7

# ALGEBRAIC EXPRESSIONS

## 7.1 INTRODUCTION

In class VI, we have learnt about literals and their addition, subtraction, multiplication and division. We have also studied about powers of literals, variables and constants. A symbol having a fixed numeric value is called a constant and a symbol which takes various numerical values is called a variable. A combination of constants and variables connected by the signs of addition, subtraction, multiplication and division is called an algebraic expression. In this chapter, we shall learn about simple algebraic expressions involving one or two variables. We shall also study about their addition and subtraction.

## 7.2 ALGEBRAIC EXPRESSION

In this section, we shall define an algebraic expression. We shall also define various types of algebraic expressions.

**DEFINITION** *A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression.*

**TERMS** *Various parts of an algebraic expression which are separated by the signs + or – are called the 'terms' of the expression.*

**ILLUSTRATION** (i) $3x + 2y - 4z$ is an algebraic expression having $3x$, $2y$ and $-4z$ as its terms.

(ii) $5x^3 - 6x^2y + 8xy^3z - 7$ is an algebraic expression consisting of four terms, namely $5x^3, -6x^2y, 8xy^3z$ and $-7$.

(iii) $4,\ 3x - 5,\ a^2 + b^2 + c^2 - ab - bc - ca,\ 2x^3 + 5x,\ xy + xz,\ xy + yz + zx$, etc. are all algebraic expressions.

### 7.2.1 TYPES OF ALGEBRAIC EXPRESSIONS

**MONOMIAL** *An algebraic expression containing only one term is called a monomial.*

For example, $3, 2x, 5x^2y, -6\,abc, \frac{3}{2}ab^2c^3$ are all monomials.

**BINOMIAL** *An algebraic expression containing two terms is called a binomial.*

For example, $x + 3, 5 - 2x, a^2 - 2abc, x^3 + 7, \frac{2}{3}x^2 + xyz^2$ are all binomials.

Note that $3 + 7$ is not a binomial, because $3 + 7 = 10$, which is a monomial.

**TRINOMIAL** *An algebraic expression containing three terms is called a trinomial.*

For example, $2x - y + 3, x^2 + y^2 + z^2, 3 + xyz + x^3$ are all trinomials.

Note that $7 + 2x + 9$ is not a trinomial, because $7 + 2x + 9 = 16 + 2x$, which is a binomial.

**QUADRINOMIAL** *An algebraic expression containing four terms is called a quadrinomial.*

For example, $a^3 + b^3 + c^3 + 3abc, a^2 + b^2 + c^2 + 5, ab + bc + ca + abc$ are all quadrinomials.

**POLYNOMIAL** *An algebraic expression containing two or more terms is called a polynomial.*

It follows from the above definition that binomial, trinomial and quadrinomial etc. are polynomials.

**ILLUSTRATION** Identify the monomials, binomials, trinomials and quadrinomials among the following algebraic expressions:

(i) $-7$ (ii) $x + y$ (iii) $4.5a$ (iv) $a^3 - b^3$

(v) $a^2 + 2ab + b^2$ (vi) $ax + by + c$ (vii) $ax + by + cz + d$

(viii) $3xyz$ (ix) $x + y + z - xyz$

*Solution* Monomials are: (i), (iii) and (viii) Binomials are: (ii) and (iv)

Trinomials are: (v) and (vi) Quadrinomials are: (vii) and (ix).

## 7.3 FACTORS AND COEFFICIENTS

**FACTORS** *Each term in an algebraic expression is a product of one or more number(s) and/or literal number(s). These number(s) and/or literal number(s) are known as the factors of that term.*

**ILLUSTRATION 1** (i) The monomial $7x$ is the product of number 7 and literal $x$. So, 7 and $x$ are factors of the monomial $7x$.

(ii) In the binomial $3xy + 7z$, $3xy$ and $7z$ are two terms. In the term $3xy$, for instance, 3, $x$ and $y$ are its factors. Clearly, number 3 is the numerical factor, and $x$ and $y$ are literal factors.

(iii) In the term $-4xyz$, the numerical factor is $-4$ whereas $x$, $y$ and $z$ are literal factors.

(iv) In the binomial expression $-xy + 3$, the term $-xy$ has $-1$ as the numerical factor while $x$ and $y$ are literal factors. The term 3 has only numerical factor. It has no literal factor.

(v) In the algebraic expression $ab - c^2 - 7$ the term $ab$ has numerical factor as 1 and literal factors are $a$ and $b$. The term $-c^2$ has numerical factor as $-1$ and literal factors are $c$ and $c^2$. The third term $-7$ has no literal factor.

**CONSTANT TERM** *A term of the expression having no literal factor is called a constant term.*

**ILLUSTRATION 2** (i) In the binomial expression $5x + 7$, the constant term is 7.

(ii) In the trinomial expression $a^2 + b^2 - \frac{3}{4}$, the constant term is $\frac{-3}{4}$.

**COEFFICIENT** *In a term of an algebraic expression, any of the factors with the sign of the term is called the coefficient of the product of the other factors.*

**ILLUSTRATION 3** (i) In the monomial $3xy$, the coefficient of $y$ is $3x$, the coefficient of $x$ is $3y$ and the coefficient of $xy$ is 3.

(ii) Consider the term $-8xy$ in the binomial $-8xy + 7$. The coefficient of $x$ in the term $-8xy$ is $-8y$, the coefficient of $y$ is $-8x$ and the coefficient of $xy$ is $-8$.

**ILLUSTRATION 4** Write down the terms of the expression: $8x^4y - 7x^3yz + \frac{4}{3}x^2yz^2 - 2.5xyz$

What is the coefficient of $x^2$ in the term $\frac{4}{3}x^2yz^2$?

*Solution* The given expression has four terms, namely $8x^4y$, $-7x^3yz$, $\frac{4}{3}x^2yz^2$ and $-2.5\,xyz$

The coefficient of $x^2$ in the term $\frac{4}{3}x^2yz^2$ is $\frac{4}{3}yz^2$.

**ILLUSTRATION 5** Write down the coefficients of $a$, $ab$ and $abc$ in the term $4a^4b^2c$ of the algebraic expression $4a^4b^2c - 3a^3b^2c + \frac{3}{2}ab^3c^2$.

*Solution* The given algebraic expression has three terms, namely, $4a^4b^2c$, $-3a^3b^2c$ and $\frac{3}{2}ab^3c^2$.

Consider the term $4a^4b^2c$

We have, $4a^4b^2c = 4a \,.\, a^3b^2c$

$\therefore$ Coefficient of $a$ in $4a^4b^2c$ is $4a^3b^2c$

Also, $4a^4b^2c = 4a^3 \,.\, ab \,.\, bc$

$\therefore$ Coefficient of $ab$ in $4a^4b^2c$ is $4a^3bc$

Again, $4a^4b^2c = 4 \,.\, abc \,.\, a^3b$

$\therefore$ Coefficient of $abc$ in $4a^4b^2c$ is $4a^3b$.

## 7.4 LIKE AND UNLIKE TERMS

**LIKE TERMS** *The terms having the same literal factors are called like or similar terms.*

**UNLIKE TERMS** *The terms not having same literal factors are called unlike or dissimilar terms.*

**ILLUSTRATION 1** (i) In the algebraic expression $5x^2y + 7xy^2 - 3xy - 4 \cdot x^2$, we have $5x^2y$ and $-4yx^2$ as like terms, whereas $7xy^2$ and $-3xy$ are unlike terms.

(ii) In the algebraic expression $a^2 - 3b^2 + 7b^2 - 9a^2 + 6ab + 5$, we have, $a^2$ and $-9a^2$ as like terms. Also, $-3b^2$ and $7b^2$ are like terms. But $a^2$, $7b^2$ and $6ab$ are unlike terms.

**ILLUSTRATION 2** In the following write down the pairs which contain like terms:

(i) $3x, -7x$ (ii) $16x, 16y$ (iii) $x^2y, -7x^2y$

(iv) $9ab, -6b$ (v) $a^2, 4b^2$ (vi) $a^2b, 3a^2bc$

*Solution*

(i) In the pair $3x, -7x$ the literal factor $x$ is the same. Hence, the pair $3x, -7x$ contains like terms.

(ii) In the pair $16x, 16y$ the literal factors $x$ and $y$ are not same. So, the pair $16x, 16y$ does not contain like terms.

(iii) Terms $x^2y$ and $-7x^2y$ have the same literal factor $x^2y$. So, the pair $x^2y$, $-7x^2y$ is the pair of like terms.

(iv) Since the terms $9ab$ and $-6b$ have literal factors as $ab$ and $b$. So, the pair $9ab, -6b$ is the pair of unlike terms.

(v) $a^2$ and $4b^2$ are unlike terms as their literal factors $a^2$ and $b^2$ are distinct.

