MATHEMATICS
WORKSHEET_190425
CHAPTER-10 HERON'S FORMULA (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : IX **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The lengths of the three sides of a triangle are 30 cm, 24 cm and 18 cm respectively. The length of the altitude of the triangle corresponding to the smallest side is
(a) 24 cm (b) 18 cm (c) 30 cm (d) 12 cm
Ans: (a) 24 cm

2. Each side of an equilateral triangle is 10 cm long. The height of the triangle is
(a) $10\sqrt{3}$ cm (b) $5\sqrt{3}$ cm (c) $10\sqrt{2}$ cm (d) 5 cm
Ans: (b) $5\sqrt{3}$ cm

$$Height\ of\ equilateral\ triangle = \frac{\sqrt{3}}{2} \times Side = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}\,cm$$

3. The area of an equilateral triangle with side $2\sqrt{3}$ cm is (use $\sqrt{3} = 1.732$)
(a) 5.196 cm² (b) 0.866 cm² (c) 3.496 cm² (d) 1.732 cm²
Ans: (a) 5.196 cm²

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (side)^2 = \frac{\sqrt{3}}{4} \times 2\sqrt{3} \times 2\sqrt{3} = 3\sqrt{3}\,cm^2 = 5.196\,cm^2$

4. If the area of an equilateral triangle is $16\sqrt{3}$ cm², then the perimeter of the triangle is
(a) 48 cm (b) 24 cm (c) 12 cm (d) 36 cm
Ans: (b) 24 cm

Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (side)^2$

$$\Rightarrow 16\sqrt{3} = \frac{\sqrt{3}}{4} \times (side)^2 \Rightarrow (side)^2 = 64 \Rightarrow side = 8\,cm$$

Perimeter = 3 x side = 3 x 8 cm = 24 cm

5. The base of an isosceles triangle is 16 cm and its area is 48 cm2. The perimeter of the triangle is
(a) 41 cm (b) 36 cm (c) 48 cm (d) 324 cm
Ans: (b) 36 cm

6. The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is
(a) 480 m² (b) 320 m² (c) 384 m² (d) 360 m2
Ans: (c) 384 m²

7. Each of the equal sides of an isosceles triangle is 13 cm and its base is 24 cm. The area of the triangle is

(a) 156 cm^2 (b) 78 cm^2 (c) 60 cm^2 (d) 120 cm^2

Ans: (c) 60 cm^2

8. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(a) 1322 cm^2 (b) 1311 cm^2 (c) 1344 cm^2 (d) 1392 cm^2

Ans: (c) 1344 cm^2

We have, a = 56 cm, b = 60 cm and c = 52 cm.

$$s = \frac{a+b+c}{2} \Rightarrow s = \frac{56+60+52}{2} = 84$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{84(84-56)(84-60)(84-52)}$

$$= \sqrt{84\times28\times24\times32} = \sqrt{3\times7\times4\times4\times7\times4\times2\times3\times16\times2} = 3\times7\times4\times2\times2\times4 = 1344\,cm^2$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. Assertion (A): Area of an equilateral triangle having each side 4 cm is $10\sqrt{3}$ cm^2

Reason (R): Area of an equilateral triangle = $\frac{\sqrt{3}}{4}\times(Side)^2$

Ans: (d) A is false but R is true.

10. Assertion (A): Area of a triangle whose sides are 9 cm, 12 cm and 15 cm is 54 cm^2.

Reason (R): Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. The base of an isosceles triangle is 10 cm and one of its equal sides is 13 cm. Find its area using Heron's formula.

Ans: Given a = 10 cm, b = c = 13 cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{10+13+13}{2} = 18$ cm

Using Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-10)(18-13)(18-13)}$

$= \sqrt{18\times8\times\underline{5\times5}}$

$= 5\sqrt{\underline{3\times3}\times\underline{2\times2}\times\underline{2\times2}}$

$= 5\times3\times2\times2 = 60\ cm^2$

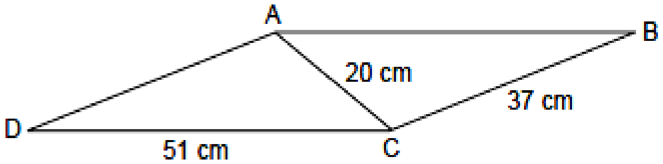
12. The length of two adjacent sides of a parallelogram are respectively 51 cm and 37 cm. One of its diagonal is 20 cm. Find the area of the parallelogram.

