2

FRACTIONS

2.1 INTRODUCTION

We have learnt about the concept of fraction in the earlier class. We have also studied about types of fractions (proper, improper and mixed), comparison of fractions, equivalent fractions, representation of fractions on the number line, addition and subtraction of fractions.

In this chapter, we will learn the multiplication and division of fractions. Let us first recall, in brief, what we have learnt about fractions in the earlier class.

2.2 RECAPITULATION

FRACTION *A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.*

The fraction representing three parts out of 7 equal parts in which the whole is divided is denoted by $\frac{3}{7}$ and is read as "*three-sevenths*". In the fraction $\frac{3}{7}$, 3 is called the numerator and 7 is called the denominator. Similarly, $\frac{2}{5}$ is a fraction with numerator 2 and denominator 5 and $\frac{12}{7}$ is a fraction with numerator 12 and denominator 7.

PROPER FRACTION *A fraction whose numerator is less than the denominator, is called a proper fraction.*

For example, $\frac{7}{9}, \frac{3}{11}, \frac{2}{5}$ etc. are proper fractions.

IMPROPER FRACTION *A fraction whose numerator is more than or equal to the deno-minator, is called an improper fraction.*

For example, $\frac{17}{5}, \frac{47}{31}, \frac{195}{111}$ etc. are improper fractions.

MIXED FRACTION *A combination of a whole number and a proper fraction is called a mixed fraction.*

For example, $2\frac{3}{5}, 7\frac{4}{15}, 21\frac{6}{29}$ etc. are mixed fractions.

In the mixed fraction $3\frac{2}{5}$, 3 is the whole number and $\frac{2}{5}$ is the proper fraction with numerator 2 and denominator 5.

In order to convert a mixed fraction into an improper fraction, we may use the following formula:

$$\text{Improper fraction} = \frac{(\text{Whole number} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$$

For example, (i) $3\frac{2}{5} = \frac{3\times5+2}{5} = \frac{15+2}{5} = \frac{17}{5}$ (ii) $9\frac{5}{4} = \frac{9\times4+5}{4} = \frac{36+5}{4} = \frac{41}{4}$

In order to express an improper fraction as a mixed fraction, we first divide the numerator by denominator and obtain the quotient and remainder and then we write the mixed fraction as

$$\text{Quotient}\ \frac{\text{Remainder}}{\text{Denominator}}$$

For example, (i) $\frac{19}{4} = 4\frac{3}{4}$ [$\because$ Quotient = 4, Remainder = 3]

(ii) $\frac{47}{15} = 3\frac{2}{15}$ [$\because$ Quotient = 3, Remainder = 2]

Remark *If the numerator and denominator of a fraction are both multiplied by the same non-zero number, then its value does not change.*

i.e., $\frac{4}{5} = \frac{4\times2}{5\times2} = \frac{4\times3}{5\times3} = \frac{4\times4}{5\times4} = \frac{4\times5}{5\times5} = \frac{4\times6}{5\times6}$ *etc.*

If the numerator and denominator of a fraction are both divided by their common factor, then its value does not change.

i.e., $\frac{6}{10} = \frac{6\div2}{10\div2} = \frac{3}{5}, \frac{9}{24} = \frac{9\div3}{24\div3} = \frac{3}{8}$ *etc.*

EQUIVALENT FRACTIONS *A given fraction and various fractions obtained by multiplying (or dividing) its numerator and denominator by the same non-zero number, are called equilvalent fractions.*

For example, $\frac{3\times2}{4\times2} = \frac{6}{8}, \frac{3\times3}{4\times3} = \frac{9}{12}, \frac{3\times4}{4\times4} = \frac{12}{16}$ etc. are equivalent fractions equivalent to the fraction $\frac{3}{4}$.

We have,

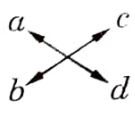
$$\frac{8}{28} = \frac{8\div4}{28\div4} = \frac{2}{7}, \frac{10}{35} = \frac{10\div5}{35\div5} = \frac{2}{7}$$

$\therefore$ $\frac{8}{28}$ and $\frac{10}{25}$ are equivalent fractions equivalent to the fraction $\frac{2}{7}$

If $\frac{a}{b}$ and $\frac{c}{d}$ are two equivalent fractions, then

$$a\times d = b\times c$$

i.e., $\frac{a}{b} = \frac{c}{d} \Leftrightarrow a\times d = b\times c$



LIKE FRACTIONS *Fractions having the same denominators are called like fractions.*

For example, $\frac{2}{15}, \frac{7}{15}, \frac{11}{15}$ etc are like fractions.