(vi) $a^2b$ and $3a^2bc$ are unlike terms as their literal factors $a^2b$ and $a^2bc$ are distinct.

**ILLUSTRATION 3** Identify the like terms and also mention the coefficients of those terms in the following terms: $4xy$, $-5x^2y$, $-3yx$, $2xy^2$

*Solution* Clearly, terms $4xy$ and $-3yx = -3xy$ have $xy$ as the same literal factor. So, $4xy$ and $-3yx$ are like terms among the given terms. The coefficients of $xy$ in these two terms are 4 and $-3$ respectively.

Remark *If the coefficient of a term in an algebraic expression is 1, then '1' is usually omitted. For instance, 1a is written as a. Similarly, if the coefficient is –1, then also we omit 1. For instance, –1a is written as –a.*

## 7.5 FINDING THE VALUE OF AN ALGEBRAIC EXPRESSION

As we have studied in the earlier sections that an algebraic expression involves one or more terms and each term contains one or more literals and some numeric coefficient. Thus, to find the numeric value of an algebraic expression, we need to know the numerical values of all the literals appearing in it.

In order to find the value of an algebraic expression for given values of the literals involved in it, we replace the literals by their numeric values to obtain an arithmetic expression and then evaluate it by usual method of arithmetic.

*The process of replacing the literals by their numeric values is called substitution.*

The following procedure can be used to find the value of an algebraic expression for the given values of literals involved in it.

**PROCEDURE**

STEP I *Write the algebraic expression.*

STEP II *Obtain the values of literals involved in the expression.*

STEP III *Replace each literal by its numeric value to obtain an arithmetical expression.*

STEP IV *Simplify the arithmetical expression obtained in step III by the usual method of arithmetic.*

STEP V *The value obtained in step IV is the required value.*

Following illustration will illustrate the above procedure.

**ILLUSTRATION 1** If $x = 1$ and $y = 2$, find the values of each of the following algebraic expressions:

(i) $2x + 3$ (ii) $3x - 5y$ (iii) $3x + 2y - 7$ (iv) $x^2$
(v) $x^2y$ (vi) $x^2 + y^2$ (vii) $-y^2 + 3 - x^2$

*Solution* Substituting $x = 1$ and $y = 2$ in each of the following expressions, we have

(i) $2x + 3 = 2 \times 1 + 3 = 2 + 3 = 5$

(ii) $3x - 5y = 3 \times 1 - 5 \times 2 = 3 - 10 = -7$

(iii) $3x + 2y - 7 = 3 \times 1 + 2 \times 2 - 7 = 3 + 4 - 7 = 7 - 7 = 0$

(iv) $x^2 = (1)^2 = 1 \times 1 = 1$

(v) $x^2y = (1)^2 \times 2 = 1 \times 2 = 2$

(vi) $x^2 + y^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

(vii) $-y^2 + 3 - x^2 = -(2)^2 + 3 - (1)^2 = -4 + 3 - 1 = -1 - 1 = -2.$

**ILLUSTRATION 2** If $x = 1, y = -2$ and $z = 3$, find the values of each of the following algebraic expressions:

(i) $x^3 + y^3 + z^3 - 3xyz$ (ii) $3xy^4 - 15x^2y + 4z$

**Solution** Substituting $x = 1, y = -2$ and $z = 3$ in the given expressions, we have

(i) $x^3 + y^3 + z^3 - 3xyz$
$= (1)^3 + (-2)^3 + (3)^3 - 3 \times 1 \times -2 \times 3$
$= 1 + (-8) + 27 - 3 \times -6$
$= 1 - 8 + 27 + 18 = 46 - 8 = 38$

(ii) $3xy^4 - 15x^2y + 4z$
$= 3 \times 1 \times (-2)^4 - 15 \times (1)^2 \times (-2) + 4 \times 3$
$= 3 \times 16 - 15 \times -2 + 4 \times 3 = 48 + 30 + 12 = 90.$

**ILLUSTRATION 3** Find the values of each of the following expressions for $a = 1, b = 2$ and $c = -1$.

(i) $a^2 + b^2 + 2ab$ (ii) $2a^2 - b^2c + 3abc$
(iii) $a^3 + b^3 + c^3 - 3abc$ (iv) $a^2 + b^2 + c^2 - ab - bc - ca$

**Solution** Substituting $a = 1, b = 2$ and $c = -1$ in the given expressions, we have

(i) $a^2 + b^2 + 2ab = (1)^2 + 2^2 + 2 \times 1 \times 2 = 1 + 4 + 4 = 9$

(ii) $2a^2 - b^2c + 3abc = 2 \times (1)^2 - 2^2 \times (-1) + 3 \times 1 \times 2 \times (-1)$
$= 2 \times 1 - 4 \times (-1) + 6 \times (-1) = 2 + 4 - 6 = 0$

(iii) $a^3 + b^3 + c^3 - 3abc = (1)^3 + (2)^3 + (-1)^3 - 3 \times 1 \times 2 \times (-1)$
$= 1 + 8 - 1 + 6 = 15 - 1 = 14$

(iv) $a^2 + b^2 + c^2 - ab - bc - ca = (1)^2 + (2)^2 + (-1)^2 - 1 \times 2 - 2 \times (-1) - (-1) \times 1$
$= 1 + 4 + 1 - 2 + 2 + 1 = 9 - 2 = 7.$

## ILLUSTRATIVE EXAMPLES

**Example 1** Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) $4x^2$ (ii) $x^2 - 1$ (iii) $x^2 - y^2$
(iv) $3x^2 + 4y^2 + 5z$ (v) $ax^2 + bx + c$ (vi) $a^2 + b^2 + c^2 - d^2$
(vii) $3ab^2$ (viii) $a^3 + b^3 - 3ab + 5$ (ix) $-xyz$
(x) $3x - 2$ (xi) $4x - 3x$

**Solution**

(i) $4x^3$ is a monomial expression as it contains one term only.
(ii) $x^2 - 1$ is a binomial expression because it contains two terms.
(iii) $x^2 - y^2$ is a binomial expression as it consists of two terms.
(iv) $3x^2 - 4y^3 + 5z$ contains three terms. So, it is trinomial.
(v) $ax^2 + bx + c$ contains three terms. So, it is trinomial.
(vi) $a^2 + b^2 + c^2 - d^2$ is quadrinomial, because it contains four terms.
(vii) $3ab^2$ is monomial as it contains just one term.
(viii) $a^3 + b^3 - 3ab + 5$ contains four terms. So, it is quadrinomial.

(ix) $-xyz$ contains just one term. So, it is a monomial.

(x) $3x-2$ contains two terms. So, it is a binomial expression.

(xi) We have, $4x-3x=x$. So. it is monomial.

**Example 2** Write all the terms of each of the following algebraic expressions:

(i) $3x^5+5y^4-7x^2y+7$ (ii) $9y^3-2z^3+7x^3y-3xyz$

(iii) $a^5-3ab-b^2+6$ (iv) $x^2-x+1$

*Solution* (i) $3x^5, 5y^4, -7x^2y$ and 7 are the terms of the algebraic expressions $3x^5+5y^4-7x^2y+7$.

(ii) Terms of the algebraic expression $9y^3-2z^3+7x^3y-3xyz$ are $9y^3, -2z^3$, $7x^3y$ and $-3xyz$.

(iii) The algebraic expression $a^5-3ab-b^2+6$ has $a^5, -3ab, -b^2$ and 6 as it terms.

(iv) Various terms of the algebraic expression $x^2-x+1$ are $x^2, -x$ and 1.

**Example 3** Write down the coefficient of $x$ in each of the following:

(i) $3x$ (ii) $-4ax$ (iii) $5xy^2$

(iv) $xyz$ (v) $-\frac{3}{2}x+5$ (vi) $-\frac{5}{2}xyz^2$

*Solution* (i) The coefficient of $x$ in $3x$ is 3.

(ii) The coefficient of $x$ in $-4ax$ is $-4a$

(iii) The coefficient of $x$ in $5xy^2$ is $5y^2$.

(iv) The coefficient of $x$ in $xyz$ is $yz$

(v) The coefficient of $x$ in $-\frac{3}{2}x+5$ is $-\frac{3}{2}$

(vi) The coefficient of $x$ in $-\frac{5}{2}xyz^2$ is $-\frac{5}{2}yz^2$

**Example 4** Write the numerical coefficient of each term of the following algebraic expressions:

(i) $x^2-7x^2y+5xy^2-2$ (ii) $-2a^3+7ab^2-6ab+8$

*Solution* (i) Various terms of the algebraic expression $x^3-7x^2y+5xy^2-2$ are $x^3$, $-7x^2y$, $5xy^2$ and $-2$. The numerical coefficients of these terms are 1, $-7$, 5 and $-2$ respectively.