Ans: Let ABCD be a parallelogram.

AD = BC = 37 cm

AB = DC = 51 cm

AC = 20 cm



In ΔABC, let

a = BC = 37 cm, b = AB = 51 cm, c = AC = 20 cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{37+51+20}{2} = 54 \text{ cm}$

Using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{54(54-37)(54-51)(54-20)}$$
$$= \sqrt{54 \times 17 \times 3 \times 34}$$
$$= \sqrt{9 \times 3 \times 2 \times 17 \times 3 \times 17 \times 2}$$
$$= \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{2 \times 2} \times \underline{17 \times 17}}$$
$$= 3 \times 3 \times 2 \times 17 = 306 \text{ cm}^2$$

Since the diagonal divides the parallelogram into two congruent triangles of equal area,

$\therefore$ Area of parallelogram ABCD = 2 × ar (ΔABC) = 2 × 306 = 612 cm^2

13. The perimeter of an equilateral triangle is 60 cm. Find its area. (Use $\sqrt{3}$ = 1.73)

Ans: Given perimeter of an equilateral triangle = 60 cm

$\Rightarrow x + x + x = 60$

$\Rightarrow 3x = 60$

$\Rightarrow x = \frac{60}{3} = 20$ cm

Therefore, each side of triangle = 20 cm

$\therefore$ Area of an equilateral triangle by Heron's formula = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (20)^2$ (Side = 20 cm)

$= 100\sqrt{3}\ cm^2 = 100 \times 1.73 = 173\ cm^2$

14. Find the area of triangle whose sides are 18 cm, 24 cm and 30 cm.

Ans: Given a = 18 cm, b = 24 cm and c = 30 cm

The semi-perimeter of the triangle,

$s = \frac{a+b+c}{2} = \frac{18+24+30}{2} = 36$

$\Rightarrow s = 36$ cm

Using Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-18)(36-24)(36-30)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{6 \times 6 \times 6 \times 3 \times 6 \times 2 \times 6}$$
$$= \sqrt{\underline{6 \times 6} \times \underline{6 \times 6} \times \underline{6 \times 6}} = 6 \times 6 \times 6 = 216 \text{ cm}^2$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the area of a triangle whose perimeter is 180 cm and its two sides are 80 cm and 18 cm. Calculate the altitude of triangle corresponding to its shortest side.

Ans:

Given a = 80 cm and b = 18 cm
Perimeter of triangle = $a + b + c$
$\Rightarrow 180 = 80 + 18 + c$
$\therefore\ c = 180 - 98 = 82$ cm
and semi-perimeter, $s = \frac{180}{2} = 90$ cm
Using Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{90(90-80)(90-18)(90-82)}$$
$$= \sqrt{90 \times 10 \times 72 \times 8} = \sqrt{10 \times 9 \times 10 \times 9 \times 8 \times 8}$$
$$= \sqrt{\underline{10 \times 10} \times \underline{9 \times 9} \times \underline{8 \times 8}} = 10 \times 9 \times 8 = 720 \text{ cm}^2$$

The shortest side of triangle = 18 cm
$\therefore$ Area of triangle = $\frac{1}{2}$ × base × altitude

$720 = \frac{1}{2} \times 18 \times h$

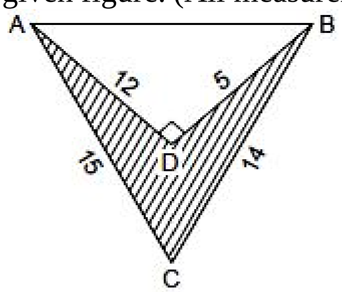
$\therefore\ h = \frac{720}{9} = 80$ cm

$\therefore$ Altitude of triangle corresponding to its shortest side (18 cm) is 80 cm.

