6

EXPONENTS

6.1 INTRODUCTION

Consider following numbers:

89,000,000,000
1,459,500,000,000
750,000,000,000,000
5,978,043,000,000,000

We find that it is not very convenient to read, understand and compare such large numbers. In order to make such large numbers easy to read, understand and compare, we use exponents. In this chapter, we shall learn about exponents and their uses.

6.2 EXPONENTS

As we know that the continued sum of a number added to itself a number of times can be written as the product of a natural number, equal to the number of times it is added, and the number itself.

For example,

$$5+5+5+5+5+5+5=7\times 5$$

$$(-2)+(-2)+(-2)+(-2)+(-2)+(-2)+(-2)+(-2)+(-2)=9\times(-2)$$

$$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=5\times\frac{3}{4} \text{ etc.}$$

Similarly, the continued product of a number multiplied with itself a number of times can be written as the number raised to the power a natural number, equal to the number of times the number is multiplied with itself.

For example,

$5\times5\times5$ can be written as 5^3 and it is read as 5 raised to the power 3 or third power of 5. In 5^3, we call 5 as the base and 3 as the exponent.

Similarly,

$(-2)\times(-2)\times(-2)\times(-2)$ is written as $(-2)^4$

$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$ is written as $\left(\frac{2}{3}\right)^5$

We have,

$5\times5\times5=125$

Also, $5\times5\times5$ is written as 5^3.

$\therefore$ $125=5^3$

Example 4 Simplify:

(i) $5^2 \times 3^3$ (ii) $2^4 \times 3^2$ (iii) $3^2 \times 10^4$ (iv) $5^3 \times 2^4$

Solution (i) We have,

$5^2 \times 3^3 = 25 \times 27$ $[\because 5^2 = 5 \times 5 = 25 \text{ and } 3^3 = 3 \times 3 \times 3 = 27]$

$= 675$

(ii) We have,

$2^4 \times 3^2 = 16 \times 9$ $[\because 2^4 = 2 \times 2 \times 2 \times 2 = 16 \text{ and } 3^2 = 3 \times 3 = 9]$

$= 144$

(iii) We have,

$3^2 \times 10^4 = 9 \times 10000$ $[\because 3^2 = 3 \times 3 = 9 \text{ and } 10^4 = 10 \times 10 \times 10 \times 10 = 10000]$

$= 90000$

(iv) We have,

$5^3 \times 2^4 = 125 \times 16$ $[\because 5^3 = 5 \times 5 \times 5 = 125 \text{ and } 2^4 = 2 \times 2 \times 2 \times 2 = 16]$

$= 2000$

Example 5 Simplify:

(i) $(-3) \times (-2)^3$ (ii) $(-3)^2 \times (-5)^2$ (iii) $(-2)^3 \times (-10)^3$ (iv) $(-2)^4 \times (-5)^2$

Solution (i) We have,

$(-3) \times (-2)^3 = (-3) \times (-8)$ $[\because (-2)^3 = (-2) \times (-2) \times (-2) = -8]$

$= 24$

(ii) We have,

$(-3)^2 \times (-5)^2 = 9 \times 25$ $[\because (-3)^2 = (-3) \times (-3) = 9 \text{ and } (-5)^2 = (-5) \times (-5) = 25]$

$= 225$

(iii) We have,

$$(-2)^3 \times (-10)^3 = (-8) \times (-1000) \quad \left[\begin{array}{l} \because \quad (-2)^3 = (-2) \times (-2) \times (-2) = -8 \\ \text{and } (-10)^3 = (-10) \times (-10) \times (-10) = -1000 \end{array}\right]$$

$= 8000$

(iv) We have,

$$(-2)^4 \times (-5)^2 = 16 \times 25 \quad \left[\begin{array}{l} \because \quad (-2)^4 = (-2) \times (-2) \times (-2) \times (-4) = 16 \\ \text{and } (-5)^2 = (-5) \times (-5) = 25 \end{array}\right]$$

$= 400$

Example 6 Simplify:

(i) $(1)^5$ (ii) $(-1)^5$ (iii) $(-1)^6$ (iv) $(-1)^7$

Solution We have,

(i) $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

(ii) $(-1)^5 = (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

$= ((-1) \times (-1)) \times ((-1) \times (-1)) \times (-1)$

$$= 1 \times 1 \times (-1) = 1 \times (-1) = -1$$

(iii) $(-1)^6 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

$$= ((-1) \times (-1)) \times ((-1) \times (-1)) \times ((-1) \times (-1)) = 1 \times 1 \times 1 = 1$$

(iv) $(-1)^7 = (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

$$= ((-1) \times (-1)) \times ((-1) \times (-1)) \times ((-1) \times (-1)) \times (-1)$$

$$= 1 \times 1 \times 1 \times (-1) = 1 \times (-1) = -1$$

Remark 1 *It follows from the above example that*

$$(-1)^{\text{An odd natural number}} = -1 \text{ and, } (-1)^{\text{An even natural number}} = 1$$

i.e., $$(-1)^n = \begin{cases} 1, & \text{if } n \text{ is an even natural number} \\ -1, & \text{if } n \text{ is an odd natural number} \end{cases}$$

Example 7 Express each of the following in the form $\frac{p}{q}$:

(i) $\left(\frac{2}{3}\right)^2$ (ii) $\left(\frac{-3}{4}\right)^3$ (iii) $\left(\frac{-2}{5}\right)^4$

Solution We have,

(i) $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$

(ii) $\left(\frac{-3}{4}\right)^3 = \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) = \frac{(-3) \times (-3) \times (-3)}{4 \times 4 \times 4} = \frac{-27}{64}$

(iii) $\left(\frac{-2}{5}\right)^4 = \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) \times \left(\frac{-2}{5}\right) = \frac{(-2) \times (-2) \times (-2) \times (-2)}{5 \times 5 \times 5 \times 5} = \frac{16}{625}$

Remark 2 *If $\frac{a}{b}$ is a rational number and n is a natural number, then*

$$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \cdots \times \frac{a}{b}}_{(n\text{-times})} = \frac{\overbrace{a \times a \times a \times a \times \cdots \times a}^{(n\text{-times})}}{\underbrace{b \times b \times b \times b \times \cdots \times b}_{(n\text{-times})}} = \frac{a^n}{b^n}$$

$$\therefore \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example 8 Identify the greater number in each of the following:

(i) 5^3 or 3^5 (ii) 2^8 or 8^2 (iii) 2^{10} or 10^2 (iv) 2^{100} or 100^2

Solution (i) We have,

$5^3 = 5 \times 5 \times 5 = 125$ and $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$\because$ $243 > 125$

$\therefore$ $3^5 > 5^3$

(ii) We have,

$$2^8 = 2\times2\times2\times2\times2\times2\times2\times2$$
$$= (2\times2)\times(2\times2)\times(2\times2)\times(2\times2)$$
$$= 4\times4\times4\times4 = (4\times4)\times(4\times4) = 16\times16 = 256$$

and, $8^2 = 8\times8 = 64$

$\because \quad 256 > 64$

$\therefore \quad 2^8 > 8^2$

(iii) We have,

$$2^{10} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2$$
$$= (2\times2)\times(2\times2)\times(2\times2)\times(2\times2)\times(2\times2)$$
$$= 4\times4\times4\times4\times4 = (4\times4)\times(4\times4)\times4 = 16\times16\times4 = 256\times4 = 1024$$

and, $10^2 = 10\times10 = 100$

$\because \quad 1024 > 100$

$\therefore \quad 2^{10} > 10^2$

(iv) In (iii) and (iv), we have seen that

$$2^8 > 8^2 \text{ and } 2^{10} > 10^2$$

Similarly, it can be seen that

$$2^{15} > 15^2, 2^{20} > 20^2, 2^{50} > 50^2 \text{ and } 2^{100} > 100^2$$

Example 9 Find the product of the cube of $\frac{-2}{3}$ and the square of $\frac{4}{-5}$.

Solution We have,

$$\left(\text{Cube of } \frac{-2}{3}\right)\times\left(\text{Square of } \frac{4}{-5}\right)$$

$$= \left(\frac{-2}{3}\right)^3 \times \left(\frac{4}{-5}\right)^2$$

$$= \left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{4}{-5}\right)\times\left(\frac{4}{-5}\right)$$

$$= \frac{(-2)\times(-2)\times(-2)}{3\times3\times3}\times\frac{4\times4}{(-5)\times(-5)} = \frac{-8}{27}\times\frac{16}{25} = \frac{-8\times16}{27\times25} = \frac{-128}{675}$$

Example 10 Express the following as a rational number:

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{-3}{5}\right)^3 \times \left(\frac{-4}{9}\right)^2$$

Solution We have,

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{-3}{5}\right)^3 \times \left(\frac{-4}{9}\right)^2$$

$$= \frac{1^3}{2^3} \times \frac{(-3)^3}{5^3} \times \frac{(-4)^2}{9^2} \qquad \left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \text{ See remark 2 on page 6.6}\right]$$

$$= \frac{1}{8} \times \frac{-27}{125} \times \frac{16}{81} = \frac{1 \times -\overset{1}{\cancel{27}} \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}} \times 125 \times \underset{3}{\cancel{81}}} = \frac{1 \times -1 \times 2}{1 \times 125 \times 3} = \frac{-2}{375}$$

Example 11 Simplify:

(i) $(-3)^2 \times \left(\frac{-5}{12}\right)^2$ (ii) $\left(\frac{-2}{5}\right)^3 \div \left(\frac{-3}{10}\right)^4$

Solution (i) We have,

$$(-3)^2 \times \left(\frac{-5}{12}\right)^2 = (-3)^2 \times \frac{(-5)^2}{12^2} \qquad \left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$$

$$= \overset{1}{\cancel{9}} \times \frac{25}{\underset{16}{\cancel{144}}} = 1 \times \frac{25}{16} = \frac{25}{16}$$

