them is a polynomial. In option (d), 0 is zero polynomial whose degree is not defined.

EXAMPLE 4 *If $x^{101}+101$ is divided by $x+1$, then the remainder is*

(a) 0 (b) 100 (c) 101 (d) 1

Ans. (b)

SOLUTION Let $f(x)=x^{101}+101$. If $f(x)$ is divided by $x+1$, then

$$\text{Remainder} = f(-1) = (-1)^{101} + 101 = -1 + 101 = 100$$

EXAMPLE 5 *If $f(x)=x^{100}+2x^{99}+k$ is divisible by $(x+1)$, then the value of k is*

(a) 1 (b) 2 (c) – 2 (d) – 3

Ans. (a)

SOLUTION If $f(x)$ is divisible by $(x+1)$, then

$$f(-1)=0 \Rightarrow (-1)^{100}+2(-1)^{99}+k=0 \Rightarrow 1-2+k=0 \Rightarrow k=1$$

EXAMPLE 6 *The remainder when $f(x)=x^3-2x^2+6x-2$ is divided by $(x-2)$, is*

(a) 5 (b) 8 (c) – 10 (d) 10

Ans. (a)

SOLUTION $f(2)$ is the remainder when $f(x)$ is divided by $x+2$.

Hence, remainder $= f(2) = 2^3 - 2\times 2^2 + 6\times 2 - 2 = 10$

EXAMPLE 7 *The remainder when $f(x)=x^3+ax^2+6x+a$ is divided by $(x+a)$, is*

(a) $-5a$ (b) $5a$ (c) $10a$ (d) 0

Ans. (a)

SOLUTION If $f(x)$ is divided by $(x+a)$, the remainder is $f(-a)$.

$\therefore$ Remainder $= f(-a) = (-a)^3 + a(-a)^2 + 6(-a) + a = -a^3 + a^3 - 6a + a = -5a$

EXAMPLE 8 *If $(x-a)$ is a factor of the polynomial $p(x)=x^3-ax^2+2x+a-6$, then the value of a is*

(a) 1 (b) – 1 (c) 2 (d) – 2

Ans. (c)

SOLUTION If $(x-a)$ is a factor of $p(x)$, then

$$p(a)=0 \Rightarrow a^3-a^3+2a+a-6=0 \Rightarrow 3a-6=0 \Rightarrow a=2$$

EXAMPLE 9 *If $f(x+3)=x^2-7x+2$, then the remainder when $f(x)$ is divided by $(x+1)$ is*

(a) 8 (b) – 4 (c) 20 (d) 46

Ans. (d)

SOLUTION We have, $f(x+3)=x^2-7x+2$. Let $x+3=\alpha$. Then, $x=\alpha-3$.

Replacing x by $\alpha-3$, we obtain

$$f(\alpha)=(\alpha-3)^2-7(\alpha-3)+2 \text{ or, } f(\alpha)=\alpha^2-13\alpha+32$$

Thus, we obtain $f(x) = x^2 - 13x + 32$.

The remainder when $f(x)$ is divided by $x + 1$ is $f(-1) = 1 + 13 + 32 = 46$.

ALITER Putting $x + 3 = -1$ i.e. $x = -4$ in $f(x+3) = x^2 - 7x + 2$, we obtain: $f(-1) = 16 + 28 + 2 = 46$.

EXAMPLE 10 *If* $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, *the remainder when* $f(x)$ *is divided by* $x - 3$ *is*

(a) 10 (b) 11 (c) 7 (d) 5

Ans. (c)

SOLUTION We have, $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

or, $f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2$ i.e. $f(u) = u^2 - 2$, where $u = x + \frac{1}{x}$

The remainder when $f(u) = u^2 - 2$ is divided by $u - 3$ is $f(3) = 3^2 - 2 = 7$.