UNLIKE FRACTIONS *Fractions with different denominators are called unlike fractions.*

For example, $\frac{2}{13}, \frac{7}{24}, \frac{9}{125}$ etc are unlike fractions.

FRACTION IN LOWEST TERMS *A fraction is in its lowest terms if its numerator and denominator have no common factor other than 1.*

In other words, a fraction is in its lowest terms or in lowest form, if the HCF of its numerator and denominator is 1.

In order to reduce a fraction to its lowest terms, we divide its numerator and denominator by their HCF.

ILLUSTRATION 1 Reduce $\frac{144}{180}$ to its lowest form.

Solution First we find the HCF of 144 and 180 by factorization method.

The factors of 144 are: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72 and 144

The factors of 180 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 18, 30, 36, 45, 60, 90 and 180

The common factors of 144 and 180 are: 1, 2, 3, 4, 6, 12, 18 and 36

$\therefore$ HCF of 144 and 180 is 36.

Now, $\frac{144}{180} = \frac{144 \div 36}{180 \div 36}$ [Dividing numerator and denominator by the HCF of 144 and 180 i.e., 36]

$\Rightarrow \frac{144}{180} = \frac{4}{5}$

2.2.1 COMPARING FRACTIONS

In order to compare fractions, we may use the following steps:

STEP I *Find the LCM of the denominators of the given fractions.*

STEP II *Convert each fraction to its equivalent fraction with denominator equal to the LCM obtained in step I.*

STEP III *Arrange the fractions in ascending or descending order by arranging numerators in ascending or descending order.*

ILLUSTRATION 1 Which is larger $\frac{3}{4}$ or $\frac{5}{12}$?

Solution Let us first find the LCM of 4 and 12.

We have,

2	4 12
2	2 6
3	1 3
	1 1

$\therefore$ LCM of 4 and 12 is $2 \times 2 \times 3 = 12$.

Now we convert the given fractions to equivalent fractions with denominator 12.

We have,

$$\frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$$

We know that $9>5$

$$\therefore \quad \frac{9}{12}>\frac{5}{12} \Rightarrow \frac{3}{4}>\frac{5}{12}$$

ILLUSTRATION 2 Arrange the following fractions in ascending order $\frac{5}{8},\frac{5}{6},\frac{7}{4},\frac{3}{5}$

Solution Let us first find the LCM of the denominators:

We have,

$$\begin{array}{r|l} 2 & 5\ 8\ 6\ 4 \\ \hline 2 & 5\ 4\ 3\ 2 \\ \hline & 5\ 2\ 3\ 1 \end{array}$$

$\therefore$ LCM $= 2\times2\times5\times2\times3=120$

Now, we convert each fraction to its equivalent fraction with denominator 120.

We have,

$$\frac{3}{5}=\frac{3\times24}{5\times24}=\frac{72}{120} \qquad [\because\ 120\div5=24]$$

$$\frac{5}{8}=\frac{5\times15}{8\times15}=\frac{75}{120} \qquad [\because\ 120\div8=15]$$

$$\frac{5}{6}=\frac{5\times20}{6\times20}=\frac{100}{120} \qquad [\because\ 120\div6=20]$$

$$\frac{7}{4}=\frac{7\times30}{4\times30}=\frac{210}{120} \qquad [\because\ 120\div4=30]$$

We know that

$72<75<100<210.$

$$\Rightarrow \quad \frac{72}{120}<\frac{75}{120}<\frac{100}{120}<\frac{210}{120}$$

$$\Rightarrow \quad \frac{3}{5}<\frac{5}{8}<\frac{5}{6}<\frac{7}{4}$$

Remark *If two fractions have the same numerator but different denominators, the fraction with greater denominator is smaller.*

For example, $\frac{9}{14}<\frac{9}{10}$. Also, $\frac{7}{25}<\frac{7}{18}<\frac{7}{11}<\frac{7}{10}<\frac{7}{8}$.