(ii) Various terms of $-2a^3+7ab^2-6ab+8$ are $-2a^3$, $7ab^2$, $-6ab$ and 8. The numerical coefficients of these terms are $-2$, 7, $-6$ and 8 respectively.

**Example 5** Identify the like terms in each of the following:

(i) $x^2, y^2, 2x^2, z^2$ (ii) $2xy, yz, 3x, \frac{yz}{2}$ (iii) $-2x^2y, x^2z, -yx^2, x^2y^2$

(iv) $cab^2, a^2bc, b^2ac, c^2ab, ab^2c, abc, acb^2$

*Solution* Recall that the terms having the same literal factors are called like terms.

(i) In $x^2, y^2, 2x^2, z^2$ we have $x^2$ and $2x^2$ as like terms.

(ii) In $2xy, yz, 3x, \frac{1}{2}yz$ like terms are $yz$ and $\frac{1}{2}yz$.

(iii) $-2x^2y, x^2z, -yx^2, x^2y^2$ like terms are $-2x^2y$ and $-yx^2$

(iv) In $cab^2, a^2bc, b^2ac, c^2ab, ab^2c, abc, acb^2$ like terms are $cab^2$, $b^2ac$, $ab^2c$ and $acb^2$

**Example 6** Identify the like terms in each of the following algebraic expressions:

(i) $x - 2y + 3z - 4x + 3xy$ (ii) $3a + 2b - c + \frac{3}{2}a - 4 + 3b$

(iii) $xy^2 + 3x^2y - 4x^2y^2 - 5y^2x - 2z^2x + 3xz^2$

**Solution** (i) In the algebraic expression $x - 2y + 3z - 4x + 3xy$ like terms are $x$ and $-4x$.

(ii) In the algebraic expression $3a + 2b - c + \frac{3}{2}a - 4 + 3b$, the groups of like terms are $3a, \frac{3}{2}a$ and $2b, 3b$.

(iii) In the algebraic expression $xy^2 + 3x^2y - 4 - x^2y^2 - 5y^2x - 2z^2x + 3xz^2$, the groups of like terms are $xy^2, -5y^2x$ and $-2z^2x, 3xz^2$.

**Example 7** Evaluate each of the following algebraic expressions for $x = 2, y = -3, z = -2, a = 2, b = 3$:

(i) $2a^2 + ab$ (ii) $2a^2 + x^2 - y^2$ (iii) $x^3 - y^3 + z^3$

(iv) $4xy^2 - 3yz^2 + 4x^2z$ (v) $x^3 + y^3 + 3xyz + ab$ (vi) $5 + 4z^3 - 6y + 7a + xy$

**Solution** Substituting $x = 2, y = -3, z = -2, a = 2$ and $b = 3$ in the given expressions, we have

(i) $2a^2 + ab = 2\times(2)^2 + 2\times3 = 2\times4 + 2\times3 = 8 + 6 = 14$

(ii) $2a^2 + x^2 - y^2 = 2\times(2)^2 + 2^2 - (-3)^2 = 2\times4 + 4 - 9 = 8 + 4 - 9 = 3$

(iii) $x^3 - y^3 + z^3 = 2^3 - (-3)^3 + (-2)^3 = 8 - (-27) + (-8) = 8 + 27 - 8 = 27$

(iv) $4xy^2 - 3yz^2 + 4x^2z = 4\times2\times(-3)^2 - 3\times(-3)\times(-2)^2 + 4\times(2)^2\times(-2)$
$= 4\times2\times9 - 3\times(-3)\times4 + 4\times4\times-2$
$= 72 + 36 - 32 = 76$

(v) $x^3 + y^3 + 3xyz + ab = 2^3 + (-3)^3 + 3\times2\times-3\times-2 + 2\times3$
$= 8 + (-27) + 36 + 6 = 8 - 27 + 36 + 6 = 23$

(vi) $5 + 4z^3 - 6y + 7a + xy = 5 + 4\times(-2)^3 - 6\times(-3) + 7\times2 + 2\times-3$
$= 5 + 4\times-8 + 6\times3 + 7\times2 - 2\times3$
$= 5 - 32 + 18 + 14 - 6 = 37 - 38 = -1$

## EXERCISE 7.1

1. Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) $a^2$ (ii) $a^2 - b^2$ (iii) $x^3 + y^3 + z^3$ (iv) $x^3 + y^3 + z^3 + 3xyz$

(v) $7 + 5$ (vi) $abc + 1$ (vii) $3x - 2 + 5$ (viii) $2x - 3y + 4$

(ix) $xy + yz + zx$ (x) $ax^3 + bx^2 + cx + d$

2. Write all the terms of each of the following algebraic expressions:
   (i) $3x$ (ii) $2x - 3$ (iii) $2x^2 - 7$ (iv) $2x^2 + y^2 - 3xy + 4$
3. Identify the like terms and also mention the numerical coefficients of those terms:
   (i) $4xy, -5x^2y, -3yx, 2xy^2$ (ii) $7a^2bc, -3ca^2b, -\frac{5}{2}abc^2, \frac{3}{2}abc^2, -\frac{4}{3}cba^2$
4. Identify the like terms in the following algebraic expressions:
   (i) $a^2 + b^2 - 2a^2 + c^2 + 4a$ (ii) $3x + 4xy - 2yz + \frac{5}{2}zy$
   (iii) $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac - 2a^2bc + 3cab^2$
5. Write the coefficient of $x$ in the following:
   (i) $-12x$ (ii) $-7xy$ (iii) $xyz$ (iv) $-7ax$
6. Write the coefficient of $x^2$ in the following:
   (i) $-3x^2$ (ii) $5x^2yz$ (iii) $\frac{5}{7}x^2z$ (iv) $-\frac{3}{2}ax^2 + yx$
7. Write the coefficient of:
   (i) $y$ in $-3y$ (ii) $a$ in $2ab$ (iii) $z$ in $-7xyz$ (iv) $p$ in $-3pqr$
   (v) $y^2$ in $9xy^2z$ (vi) $x^3$ in $x^3 + 1$ (vii) $x^2$ in $-x^2$
8. Write the numerical coefficient of each of the following:
   (i) $xy$ (ii) $-6yz$ (iii) $7abc$ (iv) $-2x^3y^2z$
9. Write the numerical coefficient of each term in the following algebraic expressions:
   (i) $4x^2y - \frac{3}{2}xy + \frac{5}{2}xy^2$ (ii) $-\frac{5}{3}x^2y + \frac{7}{4}xyz + 3$
10. Write the constant term of each of the following algebraic expressions:
    (i) $x^2y - xy^2 + 7xy - 3$ (ii) $a^3 - 3a^2 + 7a + 5$
11. Evaluate each of the following expressions for $x = -2, y = -1, z = 3$:
    (i) $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ (ii) $x^2 + y^2 + z^2 - xy - yz - zx$
12. Evaluate each of the following algebraic expressions for $x = 1, y = -1, z = 2, a = -2, b = 1, c = -2$:
    (i) $ax + by + cz$ (ii) $ax^2 + by^2 - cz^2$ (iii) $axy + byz + cxy$

## ANSWERS

1. (i) Monomial (ii) Binomial (iii) Trinomial (iv) Quadrinomial (v) Monomial (vi) Binomial (vii) Binomial (viii) Trinomial (ix) Trinomial (x) Quadrinomial
2. (i) $3x$ (ii) $2x, -3$ (iii) $2x^2, -7$ (iv) $2x^2, y^2, -3xy, 4$
3. Like terms — Coefficients

| Like terms | Coefficients |
|---|---|
| (i) $4xy, -3yx$ | $4, -3$ |
| (ii) $7a^2bc, -3ca^2b, -\frac{4}{3}cba^2, -\frac{5}{2}abc^2, \frac{3}{2}abc^2$ | $7, -3, -\frac{4}{3}, -\frac{5}{2}, \frac{3}{2}$ |

4. (i) $a^2, -2a^2$ (ii) $-2yz, \frac{5}{2}zy$ (iii) $ab^2c, 2acb^2, b^2ac, 3cab^2$
5. (i) $-12$ (ii) $-7y$ (iii) $yz$ (iv) $-7a$
6. (i) $-3$ (ii) $5yz$ (iii) $\frac{5}{7}z$ (iv) $-\frac{3}{2}a$
7. (i) $-3$ (ii) $2b$ (iii) $-7xy$ (iv) $-3qr$ (v) $9xz$ (vi) 1 (vii) $-1$
8. (i) 1 (ii) $-6$ (iii) 7 (iv) $-2$
9. (i) $4, -\frac{3}{2}, \frac{5}{2}$ (ii) $-\frac{5}{3}, \frac{7}{4}, 3$ 10. (i) $-3$ (ii) 5 11. (i) $\frac{1}{6}$ (ii) 21
12. (i) $-7$ (ii) 7 (iii) 2

## 7.6 OPERATIONS ON ALGEBRAIC EXPRESSIONS

In the previous sections, we have learnt about algebraic expressions and like and unlike terms in an algebraic expression. In this section, we shall study about the fundamental operations of addition and subtraction of algebraic expressions. Since an algebraic expression may consist of like and unlike terms. So, the operations of addition and subtraction of algebraic expressions mean addition or subtraction of like terms. We shall first study addition of like terms.