EXAMPLE 11 *If* $f(x+1) = 2x^2 + 7x + 5$, *then one of the factors of* $f(x)$ *is*

(a) $2x + 3$ (b) $3x + 2$ (c) $2x - 3$ (d) $3x - 2$

Ans. (a)

SOLUTION We have,

$$f(x+1) = 2x^2 + 7x + 5$$

$$\Rightarrow f(u) = 2(u-1)^2 + 7(u-1) + 5, \text{ where } x + 1 = u \text{ or, } x = u - 1$$

$$\Rightarrow f(u) = 2u^2 + 3u \Rightarrow f(u) = u(2u+3) \Rightarrow f(x) = x(2x+3)$$

Hence, $2x + 3$ is a factor of $f(x)$.

EXAMPLE 12 *If* $(x - 2)$ *is a factor of* $f(x) = x^2 + ax + 1$, *then the remainder when* $x^2 + ax + 1$ *is divided by* $(2x + 3)$, *is*

(a) 7 (b) 8 (c) 1 (d) 0

Ans. (a)

SOLUTION If $(x - 2)$ is a factor of $f(x)$, then

$$f(2) = 0 \Rightarrow 2^2 + 2a + 1 = 0 \Rightarrow a = -\frac{5}{2}$$

The remainder when $f(x)$ is divided by $2x + 3$ is

$$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + a\left(-\frac{3}{2}\right) + 1 = \frac{9}{4} - \frac{3}{2} \times -\frac{5}{2} + 1 = 7$$

EXAMPLE 13 *If* $(x - 3)$ *is a factor of* $f(x) = x^2 + a$, *then the remainder when* $f(x)$ *is divided by* $(x - 2)$ *is*

(a) 5 (b) -5 (c) 13 (d) -13

Ans. (b)

SOLUTION Given that $(x - 3)$ is a factor of $f(x) = x^2 + a$

$$\therefore \quad f(3) = 0 \Rightarrow 3^2 + a = 0 \Rightarrow a = -9$$

Thus, $f(x) = x^2 - 9$

The remainder when $f(x)$ is divided by $(x-2)$ is: $f(2) = 2^2 - 9 = -5$.

EXAMPLE 14 *If* $(2x-1)$ *is a factor of* $f(x) = 2z^2 + ax - 2$, *then the other factor of* $f(x)$ *is*

(a) $x-2$ (b) $x+2$ (c) $x-1$ (d) $x+1$

Ans. (b)

SOLUTION Given that $(2x-1)$ is a factor of $f(x)$

$$\therefore \quad f\left(\frac{1}{2}\right) = 0 \Rightarrow 2\left(\frac{1}{2}\right)^2 + \frac{a}{2} - 2 = 0 \Rightarrow a = 3$$

$$\therefore \quad f(x) = 2x^2 + 3x - 2 \Rightarrow f(x) = (2x-1)(x+2)$$

Hence, $(x+2)$ is the other factor of $f(x)$.

EXAMPLE 15 *If* $(x+1)$ *and* $(x-1)$ *are factors of* $f(x) = ax^3 + bx^2 + cx + d$, *then*

(a) $a+b=0$ (b) $b+c=0$ (c) $b+d=0$ (d) $a+d=0$

Ans. (c)

SOLUTION Given that $(x-1)$ and $(x+1)$ are factors of $f(x)$.

$$\therefore \quad f(1) = 0 \text{ and } f(-1) = 0$$

$$\Rightarrow \quad a+b+c+d = 0 \text{ and } -a+b-c+d = 0$$

$$\Rightarrow \quad 2(b+d) = 0 \text{ and } 2(a+c) = 0 \qquad \text{[On adding and subtracting]}$$

$$\Rightarrow \quad b+d = 0 \text{ and } a+c = 0$$

EXAMPLE 16 *When the polynomial* $p(x) = ax^2 + bx + c$ *is divided by* $(x-1)$ *and* $(x+1)$, *the remainders obtained are* 6 *and* 10 *respectively. If the value of* $p(x)$ *at* $x = 0$ *is* 5, *then* $5a - 2b + 5c =$