2.2.2 CONVERSION OF UNLIKE FRACTIONS TO LIKE FRACTIONS

In order to convert unlike fractions to like fractions, we follow the following steps:

STEP I *Find the LCM of the denominators of the given fractions.*

STEP II *Convert each of the given fractions into an equivalent fraction having denominator equal to the LCM obtained in step I.*

ILLUSTRATION 1 Convert the unlike fractions $\frac{7}{6}, \frac{5}{9}$ and $\frac{5}{12}$ into like fractions.

Solution We have,

LCM of $(6,9,12) = (3\times2\times3\times2) = 36$

Now, $\frac{7}{6} = \frac{7\times6}{6\times6} = \frac{42}{36}; \frac{5}{9} = \frac{5\times4}{9\times4} = \frac{20}{36}$ and $\frac{5}{12} = \frac{5\times3}{12\times3} = \frac{15}{36}$

3	6	9	12
2	2	3	4
	1	3	2

Clearly, $\frac{42}{36}, \frac{20}{36}$ and $\frac{15}{36}$ are like fractions.

2.2.3 ADDITION AND SUBTRACTION OF FRACTIONS

In order to add or subtract like fractions, we add or subtract their numerators and retain the common denominator.

For example,

(i) $\frac{5}{8}+\frac{2}{8}=\frac{5+2}{8}=\frac{7}{8}$

(ii) $\frac{11}{15}-\frac{7}{15}=\frac{11-7}{15}=\frac{4}{15}$

(iii) $\frac{16}{5}-\frac{3}{5}+\frac{2}{5}-\frac{9}{5}=\frac{16-3+2-9}{5}=\frac{18-12}{5}=\frac{6}{5}$

(iv) $4\frac{2}{3}+\frac{1}{3}-4\frac{1}{3}=\frac{4\times3+2}{3}+\frac{1}{3}-\frac{4\times3+1}{3}=\frac{14}{3}+\frac{1}{3}-\frac{13}{3}=\frac{14+1-13}{3}=\frac{15-13}{3}=\frac{2}{3}$

In order to add and subtract unlike fractions, we follow the following steps:

STEP I *Obtain the fractions and their denominators.*

STEP II *Find the LCM of the denominators.*

STEP III *Convert each fraction into an equivalent fraction having its denominator equal to the LCM obtained in step II.*

STEP IV *Add or subtract like fractions obtained in Step III.*

ILLUSTRATION 1 Add : (i) $\frac{7}{10}+\frac{2}{15}$ (ii) $2\frac{2}{3}+3\frac{1}{2}$

Solution (i) LCM of 10 and 15 is $(5\times2\times3) = 30$.

So, we convert the given fractions into equivalent fractions with denominator 30.

We have, $\frac{7}{10} = \frac{7\times3}{10\times3} = \frac{21}{30}$ and, $\frac{2}{15} = \frac{2\times2}{15\times2} = \frac{4}{30}$

$\therefore \quad \frac{7}{10}+\frac{2}{15}=\frac{21}{30}+\frac{4}{30}=\frac{21+4}{30}=\frac{\cancel{25}^{5}}{\cancel{30}_{6}}=\frac{5}{6}$

5	10	15
	2	3

(ii) We have,

$$2\frac{2}{3}+3\frac{1}{2}$$

$$=\frac{2\times3+2}{3}+\frac{3\times2+1}{2}$$

$$=\frac{8}{3}+\frac{7}{2}$$

$$=\frac{8\times2}{3\times2}+\frac{7\times3}{2\times3}$$ [$\because$ LCM of 3 and 2 is 6. So, convert each fraction to an equivalent fraction with denominator 6]

$$=\frac{16}{6}+\frac{21}{6}=\frac{16+21}{6}=\frac{37}{6}$$

ILLUSTRATION 2 Simplify: $\frac{15}{16}-\frac{11}{12}$

Solution We have,

LCM of 16 and 12 $=(4\times4\times3)=48$

4	16 12
	4 3

$\therefore$ $\frac{15}{16}-\frac{11}{12}$

$$=\frac{15\times3}{16\times3}-\frac{11\times4}{12\times4}$$ [Converting each fraction to an equivalent fraction with denominator 48]