16. The sides of a triangle are in the ratio 13 : 14 : 15 and its perimeter is 84 cm. Find the area of the triangle.
Ans:
Given ratio of the sides of a triangle = 13 : 14 : 15
Let $a = 13k$, $b = 14k$ and $c = 15k$
Perimeter of triangle = 84 cm
$\Rightarrow 13k + 14k + 15k = 84$
$\Rightarrow 42k = 84$
$\Rightarrow k = \frac{84}{42} = 2$
So, the sides of a triangle are 13 × 2 = 26 cm, 14 × 2 = 28 cm and 15 × 2 = 30 cm
Its semi-perimeter, $s = \frac{84}{2} = 42$ cm
Using Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42(42-26)(42-28)(42-30)}$$
$$= \sqrt{42 \times 16 \times 14 \times 12}$$
$$= \sqrt{14 \times 3 \times 4 \times 4 \times 14 \times 4 \times 3}$$
$$= \sqrt{\underline{14 \times 14} \times \underline{4 \times 4} \times \underline{3 \times 3} \times \underline{2 \times 2}}$$
$$= 14 \times 4 \times 3 \times 2 = 336 \text{ cm}^2$$

17. Find the area of shaded region in the given figure. (All measurements are in cm)



Ans: Area of right-angled ΔADB = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × BD × AD

(Base = BD, height = AD)

$= \frac{1}{2} \times 5 \times 12 = 30\ cm^2$

Using Pythagoras theorem in right-angled ΔADB, we have

$AB^2 = AD^2 + BD^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\therefore$ AB = $\sqrt{169}$ = 13 cm

Now, in ΔABC, AB = 13 cm, AC = 15 cm and BC = 14 cm

$\therefore$ Perimeter of triangle, 2s = AB + BC + AC = 13 + 14 + 15 = 42 cm

$\therefore$ Semi-perimeter, $s = \frac{42}{2}$ = 21 cm

Using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21(21-13)(21-15)(21-14)}$$

$$= \sqrt{21 \times 8 \times 6 \times 7} = \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7}$$

$$= \sqrt{\underline{7 \times 7} \times \underline{3 \times 3} \times \underline{2 \times 2} \times \underline{2 \times 2}} = 7 \times 3 \times 2 \times 2 = 84\ cm^2$$

$\therefore$ Area of shaded portion = ar(ΔABC) – ar(ΔADB) = 84 – 30 = 54 cm^2

SECTION – D

Questions 18 carry 5 marks.

18. A gardener has to put double fence all around a triangular field with sides 120 m, 80 m and 60 m. In the middle of each of the sides, there is a gate of width 10 m.

(i) Find the length of wire needed for fencing.

(ii) Find the cost of fencing at the rate of ₹ 6 per metre.

(iii) Find the area of triangular field.

Ans: Perimeter of triangular field = 120 + 80 + 60 = 260 m

(i) Length of wire needed for single fencing

= 260 – 30 (to be left for gate on each side)

= 230 m

$\therefore$ Total length of wire needed for double fencing = 2 × 230 = 460 m

(ii) Cost of fencing = ₹ 6 per metre

$\therefore$ Total cost of fencing = 460 × 6 = ₹ 2760

(iii) Given a = 120 m, b = 80 m and c = 60 m

The semi-perimeter, $s = \frac{260}{2}$ = 130 m

Using Heron's formula,

Area of triangular field

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{130(130-120)(130-80)(130-60)}$