(a) 21 (b) 40 (c) 42 (d) 44

Ans. (d)

SOLUTION It is given that

$$p(1) = 6, p(-1) = 10 \text{ and } p(0) = 5$$

$$\Rightarrow \quad a+b+c = 6, a-b+c = 10 \text{ and } c = 5 \Rightarrow a+b = 1, a-b = 5 \text{ and } c = 5$$

$$\Rightarrow \quad a = 3, b = -2 \text{ and } c = 5$$

$$\therefore \quad 5a - 2b + 5c = 15 + 4 + 24 = 44$$

EXAMPLE 17 *If* $f(x+3) = x^2 + x - 6$, *then one of the factors of* $f(x)$ *is*

(a) $x-3$ (b) $x-4$ (c) $x-5$ (d) $x-6$

Ans. (c)

SOLUTION We have, $f(x+3) = x^2 + x - 6$

Let $x + 3 = u$. Then $x = u - 3$. Putting $x = u - 3$ in $f(x) = x^2 + x - 6$, we obtain

$$f(u) = (u-3)^2 + (u-3) - 6 \Rightarrow f(u) = u^2 - 5u \text{ or, } f(u) = u(u-5)$$

Thus, we obtain $f(x) = x(x-5)$.

Hence, x and $x-5$ are factors of $f(x)$.

ALITER We have, $f(x+3)=x^2+x-6$ or, $f(x+3)=(x+3)(x-2)$

Replacing x by $x-3$, we obtain: $f(x)=x(x-5)$

EXAMPLE 18 *The ratio of the remainders when* $f(x)=x^2+ax+b$ *is divided by* $(x-2)$ *and* $(x-1)$ *respectively is* $4:3$. *If* $(x+1)$ *is a factor of* $f(x)$, *then*

(a) $a=9, b=-10$ (b) $a=-9, b=10$ (c) $a=9, b=10$ (d) $a=-9, b=-10$

Ans. (d)

SOLUTION It is given that

$$\frac{f(2)}{f(1)}=\frac{4}{3} \text{ and } f(-1)=0 \Rightarrow \frac{4+2a+b}{1+a+b}=\frac{4}{3} \text{ and } 1-a+b=0$$

$$\Rightarrow \quad 2a-b+8=0 \text{ and } -a+b+1=0 \Rightarrow a=-9, b=-10.$$

EXAMPLE 19 *If a quadratic polynomial* $f(x)$ *leaves remainders 4, 4 and 0 respectively when divided by* $(x-1), (x-2)$ *and* $(x-3)$ *respectively, then* $f(x)=$

(a) $-2x^2+6x+3$ (b) $-2x^2+6x$ (c) $-2x^2+6x+5$ (d) $-2x^2+6x-5$

Ans. (b)

SOLUTION Let $f(x)=ax^2+bx+c$. It is given that

$$f(1)=4, f(2)=4 \text{ and } f(3)=0$$

$$\Rightarrow \quad a+b+c=4, 4a+2b+c=4 \text{ and } 9a+3b+c=0 \Rightarrow a=-2, b=6, c=0$$

Hence, $f(x)=-2x^2+6x$.

EXAMPLE 20 *The remainder when* $f(x)=x^5$ *is divided by* $g(x)=x^2-9$, *is*

(a) $81x$ (b) $81x+10$ (c) $243x+81$ (d) 0

Ans. (a)

SOLUTION Since $g(x)=x^2-9$ is a quadratic polynomial. Therefore, when $f(x)$ is divided by $g(x)$ the remainder is a linear polynomial. Let $f(x)=ax+b$ be the remainder and $q(x)$ be the quotient.

$$f(x)=q(x)g(x)+r(x) \Rightarrow x^5=(x^2-9)q(x)+ax+b \quad \text{...(i)}$$

Zeroes of $g(x)$ are given by

$$g(x)=0 \Rightarrow x^2-9=0 \Rightarrow (x-3)(x+3)=0 \Rightarrow x=-3, 3$$

Putting $x=-3$ and $x=3$ successively in (i), we obtain

$$-243=-3a+b \text{ and } 243=3a+b \Rightarrow a=81 \text{ and } b=0$$

Hence, $r(x)=81x$.