$$=\frac{45}{48}-\frac{44}{48}=\frac{45-44}{48}=\frac{1}{48}$$

ILLUSTRATION 3 Simplify : $4\frac{5}{6}-2\frac{3}{8}+3\frac{7}{12}$

Solution We have,

2	6	8	12
3	3	4	6
2	1	4	2
2	1	2	1
	1	1	1

$$4\frac{5}{6}-2\frac{3}{8}+3\frac{7}{12}$$

$$=\frac{6\times4+5}{6}-\frac{2\times8+3}{8}+\frac{3\times12+7}{12}$$

$$=\frac{29}{6}-\frac{19}{8}+\frac{43}{12}$$

$$=\frac{29\times4}{6\times4}-\frac{19\times3}{8\times3}+\frac{43\times2}{12\times2}$$ [$\because$ LCM of 6, 8, 12 is $2\times3\times2\times2=24$]

$$=\frac{116}{24}-\frac{57}{24}+\frac{86}{24}=\frac{116-57+86}{24}=\frac{202-57}{24}=\frac{145}{24}$$

ILLUSTRATIVE EXAMPLES

Example 1 Arrange the following fractions in descending order:

(i) $\frac{2}{9},\frac{2}{3},\frac{8}{21}$ (ii) $\frac{1}{5},\frac{3}{7},\frac{7}{10},\frac{13}{28}$

Solution (i) First we convert the given fractions into like fractions i.e. fractions having common denominator. For this, we first find the LCM of the denominators of the given fractions. Denominators are 9, 3, 21

LCM of $(9, 3, 21) = 3 \times 3 \times 7 = 63$

3	9	3	21
3	3	1	7
7	1	1	7
	1	1	1

Now, we convert each fraction into equivalent fractions with 63 as its denominator.

We have,

$\frac{2}{9} = \frac{2\times7}{9\times7} = \frac{14}{63}$ $[\because 63 \div 9 = 7]$

$\frac{2}{3} = \frac{2\times21}{3\times21} = \frac{42}{63}$ $[\because 63 \div 3 = 21]$

$\frac{8}{21} = \frac{8\times3}{21\times3} = \frac{24}{63}$ $[\because 63 \div 21 = 3]$

We know that

$42 > 24 > 14$

$\therefore \quad \frac{42}{63} > \frac{24}{63} > \frac{14}{63} \Rightarrow \frac{2}{3} > \frac{8}{21} > \frac{2}{9}$

(ii) Denominators of the given fractions are : 5, 7, 10, 28

LCM of denominators $= 5 \times 7 \times 2 \times 2 = 140$

We now convert each fraction into an equivalent fraction with 140 as its denominator.

5	5	7	10	28
7	1	7	2	28
2	1	1	2	4
	1	1	1	2

$\therefore \quad \frac{1}{5} = \frac{1\times28}{5\times28} = \frac{28}{140}$ $[\because 140 \div 5 = 28]$

$\frac{3}{7} = \frac{3\times20}{7\times20} = \frac{60}{140}$ $[\because 140 \div 7 = 20]$

$\frac{7}{10} = \frac{7\times14}{10\times14} = \frac{98}{140}$ $[\because 140 \div 10 = 14]$

$\frac{13}{28} = \frac{13\times5}{28\times5} = \frac{65}{140}$ $[\because 140 \div 28 = 5]$

$\because \quad 98 > 65 > 60 > 28$

$\therefore \quad \frac{98}{140} > \frac{65}{140} > \frac{60}{140} > \frac{28}{140} \Rightarrow \frac{7}{10} > \frac{13}{28} > \frac{3}{7} > \frac{1}{5}$

Remark *We can also arrange the given fractions in descending order by making their numerators the same and using the result that the fraction having greater denominator is smaller, provided that they have the same numerator.*

Example 2 Ramesh solved $\frac{2}{7}$ part of an exercise while Seema solved $\frac{4}{5}$ of it. Who solved less?

Solution In order to know who solved less part of the exercise, we will compare $\frac{2}{7}$ and $\frac{4}{5}$.