(ii) We have

$$\left(\frac{-2}{5}\right)^3 \div \left(\frac{-3}{10}\right)^4 = \frac{\left(\frac{-2}{5}\right)^3}{\left(\frac{-3}{10}\right)^4}$$

$$= \frac{\frac{(-2)^3}{5^3}}{\frac{(-3)^4}{10^4}} \qquad \left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$$

$$= \frac{\frac{-8}{125}}{\frac{81}{10000}} \quad [\because (-2)^5 = -8, 5^3 = 125, (-3)^4 = 81 \text{ and } 10^4 = 10000]$$

$$= \frac{-8}{125} \times \frac{10000}{81} = \frac{-8 \times \overset{80}{\cancel{10000}}}{\underset{1}{\cancel{125}} \times 81} = \frac{-8 \times 80}{1 \times 81} = \frac{-640}{81}$$

Example 12 Simplify:

(i) $\left\{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^3\right\} \times 2^3$ (ii) $\left\{(3^2 - 2^2) \div \left(\frac{1}{5}\right)^2\right\}$

Solution (i) We have,

$$\left\{\left(\frac{1}{2}\right)^2-\left(\frac{1}{4}\right)^3\right\}\times 2^3$$

$$=\left\{\frac{1^2}{2^2}-\frac{1^3}{4^3}\right\}\times 2^3 \qquad \left[\because \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\right]$$

$$=\left(\frac{1}{4}-\frac{1}{64}\right)\times 8$$

$$=\left(\frac{16-1}{64}\right)\times 8=\frac{15}{64}\times 8=\frac{15\times \overset{1}{\cancel{8}}}{\underset{8}{\cancel{64}}}=\frac{15\times 1}{8}=\frac{15}{8}$$

(ii) We have,

$$\left\{(3^2-2^2)\div\left(\frac{1}{5}\right)^2\right\}=\left\{(9-4)\div\left(\frac{1}{5}\right)^2\right\}$$

$$=5\div\left(\frac{1}{5}\right)^2=\frac{5}{\left(\frac{1}{5}\right)^2}=\frac{5}{\frac{1^2}{5^2}}=\frac{5}{\frac{1}{25}}=5\times\frac{25}{1}=125$$

Example 13 If $a=2$ and $b=3$, then find the values of each of the following:

(i) a^a+b^b (ii) a^b+b^a (iii) $\left(\frac{a}{b}\right)^a$ (iv) $\left(\frac{1}{a}+\frac{1}{b}\right)^a$

Solution We have,

(i) $a^a+b^b=2^2+3^3=2\times 2+3\times 3\times 3=4+27=31$

(ii) $a^b+b^a=2^3+3^2=2\times 2\times 2+3\times 3=8+9=17$

(iii) $\left(\frac{a}{b}\right)^a=\left(\frac{2}{3}\right)^2=\frac{2}{3}\times\frac{2}{3}=\frac{2\times 2}{3\times 3}=\frac{4}{9}$

(iv) $\left(\frac{1}{a}+\frac{1}{b}\right)^a=\left(\frac{1}{2}+\frac{1}{3}\right)^2=\left(\frac{3+2}{6}\right)^2=\left(\frac{5}{6}\right)^2=\frac{5}{6}\times\frac{5}{6}=\frac{5\times 5}{6\times 6}=\frac{25}{36}$

Example 14 Simplify and express each of the following as power of a rational number:

(i) $\left(\frac{2}{3}\right)^3\times\left(-\frac{6}{7}\right)^2\times\left(-\frac{7}{4}\right)\times\frac{3}{2}$ (ii) $-\left(\frac{2}{5}\right)^2\times\left(\frac{5}{7}\right)^2\times\frac{49}{5}+\left(-\frac{4}{5}\right)^3\times\frac{5}{4}\times\frac{3}{4}$

Solution (i) We have,

$$\left(\frac{2}{3}\right)^3\times\left(-\frac{6}{7}\right)^2\times\left(-\frac{7}{4}\right)\times\frac{3}{2}$$

$$=\left(\frac{2}{3}\right)^3\times(-1)^2\times\left(\frac{6}{7}\right)^2\times\left(-\frac{7}{4}\right)\times\frac{3}{2}$$

$$= \frac{2^3}{3^3} \times (-1)^2 \times \frac{6^2}{7^2} \times -\frac{7}{4} \times \frac{3}{2}$$

$$= \frac{8}{27} \times 1 \times \frac{36}{49} \times -\frac{7}{4} \times \frac{3}{2}$$

$$= \frac{8}{27} \times \frac{36}{49} \times \frac{-7}{4} \times \frac{3}{2} = \frac{8 \times 36 \times -7 \times 3}{27 \times 49 \times 4 \times 2}$$

$$= \frac{1 \times 36 \times -1 \times 1}{9 \times 7 \times 1 \times 1} = \frac{-36}{9 \times 7} = \frac{-4}{7} = \left(-\frac{4}{7}\right)^1 \qquad [\because a^1 = a]$$

(ii) We have,

$$-\left(\frac{2}{5}\right)^2 \times \left(\frac{5}{7}\right)^2 \times \frac{49}{5} + \left(-\frac{4}{5}\right)^3 \times \frac{5}{4} \times \frac{3}{4}$$

$$= -\frac{2^2}{5^2} \times \frac{5^2}{7^2} \times \frac{49}{5} + (-1)^3 \times \left(\frac{4}{5}\right)^3 \times \frac{5}{4} \times \frac{3}{4}$$

$$= -\frac{2^2}{5^2} \times \frac{5^2}{7^2} \times \frac{49}{5} + (-1) \times \frac{4^3}{5^3} \times \frac{5}{4} \times \frac{3}{4}$$

$$= -\frac{4}{25} \times \frac{25}{49} \times \frac{49}{5} + (-1) \times \frac{64}{125} \times \frac{5}{4} \times \frac{3}{4}$$

$$= -\frac{4}{1} \times \frac{1}{1} \times \frac{1}{5} + (-1) \times \frac{4}{25} \times \frac{1}{1} \times \frac{3}{1}$$

$$= -\frac{4}{5} - \frac{12}{25} = -\frac{4}{5} - \frac{12}{25} = -\frac{20}{25} - \frac{12}{25} = -\frac{32}{25} = \left(\frac{-32}{25}\right)^1$$

Type II **ON EXPRESSING NUMBERS IN THE EXPONENTIAL FORM**

Example 15 Express each of the following in exponential form:

(i) $(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4)$ (ii) $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

Solution We have,

(i) $(-4) \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) = (-4)^6$

(ii) $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \left(\frac{3}{5}\right)^4$

Example 16 Express each of the following in exponential form:

(i) $2 \times 2 \times 2 \times a \times a$ (ii) $a \times a \times a \times a \times b \times b \times c \times c \times c \times c \times c$

(iii) $a \times a \times a \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right)$

Solution We have,

(i) $2 \times 2 \times 2 \times a \times a = 2^3 \times a^2$

(ii) $a \times a \times a \times a \times b \times b \times c \times c \times c \times c \times c = a^4 b^2 c^5$

(iv) $a \times a \times a \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) = a^3 \times \left(\frac{-2}{3}\right)^2$

Example 17 Express each of the following numbers in exponential form:

(i) 128 (ii) 243 (iii) 3125

Solution (i) We have,

$128 = 2\times2\times2\times2\times2\times2\times2$

$\therefore \quad 128 = 2^7$

2	128
2	64
2	32
2	16
2	8
2	4
	2

(ii) We have,

$243 = 3\times3\times3\times3\times3$

$\therefore \quad 243 = 3^5$

3	243
3	81
3	27
3	9
	3

(iii) We have,

$3125 = 5\times5\times5\times5\times5$

$\therefore \quad 3125 = 5^5$

5	3125
5	625
5	125
5	25
	5

Example 18 Express each of the following numbers as a product of powers of their prime factors:

(i) 432 (ii) 648 (iii) 540

Solution (i) Using prime factorisation of 432, we have

$432 = 2\times2\times2\times2\times3\times3\times3$

$\therefore \quad 432 = 2^4\times3^3$

2	432
2	216
2	108
2	54
3	27
3	9
	3

(ii) Using prime factorisation of 648, we have

$648 = 2\times2\times2\times3\times3\times3\times3$

$\therefore \quad 648 = 2^3\times3^4$

2	648
2	324
2	162
3	81
3	27
3	9
	3

(ii) Using prime factorisation of 540, we have

$540 = 2\times2\times3\times3\times3\times5$

$\therefore \quad 540 = 2^2\times3^3\times5^1$

2	540
2	270
3	135
3	45
3	15
	5

Example 19 Express the following numbers as product of powers of their prime factors:

(i) 1000 (ii) 16000 (iii) 3600

Solution (i) Expressing 1000 as the product of prime factors, we have

$1000 = 2\times2\times2\times5\times5\times5$

$\therefore \quad 1000 = 2^3 \times 5^3$

2	1000
2	500
2	250
5	125
5	25
	5

Aliter We have,

$1000 = 10\times10\times10$

$= (2\times5)\times(2\times5)\times(2\times5)$

$= 2\times2\times2\times5\times5\times5 = 2^3\times5^3$

(ii) Expressing 16000 as the product of prime factors, we have

$16000 = 2\times2\times2\times2\times2\times2\times2\times5\times5\times5$

$= 2^7 \times 5^3$

2	16000
2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
	5

Aliter We have,

$16000 = 16\times1000$

$= 16\times10\times10\times10$

$= (2\times2\times2\times2)\times(2\times5)\times(2\times5)\times(2\times5)$

$= (2\times2\times2\times2\times2\times2\times2)\times(5\times5\times5)$

$= 2^7\times5^3$

(iii) Expressing 3600 as the product of prime factors, we have

$3600 = 2\times2\times2\times2\times3\times3\times5\times5$

$\therefore \quad 3600 = 2^4\times3^2\times5^2$

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
	5

Aliter We have,

$3600 = 36\times100$

$= 4\times9\times10\times10$

$= (2\times2)\times(3\times3)\times(2\times5)\times(2\times5)$

$= (2\times2\times2\times2)\times(3\times3)\times(5\times5) = 2^4\times3^2\times5^2$

Example 20 Express each of the following rational numbers in exponential form:

(i) $\frac{27}{64}$ (ii) $\frac{-27}{125}$ (iii) $\frac{-1}{243}$

Solution (i) We have,

$\frac{27}{64} = \frac{3^3}{4^3}$ $\quad [\because 27 = 3^3 \text{ and } 64 = 4^3]$

$= \left(\frac{3}{4}\right)^3$ $\quad \left[\because \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n\right]$

Similarly,

$$(-2)\times(-2)\times(-2)\times(-2)=16$$

$$\Rightarrow \qquad (-2)^4=16$$

5^3 is known as the exponential form or power notation of 125. Here, 5 is the base and 3 is the exponent.