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 21 Statement-1 (Assertion): *If $a \neq 0$ and $ax^2 + bx + a$ is exactly divisible by $(x - a)$, then $a^2 + b + 1 = 0$.*

Statement-2 (Reason): *If $(x - a)$ is a factor of a polynomial $f(x)$, then $f(a) = 0$.*

Ans. (a)

SOLUTION Statement-2 is true (see Factor Theorem). Using statement-2, if $f(x) = ax^2 + bx + a$ is exactly divisible by $(x - a)$, then

$$f(a) = 0 \Rightarrow a^3 + ba + a = 0 \Rightarrow a^2 + b + 1 = 0 \qquad [\because a \neq 0]$$

So, statement-1 is also true and statement-2 is a correct explanation for statement-1.

Hence, option (a) is correct.

EXAMPLE 22 Statement-1 (Assertion): *If $x + 7$ is a factor of $f(x) = x^2 + 11x - 2a$, then $a = -14$.*

Statement-2 (Reason): *If $x + a$ is a factor of a polynomial, then $f(a) = 0$.*

Ans. (c)

SOLUTION If $x + 7$ is a factor of $f(x) = x^2 + 11x - 2a$, then $f(-7) = 0$.

$$\therefore \quad (-7)^2 + 11(-7) - 2a = 0 \Rightarrow 49 - 77 - 2a = 0 \Rightarrow -28 - 2a = 0 \Rightarrow a = -14$$

Thus, statement-1 is true but statement-2 is not true. Hence, option (c) is correct.

EXAMPLE 23 Statement-1 (Assertion): *If the polynomial $f(x) = 2x^3 + 3x^2 - 5x + a$ when divided by $x + 2$ leaves the remainder $3a + 2$, then $a = 2$.*

Statement-2 (Reason): *The remainder when a polynomial $p(x)$ is divided by $(x - a)$ is given by $p(a)$.*

Ans. (a)

SOLUTION Statement-2 is the remainder theorem. So, it is true. Using statement-2, the remainder when $f(x) = 2x^3 + 3x^2 - 5x + a$ is divided by $x + 2$, is $f(-2)$. But, it is given that the remainder is $3a + 2$

$$\therefore \quad f(-2) = 3a + 2$$

$$\Rightarrow \quad 2 \times (-2)^3 + 3(-2)^2 - 5(-2) + a = 3a + 2$$

$$\Rightarrow \quad -16 + 12 + 10 + a = 3a + 2 \Rightarrow a + 6 = 3a + 2 \Rightarrow a = 2$$

Thus, both the statements are true and statement-2 is a correct explanation for statement-1.

EXAMPLE 24 Statement-1 (Assertion): *If sum of all the coefficients, including the constant term, of a polynomial is zero, then $(x - 1)$ is one of its factors.*

Statement-2 (Reason): *If a polynomial $f(x)$ is divisible by $(x - \alpha)$, then $f(\alpha) = 0$.*

Ans. (a)

SOLUTION If a polynomial $f(x)$ is divisible by $(x - \alpha)$, then $(x - \alpha)$ is one of its factors. Therefore, $f(\alpha) = 0$. So, statement-2 is true.

Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$ be a polynomial such that the sum of all the coefficients is zero.

i.e. $\quad a_0 + a_1 + a_2 + \ldots + a_{n-2} + a_{n-1} + a_n = 0$

$\Rightarrow \quad f(1) = 0$

$\Rightarrow \quad (x - 1)$ is a factor of $f(x)$ or, $f(x)$ is divisible by $(x - 1)$.

So, statement-1 is true.