We have,

LCM of denominators (i.e., 7 and 5) $= 7 \times 5 = 35$

Converting each fraction in to an equivalent fraction having 35 as its denominator, we have

$$\frac{2}{7}=\frac{2\times5}{7\times5}=\frac{10}{35} \text{ and } \frac{4}{5}=\frac{4\times7}{5\times7}=\frac{28}{35}$$

$\because \quad 10<28$

$\therefore \quad \frac{10}{35}<\frac{28}{35} \Rightarrow \frac{2}{7}<\frac{4}{5}$

Hence, Ramesh solved lesser part than Seema.

Example 3 Michael finished colouring a picture in $\frac{7}{12}$ hour. Vaibhav finished colouring the same picture in $\frac{3}{4}$ hour. Who worked longer? By what fraction was it longer?

Solution In order to know who worked longer, we will compare fractions $\frac{7}{12}$ and $\frac{3}{4}$.

We have,

4	12	4
	3	1

(LCM of 12 and 4) = 12

Converting each fraction into an equivalent fraction with 12 as denominator, we have

$$\frac{7}{12}=\frac{7\times1}{12\times1}=\frac{7}{12} \text{ and } \frac{3}{4}=\frac{3\times3}{4\times3}=\frac{9}{12}$$

$\because \quad 7<9$

$\therefore \quad \frac{7}{12}<\frac{9}{12} \Rightarrow \frac{7}{12}<\frac{3}{4}$

Thus, Vaibhav finished colouring in longer time.

Now, $\frac{3}{4}-\frac{7}{12}=\frac{9}{12}-\frac{7}{12}=\frac{9-7}{12}=\frac{2}{12}=\frac{1}{6}$

Hence, Vaibhav finished colouring in $\frac{1}{6}$ hour more time than Michael.

Example 4 Simplify :

(i) $2-\frac{3}{5}$ (ii) $4+\frac{7}{8}$ (iii) $\frac{9}{11}-\frac{4}{15}$ (iv) $8\frac{1}{2}-3\frac{5}{8}$

Solution (i) We have,

$$\therefore \quad 2-\frac{3}{5}=\frac{2}{1}-\frac{3}{5} \qquad \left[\because 2=\frac{2}{1}\right]$$

$$=\frac{2\times5}{1\times5}-\frac{3\times1}{5\times1} \qquad [\because \text{LCM of 1 and 5 is 5}]$$

$$=\frac{10}{5}-\frac{3}{5}=\frac{10-3}{5}=\frac{7}{5}$$

(ii) We have,

$$\therefore \quad 4+\frac{7}{8}=\frac{4}{1}+\frac{7}{8} \qquad \left[\because 4=\frac{4}{1}\right]$$

$$=\frac{4\times8}{1\times8}+\frac{7\times1}{8\times1} \qquad [\because \text{LCM of 1 and 8 is 8}]$$

$$= \frac{32}{8} + \frac{7}{8} = \frac{32+7}{8} = \frac{39}{8} = 4\frac{7}{8}$$

(iii) The LCM of 11 and 15 is $11 \times 15 = 165$.

$$\therefore \quad \frac{9}{11} - \frac{4}{15} = \frac{9\times15}{11\times15} - \frac{4\times11}{15\times11} = \frac{135}{165} - \frac{44}{165} = \frac{135-44}{165} = \frac{91}{165}$$

(iv) We have,

$$8\frac{1}{2} - 3\frac{5}{8}$$

$$= \frac{17}{2} - \frac{29}{8}$$

$$= \frac{17\times4}{2\times4} - \frac{29\times1}{8\times1} \qquad [\because \text{LCM of 2 and 8 is 8}]$$

$$= \frac{68}{8} - \frac{29}{8} = \frac{68-29}{8} = \frac{39}{8} = 4\frac{7}{8}$$

Aliter We have,

$$8\frac{1}{2} - 3\frac{5}{8}$$

$$= \left(8 + \frac{1}{2}\right) - \left(3 + \frac{5}{8}\right)$$

$$= 8 + \frac{1}{2} - 3 - \frac{5}{8}$$

$$= 8 - 3 + \frac{1}{2} - \frac{5}{8}$$

$$= 5 + \frac{1\times4}{2\times4} - \frac{5\times1}{8\times1}$$

$$= 5 + \frac{4}{8} - \frac{5}{8}$$

$$= 5 + \frac{4-5}{8}$$

$$= 5 + \frac{(-1)}{8}$$

$$= \frac{5}{1} - \frac{1}{8} = \frac{5\times8}{1\times8} - \frac{1\times1}{8\times1} = \frac{40}{8} - \frac{1}{8} = \frac{40-1}{8} = \frac{39}{8} = 4\frac{7}{8}$$

Example 5 Simplify: $4\frac{2}{3} - 3\frac{1}{4} + 2\frac{1}{6}$.