$= \sqrt{130 \times 10 \times 50 \times 70}$

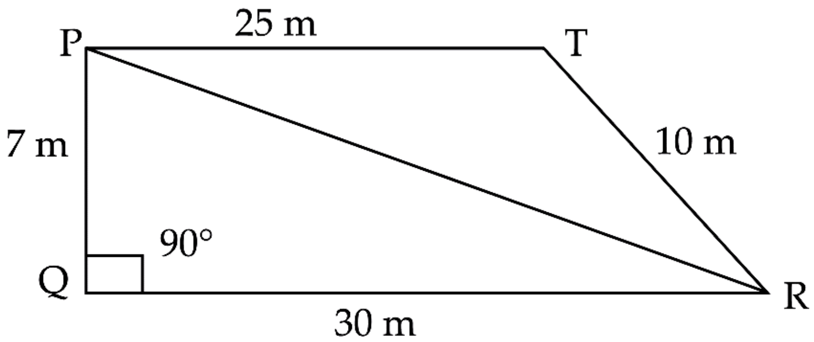
$= 100\sqrt{13 \times 5 \times 7}$

$= 100\sqrt{455} = 100 \times 21.33 = 2133\ m^2$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Under Swachh Bharat Mission, a school management suggested teachers as well as students to organize Marathon Running on 2nd October in memory of Mahatma Gandhi. Both teachers and students of school dramatically made a gathering for spotlessness drive. They walked throughout the following paths in two groups. One group walked through the paths PQ QR and RP whereas the other through PR, RT and TP (As shown in figure). Then they ran over the area enclosed within their paths. If PQ = 7m, QR = 30m, RT =10m, TP = 25m and $\angle Q = 90°$.



Analyze the above information answer the following questions:

(i) What is the value of longest path which is covered by Marathon participants? (1)

(ii) Find the value area of triangle PQR in which first group is running Marathon? (1)

(iii) How much area is covered by Group 2 of triangles PRT? (2)

Ans: (i) In ΔPQR, by Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$\Rightarrow PR^2 = 7^2 + 30^2$

$\Rightarrow PR^2 = 49 + 900 = 949$

$\Rightarrow$ PR = 30.80 m

(ii) Area of ΔPQR = $\frac{1}{2}$ x 30 x 7

$\Rightarrow$ Area of ΔPQR = 15 x 7

= 105 m^2

(iii) s = (25 + 10 + 30.8)/2

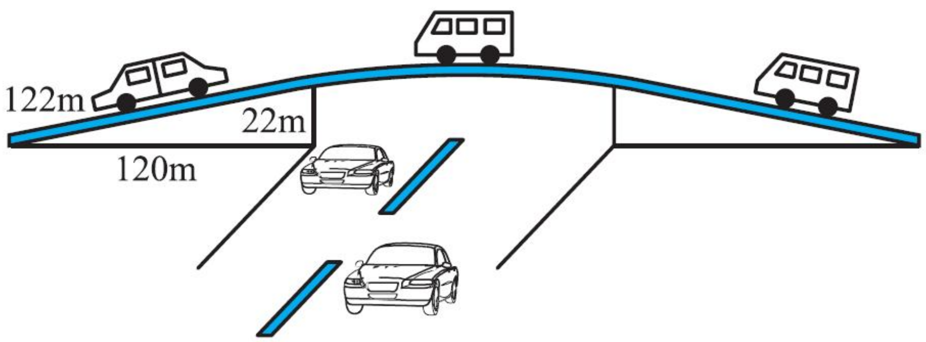
$\Rightarrow$s = 32.9 m

Area of ΔPRT = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{32.9(32.9-25)(32.9-10)(32.9-30.8)}$

$= \sqrt{32.9 \times 7.9 \times 22.9 \times 2.1} = \sqrt{12499.07} = 111.8 \;\; m^2$

20. There is a road running across the city, which is also a connecting road between the 2 towns. Due to this busy road, lot of traffic generally occurs on this road. To get rid of it a flyover was made on it. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122m, 22m and 120 m.



(i) What type of triangle is the side wall of the flyover? (1)

(ii) If there are 2 walls for the advertisement what is the total area of the 2 walls? (1)

(iii) If the advertisements yields an earning of Rs. 6,000 m^2 per year. What is the monthly rent for 2 walls? (1)

(iv) If a company hires these 2 walls for 4 months, how much rent they need to pay? (1)

Ans: (i) $122^2 = 14884$

$22^2 = 484$

$120^2 = 14400$

$\Rightarrow 122^2 = 120^2 + 22^2$

$\Rightarrow$ By Pythagoras theorem, the triangle is right angled triangle.

(ii) Area of two walls = $2\times\frac{1}{2}\times b\times h = b\times h$ = 120 x 22 = 2640 m^2

(iii) Yearly rent = Rs. 6000 per m2

∴ Monthly rent = Rs. 6000 / 12 = Rs. 500 per m^2.

(iv) Monthly rent of 2 walls = Rs. 500 x 2640 = Rs. 13,20,000

∴ Total rent paid for 4 years = Rs. 13,20,000 x 4 = Rs. 52,80,000