Thus, both the statements are true and statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 25 *Statement-1 (Assertion): If $f(x+2) = 2x^2 + x - 3$ is divided by $(x-1)$, the remainder is 2.*

Statement-2 (Reason): If $f(x)$ is divided by $(2-3x)$, the remainder is $f(2/3)$.

Ans. (d)

SOLUTION Using remainder theorem, we find that if $f(x)$ is divided by $(2-3x)$, the remainder is $f(2/3)$. So, statement-2 is true.

Now, $f(x+2) = 2x^2 + x - 3$

$\Rightarrow$ $f(u) = 2(u-2)^2 + (u-2) - 3$, where $u = x + 2$

$\Rightarrow$ $f(u) = 2u^2 - 7u + 3 \Rightarrow f(x) = 2x^2 - 7x + 3$

So, the remainder when $f(x)$ is divided by $(x-1)$ is $f(1) = 2 - 7 + 3 = -2$.

ALITER We have, $f(x+2) = 2x^2 + x - 3 = 2(x+2)^2 - 7(x+2) + 3$

Replacing $x + 2$ by -1, we obtain: $f(-1) = 2 - 7 + 3 = -2$.

So, statement-1, is not true. Hence, option (d) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. Which one of the following is a polynomial?

   (a) $\frac{x^2}{2} - \frac{2}{x^2}$ (b) $\sqrt{2x} - 1$ (c) $x^2 + \frac{3x^{3/2}}{\sqrt{x}}$ (d) $\frac{x-1}{x+1}$

   [NCERT EXEMPLAR]

2. Degree of the polynomial $f(x) = 4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

   (a) 4 (b) 5 (c) 3 (d) 7

   [NCERT EXEMPLAR]

3. Degree of the zero polynomial is

   (a) 0 (b) 1 (c) any natural number (d) not defined

   [NCERT EXEMPLAR]

4. $\sqrt{2}$ is a polynomial of degree

   (a) 2 (b) 0 (c) 1 (d) $\frac{1}{2}$

   [NCERT EXEMPLAR]

5. Zero of the zero polynomial is

   (a) 0 (b) 1 (c) any real number (d) not defined

   [NCERT EXEMPLAR]

6. If $f(x) = x + 3$, then $f(x) + f(-x)$ is equal to

   (a) 3 (b) $2x$ (c) 0 (d) 6

   [NCERT EXEMPLAR]

7. Zero of the polynomial $f(x) = 3x + 7$ is

(a) $\frac{7}{3}$ (b) $\frac{-3}{7}$ (c) $-\frac{7}{3}$ (d) -7

8. One of the zeros of the polynomial $f(x) = 2x^2 + 7x - 4$ is

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

9. If $f(x) = x^2 - 2\sqrt{2}x + 1$, then $f(2\sqrt{2})$ is equal to

(a) 0 (b) 1 (c) $4\sqrt{2}$ (d) $8\sqrt{2} + 1$

[NCERT EXEMPLAR]

10. $x + 1$ is a factor of the polynomial

(a) $x^3 + x^2 - x + 1$ (b) $x^3 + x^2 + x + 1$

(c) $x^4 + x^3 + x^2 + 1$ (d) $x^4 + 3x^3 + 3x^2 + x + 1$

[NCERT EXEMPLAR]

11. If $x^2 + kx + 6 = (x + 2)(x + 3)$ for all x, then the value of k is

(a) 1 (b) -1 (c) 5 (d) 3

[NCERT EXEMPLAR]

12. If $x - 2$ is a factor of $x^2 + 3ax - 2a$, then $a =$

(a) 2 (b) -2 (c) 1 (d) -1

13. If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $x + 2$, then $k =$

(a) -6 (b) -7 (c) -8 (d) -10

14. If $x - a$ is a factor of $x^3 - 3x^2a + 2a^2x + b$, then the value of b is

(a) 0 (b) 2 (c) 1 (d) 3

15. If $x^{140} + 2x^{151} + k$ is divisible by $x + 1$, then the value of k is

(a) 1 (b) -3 (c) 2 (d) -2

16. If $x + 2$ is a factor of $x^2 + mx + 14$, then $m =$

(a) 7 (b) 2 (c) 9 (d) 14

17. If $x - 3$ is a factor of $x^2 - ax - 15$, then $a =$

(a) -2 (b) 5 (c) -5 (d) 3

18. If $x^{51} + 51$ is divided by $x + 1$, the remainder is

(a) 0 (b) 1 (c) 49 (d) 50

[NCERT EXEMPLAR]

19. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then $k =$

(a) -2 (b) -3 (c) 4 (d) 2

[NCERT EXEMPLAR]

20. If $x + a$ is a factor of $x^4 - a^2x^2 + 3x - 6a$, then $a =$

(a) 0 (b) –1 (c) 1 (d) 2

21. The value of k for which $x-1$ is a factor of $4x^3+3x^2-4x+k$, is

(a) 3 (b) 1 (c) -2 (d) -3

22. If $x+2$ and $x-1$ are the factors of x^3+10x^2+mx+n, then the values of m and n are respectively

(a) 5 and -3 (b) 17 and -8 (c) 7 and -18 (d) 23 and -19

23. Let $f(x)$ be a polynomial such that $f\left(-\frac{1}{2}\right)=0$, then a factor of $f(x)$ is

(a) $2x-1$ (b) $2x+1$ (c) $x-1$ (d) $x+1$

24. When x^3-2x^2+ax-b is divided by x^2-2x-3, the remainder is $x-6$. The values of a and b are respectively

(a) $-2, -6$ (b) 2 and -6 (c) -2 and 6 (d) 2 and 6

25. One factor of x^4+x^2-20 is x^2+5. The other factor is

(a) x^2-4 (b) $x-4$ (c) x^2-5 (d) $x+4$

26. If $(x-1)$ is a factor of polynomial $f(x)$ but not of $g(x)$, then it must be a factor of

(a) $f(x)\,g(x)$ (b) $-f(x)+g(x)$ (c) $f(x)-g(x)$ (d) $\{f(x)+g(x)\}\,g(x)$

27. $(x+1)$ is a factor of x^n+1 only if

(a) n is an odd integer (b) n is an even integer
(c) n is a negative integer (d) n is a positive integer

28. If x^2+x+1 is a factor of the polynomial $3x^3+8x^2+8x+3+5k$, then the value of k is

(a) 0 (b) 2/5 (c) 5/2 (d) -1

29. If $(3x-1)^7=a_7x^7+a_6x^6+a_5x^5+\cdots+a_1x+a_0$, then $a_7+a_6+a_5+\cdots+a_1+a_0=$

(a) 0 (b) 1 (c) 128 (d) 64

30. If both $x-2$ and $x-\frac{1}{2}$ are factors of px^2+5x+r, then

(a) $p=r$ (b) $p+r=0$ (c) $2p+r=0$ (d) $p+2r=0$

31. If x^2-1 is a factor of $ax^4+bx^3+cx^2+dx+e$, then

(a) $a+c+e=b+d$ (b) $a+b+e=c+d$ (c) $a+b+c=d+e$ (d) $b+c+d=a+e$

32. If $f(x+3)=x^2-7x+2$, then the remainder when $f(x)$ is divided by $(x+1)$, is

(a) 8 (b) -4 (c) 20 (d) 46

33. If $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}$, then the remainder when $f(x)$ is divided by $(2x+1)$, is

(a) $-\frac{7}{4}$ (b) $-\frac{9}{4}$ (c) $\frac{9}{4}$ (d) $\frac{11}{4}$

34. If $f\left(x-\frac{1}{x}\right)=x^2+\frac{1}{x^2}$, then the remainder when $f(x)$ is divided by $(x-3)$, is

(a) 10 (b) 11 (c) 7 (d) $\frac{82}{9}$

35. If $f(x-2) = 2x^2 - 3x + 4$, then the remainder when $f(x)$ is divided by $(x-1)$, is
(a) 3 (b) 9 (c) 13 (d) -13

36. When the polynomial $p(x) = ax^2 + bx + c$ is divided by x, $x-2$ and $x+3$, the remainders obtained are 7, 9 and 49 respectively. The value of $3a + 5b + 2c$ is
(a) -5 (b) 5 (c) 2 (d) -2