Solution We have,

$$4\frac{2}{3} - 3\frac{1}{4} + 2\frac{1}{6}$$

$$= \frac{14}{3} - \frac{13}{4} + \frac{13}{6}$$

$$= \frac{14\times4}{3\times4} - \frac{13\times3}{4\times3} + \frac{13\times2}{6\times2}$$ [∵ LCM of 3, 4 and 6 is 12, so we convert each fraction into an equivalent fraction with denominator 12]

$$= \frac{56}{12} - \frac{39}{12} + \frac{26}{12} = \frac{56-39+26}{12} = \frac{82-39}{12} = \frac{43}{12} = 3\frac{7}{12}$$

Example 6 Sameera purchased $3\frac{1}{2}$ kg apples and $4\frac{3}{4}$ kg oranges. What is the total weight of fruits purchased by her?

Solution Total weight of the fruits purchased by Sameera is $\left(3\frac{1}{2}+4\frac{3}{4}\right)$ kg.

Now, $3\frac{1}{2}+4\frac{3}{4}$

$$= \frac{7}{2} + \frac{19}{4}$$

$$= \frac{7\times2}{2\times2} + \frac{19\times1}{4\times1}$$

$$= \frac{14}{4} + \frac{19}{4} = \frac{14+19}{4} = \frac{33}{4} = 8\frac{1}{4}$$

Hence, total weight is $8\frac{1}{4}$ kg.

Example 7 Ritu ate $\frac{3}{5}$ part of an apple and the remaining apple was eaten by her brother Somu. How much part of the apple did Somu eat? Who had the larger share? By how much?

Solution We have, Part of an apple eaten by Ritu $= \frac{3}{5}$

$\therefore$ Part of an apple eaten by Somu

$$= 1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$$

Clearly, $\frac{3}{5} > \frac{2}{5}$. So, Ritu had the larger share.

Now,

$$\frac{3}{5} - \frac{2}{5} = \frac{3-2}{5} = \frac{1}{5}$$

$\therefore$ Ritu had $\frac{1}{5}$ part more than Somu.

Example 8 In the adjoining Fig. 1, find the perimeters of (i) ΔABE (ii) the rectangle $BCDE$. Whose perimeter is greater?

Solution (i) We have,

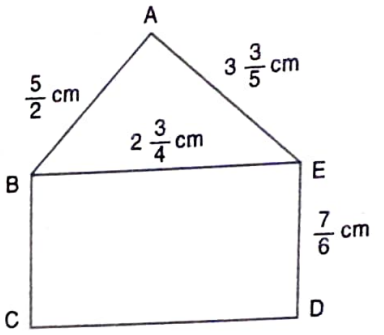


Fig. 1

Perimeter of ΔABE

$= AB + BE + EA$

$= \left(\frac{5}{2} + 2\frac{3}{4} + 3\frac{3}{5}\right)$ cm

$= \left(\frac{5}{2} + \frac{11}{4} + \frac{18}{5}\right)$ cm

$= \left(\frac{5\times10}{2\times10} + \frac{11\times5}{4\times5} + \frac{18\times4}{5\times4}\right)$ cm [$\because$ LCM of 2, 4 and 5 is 20]

$= \left(\frac{50}{20} + \frac{55}{20} + \frac{72}{20}\right)$ cm $= \frac{50+55+72}{20}$ cm $= \frac{177}{20}$ cm $= 8\frac{17}{20}$ cm

(ii) We have,

Perimeter of rectangle $BCDE$

$= BC + CD + DE + EB$

$= \left(\frac{7}{6} + 2\frac{3}{4} + \frac{7}{6} + 2\frac{3}{4}\right)$ cm

$= \left(\frac{7}{6} + \frac{11}{4} + \frac{7}{6} + \frac{11}{4}\right)$ cm

$= \left(\frac{7\times2}{6\times2} + \frac{11\times3}{4\times3} + \frac{7\times2}{6\times2} + \frac{11\times3}{4\times3}\right)$ cm [$\because$ LCM of 6 and 4 is 12]

$= \left(\frac{14}{12} + \frac{33}{12} + \frac{14}{12} + \frac{33}{12}\right)$ cm $= \frac{14+33+14+33}{12}$ cm $= \frac{94}{12}$ cm $= 7\frac{10}{12}$ cm $= 7\frac{5}{6}$ cm

Thus, perimeters of triangle ABE and rectangle $BCDE$ are $\frac{177}{20}$ cm and $\frac{94}{12}$ cm respectively.