### 7.6.1 ADDITION OF POSITIVE LIKE TERMS

To add several positive like terms, we proceed as follows:

STEP I *Obtain all like terms.*

STEP II *Find the sum of the numerical coefficients of all terms.*

STEP III *Write the required sum as a like term whose numerical coefficient is the numerical obtained in step II and literal factor is same as the literal factors of the given like terms.*

**ILLUSTRATION 1** Add $4xy$, $12xy$ and $3xy$.

*Solution* The sum of the numerical coefficients of the given like terms is $4 + 12 + 3 = 19$.

Thus, the sum of the given like terms is another like term whose numerical coefficient is 19.

Hence, $4xy + 12xy + 3xy = 19xy$.

Aliter The sum of the given like terms can also be obtained by using the distributive property of multiplication over addition as discussed below:

$4xy + 12xy + 3xy = (4 + 12 + 3)xy = 19xy$.

**ILLUSTRATION 2** Add $3a^2b$, $2a^2b$, $13a^2b$ and $a^2b$.

*Solution* The sum of the numerical coefficients of the given like terms is

$3 + 2 + 13 + 1 = 19$

So, the sum of the given like terms is another like term whose numerical coefficient is 19.

Hence, $3a^2b + 2a^2b + 13a^2b + a^2b = 19a^2b$.

Aliter By using the distributive property of multiplication over addition, we have

$3a^2b + 2a^2b + 13a^2b + a^2b = (3 + 2 + 13 + 1)a^2b = 19a^2b$.

### 7.6.2 ADDITION OF NEGATIVE LIKE TERMS

To add negative like terms, we proceed as follows:

STEP I *Obtain all like terms.*

STEP II *Obtain the sum of the numerical coefficients (without negative sign) of all like terms.*

STEP III *Write an expression as a product of the number obtained is step II, with all the literal coefficients preceded by a minus sign.*

STEP IV *The expression obtained in step III is the required sum.*

**ILLUSTRATION 1** Add $-7xy$, $-3xy$ and $-9xy$.

*Solution* The numerical coefficients (without negative sign) of the given like terms are 7, 3, 9.

$\therefore$ Sum of the numerical coefficients $= 7 + 3 + 9 = 19$.

So, the sum of the given like terms is another like term whose numerical coefficient is 19.

Hence, $-7xy - 3xy - 9xy = -19xy$.

Aliter Using the distributive property of multiplication over addition, we have

$-7xy - 3xy - 9xy = (-7 - 3 - 9)xy = -19xy.$

**ILLUSTRATION 2** Add $-4a^2y$, $-7a^2y$, $-10a^2y$ and $-3a^2y$.

*Solution* The sum of the numerical coefficients (without negative sign) is

$4 + 7 + 10 + 3 = 24$

Hence, $-4a^2y - 7a^2y - 10a^2y - 3a^2y = -24a^2y$.

### 7.6.3 ADDITION OF POSITIVE AND NEGATIVE LIKE TERMS

To add positive and negative like terms, we proceed as follows:

STEP I *Collect all positive like terms and find their sum.*

STEP II *Collect all negative like terms and find their sum.*

STEP III *Obtain the numerical coefficients (without negative sign) of like terms obtained in steps I and II.*

STEP IV *Subtract the numerical coefficient in step II from the numerical coefficient in step I. Write the answer as a product of this number and all the literal coefficients.*

**ILLUSTRATION 1** Add $4x^2y$, $8x^2y$ and $-2x^2y$.

*Solution* We have,

$4x^2y + 8x^2y - 2x^2y$

$= (4x^2y + 8x^2y) - 2x^2y$ [Collecting + ive and – ive like terms together]

$= 12x^2y - 2x^2y$

$= 10x^2y$ [$\because 12 - 2 = 10$]

Aliter $4x^2y + 8x^2y - 2x^2y = (4 + 8 - 2)x^2y = 10x^2y.$

**ILLUSTRATION 2** Add $4ab, -7ab, -10ab$ and $3ab$.

*Solution* We have,

$4ab - 7ab - 10ab + 3ab$

$= 4ab + 3ab - 7ab - 10ab$ [Collecting + ive and – ive like terms together]

$= 7ab - 17ab$ [$\because 4ab + 3ab = 7ab$ and $-7ab - 10ab = -17ab$]

$= -10ab$ [$\because 7 - 17 = -10$]

Aliter $4ab - 7ab - 10ab + 3ab = (4 - 7 - 10 + 3)\,ab = -10ab.$

## 7.7 ADDITION OF ALGEBRAIC EXPRESSIONS WITH LIKE AND UNLIKE TERMS

In adding algebraic expressions containing like and unlike terms, we collect different groups of like terms and find the sum of like terms in each group by the methods discussed above. The collection of like terms can be done by any one of the following two methods:

**(i) HORIZONTAL METHOD**

In this method, all expressions are written in a horizontal line and then the terms are arranged to collect all the groups of like terms and then added.

**(ii) COLUMN METHOD**

In this method, each expression is written in a separate row such that their like terms are arranged one below the other in a column. Then the addition of terms is done columnwise. Following illustrations will illustrate these methods.

**ILLUSTRATION 1** Add the following:

(i) $3x + 2y$ and $x + y$ (ii) $x + y + 3$ and $3x + 2y + 5$

(iii) $2x + 3y + z$ and $2x - y - z$

**Solution**

(i) *Horizontal Method*

$(3x + 2y) + (x + y)$
$= (3 + 1)x + (2 + 1)y$
$= 4x + 3y$

*Column Method*

$$\begin{array}{r} 3x + 2y \\ + \quad x + y \\ \hline 4x + 3y \end{array}$$

(ii) *Horizontal Method*

$(x + y + 3) + (3x + 2y + 5)$
$= (x + 3x) + (y + 2y) + (3 + 5)$
$= (1 + 3)x + (1 + 2)y + (3 + 5)$
$= 4x + 3y + 8$

*Column Method*

$$\begin{array}{r} x + y + 3 \\ + \; 3x + 2y + 5 \\ \hline 4x + 3y + 8 \end{array}$$

(iii) *Horizontal Method*

$(2x + 3y + z) + (2x - y - z)$
$= (2x + 2x) + (3y - y) + (z - z)$
$= (2 + 2)x + (3 - 1)y + (1 - 1)z$
$= 4x + 2y + 0z$
$= 4x + 2y.$

*Column Method*

$$\begin{array}{r} 2x + 3y + z \\ + \; 2x - y - z \\ \hline 4x + 2y \end{array}$$

**ILLUSTRATION 2** Add:

(i) $xy^2 + 4x^2y - 7x^2y - 3xy^2 + 3$ and $x^2y + xy^2$

(ii) $5x^2 + 7y - 6z^2, 4y + 3x^2, 9x^2 + 2z^2 - 9y$ and $2y - 2x^2$

**Solution**

(i) We have,

$xy^2 + 4x^2y - 7x^2y - 3xy^2 + 3$
$= xy^2 - 3xy^2 + 4x^2y - 7x^2y + 3$
$= (1 - 3)xy^2 + (4 - 7)x^2y + 3$
$= -2xy^2 - 3x^2y + 3$ ...(i)

*Horizontal Method*

$= (xy^2 + 4x^2y - 7x^2y - 3xy^2 + 3) + x^2y + xy^2$
$= -2xy^2 - 3x^2y + 3 + x^2y + xy^2$ [Using (i)]
$= -2xy^2 + xy^2 - 3x^2y + x^2y + 3$

*Column Method*

$$\begin{array}{r} -2xy^2 - 3x^2y + 3 \\ + \quad xy^2 + x^2y \\ \hline - xy^2 - 2x^2y + 3 \end{array}$$

$= (-2 + 1)xy^2 + (-3 + 1)x^2y + 3$

$= -xy^2 - 2x^2y + 3$

(ii) *Horizontal Method*

$(5x^2 + 7y - 6z^2) + (4y + 3x^2) + (9x^2 + 2z^2 - 9y) + (2y - 2x^2)$

$= (5x^2 + 3x^2 + 9x^2 - 2x^2) + (7y + 4y - 9y + 2y) + (-6z^2 + 2z^2)$

$= (5 + 3 + 9 - 2)x^2 + (7 + 4 - 9 + 2)y + (-6 + 2)z^2$

$= 15x^2 + 4y - 4z^2$

*Column Method*

$$\begin{array}{r} 5x^2 + 7y - 6z^2 \\ +\ 3x^2 + 4y \\ +\ 9x^2 - 9y + 2z^2 \\ -\ 2x^2 + 2y \\ \hline 15x^2 + 4y - 4z^2 \end{array}$$

## 7.8 SUBTRACTION OF ALGEBRAIC EXPRESSIONS

To subtract an algebraic expression from another, we should change the signs (from '+' to '−' or from '−' to '+') of all the terms of the expression which is to be subtracted and then the two expressions are added.