Similarly, $(-2)^4$ is the exponential form or power notation of 16 with base -2 and exponent 4.

We have,

$$100=10\times 10$$
$$1000=10\times 10\times 10$$
$$10000=10\times 10\times 10\times 10$$
$$100000=10\times 10\times 10\times 10\times 10 \text{ etc.}$$

So, the exponential forms of 100, 1000, 10000 and 100000 are as given below:

$$100=10^2 \text{ (read as 10 raised to the power 2)}$$
$$1000=10^3 \text{ (read as 10 raised to the power 3)}$$
$$10000=10^4 \text{ (read as 10 raised to the power 4)}$$
$$100000=10^5 \text{ (read as 10 raised to the power 5)}$$

Some powers have special names. For example,

5^2, which is 5 raised to the power 2, is also read as '5 squared'.

5^3, which is 5 raised to the power 3, is also read as '5 cubed'.

Similarly, 10^2 is read as '10 squared' and 10^3 is read as '10 cubed'.

It follows from the above discussion that for any rational number a, we have

$a\times a=a^2$ (read as 'a squared' or 'a raised to the power 2')

$a\times a\times a=a^3$ (read as 'a cubed' or 'a raised to the power 3')

$a\times a\times a\times a=a^4$ (read as a raised to the power 4 or the 4^{th} power of a)

$a\times a\times a\times a\times a=a^5$ (read as a raised to the power 5 or the 5^{th} power of a)

and so on.

In general, if n is a natural number, then

$$\underbrace{a\times a\times a\times a\times\cdots\times a}_{n\text{-times}}=a^n$$

a^n is called the nth power of a and is also read as 'a raised to the power n'.

NOTE 1 *It is evident from the above discussion that*

$a\times a\times a\times b\times b$ *is written as* a^3b^2 *(read as a cubed into b squared)*

$a\times a\times b\times b\times b\times b$ *is written* a^2b^4 *(read as a squared into b raised to the power 4)*

NOTE 2 *For any non-zero rational number a, we define*

(i) $a^1 = a$ (ii) $a^0 = 1$.

ILLUSTRATIVE EXAMPLES

Type I ON FINDING THE VALUE OF A NUMBER GIVEN IN EXPONENTIAL FORM

Example 1 Find the value of each of the following:

(i) 11^2 (ii) 9^3 (iii) 5^4

Solution (i) We have,

$$11^2 = 11\times11 = 121$$

(ii) We have,

$$9^3 = 9\times9\times9 = (9\times9)\times9 = 81\times9 = 729$$

(iii) We have,

$$5^4 = 5\times5\times5\times5$$
$$= (5\times5)\times5\times5 = 25\times5\times5 = (25\times5)\times5 = 125\times5 = 625$$

Example 2 Find the value of each of the following:

(i) $(-3)^2$ (ii) $(-4)^3$ (iii) $(-5)^4$

Solution (i) We have,

$$(-3)^2 = (-3)\times(-3) = 9$$

(ii) We have,

$$(-4)^3 = (-4)\times(-4)\times(-4) = \big((-4)\times(-4)\big)\times(-4) = 16\times(-4) = -64$$

(iii) We have,

$$(-5)^4 = (-5)\times(-5)\times(-5)\times(-5)$$
$$= \big((-5)\times(-5)\big)\times(-5)\times(-5)$$
$$= 25\times(-5)\times(-5)$$
$$= \big(25\times(-5)\big)\times(-5)$$
$$= (-125)\times(-5) = 125\times5 = 625$$

Example 3 Simplify:

(i) 2×10^3 (ii) $7^2\times2^2$ (iii) $2^3\times5$ (iv) 0×10^2

Solution (i) We have,

$$2\times10^3 = 2\times1000 = 2000 \qquad [\because\ 10^3 = 10\times10\times10 = 1000]$$

(ii) We have,

$$7^2\times2^2 = 49\times4 = 196 \qquad [\because\ 7^2 = 49,\ 2^2 = 4]$$

(iii) We have,

$$2^3\times5 = 8\times5 = 40 \qquad [\because\ 2^3 = 2\times2\times2 = 8]$$

(iv) We have,

$$0\times10^2 = 0\times100 = 0 \qquad [\because\ 10^2 = 10\times10 = 100]$$

(ii) We have,

$$\frac{-27}{125}=\frac{(-3)^3}{5^3} \quad [\because -27=(-3)^3 \text{ and } 125=5^3]$$

$$=\left(\frac{-3}{5}\right)^3$$

(iii) We have,

$$\frac{-1}{243}=\frac{(-1)^5}{3^5} \quad [\because (-1)^5=-1 \text{ and } 3^5=243]$$

$$=\left(\frac{-1}{3}\right)^5$$

EXERCISE 6.1

1. Find the value of each of the following:
   (i) 13^2 (ii) 7^3 (iii) 3^4
2. Find the value of each of the following:
   (i) $(-7)^2$ (ii) $(-3)^4$ (iii) $(-5)^5$
3. Simplify:
   (i) 3×10^2 (ii) $2^2\times5^3$ (iii) $3^3\times5^2$
4. Simplify:
   (i) $3^2\times10^4$ (ii) $2^4\times3^2$ (iii) $5^2\times3^4$
5. Simplify:
   (i) $(-2)\times(-3)^3$ (ii) $(-3)^2\times(-5)^3$ (iii) $(-2)^5\times(-10)^2$
6. Simplify:
   (i) $\left(\frac{3}{4}\right)^2$ (ii) $\left(\frac{-2}{3}\right)^4$ (iii) $\left(\frac{-4}{5}\right)^5$
7. Identify the greater number in each of the following:
   (i) 2^5 or 5^2 (ii) 3^4 or 4^3 (iii) 3^5 or 5^3
8. Express each of the following in exponential form:
   (i) $(-5)\times(-5)\times(-5)$ (ii) $\frac{-5}{7}\times\frac{-5}{7}\times\frac{-5}{7}\times\frac{-5}{7}$ (iiii) $\frac{4}{3}\times\frac{4}{3}\times\frac{4}{3}\times\frac{4}{3}\times\frac{4}{3}$
9. Express each of the following in exponential form:
   (i) $x\times x\times x\times x\times a\times a\times b\times b\times b$ (ii) $(-2)\times(-2)\times(-2)\times(-2)\times a\times a\times a$
   (iiii) $\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times x\times x\times x$
10. Express each of the following numbers in exponential form:
    (i) 512 (ii) 625 (iii) 729
11. Express each of the following numbers as a product of powers of their prime factors:
    (i) 36 (ii) 675 (iii) 392

12. Express each of the following numbers as a product of powers of their prime factors:
(i) 450 (ii) 2800 (iii) 24000

13. Express each of the following as a rational number of the form $\frac{p}{q}$:
(i) $\left(\frac{3}{7}\right)^2$ (ii) $\left(\frac{7}{9}\right)^3$ (iii) $\left(\frac{-2}{3}\right)^4$

14. Express each of the following rational numbers in power notation:
(i) $\frac{49}{64}$ (ii) $-\frac{64}{125}$ (iii) $-\frac{1}{216}$

15. Find the value of each of the following:
(i) $\left(\frac{-1}{2}\right)^2 \times 2^3 \times \left(\frac{3}{4}\right)^2$ (ii) $\left(\frac{-3}{5}\right)^4 \times \left(\frac{4}{9}\right)^4 \times \left(\frac{-15}{18}\right)^2$

16. If $a=2$ and $b=3$, then find the values of each of the following:
(i) $(a+b)^a$ (ii) $(ab)^b$ (iii) $\left(\frac{b}{a}\right)^b$ (iv) $\left(\frac{a}{b}+\frac{b}{a}\right)^a$

ANSWERS

1. (i) 169 (ii) 343 (iii) 81
2. (i) 49 (ii) 81 (iii) –3125
3. (i) 300 (ii) 500 (iii) 675
4. (i) 90000 (ii) 144 (iii) 2025
5. (i) 54 (ii) –1125 (iii) –3200
6. (i) $\frac{9}{16}$ (ii) $\frac{16}{81}$ (iii) $\frac{-1024}{3125}$
7. (i) 2^5 (ii) 3^4 (iii) 3^5
8. (i) $(-5)^3$ (ii) $\left(\frac{-5}{7}\right)^4$ (iii) $\left(\frac{4}{3}\right)^5$
9. (i) $x^4a^2b^3$ (ii) $(-2)^4 \times a^3$ (iii) $\left(\frac{-2}{3}\right)^2 \times x^3$
10. (i) 2^9 (ii) 5^4 (iii) 3^6
11. (i) $2^2 \times 3^2$ (ii) $3^3 \times 5^2$ (iii) $2^3 \times 7^2$
12. (i) $2 \times 3^2 \times 5^2$ (ii) $2^4 \times 5^2 \times 7$ (iii) $2^6 \times 3^1 \times 5^3$
13. (i) $\frac{9}{49}$ (ii) $\frac{343}{729}$ (iii) $\frac{16}{81}$
14. (i) $\left(\frac{7}{8}\right)^2$ (ii) $\left(-\frac{4}{5}\right)^3$ (iii) $\left(\frac{-1}{6}\right)^3$
15. (i) $\frac{9}{8}$ (ii) $\frac{64}{18225}$
16. (i) 25 (ii) 216 (iii) $\frac{27}{8}$ (iv) $\frac{169}{36}$

6.3 LAWS OF EXPONENTS

In this section, we will learn about various laws of exponents:

6.3.1 MULTIPLYING POWERS WITH THE SAME BASE

Consider the following product of two powers with the same base:

$$3^2 \times 3^4 = (3\times3)\times(3\times3\times3\times3) = 3\times3\times3\times3\times3\times3 = 3^6 = 3^{2+4}$$

In this product LHS is the product of two powers with the same base and exponents 2 and 4 and RHS is the power with the same base and exponent equal to 2 + 4 *i.e.*, the sum of the exponents 2 and 4.