37. If $(x-a)$ and $(x-b)$ are factors of $x^2 + ax + b$, then
(a) $a = 1, b = -2$ (b) $a = -2, b = 1$ (c) $a = 2, b = -3$ (d) $a = -\frac{1}{3}, b = -\frac{2}{3}$

38. If $(x-a)$ and $(x-b)$ are factors of $x^2 + ax - b$, then
(a) $a = -1, b = -2$ (b) $a = 0, b = 1$ (c) $a = -\frac{1}{2}, b = \frac{1}{2}$ (d) $a = -1, b = 2$

39. The ratio of remainders when $f(x) = x^2 + ax + b$ is divided by $(x-2)$ and $(x-3)$ respectively is 5 : 4. If $(x-1)$ is a factor of $f(x)$, then
(a) $a = -\frac{11}{3}, b = \frac{14}{3}$ (b) $a = -\frac{14}{3}, b = \frac{11}{3}$ (c) $a = \frac{14}{3}, b = -\frac{11}{3}$ (d) $a = -\frac{14}{3}, b = -\frac{11}{3}$

40. The remainder when $f(x) = x^{45} + x^{25} + x^{14} + x^9 + x$ is divided by $g(x) = x^2 - 1$, is
(a) $4x - 1$ (b) $4x + 2$ (c) $4x + 1$ (d) $4x - 2$

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

41. Statement-1 (Assertion): If the polynomial $p(x) = x^3 + ax^2 - 2x + a + 4$ has $(x+a)$ as one of its factors, then $a = -\frac{4}{3}$.
Statement-2 (Reason): If $f(x) = ax^2 + b + c$ is exactly divisible by $2x - 3$ then $4a + 6b + 9c = 0$.

42. Statement-1 (Assertion): If the polynomial $f(x) = 3x^4 - 11x^2 + 6x + k$ when divided by $(x-3)$ leaves remainder 7, then $k = -155$.
Statement-2 (Reason): If a polynomial is divided by $(x-a)$, the remainder is $f(a)$.

43. Statement-1 (Assertion): If $f(x+2) = 2x^2 + 7x + 5$, then the remainder when $f(x)$ is divided by $(x-1)$ is 0.
Statement-2 (Reason): If a polynomial $f(x)$ is divided by $(ax+b)$, then the remainder is $f(b/a)$.

44. Statement-1 (Assertion): If $x + 1$ is a factor of $f(x) = px^2 + 5x + r$, then $p + r + 5 = 0$.
Statement-2 (Reason): If $x - 2$ and $2x - 1$ are factors of $f(x) = px^2 + 5x + r$, then $p = r$.

45. Statement-1 (Assertion): If $x + 2a$ is a factor of $f(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$, then $2a - 3 = 0$.
Statement-2 (Reason): If $f(x)$ is divisible by $(ax+b)$, then $f\left(-\frac{b}{a}\right) = 0$.

ANSWERS

1. (c)	2. (a)	3. (d)	4. (b)	5. (c)	6. (d)	7. (c)
8. (b)	9. (b)	10. (b)	11. (c)	12. (d)	13. (c)	14. (a)
15. (a)	16. (c)	17. (a)	18. (d)	19. (d)	20. (a)	21. (d)
22. (c)	23. (b)	24. (c)	25. (a)	26. (a)	27. (a)	28. (b)
29. (c)	30. (a)	31. (a)	32. (d)	33. (a)	34. (b)	35. (c)
36. (d)	37. (a)	38. (d)	39. (b)	40. (c)	41. (b)	42. (a)
43. (c)	44. (d)	45. (a)				