In the subtraction of two expressions by column method, we indicate the change of sign of every term in the expression to be subtracted below the original sign of each term.

Following examples will illustrate the procedure.

### ILLUSTRATIVE EXAMPLES

**Example 1** Subtract:

(i) $5x$ from $9x$ (ii) $-7x$ from $5x$ (iii) $-8a$ from $-3a$

**Solution** (i) $9x - 5x = (9 - 5)x = 4x$

(ii) $5x - (-7x) = 5x + 7x = (5 + 7)x = 12x$

(iii) $-3a - (-8a) = -3a + 8a = (-3 + 8)a = 5a.$

**Example 2** Subtract: $a^2 - 3ab$ from $2a^2 - 7ab$

*Solution* *Horizontal Method*

$(2a^2 - 7ab) - (a^2 - 3ab)$

$= 2a^2 - 7ab - a^2 + 3ab$

$= (2a^2 - a^2) - 7ab + 3ab$

$= a^2 - 4ab.$

*Column Method*

$$\begin{array}{r} 2a^2 - 7ab \\ a^2 - 3ab \\ - \quad + \\ \hline a^2 - 4ab \end{array}$$

**Example 3** Subtract: $x^2 - 3xy + 7y^2 - 2$ from $6xy - 4x^2 - y^2 + 5$

*Solution* *Horizontal Method*

$(6xy - 4x^2 - y^2 + 5) - (x^2 - 3xy + 7y^2 - 2)$

$= 6xy - 4x^2 - y^2 + 5 - x^2 + 3xy - 7y^2 + 2$

$= (6xy + 3xy) - 4x^2 - x^2 - y^2 - 7y^2 + 5 + 2$

$= 9xy - 5x^2 - 8y^2 + 7$

*Column Method*

$$\begin{array}{rrrr} 6xy & -\ 4x^2 & -\ y^2 & +\ 5 \\ -3xy & +\ x^2 & +\ 7y^2 & -\ 2 \\ + & - & - & + \\ \hline 9xy & -\ 5x^2 & -\ 8y^2 & +\ 7 \end{array}$$

**Example 4** From the sum of $4x^4 - 3x^3 + 6x^2$, $4x^3 + 4x - 3$ and $-3x^4 - 5x^2 + 2x$ subtract $5x^4 - 7x^3 - 3x + 4$.

*Solution* The sum of $4x^4 - 3x^3 + 6x^2$, $4x^3 + 4x - 3$ and $-3x^4 - 5x^2 + 2x$ is given by

$(4x^4 - 3x^3 + 6x^2) + (4x^3 + 4x - 3) + (-3x^4 - 5x^2 + 2x)$

$= 4x^4 - 3x^4 - 3x^3 + 4x^3 + 6x^2 - 5x^2 + 4x + 2x - 3$

$= x^4 + x^3 + x^2 + 6x - 3$

Now, we have to subtract $5x^4 - 7x^3 - 3x + 4$ from $x^4 + x^3 + x^2 + 6x - 3$.

$\therefore$ Required expression $= (x^4 + x^3 + x^2 + 6x - 3) - (5x^4 - 7x^3 - 3x + 4)$

$= x^4 + x^3 + x^2 + 6x - 3 - 5x^4 + 7x^3 + 3x - 4$

$= (x^4 - 5x^4) + (x^3 + 7x^3) + x^2 + 6x + 3x - 3 - 4$

$= -4x^4 + 8x^3 + x^2 + 9x - 7.$

**Example 5** What should be added to $a^2 + 2ab + b^2$ to obtain $4ab + b^2$?

*Solution* Required expression is equal to the subtraction of $a^2 + 2ab + b^2$ from $4ab + b^2$.

Hence, required expression $= (4ab + b^2) - (a^2 + 2ab + b^2)$

$= 4ab + b^2 - a^2 - 2ab - b^2$

$= 4ab - 2ab + b^2 - b^2 - a^2$

$= 2ab - a^2.$

**Example 6** What should be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$?

*Solution* Let $P$ denote the required expression. Then,

$(a^3 - 4a^2 + 5a - 6) - P = a^2 - 2a + 1$

Hence, required expression $P = (a^3 - 4a^2 + 5a - 6) - (a^2 - 2a + 1)$

$= a^3 - 4a^2 + 5a - 6 - a^2 + 2a - 1$

$= a^3 - 4a^2 - a^2 + 5a + 2a - 7$

$= a^3 - 5a^2 + 7a - 7.$

**Example 7** How much is $x^3 - 2x^2 + x + 4$ greater than $2x^3 + 7x^2 - 5x + 6$?

*Solution* Required expression $= (x^3 - 2x^2 + x + 4) - (2x^3 + 7x^2 - 5x + 6)$

$= x^3 - 2x^2 + x + 4 - 2x^3 - 7x^2 + 5x - 6$

$= x^3 - 2x^3 - 2x^2 - 7x^2 + x + 5x + 4 - 6$

$= -x^3 - 9x^2 + 6x - 2.$

**Example 8** How much is $2a^2 - 7a + 5$ less than $a^3 - 3a^2 + 2a - 3$?

*Solution* Required expression $= (a^3 - 3a^2 + 2a - 3) - (2a^2 - 7a + 5)$

$= (a^3 - 3a^2 + 2a - 3) - 2a^2 + 7a - 5$

$= a^3 - 3a^2 - 2a^2 + 2a + 7a - 3 - 5$

$= a^3 - 5a^2 + 9a - 8.$

**Example 9** How much does $2a^2 - 5a + 4$ exceed $3a^3 - 5a^2 + 7a - 9$?

*Solution* Required expression $= (2a^2 - 5a + 4) - (3a^3 - 5a^2 + 7a - 9)$

$= 2a^2 - 5a + 4 - 3a^3 + 5a^2 - 7a + 9$

$= -3a^3 + 2a^2 + 5a^2 - 5a - 7a + 4 + 9$

$= -3a^3 + 7a^2 - 12a + 13.$

## EXERCISE 7.2

1. Add the following:
   (i) $3x$ and $7x$  (ii) $-5xy$ and $9xy$
2. Simplify each of the following:
   (i) $7x^3y + 9yx^3$  (ii) $12a^2b + 3ba^2$
3. Add the following:
   (i) $7abc, -5abc, 9abc, -8abc$  (ii) $2x^2y, -4x^2y, 6x^2y, -5x^2y$

4. Add the following expressions:

(i) $x^3 - 2x^2y + 3xy^2 - y^3$, $2x^3 - 5xy^2 + 3x^2y - 4y^3$

(ii) $a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4$, $-2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4$

5. Add the following expressions:

(i) $8a - 6ab + 5b$, $-6a - ab - 8b$ and $-4a + 2ab + 3b$

(ii) $5x^3 + 7 + 6x - 5x^2$, $2x^2 - 8 - 9x$, $4x - 2x^2 + 3x^3$, $3x^3 - 9x - x^2$ and $x - x^2 - x^3 - 4$

6. Add the following:

(i)
$x - 3y - 2z$
$5x + 7y - 8z$
$3x - 2y + 5z$

(ii)
$4ab - 5bc + 7ca$
$-3ab + 2bc - 3ca$
$5ab - 3bc + 4ca$

7. Add $2x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $3x + 7$.

8. Add $x^2 + 2xy + y^2$ to the sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$.

9. Add $a^3 + b^3 - 3$ to the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$.

10. Subtract:

(i) $7a^2b$ from $3a^2b$ (ii) $4xy$ from $-3xy$

11. Subtract:

(i) $-4x$ from $3y$ (ii) $-2x$ from $-5y$

12. Subtract:

(i) $6x^3 - 7x^2 + 5x - 3$ from $4 - 5x + 6x^2 - 8x^3$

(ii) $-x^2 - 3z$ from $5x^2 - y + z + 7$ (ii) $x^3 + 2x^2y + 6xy^2 - y^3$ from $y^3 - 3xy^2 - 4x^2y$

13. From

(i) $p^3 - 4 + 3p^2$, take away $5p^2 - 3p^3 + p - 6$

(ii) $7 + x - x^2$, take away $9 + x + 3x^2 + 7x^3$

(iii) $1 - 5y^2$, take away $y^3 + 7y^2 + y + 1$

(iv) $x^3 - 5x^2 + 3x + 1$, take away $6x^2 - 4x^3 + 5 + 3x$

14. From the sum of $3x^2 - 5x + 2$ and $-5x^2 - 8x + 9$ subtract $4x^2 - 7x + 9$.

15. Subtract the sum of $13x - 4y + 7z$ and $-6z + 6x + 3y$ from the sum of $6x - 4y - 4z$ and $2x + 4y - 7$.

16. From the sum of $x^2 + 3y^2 - 6xy$, $2x^2 - y^2 + 8xy$, $y^2 + 8$ and $x^2 - 3xy$ subtract $-3x^2 + 4y^2 - xy + x - y + 3$.