Let us now consider some more products:

$$(-2)^4 \times (-2)^3 = ((-2)\times(-2)\times(-2)\times(-2))\times((-2)\times(-2)\times(-2))$$

$$= (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2) = (-2)^7 = (-2)^{4+3}$$

and, $$a^3 \times a^5 = (a\times a\times a)\times(a\times a\times a\times a\times a)$$

$$= a\times a\times a\times a\times a\times a\times a\times a = a^8 = a^{3+5}$$

In these products also LHS of each product is the product of two powers with the same base and RHS is the power with the same base and exponent equal to the sum of the exponents of powers on LHS.

The above discussion suggests us the following law of exponents:

FIRST LAW *If a is any non-zero rational number and m, n are natural numbers, then*

$$a^m \times a^n = a^{m+n}$$

Following is the generalised form of the above law:

Generalisation: If a is a non-zero rational number and m, n, p are natural numbers, then

$$a^m \times a^n \times a^p = a^{m+n+p}$$

ILLUSTRATION 1 Simplify and write the answer of each of the following in exponential form:

(i) $5^2 \times 5^3$ (ii) $3^2 \times 3^4 \times 3^8$ (iii) $7^x \times 7^2$ (iv) $\left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^5$

Solution We have,

(i) $5^2 \times 5^3 = 5^{2+3} = 5^5$

(ii) $3^2 \times 3^4 \times 3^8 = 3^{2+4+8} = 3^{14}$

(iii) $7^x \times 7^2 = 7^{x+2}$

(iv) $\left(\frac{3}{2}\right)^2 \times \left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{2+5} = \left(\frac{3}{2}\right)^7$

6.3.2 DIVIDING POWERS WITH THE SAME BASE

In the above discussion, we have learnt about multiplication of powers with the same base. What happens if powers with the same base are divided. To understand this, let us consider the following division of powers:

$$5^7 \div 5^4 = \frac{5^7}{5^4} = \frac{5\times5\times5\times5\times5\times5\times5}{5\times5\times5\times5} = 5\times5\times5 = 5^3 = 5^{7-4}$$

We observe that 5^7 and 5^4 have the same base and $5^7 \div 5^4$ is equal to 5^{7-4}.

Let us now compute some more division of powers having the same base.

We have,

$$(-3)^6 \div (-3)^2 = \frac{(-3)^6}{(-3)^2} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)}$$

$$= (-3)\times(-3)\times(-3)\times(-3) = (-3)^4 = (-3)^{6-2}$$

and, $b^9 \div b^3 = \frac{b^9}{b^3} = \frac{b\times b\times b\times b\times b\times b\times b\times b\times b}{b\times b\times b} = b\times b\times b\times b\times b\times b = b^6 = b^{9-3}$

In all the above divisions of powers with the same base, we find that the divisions of powers with the same base is equal to a power of the same base whose exponent is equal to the difference of exponents of numerator and denominator.

Thus, we have the following law for the division of powers with the same base.

SECOND LAW *If a is any non-zero rational number and m and n are natural numbers such that m>n, then*

$$a^m \div a^n = a^{m-n} \text{ or, } \frac{a^m}{a^n} = a^{m-n}$$

ILLUSTRATION 2 Simplify and write each of the following in exponential form:

(i) $9^{11} \div 9^7$ (ii) $(-7)^{13} \div (-7)^9$ (iii) $\left(\frac{-3}{4}\right)^7 \div \left(\frac{-3}{4}\right)^5$

Solution Using second law of exponents, we have

(i) $9^{11} \div 9^7 = \frac{9^{11}}{9^7} = 9^{11-7} = 9^4$

(ii) $(-7)^{13} \div (-7)^9 = \frac{(-7)^{13}}{(-7)^9} = (-7)^{13-9} = (-7)^4$

(iii) $\left(\frac{-3}{4}\right)^7 \div \left(\frac{-3}{4}\right)^5 = \frac{\left(\frac{-3}{4}\right)^7}{\left(\frac{-3}{4}\right)^5} = \left(\frac{-3}{4}\right)^{7-5} = \left(\frac{-3}{4}\right)^2$

6.3.3 POWER WITH EXPONENT ZERO

Let us now find the value of a power of a non-zero rational number when its exponent is zero.

We know that

$$\frac{3^4}{3^4} = \frac{3\times3\times3\times3}{3\times3\times3\times3} = 1 \qquad \text{...(i)}$$

Also, $\frac{3^4}{3^4} = 3^{4-4}$ [Using second law of exponents]

$= 3^0$ ...(ii)

We find that the left hand sides of (i) and (ii) are same.

So, their right hand sides are equal

$\therefore \quad 3^0 = 1$

Similarly,

$$\frac{7^3}{7^3} = \frac{7\times7\times7}{7\times7\times7} = 1 \text{ and } \frac{7^3}{7^3} = 7^{3-3} = 7^0$$

$$\therefore \quad 7^0 = 1$$

Also for any non-zero rational number a, we have

$$\frac{a^5}{a^5} = \frac{a\times a\times a\times a\times a}{a\times a\times a\times a\times a} = 1 \text{ and } \frac{a^5}{a^5} = a^{5-5} = a^0$$

$$\therefore \quad a^0 = 1$$

It follows from the above discussion that for any non-zero rational number a, we have

$$a^0 = 1$$

6.3.4 POWER OF A POWER

Let us now find what is the power of a power. In order to find it, let us consider the following powers:

$(3^2)^5 = 3^2\times3^2\times3^2\times3^2\times3^2$ [Using definition of power]

$= 3^{2+2+2+2+2}$ [Using first law of exponents]

$= 3^{10} = 3^{2\times5}$

$\left\{\left(\frac{3}{2}\right)^3\right\}^2 = \left(\frac{3}{2}\right)^3 \times \left(\frac{3}{2}\right)^3$ [Using definition of power]

$= \left(\frac{3}{2}\right)^{3+3}$ [Using first law of exponents]

$= \left(\frac{3}{2}\right)^6 = \left(\frac{3}{2}\right)^{3\times2}$

$(a^3)^4 = a^3\times a^3\times a^3\times a^3$ [Using definition of power]

$= a^{3+3+3+3}$ [Using first law of exponents]

$= a^{12} = a^{3\times4}$

It follows from the above discussion that the power of a power is equal to the power with the same base and exponent equal to the product of the exponents.

This suggests us the following law.

THIRD LAW *If a is any rational number different from zero and m, n are natural numbers, then*

$$(a^m)^n = a^{m\times n} = (a^n)^m$$

ILLUSTRATION 3 Simplify and write each of the following in exponential form:

(i) $(2^3)^4$ (ii) $((-3)^5)^3$

(iii) $\left\{\left(\frac{2}{3}\right)^2\right\}^5$ (iv) $(3^2)^5\times(3^4)^2$

Solution Using the third law of exponents, we have

(i) $(2^3)^4 = 2^{3\times 4} = 2^{12}$

(ii) $\left((-3)^5\right)^3 = (-3)^{5\times 3} = (-3)^{15}$

(iii) $\left\{\left(\frac{2}{3}\right)^2\right\}^5 = \left(\frac{2}{3}\right)^{2\times 5} = \left(\frac{2}{3}\right)^{10}$

(iv) $(3^2)^5 \times (3^4)^2 = (3^{2\times 5}) \times (3^{4\times 2})$

$= 3^{10} \times 3^8 = 3^{10+8}$ [Using first law of exponents]

$= 3^{18}$

6.3.5 MULTIPLYING POWERS WITH THE SAME EXPONENTS

In the first law of exponents, we have learnt about the product (or multiplication) of powers with the same base. Now, let us find the product of powers with different bases and same exponents.

Consider the following products:

$$2^3 \times 3^3 = (2\times 2\times 2)\times(3\times 3\times 3)$$
$$= (2\times 3)\times(2\times 3)\times(2\times 3) = 6\times 6\times 6 = 6^3 = (2\times 3)^3$$

$$3^4 \times 5^4 = (3\times 3\times 3\times 3)\times(5\times 5\times 5\times 5)$$
$$= (3\times 5)\times(3\times 5)\times(3\times 5)\times(3\times 5) = 15\times 15\times 15\times 15 = 15^4 = (3\times 5)^4$$

$$2^3 \times b^3 = (2\times 2\times 2)\times(b\times b\times b) = (2\times b)\times(2\times b)\times(2\times b) = (2\times b)^3$$

$$x^5 \times y^5 = (x\times x\times x\times x\times x)\times(y\times y\times y\times y\times y)$$
$$= (x\times y)\times(x\times y)\times(x\times y)\times(x\times y)\times(x\times y)$$
$$= (xy)\times(xy)\times(xy)\times(xy)\times(xy) \quad [\because x\times y = xy]$$
$$= (xy)^5$$

It is evident from the above computations that the product of powers with different bases and same exponents is equal to the power whose base is equal to the product of different bases and exponent equal to the common exponent.

Thus, we have the following law:

FOURTH LAW *If a, b are non-zero rational numbers and n is a natural number, then*

$$a^n \times b^n = (ab)^n$$

Generalisation: If a, b, c are non-zero rational numbers and n is a natural number, then

$$a^n \times b^n \times c^n = (abc)^n$$

ILLUSTRATION 4 Express each of the following products of powers as the exponent of a rational number:

(i) $2^5 \times 3^5$ (ii) $(-4)^3 \times (-2)^3$

(iii) $3^7 \times (-2)^7$ (iv) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^3$

(v) $2^3 \times x^3 \times y^3$

Solution Using fourth law of exponents, we have

(i) $2^5 \times 3^5 = (2 \times 3)^5 = 6^5$

(ii) $(-4)^3 \times (-2)^3 = \{(-4) \times (-2)\}^3 = 8^3$

(iii) $3^7 \times (-2)^7 = \{3 \times (-2)\}^7 = (-6)^7$

(iv) $\left(\frac{2}{3}\right)^3 \times \left(\frac{3}{5}\right)^3 = \left(\frac{2}{3} \times \frac{3}{5}\right)^3 = \left(\frac{2}{5}\right)^3$

(v) $2^3 \times x^3 \times y^3 = (2 \times x \times y)^3 = (2xy)^3$

ILLUSTRATION 5 Express each of the following powers as the product of powers:

(i) $(3a)^5$ (ii) $(-4)^7$

Solution We have,

(i) $(3a)^5 = (3 \times a)^5 = 3^5 \times a^5$

(ii) $(-4)^7 = (-1 \times 4)^7 = (-1)^7 \times 4^7$

6.3.6 DIVIDING POWERS WITH THE SAME EXPONENTS

Second law of exponents tells us about the division of powers with the same bases. Let us now find the value of division of powers with different bases and same exponents.