In order to determine whose perimeter is greater, we convert $\frac{177}{20}$ and $\frac{94}{12}$ into equivalent fractions with denominators as the LCM of 20 and 12.

We have,

4	20	12
	5	3

LCM of 20 and 12 $= (4\times5\times3) = 60$

$\therefore \quad \frac{177}{20} = \frac{177\times3}{20\times3} = \frac{531}{60}$ and $\frac{94}{12} = \frac{94\times5}{12\times5} = \frac{470}{60}$

$\because \quad 531 > 470$

$$\therefore \quad \frac{531}{60} > \frac{470}{60} \Rightarrow \frac{177}{20} > \frac{94}{12}$$

Hence, triangle *ABE* has greater perimeter.

Example 9 Salil wants to put a picture in a frame. The picture is $7\frac{3}{5}$ cm wide. To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much should the picture be trimmed?

Solution We have,

$$\text{Actual width of the picture} = 7\frac{3}{5}\text{cm} = \frac{38}{5}\text{cm}$$

$$\text{Required width of the picture} = 7\frac{3}{10}\text{cm} = \frac{73}{10}\text{cm}$$

$$\therefore \quad \text{Extra width} = \left(\frac{38}{5} - \frac{73}{10}\right)\text{cm} = \left(\frac{38\times 2}{5\times 2} - \frac{73\times 1}{10\times 1}\right)\text{cm} = \left(\frac{76}{10} - \frac{73}{10}\right)\text{cm}$$

$$= \frac{76-73}{10}\text{cm} = \frac{3}{10}\text{cm}$$

Hence, $\frac{3}{10}$ cm width of the picture should be trimmed.

EXERCISE 2.1

1. Compare the following fractions by using the symbol > or < or =:

   (i) $\frac{7}{9}$ and $\frac{8}{13}$ (ii) $\frac{11}{9}$ and $\frac{5}{9}$ (iii) $\frac{37}{41}$ and $\frac{19}{30}$ (iv) $\frac{17}{15}$ and $\frac{119}{105}$

2. Arrange the following fractions in ascending order:

   (i) $\frac{3}{8}, \frac{5}{6}, \frac{6}{8}, \frac{2}{4}, \frac{1}{3}$ (ii) $\frac{4}{6}, \frac{3}{8}, \frac{6}{12}, \frac{5}{16}$

3. Arrange the following fractions in descending order:

   (i) $\frac{4}{5}, \frac{7}{10}, \frac{11}{15}, \frac{17}{20}$ (ii) $\frac{2}{7}, \frac{11}{35}, \frac{9}{14}, \frac{13}{28}$

4. Write five equivalent fractions of $\frac{3}{5}$.

5. Find the sum:

   (i) $\frac{5}{8} + \frac{3}{10}$ (ii) $4\frac{3}{4} + 9\frac{2}{5}$ (iiii) $\frac{5}{6} + 3 + \frac{3}{4}$ (iv) $2\frac{3}{5} + 4\frac{7}{10} + 2\frac{4}{15}$

6. Find the difference of

   (i) $\frac{13}{24}$ and $\frac{7}{16}$ (ii) $\frac{23}{3}$ and 6 (iii) $\frac{18}{20}$ and $\frac{21}{25}$ (iv) $3\frac{3}{10}$ and $2\frac{7}{15}$

7. Find the difference:

(i) $\frac{6}{7}-\frac{9}{11}$ (ii) $8-\frac{5}{9}$ (iii) $9-5\frac{2}{3}$ (iv) $4\frac{3}{10}-1\frac{2}{15}$

8. Simplify:

(i) $\frac{2}{3}+\frac{1}{6}-\frac{2}{9}$ (ii) $12-3\frac{1}{2}$ (iii) $7\frac{5}{6}-4\frac{3}{8}+2\frac{7}{12}$

9. What should be added to $5\frac{3}{7}$ to get 12?

10. What should be added to $5\frac{4}{15}$ to get $12\frac{3}{5}$?

11. Suman studies for $5\frac{2}{3}$ hours daily. She devotes $2\frac{4}{5}$ hours of her time for Science and Mathematics. How much time does she devote for other subjects?