17. What should be added to $xy - 3yz + 4zx$ to get $4xy - 3zx + 4yz + 7$?

18. What should be subtracted from $x^2 - xy + y^2 - x + y + 3$ to obtain $-x^2 + 3y^2 - 4xy + 1$?

19. How much is $x - 2y + 3z$ greater than $3x + 5y - 7$?

20. How much is $x^2 - 2xy + 3y^2$ less than $2x^2 - 3y^2 + xy$?

21. How much does $a^2 - 3ab + 2b^2$ exceed $2a^2 - 7ab + 9b^2$?

22. What must be added to $12x^3 - 4x^2 + 3x - 7$ to make the sum $x^3 + 2x^2 - 3x + 2$?

23. If $P = 7x^2 + 5xy - 9y^2$, $Q = 4y^2 - 3x^2 - 6xy$ and $R = -4x^2 + xy + 5y^2$, show that $P + Q + R = 0$.

24. If $P = a^2 - b^2 + 2ab$, $Q = a^2 + 4b^2 - 6ab$, $R = b^2 + b$, $S = a^2 - 4ab$ and $T = -2a^2 + b^2 - ab + a$. Find $P + Q + R + S - T$.

## ANSWERS

1. (i) $10x$ (ii) $4xy$ 2. (i) $16x^3y$ (ii) $15a^2b$

3. (i) $3abc$ (ii) $-x^2y$

4. (i) $3x^3 + x^2y - 2xy^2 - 5y^3$ (ii) $-a^4 + 5a^3b - 2a^2b^2 - 2ab^3 + 4b^4$
5. (i) $-2a - 5ab$ (ii) $10x^3 - 7x^2 - 7x - 5$
6. (i) $9x + 2y - 5z$ (ii) $6ab - 6bc + 8ca$
7. $5x^2 - 2x + 8$ 8. $4x^2 + 2xy - 3y^2 + 9$ 9. $2a^3 - b^3 - 5$
10. (i) $-4a^2b$ (ii) $-7xy$
11. (i) $3y + 4x$ (ii) $-5y + 2x$
12. (i) $-14x^3 + 13x^2 - 10x + 7$ (ii) $6x^2 - y + 4z + 7$ (iii) $2y^3 - 9xy^2 - 6x^2y - x^3$
13. (i) $4p^3 - 2p^2 - p + 2$ (ii) $-7x^3 - 4x^2 - 2$ (iii) $-y^3 - 12y^2 - y$
(iv) $5x^3 - 11x^2 - 4$ 14. $-6x^2 - 6x + 2$
15. $-11x + y - 5z - 7$ 16. $7x^2 - y^2 - x + y + 5$ 17. $3xy + 7yz - 7zx + 7$
18. $2x^2 + 3xy - 2y^2 - x + y + 2$ 19. $-2x - 7y + 3z + 7$ 20. $x^2 + 3xy - 6y^2$
21. $-a^2 + 4ab - 7b^2$ 22. $-11x^3 + 6x^2 - 6x + 9$ 24. $5a^2 + 3b^2 - 7ab - a + b$

## 7.9 THE USE OF GROUPING SYMBOLS (OR BRACKETS) IN WRITING ALGEBRAIC EXPRESSIONS

In dealing with algebraic expressions, sometimes it becomes necessary to consider an expression consisting of two or more terms as a single term. For example, if we say that the sum of $2x - y + 3$ and $x + 2y + 1$ is to be subtracted from the sum of $x + y$ and $3x - 4y + 5$. In this case, the sum of $2x - y + 3$ and $x + 2y + 1$ is taken as one term and the sum of $x + y$ and $3x - 4y + 5$ is also taken as one term. By using brackets (grouping symbols), the above statement can be written as

$$\{(2x - y + 3) + (x + 2y + 1)\} - \{(x + y) + (3x - 4y + 5)\}$$

Thus, we need to insert the brackets (or grouping symbols) to perform algebraic operations. We have already learnt in chapter 1 about the use of brackets in expressions consisting of numerals. Exactly in the same way we use brackets in performing operations on algebraic expressions. The different types of brackets which are used in operations on algebraic expressions are:

(i) Parentheses ( )
(ii) Curly bracket or braces { }
(iii) Square bracket [ ]

We shall now illustrate the use of these brackets through following examples.

### ILLUSTRATIVE EXAMPLES

**Example 1** Put the last two terms of each of the following expressions in the parentheses preceded by a minus sign:

(i) $2x + 3y - 4z + 7$ (ii) $3a - 2b - 7c - 4d$ (iii) $7xy - 4yz + 3zx - 5$

**Solution** We have,

(i) $2x + 3y - 4z + 7 = 2x + 3y - (4z - 7)$

(ii) $3a - 2b - 7c - 4d = 3a - 2b - (7c + 4d)$

(iii) $7xy - 4yz + 3zx - 5 = 7xy - 4yz - (-3zx + 5)$.

**Example 2** Write each of the following statements by using appropriate grouping symbols:

(i) The sum of $x + y$ and $2xy - 3x + 2y$ is subtracted from $xy - x + y$.

(ii) The subtraction of $x + y - 3$ from $3x - 2y + 9$ is subtracted from the sum of $4x + 3y - 9$ and $2x - y + z$.

(iii) The subtraction of $y - 1$ from $x$ is added to $3y$ and its difference from $y$ is subtracted from $x$.

*Solution* We have,

(i) The sum of $x + y$ and $2xy - 3x + 2y$ is $(x + y) + (2xy - 3x + 2y)$.
This is subtracted from $xy - x + y$. Therefore, required expression is

$$(xy - x + y) - \{(x + y) + (2xy - 3x + 2y)\}.$$

(ii) The subtraction of $x + y - 3$ from $3x - 2y + 9$ is

$(3x - 2y + 9) - (x + y - 3)$ ...(i)

The sum of $4x + 3y - 9$ and $2x - y + z$ is

$(4x + 3y - 9) + (2x - y + z)$ ...(ii)

Subtracting (i) from (ii), we obtain

$$\{(4x + 3y - 9) + (2x - y + z)\} - \{(3x - 2y + 9) - (x + y - 3)\}$$

(iii) The subtraction of $y - 1$ from $x$ is given by $x - (y - 1)$.

When this is added to $3y$, we get

$x - (y - 1) + 3y$

The difference of this from $y$ is

$$y - \{x - (y - 1) + 3y\}$$

When this is subtracted from $x$, we obtain

$$x - [y - \{x - (y - 1) + 3y\}].$$

**Example 3** Place the last two terms in each of the following expressions in parentheses preceded by a '–' sign:

(i) $9a + 5xy - 7x^2 + 8y - 6$ (ii) $-y + z + x^2 - y^2 - a^2$

(iii) $x + y + z - xy - yz - zx$ (iv) $xy^2 + yz^2 + zx^2$

*Solution* (i) $9a + 5xy - 7x^2 + 8y - 6 = 9a + 5xy - 7x^2 - (-8y + 6)$

(ii) $-y + z + x^2 - y^2 - a^2 = -y + z + x^2 - (y^2 + a^2)$

(iii) $x + y + z - xy - yz - zx = x + y + z - xy - z(y + x)$

(iv) $xy^2 + yz^2 + zx^2 = xy^2 - z(-yz - x^2)$.

## EXERCISE 7.3

1. Place the last two terms of the following expressions in parentheses preceded by a minus sign:

   (i) $x + y - 3z + y$ (ii) $3x - 2y - 5z - 4$

   (iii) $3a - 2b + 4c - 5$ (iv) $7a + 3b + 2c + 4$

   (v) $2a^2 - b^2 - 3ab + 6$ (vi) $a^2 + b^2 - c^2 + ab - 3ac$

2. Write each of the following statements by using appropriate grouping symbols:

   (i) The sum of $a - b$ and $3a - 2b + 5$ is subtracted from $4a + 2b - 7$.