Let us consider the following simplifications:

$$\frac{3^5}{4^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4} = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^5$$

$$\frac{(-2)^3}{3^3} = \frac{(-2) \times (-2) \times (-2)}{3 \times 3 \times 3} = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) = \left(\frac{-2}{3}\right)^3$$

It is evident from the above simplifications that the division of powers with same exponents is equal to the power whose base is equal to the division of bases and exponent equal to the same exponent.

Thus, we have the following law of exponents:

FIFTH LAW *If a and b are non-zero rational numbers and n is a natural number, then*

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

ILLUSTRATION 6 Write each of the following in the form $\frac{p}{q}$:

(i) $\left(\frac{3}{5}\right)^3$ (ii) $\left(\frac{-2}{3}\right)^5$

Solution Using fifth law of exponents, we have

(i) $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{3\times3\times3}{5\times5\times5} = \frac{27}{125}$

(ii) $\left(\frac{-2}{3}\right)^5 = \frac{(-2)^5}{3^5} = \frac{(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{3\times3\times3\times3\times3} = \frac{-32}{243}$

Let us now discuss some examples on simplification of expressions by using the laws of exponents.

ILLUSTRATIVE EXAMPLES

Example 1 Using laws of exponents, simplify and write the answer in exponential form:

(i) $2^5\times2^3$ (ii) $3^2\times3^4\times3^8$ (iii) $6^{15}\div6^{10}$ (iv) $(5^3)^2$

(v) $(7^2)^3\div7^3$ (vi) $2^5\times3^5$ (vii) $a^4\times b^4$ (viii) $(2^{20}\div2^{15})\times2^3$

Solution Using the laws of exponents, we have

(i) $2^5\times2^3 = 2^{5+3} = 2^8$ [Using first law]

(ii) $3^2\times3^4\times3^8 = 3^{2+4+8} = 3^{14}$ [Using first law]

(iii) $6^{15}\div6^{10} = \frac{6^{15}}{6^{10}} = 6^{15-10} = 6^5$ [Using second law]

(iv) $(5^3)^2 = 5^{3\times2} = 5^6$ [Using third law]

(v) $(7^2)^3\div7^3 = \frac{(7^2)^3}{7^3} = \frac{7^{2\times3}}{7^3}$ [Using third law]

$= \frac{7^6}{7^3} = 7^{6-3} = 7^3$ [Using second law]

(vi) $2^5\times3^5 = (2\times3)^5 = 6^5$ [Using fourth law]

(vii) $a^4\times b^4 = (a\times b)^4 = (ab)^4$

(viii) $(2^{20}\div2^{15})\times2^3 = \left(\frac{2^{20}}{2^{15}}\right)\times2^3 = 2^{20-15}\times2^3 = 2^5\times2^3 = 2^{5+3} = 2^8$

Example 2 Simplify and express each of the following in exponential form:

(i) $\frac{3^7}{3^4\times3^3}$ (ii) $\{(5^2)^3\times5^4\}\div5^7$ (iii) $\left(\frac{3^7}{3^2}\right)\times3^5$ (iv) $\frac{4^5\times a^8b^3}{4^5\times a^5b^2}$

Solution (i) We have,

$\frac{3^7}{3^4\times3^3} = \frac{3^7}{3^{4+3}}$ [Using first law]

$= \frac{3^7}{3^7} = 3^{7-7} = 3^0 = 1$

(ii) We have,

$$\{(5^2)^3 \times 5^4\} \div 5^7 = \frac{\{(5^2)^3 \times 5^4\}}{5^7}$$

$$= \frac{5^{2\times3} \times 5^4}{5^7} \qquad [\because (a^m)^n = a^{mn}]$$

$$= \frac{5^6 \times 5^4}{5^7} = \frac{5^{6+4}}{5^7} = \frac{5^{10}}{5^7} = 5^{10-7} = 5^3$$

(iii) We have,

$$\left(\frac{3^7}{3^2}\right) \times 3^5 = 3^{7-2} \times 3^5 = 3^5 \times 3^5 = 3^{5+5} = 3^{10}$$

(iv) We have,

$$\frac{4^5 \times a^8 b^3}{4^5 \times a^5 b^2} = \left(\frac{4^5}{4^5}\right) \times \left(\frac{a^8}{a^5}\right) \times \left(\frac{b^3}{b^2}\right)$$

$$= 4^{5-5} \times a^{8-5} \times b^{3-2}$$

$$= 4^0 \times a^3 \times b^1$$

$$= 1 \times a^3 \times b \qquad [\because 4^0 = 1 \text{ and } b^1 = b]$$

$$= a^3 b$$

Example 3 Simplify and express each of the following in exponential form:

(i) $2^5 \times 5^5$ (ii) $2^3 \times 2^2 \times 5^5$ (iii) $\{(2^2)^3 \times 3^6\} \times 5^6$ (iv) $\left(\frac{a}{b}\right)^5 \times b^{10}$

Solution We have,

(i) $2^5 \times 5^5 = (2 \times 5)^5 = 10^5$ $[\because a^n \times b^n = (ab)^n]$

(ii) $2^3 \times 2^2 \times 5^5 = (2^3 \times 2^2) \times 5^5 = 2^{3+2} \times 5^5 = 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(iii) $\{(2^2)^3 \times 3^6\} \times 5^6 = \{2^{2\times3} \times 3^6\} \times 5^6$ $[\because (a^m)^n = a^{mn} \therefore (2^2)^3 = 2^{2\times3}]$

$$= (2^6 \times 3^6) \times 5^6$$

$$= (2 \times 3)^5 \times 5^6 \qquad [\because a^n \times b^n = (ab)^n]$$

$$= 6^6 \times 5^6$$

$$= (6 \times 5)^6 \qquad [\because a^n \times b^n = (ab)^n]$$

$$= 30^6$$

(iv) $\left(\frac{a}{b}\right)^5 \times b^{10} = \frac{a^5}{b^5} \times b^{10}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

$$= a^5 \times \frac{b^{10}}{b^5} = a^5 \times b^{10-5} = a^5 \times b^5 = (ab)^5$$

Example 4 Write exponential form for $8 \times 8 \times 8 \times 8$ taking base as 2.

Solution We have,

$$8 \times 8 \times 8 \times 8 = 8^4$$

$$= (2^3)^4 \qquad [\because\ 8 = 2 \times 2 \times 2 = 2^3]$$

$$= 2^{3 \times 4} \qquad [\because\ (a^m)^n = a^{mn}]$$

$$= 2^{12}$$

Example 5 Simplify and write each of the following in exponential form:

(i) $8^2 \div 2^3$ (ii) $25^4 \div 5^3$ (iii) $\dfrac{2^8 \times a^5}{4^3 \times a^3}$ (iv) $\dfrac{2^3 \times 3^4 \times 4}{3 \times 32}$

Solution We have,

(i) $$8^2 \div 2^3 = \frac{8^2}{2^3} = \frac{(2^3)^2}{2^3} \qquad [\because\ 8 = 2 \times 2 \times 2 = 2^3]$$

$$= \frac{2^{3 \times 2}}{2^3} \qquad [\because\ (a^m)^n = a^{mn} \ \therefore\ (2^3)^2 = 2^{3 \times 2}]$$

$$= \frac{2^6}{2^3} = 2^{6-3} = 2^3$$

(ii) $$25^4 \div 5^3 = \frac{25^4}{5^3} = \frac{(5^2)^4}{5^3} \qquad [\because\ 25 = 5 \times 5 = 5^2]$$

$$= \frac{5^{2 \times 4}}{5^3} \qquad [\because\ (a^m)^n = a^{mn} \ \therefore\ (5^2)^4 = 5^{2 \times 4}]$$

$$= \frac{5^8}{5^3} = 5^{8-3} = 5^5$$

(iii) $$\frac{2^8 \times a^5}{4^3 \times a^3} = \frac{2^8 \times a^5}{(2^2)^3 \times a^3} \qquad [\because\ 4 = 2 \times 2 = 2^2]$$

$$= \frac{2^8 \times a^5}{2^{2 \times 3} \times a^3} \qquad [\because\ (2^2)^3 = 2^{2 \times 3}]$$

$$= \left(\frac{2^8}{2^6}\right) \times \left(\frac{a^5}{a^3}\right) = 2^{8-6} \times a^{5-3} = 2^2 \times a^2 = (2 \times a)^2 = (2a)^2$$

(iv) $$\frac{2^3 \times 3^4 \times 4}{3 \times 32} = \frac{2^3 \times 3^4 \times 2^2}{3 \times 2^5} \qquad [\because\ 4 = 2 \times 2 = 2^2 \text{ and } 32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5]$$

$$= \frac{2^3 \times 2^2 \times 3^4}{2^5 \times 3} = \frac{2^{3+2} \times 3^4}{2^5 \times 3}$$

$$= \left(\frac{2^5}{2^5}\right) \times \left(\frac{3^4}{3^1}\right) = 2^{5-5} \times 3^{4-1} = 2^0 \times 3^3 = 1 \times 3^3 = 3^3$$

Example 6 Simplify:

(i) $2^{55} \times 2^{60} - 2^{97} \times 2^{18}$ (ii) $2^3 \times a^3 \times 5a^4$

(iii) $\dfrac{3^n + 3^{n+1}}{3^{n+1} - 3^n}$, where n is a natural number.