   (ii) Three times the sum of $2x + y - \{5 - (x - 3y)\}$ and $7x - 4y + 3$ is subtracted from $3x - 4y + 7$.

   (iii) The subtraction of $x^2 - y^2 + 4xy$ from $2x^2 + y^2 - 3xy$ is added to $9x^2 - 3y^2 - xy$.

## ANSWERS

1. (i) $x+y-(3z-y)$ (ii) $3x-2y-(5z+4)$ (iii) $3a-2b-(-4c+5)$
(iv) $7a+3b-(-2c-4)$ (v) $2a^2-b^2-(3ab-6)$ (vi) $a^2+b^2-c^2-(-ab+3ac)$

2. (i) $(4a+2b-7)-\{(a-b)+(3a-2b+5)\}$

(ii) $(3x-4y+7)-3\left[(2x+y)-\{5-(x-3y)+(7x-4y+3)\}\right]$

(iii) $\left\{(2x^2+y^2-3xy)-(x^2-y^2+4xy)\right\}+(9x^2-3y^2-xy)$

## 7.10 REMOVAL OF BRACKETS

In the previous section, we have seen that when we make operations on two or more algebraic expressions, we use the symbols of groupings, i.e., parentheses, braces and brackets. In simplifying such expressions, we first remove the grouping symbols by using the following rules:

(i) If a ' + ' sign precedes a symbol of grouping, the grouping symbol may be removed without any change in the sign of the terms.

(ii) If a '−' sign precedes a symbol of grouping, the grouping symbol may be removed and the sign of each term is changed.

(iii) If more than one grouping symbol is present in an expression, we remove the inner-most grouping symbol first and collect and combine like terms, if any. We continue this process outwards until all the grouping symbols have been removed.

Following examples will illustrate these rules.

### ILLUSTRATIVE EXAMPLES

**Example 1** Simplify each of the following algebraic expressions:

(i) $(a^2+b^2+2ab)+(a^2+b^2-2ab)$ (ii) $(a^2+b^2+2ab)-(a^2+b^2-2ab)$

*Solution* (i) Since ' + ' sign precedes the second parentheses, so we remove it as it is.

$\therefore\ (a^2+b^2+2ab)+(a^2+b^2-2ab)$
$=a^2+b^2+2ab+a^2+b^2-2ab$
$=a^2+a^2+b^2+b^2+2ab-2ab$
$=(1+1)a^2+(1+1)b^2+(2-2)ab$
$=2a^2+2b^2+0ab$
$=2a^2+2b^2$ $[\therefore 0ab=0]$

(ii) Since ' − ' sign precedes the second parentheses, so we remove it and change the sign of each term.

$\therefore\ (a^2+b^2+2ab)-(a^2+b^2-2ab)$
$=a^2+b^2+2ab-a^2-b^2+2ab$
$=a^2-a^2+b^2-b^2+2ab+2ab$
$=(1-1)a^2+(1-1)b^2+(2+2)ab$
$=0a^2+0b^2+4ab$
$=4ab$ $[0\,a^2=0 \text{ and } 0\,b^2=0]$

**Example 2** Simplify each of the following:

(i) $-5(a+b)+2(2a-b)+4a-7$ (ii) $-3(a+b)+4(2a-3b)-(2a-b)$

*Solution* We have,

(i) $-5(a+b)+2(2a-b)+4a-7$

$= -5a-5b+2\times 2a-2b+4a-7$

$= -5a-5b+4a-2b+4a-7$

$= -5a+4a+4a-5b-2b-7$

$= (-5+4+4)a+(-5-2)b-7$

$= 3a-7b-7$

(ii) $-3(a+b)+4(2a-3b)-(2a-b)$

$= -3a-3b+8a-12b-2a+b$

$= -3a+8a-2a-3b-12b+b$

$= (-3+8-2)a+(-3-12+1)b$

$= 3a-14b.$

**Example 3** Simplify each of the following:

(i) $2x-\{5y-(x-2y)\}$ (ii) $2x-[3y-\{2x-(y-x)\}]$

(iii) $-m-[m+\{m+n-2m-(m-2n)\}-n]$

(iv) $3x^2z-4yz+3xy-\{x^2z-(x^2z-3yz)-4yz-7z\}$

*Solution* (i) We first remove the innermost grouping symbol ( ) and then braces { }. Thus, we have

$2x-\{5y-(x-2y)\}$

$= 2x-\{5y-x+2y\}$ [Removing ( )]

$= 2x-\{5y+2y-x\}$

$= 2x-\{7y-x\}$

$= 2x-7y+x$ [Removing { }]

$= 2x+x-7y$

$= 3x-7y$

(ii) We first remove the innermost grouping symbol (), then { } and then [ ]. Thus, we have

$2x-[3y-\{2x-(y-x)\}]$

$= 2x-[3y-\{2x-y+x\}]$ [Removing ( )]

$= 2x-[3y-\{3x-y\}]$

$= 2x-[3y-3x+y]$ [Removing { }]

$= 2x-[4y-3x]$

$= 2x-4y+3x$ [Removing [ ]]

$= 2x+3x-4y$

$= 5x-4y$

(iii) We first remove the innermost grouping symbol (), then {} and then []. Thus, we have,

$-m-[m+\{m+n-2m-(m-2n)\}-n]$

$$= -m - \left[m + \{m + n - 2m - m + 2n\} - n\right] \qquad \text{[Removing ( )]}$$

$$= -m - \left[m + \{m - 2m - m + n + 2n)\} - n\right]$$

$$= -m - \left[m + \{-2m + 3n\} - n\right]$$

$$= -m - [m - 2m + 3n - n] \qquad \text{[Removing \{ \}]}$$

$$= -m - [-m + 2n]$$

$$= -m + m - 2n \qquad \text{[Removing [ ]]}$$

$$= -2n$$

(iv) We first remove the innermost grouping symbol () and then {}. Thus, we have

$$3x^2z - 4yz + 3xy - \left\{x^2z - (x^2z - 3yz) - 4yz - 7z\right\}$$

$$= 3x^2z - 4yz + 3xy - \left\{x^2z - x^2z + 3yz - 4yz - 7z\right\}$$

$$= 3x^2z - 4yz + 3xy - \{-yz - 7z\}$$

$$= 3x^2z - 4yz + 3xy + yz + 7z \qquad \text{[Removing \{ \}]}$$

$$= 3x^2z - 3yz + 3xy + 7z$$

**Example 4** Simplify: $15x - \left[8x^3 + 3x^2 - \left\{8x^2 - (4 - 2x - x^3) - 5x^3\right\} - 2x\right]$

*Solution* We first simplify the innermost grouping symbol ( ), then { } and then [ ]. Thus, we have

$$15x - \left[8x^3 + 3x^2 - \left\{8x^2 - (4 - 2x - x^3) - 5x^3\right\} - 2x\right]$$

$$= 15x - \left[8x^3 + 3x^2 - \left\{8x^2 - 4 + 2x + x^3 - 5x^3\right\} - 2x\right]$$

$$= 15x - \left[8x^3 + 3x^2 - \left\{8x^2 - 4 + 2x - 4x^3\right\} - 2x\right]$$

$$= 15x - \left[8x^3 + 3x^2 - 8x^2 + 4 - 2x + 4x^3 - 2x\right]$$

$$= 15x - \left[8x^3 + 4x^3 + 3x^2 - 8x^2 - 2x - 2x + 4\right]$$

$$= 15x - \left[12x^3 - 5x^2 - 4x + 4\right]$$

$$= 15x - 12x^3 + 5x^2 + 4x - 4$$

$$= -12x^3 + 5x^2 + 15x + 4x - 4 = -12x^3 + 5x^2 + 19x - 4.$$

**Example 5** Simplify : $5 + \left[x - \left\{2y - (6x + y - 4) + 2x^2\right\} - (x^2 - 2y)\right]$

*Solution* We first remove the innermost grouping symbol (), then { } and then [ ]. Thus, we have

$$5 + \left[x - \left\{2y - (6x + y - 4) + 2x^2\right\} - (x^2 - 2y)\right]$$

$$= 5 + \left[x - \left\{2y - 6x - y + 4 + 2x^2\right\} - (x^2 - 2y)\right]$$

$$= 5 + \left[x - \{2y - y - 6x + 4 + 2x^2\} - (x^2 - 2y)\right]$$

$$= 5 + \left[x - \{y - 6x + 4 + 2x^2\} - x^2 + 2y\right]$$

$$= 5 + \left[x - y + 6x - 4 - 2x^2 - x^2 + 2y\right]$$

$$= 5 + \left[x + 6x - y + 2y - 2x^2 - x^2 - 4\right]$$

$$= 5 + \left[(1 + 6)x + y(-1 + 2) + x^2(-2 - 1) - 4\right]$$

$$= 5 + \left[7x + y - 3x^2 - 4\right] = 5 + 7x + y - 3x^2 - 4 \ = 1 + 7x + y - 3x^2.$$