Solution (i) We have,

$$2^{55} \times 2^{60} - 2^{97} \times 2^{18} = 2^{55+60} - 2^{97+18} = 2^{115} - 2^{115} = 0$$

(ii) We have,

$$2^3 \times a^3 \times 5a^4 = 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4} = 40 \times a^7 = 40a^7$$

(iii) We have,

$$\frac{3^n + 3^{n+1}}{3^{n+1} - 3^n} = \frac{3^n + 3^n \times 3^1}{3^n \times 3^1 - 3^n} \qquad [\because 3^{n+1} = 3^n \times 3^1]$$

$$= \frac{3^n \times 1 + 3^n \times 3}{3^n \times 3 - 3^n \times 1}$$

$$= \frac{3^n(1+3)}{3^n(3-1)} = \frac{3^n \times 4}{3^n \times 2} = \frac{3^n \times 2^2}{3^n \times 2}$$

$$= \frac{3^n}{3^n} \times \frac{2^2}{2} = 3^{n-n} \times 2^{2-1} = 3^0 \times 2^1 = 1 \times 2 = 2.$$

Example 7 Simplify:

(i) $\dfrac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$ (ii) $\dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

Solution (i) We have,

$$\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3} \qquad \left[\because 12 = 2^2 \times 3, 9 = 3^2, 4 = 2^2, 6 = 2 \times 3, 8 = 2^3 \text{ and } 27 = 3^3\right]$$

$$= \frac{(2^2)^4 \times 3^4 \times (3^2)^3 \times 2^2}{(2^3 \times 3^3) \times (2^3)^2 \times 3^3} \qquad [\because (ab)^n = a^n \times b^n]$$

$$= \frac{2^8 \times 3^4 \times 3^6 \times 2^2}{2^3 \times 3^3 \times 2^6 \times 3^3} \qquad [\because (a^m)^n = a^{mn}]$$

$$= \frac{(2^8 \times 2^2) \times (3^4 \times 3^6)}{(2^3 \times 2^6) \times (3^3 \times 3^3)}$$

$$= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6} = \left(\frac{2^{10}}{2^9}\right) \times \left(\frac{3^{10}}{3^6}\right)$$

$$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4 = 2 \times 81 = 162$$

(ii) We have,

$$\frac{25\times5^2\times t^8}{10^3\times t^4}=\frac{5^2\times5^2\times t^8}{(2\times5)^3\times t^4}=\frac{5^{2+2}\times t^8}{2^3\times5^3\times t^4} \qquad [\because (2\times5)^3=2^3\times5^3]$$

$$=\frac{5^4\times t^8}{5^3\times2^3\times t^4}=\frac{5^{4-3}\times t^{8-4}}{2^3}=\frac{5^1\times t^4}{2^3}=\frac{5t^4}{8}$$

(iii) We have,

$$\frac{3^5\times10^5\times25}{5^7\times6^5}=\frac{3^5\times(2\times5)^5\times5^2}{5^7\times(2\times3)^5}$$

$$=\frac{3^5\times2^5\times5^5\times5^2}{5^7\times2^5\times3^5}$$

$$=\frac{3^5\times2^5\times5^7}{3^5\times2^5\times5^7}=3^{5-5}\times2^{5-5}\times5^{7-7}=3^0\times2^0\times5^0=1\times1\times1=1$$

Example 8 Express each of the following as a product of prime factors only in exponential form:

(i) 108×192 (ii) 729×64

Solution (i) Using prime factorisations of 108 and 192, we have

$108=2\times2\times3\times3\times3=2^2\times3^3$

and, $192=2\times2\times2\times2\times2\times2\times3=2^6\times3$

2	108
2	54
3	27
3	9
	3

2	192
2	96
2	48
2	24
2	12
2	6
	3

$\therefore \quad 108\times192=(2^2\times3^3)\times(2^6\times3)$

$=(2^2\times2^6)\times(3^3\times3^1)$

$=2^{2+6}\times3^{3+1}$

$=2^8\times3^4$

(ii) Using prime factorisations of 729 and 64, we have

729×64

$=(3\times3\times3\times3\times3\times3)\times(2\times2\times2\times2\times2\times2)$

$=3^6\times2^6$

3	729
3	243
3	81
3	27
3	9
	3

2	64
2	32
2	16
2	8
2	4
	2

Example 9 Compare the following numbers:

(i) 2.7×10^{12} and 1.5×10^8 (ii) 4×10^{14} and 3×10^{17}

Solution (i) We have,

$2.7\times10^{12}=2.7\times10\times10^{11}=27\times10^{11}$

and, $1.5\times10^8=1.5\times10\times10^7=15\times10^7$

$\therefore \quad 2.7\times10^{12}-1.5\times10^8=27\times10^{11}-15\times10^7$

$=27\times10^7\times10^4-15\times10^7 \qquad [\because 10^{11}=10^{7+4}=10^7\times10^4]$

$=3\times10^7(9\times10^4-5)$

$$= 3 \times 10^7 (90000 - 5)$$
$$= 3 \times 10^7 \times 89995 > 0$$

Hence, $2.7 \times 10^{12} > 1.5 \times 10^8$

(ii) We have,

$$4 \times 10^{14} - 3 \times 10^7$$
$$= 4 \times 10^7 \times 10^7 - 3 \times 10^7 \quad [\because 10^{14} = 10^{7+7} = 10^7 \times 10^7]$$
$$= (4 \times 10^7 - 3) \times 10^7 > 0 \quad [\because 4 \times 10^7 - 3 > 0]$$
$$\therefore \quad 4 \times 10^{14} > 3 \times 10^7$$

Example 10 Find the values of n in each of the following:

(i) $(2^2)^n = (2^3)^4$ (ii) $2^{5n} \div 2^n = 2^4$ (iii) $2^{n-5} \times 5^{n-4} = 5$

(iv) $2^{n-7} \times 5^{n-4} = 1250$ (v) $5^{n-2} \times 3^{2n-3} = 135$

Solution (i) We have,

$$(2^2)^n = (2^3)^4$$
$$\Rightarrow \quad 2^{2n} = 2^{3\times4} \quad [\because (a^m)^n = a^{mn}]$$
$$\Rightarrow \quad 2^{2n} = 2^{12}$$
$$\Rightarrow \quad 2n = 12 \quad \text{[On equating the exponents]}$$
$$\Rightarrow \quad n = \frac{12}{2} = 6$$

(ii) We have,

$$2^{5n} \div 2^n = 2^4$$
$$\Rightarrow \quad \frac{2^{5n}}{2^n} = 2^4$$
$$\Rightarrow \quad 2^{5n-n} = 2^4 \quad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$
$$\Rightarrow \quad 2^{4n} = 2^4$$
$$\Rightarrow \quad 4n = 4 \quad \text{[On equating the exponents]}$$
$$\Rightarrow \quad n = \frac{4}{4} = 1$$

(iii) We have,

$$2^{n-5} \times 5^{n-4} = 5$$
$$\Rightarrow \quad \frac{2^n}{2^5} \times \frac{5^n}{5^4} = 5 \quad \left[\because 2^{n-5} = \frac{2^n}{2^5} \text{ and } 5^{n-4} = \frac{5^n}{5^4}\right]$$
$$\Rightarrow \quad \frac{2^n \times 5^n}{2^5 \times 5^4} = 5$$
$$\Rightarrow \quad (2^n \times 5^n) = 2^5 \times 5^4 \times 5 \quad \text{[Using cross-multiplication]}$$
$$\Rightarrow \quad 2^n \times 5^n = 2^5 \times 5^{4+1}$$
$$\Rightarrow \quad 2^n \times 5^n = 2^5 \times 5^5$$

$\Rightarrow \quad (2\times5)^n = (2\times5)^5$ $\quad [\because a^n \times b^n = (ab)^n]$

$\Rightarrow \quad 10^n = 10^5$

$\Rightarrow \quad n = 5$ $\quad$ [On equating the exponents]

Aliter We have,

$2^{n-5}\times5^{n-4} = 5$

$\Rightarrow \quad 2(2^{n-5}\times5^{n-4}) = 2\times5$ $\quad$ [Multiplying both sides by 2]

$\Rightarrow \quad 2^{n-5+1}\times5^{n-4} = 2\times5$

$\Rightarrow \quad 2^{n-4}\times5^{n-4} = 2\times5$

$\Rightarrow \quad (2\times5)^{n-4} = 2\times5$

$\Rightarrow \quad 10^{n-4} = 10^1$

$\Rightarrow \quad n-4 = 1 \Rightarrow n = 5$

(iv) We have,

$2^{n-7}\times5^{n-4} = 1250$

$\Rightarrow \quad \frac{2^n}{2^7}\times\frac{5^n}{5^4} = 2\times5^4$ $\quad [\because 1250 = 2\times5^4]$

$\Rightarrow \quad \frac{2^n\times5^n}{2^7\times5^4} = 2\times5^4$

$\Rightarrow \quad 2^n\times5^n = 2^7\times5^4\times2\times5^4$ $\quad$ [Using cross-multiplication]

$\Rightarrow \quad 2^n\times5^n = 2^{7+1}\times5^{4+4}$ $\quad [\because a^m\times a^n = a^{m+n}]$

$\Rightarrow \quad 2^n\times5^n = 2^8\times5^8$

$\Rightarrow \quad (2\times5)^n = (2\times5)^8$ $\quad [\because a^n\times b^n = (a\times b)^n$

$\Rightarrow \quad 10^n = 10^8$

$\Rightarrow \quad n = 8$

(v) We have,

$5^{n-2}\times3^{2n-3} = 135$

$\Rightarrow \quad \frac{5^n}{5^2}\times\frac{3^{2n}}{3^3} = 3^3\times5$ $\quad [\because 135 = 3^3\times5]$

$\Rightarrow \quad \frac{5^n\times3^{2n}}{5^2\times3^3} = 3^3\times5$

$\Rightarrow \quad 5^n\times(3^2)^n = 5^2\times3^3\times3^3\times5$

$\Rightarrow \quad 5^n\times(3^2)^n = 3^{3+3}\times5^{2+1}$

$\Rightarrow \quad 5^n\times(3^2)^n = 3^6\times5^3 = (3^2)^3\times5^3 = 5^3\times(3^2)^3$

$\Rightarrow \quad (5\times3^2)^n = (5\times3^2)^3$

$\Rightarrow \quad (5\times9)^n = (5\times9)^3$

$\Rightarrow \quad 45^n = 45^3$

$\Rightarrow \quad n = 3$

Example 11 If $25^{n-1}+100=5^{2n-1}$, find the value of n.