**Example 6** Simplify and find the value of the following expression when $a = 3$ and $b = 1$:

$$4(a^2 + b^2 + 2ab) - \left[4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}\right]$$

*Solution* Proceeding outward from the innermost bracket, we obtain

$$4(a^2 + b^2 + 2ab) - \left[4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}\right]$$

$$= 4(a^2 + b^2 + 2ab) - \left[4(a^2 + b^2 - 2ab) - \{-b^3 + 4a - 12\}\right]$$

$$= 4a^2 + 4b^2 + 8ab - \left[4a^2 + 4b^2 - 8ab + b^3 - 4a + 12\right]$$

$$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 + 8ab - b^3 + 4a - 12$$

$$= 4a^2 - 4a^2 + 4b^2 - 4b^2 + 8ab + 8ab - b^3 + 4a - 12$$

$$= (4 - 4)a^2 + (4 - 4)b^2 + (8 + 8)ab - b^3 + 4a - 12 = 16ab - b^3 + 4a - 12$$

The value of this expression for $a = 3$ and $b = 1$ is

$$16 \times 3 \times 1 - (1)^3 + 4 \times 3 - 12 = 48 - 1 + 12 - 12 = 47$$

## EXERCISE 7.4

Simplify each of the following algebraic expressions by removing grouping symbols.

1. $2x + (5x - 3y)$
2. $3x - (y - 2x)$
3. $5a - (3b - 2a + 4c)$
4. $-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$
5. $3x + 2y - \{x - (2y - 3)\}$
6. $5a - \{3a - (2 - a) + 4\}$
7. $a - \left[b - \{a - (b - 1) + 3a\}\right]$
8. $a - \left[2b - \{3a - (2b - 3c)\}\right]$
9. $-x + \left[5y - \{2x - (3y - 5x)\}\right]$
10. $2a - \left[4b - \{4a - 3(2a - b)\}\right]$
11. $-a - \left[a + \{a + b - 2a - (a - 2b)\} - b\right]$
12. $2x - 3y - \left[3x - 2y - \{x - z - (x - 2y)\}\right]$
13. $5 + \left[x - \{2y - (6x + y - 4) + 2x\} - \{x - (y - 2)\}\right]$
14. $x^2 - \left[3x + \{2x - (x^2 - 1)\} + 2\right]$
15. $20 - \left[5xy + 3\{x^2 - (xy - y) - (x - y)\}\right]$
16. $85 - \left[12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}\right]$
17. $xy - \left[yz - zx - \{yx - (3y - xz) - (xy - zy)\}\right]$

## ANSWERS

1. $7x - 3y$
2. $5x - y$
3. $7a - 3b - 4c$
4. $-5x^2 - y^2 + xy$
5. $2x + 4y - 3$
6. $a - 2$
7. $5a - 2b + 1$
8. $4a - 4b + 3c$
9. $-8x + 8y$
10. $-b$
11. $-2b$
12. $-x + y - z$
13. $4x - 1$
14. $2x^2 - 5x - 3$
15. $-3x^2 - 2xy - 6y + 3x + 20$
16. $44 + 104x$
17. $xy + 2zx - 3y$

## OBJECTIVE TYPE QUESTIONS

*Mark the correct alternative in each of the following:*

1. Which of the following pairs is/are like terms?
   (1) x (2) $x^2$ (3) $3x^3$ (4) $4x^3$
   (a) 1, 2 (b) 2, 3 (c) 3, 4 (d) None of these
2. Which of the following is not a monomial?
   (a) $2x^2 + 1$ (b) $3x^4$ (c) $ab$ (d) $x^2y$
3. The sum of the coefficients in the monomials $3a^2b$ and $-2ab^2$ is
   (a) 5 (b) $-1$ (c) 1 (d) $-6$
4. The coefficient of $x^2$ in $-\frac{5}{3}x^2y$ is equal to
   (a) $-\frac{5}{3}$ (b) $-\frac{5}{3}y$ (c) $\frac{5}{3}$ (d) $\frac{5}{3}y$
5. If *a*, *b* and *c* are respectively the coefficients of $x^2$ in $-x^2$, $2x^2 + x$ and $2x - x^2$ respectively, then $a + b + c =$
   (a) 0 (b) $-2$ (c) 2 (d) $-1$
6. The sum of the coefficients in the terms of $2x^2y - 3xy^2 + 4xy$ is
   (a) $-3$ (b) 3 (c) 9 (d) 5
7. The product of the coefficients of terms in $-\frac{4}{3}ab^2 + \frac{1}{4}bc^2 + 3ca^2$ is
   (a) 1 (b) $\frac{1}{2}$ (c) $-1$ (d) 3
8. If *a* and *b* are respectively the sum and product of coefficients of terms in the expression $x^2 + y^2 + z^2 - xy - yz - zx$, then $a + 2b =$
   (a) 0 (b) 2 (c) $-2$ (d) $-1$
9. If $P = 3x^3 + 3x^2 + 3x + 3$ and $Q = 3x^2 - 3x + 3$, then $P - Q =$
   (a) $3x^3$ (b) $3x^3 + 6x^2 + 6x + 6$ (c) $6x^2 + 6x + 6$ (d) $3x^3 + 6x$
10. The sum of the values of the expression $2x^2 = 2x + 2$ when $x = -1$ and $x = 1$ is
    (a) 6 (b) 8 (c) 4 (d) 2
11. What should be added to $3x^2 + 4$ to get $9x^2 - 7$?
    (a) $6x^2 - 11$ (b) $6x^2 + 11$ (c) $12x^2 - 11$ (d) $12x^2 + 11$
12. How much is $a^2 - 3a$ greater than $2a^2 + 4a$?
    (a) $a^2 - 7a$ (b) $a^2 + 7a$ (c) $-a^2 - 7a$ (d) $-a^2 + 7a$
13. How much is $-2x^2 + x + 1$ less than $x^2 + 2x - 3$?
    (a) $-x^2 + 3x - 2$ (b) $3x^2 + x - 4$ (c) $-3x^2 - x + 4$ (d) $3x^2 + 3x - 4$

14. What should be added to $xy + yz + zx$ to get $-xy - yz - zx$ ?

(a) $-2xy - 2yz - 2zx$ (b) $-3xy - yz - zx$

(c) $-3xy - 3yz - 3zx$ (d) $2xy + 2yz + 2zx$

## ANSWERS

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (c) | 2. (a) | 3. (c) | 4. (b) | 5. (a) | 6. (b) | 7. (c) |
| 8. (c) | 9. (d) | 10. (b) | 11. (a) | 12. (c) | 13. (b) | 14. (a) |

## THINGS TO REMEMBER

1. *The letters which are used to represent numbers are called literal numbers or literals.*
2. *The literal numbers themselves as well as the combinations of literal numbers and numbers obey all the rules (and signs) of addition, subtraction, multiplication and division of numbers along with the properties of these operations.*
3. $x \times y = xy$, $5 \times x = 5x$, $1 \times x = x$, $x \times 4 = 4x$.
4. $a \times a \times a \times \ldots \times$ 12 times $= a^{12}$, $y \times y \times y \times \ldots \times$ 15 times $= y^{15}$.
5. *In $x^9$, 9 is called the index or exponent and $x$ is called the base. In $a^5$, the index or exponent is 5 and the base is a.*
6. *A symbol having a fixed numerical value is called a constant.*
7. *A symbol which takes various numerical values is called a variable.*
8. *A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression.*
9. *Various parts of an algebraic expression which are separated by the signs of '+' or '−' are called the terms of the expression.*
10. *An algebraic expression is called a monomial, a binomial, a trinomial, a quadrinomial according as it contains one term, two terms, three terms and four terms respectively.*
11. *Each term in an algebraic expression is a product of one or more number(s) and/or literal number(s). These number(s) and/or literal number(s) are known as the factors of that term.*
12. *A term of the expression having no literal factor is called a constant term.*
13. *In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the product of the factors.*
14. *The terms having the same literal factors are called like or similar terms.*
15. *The terms not having same literal factors are called unlike or dissimilar terms.*
16. *The sum or difference of several like terms is another like term whose coefficient is the sum or difference of those like terms.*
17. *In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.*
18. *To subtract an expression from another, we change the sign (from '+' to '−' and from '−' to '+') of each term of the expression to be subtracted and then add the two expressions.*
19. *When a grouping symbol preceded by '−' sign is removed or inserted, then the sign of each term of the corresponding expression is changed (from '+' to '−' and from '−' to '+').*