Solution We have,

$$25^{n-1}+100=5^{2n-1}$$

$$\Rightarrow \quad 5^{2n-1}-25^{n-1}=100$$

$$\Rightarrow \quad \frac{5^{2n}}{5}-\frac{25^{n}}{25}=10^{2}$$

$$\Rightarrow \quad \frac{5^{2n}}{5}-\frac{(5^{2})^{n}}{25}=(2\times 5)^{2}$$

$$\Rightarrow \quad \frac{5^{2n}}{5}-\frac{5^{2n}}{25}=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times\frac{1}{5}-\frac{5^{2n}}{25}=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times\left(\frac{1}{5}-\frac{1}{25}\right)=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times\left(\frac{5-1}{25}\right)=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times\frac{4}{25}=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times\frac{2^{2}}{5^{2}}=2^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}\times 2^{2}=2^{2}\times 5^{2}\times 5^{2}$$

$$\Rightarrow \quad 5^{2n}=\frac{2^{2}\times 5^{2}\times 5^{2}}{2^{2}}$$

$$\Rightarrow \quad 5^{2n}=2^{2-2}\times 5^{2+2}$$

$$\Rightarrow \quad 5^{2n}=2^{0}\times 5^{4} \Rightarrow 5^{2n}=5^{4} \Rightarrow 2n=4 \Rightarrow n=\frac{4}{2}=2$$

Example 12 Find n such that

(i) $\left(\frac{2}{3}\right)^{3}\times\left(\frac{2}{3}\right)^{5}=\left(\frac{2}{3}\right)^{n-2}$ (ii) $\left(\frac{125}{8}\right)^{5}\times\left(\frac{125}{8}\right)^{n}=\left(\frac{5}{2}\right)^{18}$

Solution (i) We have,

$$\left(\frac{2}{3}\right)^{3}\times\left(\frac{2}{3}\right)^{5}=\left(\frac{2}{3}\right)^{n-2}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)^{3+5}=\left(\frac{2}{3}\right)^{n-2}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)^{8} = \left(\frac{2}{3}\right)^{n-2}$$

$\Rightarrow \quad 8 = n - 2$ [On equating the coefficients]

$\Rightarrow \quad n = 8 + 2 = 10$

(ii) We have,

$$\left(\frac{125}{8}\right)^{5} \times \left(\frac{125}{8}\right)^{n} = \left(\frac{5}{2}\right)^{18}$$

$$\Rightarrow \quad \left\{\left(\frac{5}{2}\right)^{3}\right\}^{5} \times \left\{\left(\frac{5}{2}\right)^{3}\right\}^{n} = \left(\frac{5}{2}\right)^{18} \qquad \left[\because \frac{125}{8} = \frac{5\times5\times5}{2\times2\times2} = \frac{5^3}{2^3} = \left(\frac{5}{2}\right)^{3}\right]$$

$$\Rightarrow \quad \left(\frac{5}{2}\right)^{3\times5} \times \left(\frac{5}{2}\right)^{3n} = \left(\frac{5}{2}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{2}\right)^{15} \times \left(\frac{5}{2}\right)^{3n} = \left(\frac{5}{2}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{2}\right)^{15+3n} = \left(\frac{5}{2}\right)^{18}$$

$$\Rightarrow \quad 15 + 3n = 18 \Rightarrow 3n = 18 - 15 \Rightarrow 3n = 3 \Rightarrow n = \frac{3}{3} = 1$$

Example 13 If $\frac{p}{q} = \left(\frac{2}{3}\right)^{2} \div \left(\frac{6}{7}\right)^{0}$, find the value of $\left(\frac{q}{p}\right)^{3}$.

Solution We have,

$$\left(\frac{p}{q}\right) = \left(\frac{2}{3}\right)^{2} \div \left(\frac{6}{7}\right)^{0}$$

$$\Rightarrow \quad \frac{p}{q} = \left(\frac{2}{3}\right)^{2} \div 1 \qquad \left[\because \left(\frac{6}{7}\right)^{0} = 1\right]$$

$$\Rightarrow \quad \frac{p}{q} = \left(\frac{2}{3}\right)^{2}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{2^2}{3^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{q}{p} = \frac{9}{4}$$

$$\Rightarrow \quad \frac{q}{p} = \frac{3^2}{2^2}$$

$$\Rightarrow \quad \frac{q}{p} = \left(\frac{3}{2}\right)^{2}$$

$$\Rightarrow \quad \left(\frac{q}{p}\right)^3 = \left\{\left(\frac{3}{2}\right)^2\right\}^3$$

$$\Rightarrow \quad \left(\frac{q}{p}\right)^3 = \left(\frac{3}{2}\right)^{2\times 3} = \left(\frac{3}{2}\right)^6 = \frac{3^6}{2^6} = \frac{729}{64}$$

Example 14 Find the value of m so that

$(-3)^{m+1} \times (-3)^5 = (-3)^7$

Solution We have,

$(-3)^{m+1} \times (-3)^5 = (-3)^7$

$\Rightarrow \quad (-3)^{m+1+5} = (-3)^7$

$\Rightarrow \quad (-3)^{m+6} = (-3)^7$

$\Rightarrow \quad m+6=7 \quad \Rightarrow \quad m = 7-6 = 1$

EXERCISE 6.2

1. Using laws of exponents, simplify and write the answer in exponential form:

   (i) $2^3 \times 2^4 \times 2^5$ (ii) $5^{12} \div 5^3$ (iii) $(7^2)^3$ (iv) $(3^2)^5 \div 3^4$

   (v) $3^7 \times 2^7$ (vi) $(5^{21} \div 5^{13}) \times 5^7$

2. Simplify and express each of the following in exponential form:

   (i) $\{(2^3)^4 \times 2^8\} \div 2^{12}$ (ii) $(8^2 \times 8^4) \div 8^3$ (iii) $\left(\frac{5^7}{5^2}\right) \times 5^3$ (iv) $\frac{5^4 \times x^{10} y^5}{5^4 \times x^7 y^4}$

3. Simplify and express each of the following in exponential form:

   (i) $\{(3^2)^3 \times 2^6\} \times 5^6$ (ii) $\left(\frac{x}{y}\right)^{12} \times y^{24} \times (2^3)^4$ (iii) $\left(\frac{5}{2}\right)^6 \times \left(\frac{5}{2}\right)^2$ (iv) $\left(\frac{2}{3}\right)^5 \times \left(\frac{3}{5}\right)^5$

4. Write $9 \times 9 \times 9 \times 9 \times 9$ in exponential form with base 3.

5. Simplify and write each of the following in exponential form:

   (i) $(25)^3 \div 5^3$ (ii) $(81)^5 \div (3^2)^5$ (iii) $\frac{9^8 \times (x^2)^5}{(27)^4 \times (x^3)^2}$ (iv) $\frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3}$

6. Simplify:

   (i) $(3^5)^{11} \times (3^{15})^4 - (3^5)^{18} \times (3^5)^5$ (ii) $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

   (iii) $\frac{10 \times 5^{n+1} + 25 \times 5^n}{3 \times 5^{n+2} + 10 \times 5^{n+1}}$ (iv) $\frac{(16)^7 \times (25)^5 \times (81)^3}{(15)^7 \times (24)^5 \times (80)^3}$

7. Find the values of n in each of the following:

   (i) $5^{2n} \times 5^3 = 5^{11}$ (ii) $9 \times 3^n = 3^7$ (iii) $8 \times 2^{n+2} = 32$ (iv) $7^{2n+1} \div 49 = 7^3$

   (v) $\left(\frac{3}{2}\right)^4 \times \left(\frac{3}{2}\right)^5 = \left(\frac{3}{2}\right)^{2n+1}$ (vi) $\left(\frac{2}{3}\right)^{10} \times \left\{\left(\frac{3}{2}\right)^2\right\}^5 = \left(\frac{2}{3}\right)^{2n-2}$

8. If $\frac{9^n \times 3^2 \times 3^n - (27)^n}{(3^3)^5 \times 2^3} = \frac{1}{27}$, find the value of n.

ANSWERS

1. (i) 2^{12} (ii) 5^9 (iii) 7^6 (iv) 3^6 (v) 6^7 (vi) 5^{15}
2. (i) 2^8 (ii) 2^9 (iii) 5^8 (iv) x^3y
3. (i) 30^6 (ii) $(2xy)^{12}$ (iii) $\left(\frac{5}{2}\right)^8$ (iv) $\left(\frac{2}{5}\right)^5$
4. 3^{10} 5. (i) 5^3 (ii) 3^{10} (iii) $(3x)^4$ (iv) 91^3
6. (i) 0 (ii) $\frac{1}{2}$ (iii) $\frac{3}{5}$ (iv) 2
7. (i) 4 (ii) 5 (iii) 0 (iv) 2 (v) 4 (vi) 1
8. 4

6.4 USE OF EXPONENTS IN EXPRESSING LARGE NUMBERS IN STANDARD FORM

In science and engineering and also in many other situations, we often come across numbers which are very large. For example, the mass of the Earth, the distance of Sun from Earth, number of stars in our Galaxy, speed of light etc. are numbers which are very large. Such large numbers are normally approximate, and not exact numbers. For the sake of convenience to read, write and remember such large numbers we write them as a certain number followed by a number of zeros. For example, the speed of light in vacuum is 299792.5 km per second. It is approximated as 300000 km per second or as 300,000,000 metre per second. Similarly, the mass of the earth is 5,976,000,000,000,000,000,000,000 kg. These numbers are not convenient to write and read. To make it convenient, we write these numbers by using exponents with base 10. For example, the speed of light in vacuum may be written as 3×10^8 metre per second or 30×10^7 metre per second or, 300×10^6 metre per second. Thus, every large number can be expressed as $k \times 10^n$, where k is some natural number. However, for the sake of uniformity, we write the numbers in the form $k \times 10^n$, where k is a terminating decimal number greater than or equal to 1 and less than 10 and n is a natural number. Using this notation the speed of light is written as 3×10^8 metre per second, the mass of the Earth is written as 5.976×10^{24} kg, etc.

Such a form of a number is known as its standard form as defined below:

STANDARD FORM *A number is said to be in the standard form, if it is expressed as the product of a number between 1 and 10 (including 1 but excluding 10) and a positive integer power of 10.*

The standard form of a number is also known as Scientific notation.

In order to write large numbers in the standard form, we may use the following steps:

STEP I *Obtain the number and move the decimal point to the left till you get just one digit to the left of the decimal point.*

STEP II *Write the given number as the product of the number so obtained and 10^n, where n is the number of places the decimal point has been moved to the left. If the given number is between 1 and 10, then write it as the product of the number itself and 10^0.*

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Express the following numbers in the standard form:

(i) 3,90,878 (ii) 3,186,500,000 (iii) 65,950,000

Solution (i) We have,

3,90,878 = 390878.00

Clearly, the decimal point is moved through five places to obtain a number in which there is just one digit to the left of the decimal point.

$\therefore \quad 390878.00 = 3.90878 \times 10^5$

(ii) We have,

$$3{,}186{,}500{,}000 = 3.186500000 \times 10^9$$
$$= 3.1865 \times 10^9$$

(iii) We have,

$$65{,}950{,}000 = 65{,}950{,}000.00$$
$$= 6.5950000 \times 10^7$$
$$= 6.595 \times 10^7$$

Example 2 Write the following numbers in the usual form:

(i) 7.54×10^6 (ii) 9.325×10^{12} (iii) 8.4×10^2

Solution We have,

(i) $7.54 \times 10^6 = 7{,}540{,}000$

(ii) $9.325 \times 10^{12} = 9{,}325{,}000{,}000{,}000$

(iii) $8.4 \times 10^2 = 840$

EXERCISE 6.3

1. Express the following numbers in the standard form:
   (i) 3908.78 (ii) 5,00,00,000 (iii) 3,18,65,00,000 (iv) 846×10^7
   (v) 723×10^9
2. Write the following numbers in the usual form:
   (i) 4.83×10^7 (ii) 3.21×10^5 (iii) 3.5×10^3
3. Express the numbers appearing in the following statements in the standard form:
   (i) The distance between the Earth and the Moon is 384,000,000 metres.
   (ii) Diameter of the Earth is 1,27,56,000 metres.
   (iii) Diameter of the Sun is 1,400,000,000 metres.
   (iv) The universe is estimated to be about 12,000,000,000 years old.

ANSWERS

1. (i) 3.90878×10^3 (ii) 5×10^7 (iii) 3.1865×10^9 (iv) 8.46×10^9
   (v) 7.23×10^{11}

2. (i) 4,83,00,000 (ii) 3,21,000 (iii) 3,500
3. (i) 3.84×10^{8} m (ii) 1.2756×10^{7} m (iii) 1.4×10^{9} m (iv) 1.2×10^{10} years old

6.5 DECIMAL NUMBER SYSTEM

In earlier classes, we have learnt about place value and face value of a digit in a number. We have learnt that a natural number can be written as the sum of the place values of all digits of the numbers. For example

$$6847 = 6\times1000 + 8\times100 + 4\times10 + 7\times1$$

Such a form of a natural number is known as its expanded form.

The expanded form of a number can also be expressed in terms of powers of 10 by using

$$10^0 = 1, 10^1 = 10, 10^2 = 100, 10^3 = 1000, 10^4 = 10000 \text{ etc.}$$

For example,

$$6847 = 6\times1000 + 8\times100 + 4\times10 + 7\times1$$

$$\Rightarrow 6847 = 6\times10^3 + 8\times10^2 + 4\times10^1 + 7\times10^0$$

Similarly, we have

$$7504289 = 7\times1000000 + 5\times100000 + 0\times10000 + 4\times1000 + 2\times100 + 8\times10 + 9\times1$$

$$\Rightarrow 7504289 = 7\times10^6 + 5\times10^5 + 0\times10^4 + 4\times10^3 + 2\times10^2 + 8\times10^1 + 9\times10^0.$$

Clearly, each digit of the natural number is multiplied by 10^n, where n is the number of digits to its right and then they are added.

EXERCISE 6.4

1. Write the following numbers in the expanded exponential forms:
   (i) 20068 (ii) 420719 (iii) 7805192 (iv) 5004132
   (v) 927303
2. Find the number from each of the following expanded forms:
   (i) $7\times10^4 + 6\times10^3 + 0\times10^2 + 4\times10^1 + 5\times10^0$
   (ii) $5\times10^5 + 4\times10^4 + 2\times10^3 + 3\times10^0$
   (iii) $9\times10^5 + 5\times10^2 + 3\times10^1$
   (iv) $3\times10^4 + 4\times10^2 + 5\times10^0$

ANSWERS

1. (i) $2\times10^4 + 0\times10^3 + 0\times10^2 + 6\times10^1 + 8\times10^0$
   (ii) $4\times10^5 + 2\times10^4 + 0\times10^3 + 7\times10^2 + 1\times10^1 + 9\times10^0$
   (iii) $7\times10^6 + 8\times10^5 + 0\times10^4 + 5\times10^3 + 1\times10^2 + 9\times10^1 + 2\times10^0$
   (iv) $5\times10^6 + 0\times10^5 + 0\times10^4 + 4\times10^3 + 1\times10^2 + 3\times10^1 + 2\times10^0$
   (v) $9\times10^5 + 2\times10^4 + 7\times10^3 + 3\times10^2 + 0\times10^1 + 3\times10^0$
2. (i) 76045 (ii) 542003 (iii) 900530 (iv) 30405

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. $\left(6^{-1} - 8^{-1}\right)^{-1} =$

   (a) $\frac{1}{24}$ (b) 24 (c) -24 (d) $-\frac{1}{24}$

2. $2^{3^2} =$

   (a) 64 (b) 32 (c) 256 (d) 512

3. $\left(3^{-1} \times 5^{-1}\right)^{-1} =$

   (a) $\frac{1}{15}$ (b) $-\frac{1}{15}$ (c) 15 (d) -15

4. $\left(-\frac{3}{5}\right)^{-1} =$

   (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $-\frac{5}{3}$ (d) $-\frac{3}{5}$

5. $(-1)^{301} + (-1)^{302} + (-1)^{303} + \ldots + (-1)^{400}$

   (a) 1 (b) 101 (c) 100 (d) 0

6. If $a = 25$, then $a^{25^0} + a^{0^{25}} =$

   (a) 25 (b) 26 (c) 24 (d) 0

7. $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3} =$

   (a) $\frac{19}{64}$ (b) $\frac{64}{19}$ (c) $\frac{27}{16}$ (d) $\frac{-19}{64}$

8. $\left(25^2 - 15^2\right)^{3/2} =$

   (a) 4000 (b) 8000 (c) 3125 (d) 1024

9. If $\left(\frac{5}{3}\right)^{-5} \times \left(\frac{5}{3}\right)^{11} = \left(\frac{5}{3}\right)^{8x}$, then $x = ?$

   (a) $-\frac{1}{2}$ (b) $-\frac{3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

10. $\left[\left\{\left(-\frac{1}{3}\right)^2\right\}^{-2}\right]^{-1} =$

    (a) $\frac{1}{81}$ (b) $\frac{1}{9}$ (c) $-\frac{1}{81}$ (d) $-\frac{1}{9}$

11. $\frac{(144)^{1/2} + (256)^{1/2}}{3^2 - 2} =$

    (a) 8 (b) 4 (c) -4 (d) -8

12. $(1+3+5+7+9+11)^{3/2} =$

(a) 36 (b) 216 (c) 256 (d) None of these

13. If $abc = 0$, then $\dfrac{\{(x^a)^b\}^c}{\{(x^b)^c\}^a} =$

(a) 3 (b) 0 (c) -1 (d) 1

14. $(2^3)^4 =$

(a) 2^{4^3} (b) 2^{3^4} (c) $(2^4)^3$ (d) None of these

15. $\{(33)^2 - (31)^2\}^{5/7} =$

(a) 64 (b) 16 (c) 32 (d) 4

16. If $abc = 0$, then find the value of $\{(x^a)^b\}^c$

(a) 1 (b) a (c) b (d) c

17. If $a = 3^{-3} - 3^3$ and $b = 3^3 - 3^{-3}$, then $\dfrac{a}{b} - \dfrac{b}{a}$

(a) 0 (b) 1 (c) -1 (d) 2

18. What should be multiplied to 6^{-2} so that the product may be equal to 216?

(a) 6^4 (b) 6^5 (c) 6^3 (d) 6

19. If $xyz = 0$, then find the value of $(a^x)^{yz} + (a^y)^{zx} + (a^z)^{xy} =$

(a) 3 (b) 2 (c) 1 (d) 0

20. If $2^n = 4096$, then $2^{n-5} =$

(a) 128 (b) 64 (c) 256 (d) 32

21. The number 4,70,394 is standard form is written as

(a) 4.70394×10^5 (b) 4.70394×10^4 (c) 47.0394×10^4 (d) 4703.94×10^2

22. The number 2.35×10^4 in the usual form is written as

(a) 2.35×10^3 (b) 23500 (c) 2350000 (d) 235×10^4

23. If $3^x = 6561$, then $3^{x-3} =$

(a) 81 (b) 243 (c) 729 (d) 27

24. If $2^n = 1024$, then $2^{\frac{n}{2}+2} =$

(a) 64 (b) 128 (c) 256 (d) 512

25. $(8^4 + 8^2)^{1/2} =$

(a) 84 (b) $8\sqrt{77}$ (c) 72 (d) $8\sqrt{65}$

ANSWSERS

1. (b)	2. (d)	3. (c)	4. (c)	5. (d)	6. (b)	7. (a)
8. (b)	9. (c)	10. (a)	11. (b)	12. (b)	13. (d)	14. (c)
15. (c)	16. (a)	17. (a)	18. (b)	19. (a)	20. (a)	21. (a)
22. (b)	23. (b)	24. (b)	25. (d)			

THINGS TO REMEMBER

1. *If a is a non-zero rational number and n is a natural number, then the product*

$$\underbrace{a \times a \times a \times \cdots \times a}_{(n\text{-times})}$$

is denoted by a^n and is read as 'a raised to the power n'. Rational number 'a' is called the base and natural number n is known as the exponent. Also, a^n is known as the exponential form of $\underbrace{a \times a \times a \times \cdots \times a}_{(n\text{-times})}$.

2. *For any non-zero rational number, we have*

$$a^0 = 1 \text{ and } a^1 = a$$

3. *If a and b are non-zero rational numbers and m and n are natural numbers, then following are the laws of exponents:*

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$, where $m > n$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$

(iv) $(a \times b)^n = a^n b^n$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$