12. A piece of wire is of length $12\frac{3}{4}$m. If it is cut into two pieces in such a way that the length of one piece is $5\frac{1}{4}$m, what is the length of the other piece?

13. A rectangular sheet of paper is $12\frac{1}{2}$ cm long and $10\frac{2}{3}$ cm wide. Find its perimeter.

14. In a "magic square", the sum of the numbers in each row, in each column and along the diagonal is the same. Is this a magic square? Yes

$\frac{4}{11}$	$\frac{9}{11}$	$\frac{2}{11}$
$\frac{3}{11}$	$\frac{5}{11}$	$\frac{7}{11}$
$\frac{8}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

15. The cost of Mathematics book is ₹ $25\frac{3}{4}$ and that of Science book is ₹ $20\frac{1}{2}$. Which costs more and by how much?

16. (i) Provide the number in box □ and also give its simplest form in each of the following:

(i) $\frac{2}{3}\times\square=\frac{10}{30}$ (5) (ii) $\frac{3}{5}\times\square=\frac{24}{75}$. (8, 15)

ANSWER

1. (i) $\frac{7}{9}>\frac{8}{13}$ (ii) $\frac{11}{9}>\frac{5}{9}$ (iii) $\frac{37}{41}>\frac{19}{30}$ (iv) $\frac{17}{15}=\frac{119}{105}$

2. (i) $\frac{1}{3}<\frac{3}{8}<\frac{2}{4}<\frac{6}{8}<\frac{5}{6}$ (ii) $\frac{5}{16}<\frac{3}{8}<\frac{6}{12}<\frac{4}{6}$

3. (i) $\frac{17}{20} > \frac{4}{5} > \frac{11}{15} > \frac{7}{10}$ (ii) $\frac{9}{14} > \frac{13}{28} > \frac{11}{35} > \frac{2}{7}$ 4. $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}$

5. (i) $\frac{37}{40}$ (ii) $\frac{283}{20}$ (iii) $\frac{55}{12}$ (iv) $\frac{287}{30}$

6. (i) $\frac{5}{48}$ (ii) $\frac{5}{3}$ (iii) $\frac{3}{50}$ (iv) $\frac{5}{6}$

7. (i) $\frac{3}{77}$ (ii) $\frac{67}{9}$ (iii) $\frac{10}{3}$ (iv) $\frac{19}{6}$

8. (i) $\frac{11}{18}$ (ii) $\frac{17}{2}$ (iii) $\frac{145}{24}$

9. $\frac{46}{7}$ 10. $\frac{22}{3}$ 11. $2\frac{13}{15}$ hours 12. $7\frac{1}{2}$ m

13. $46\frac{1}{3}$ cm 14. Yes 15. Mathematics book by ₹ $5\frac{1}{4}$ 16. (i) $\frac{5}{10}$ (ii) $\frac{8}{15}$

2.3 MULTIPLICATION OF FRACTIONS

Let there be a rectangle of length $7\frac{3}{4}$ cm and breadth $5\frac{1}{2}$ cm. In the previous section, we have learnt how to find the perimeter of the rectangle by using addition of fractions. If we want to calculate the area of the rectangle, we will have to find the product of its length and breadth i.e., $7\frac{3}{4} \times 5\frac{1}{2}$ or, $\frac{31}{4} \times \frac{11}{2}$. This can be calculated if we know how to multiply two fractions. So, we define the multiplication of fractions as follows:

$$\text{Product of two fractions} = \frac{\text{Prodcut of their numerators}}{\text{Product of their denominators}}$$

i.e., $\frac{a}{b} \times \frac{c}{d} = \frac{(a \times c)}{(b \times d)}$

For example,

(i) $\frac{3}{7} \times \frac{4}{5} = \frac{3 \times 4}{7 \times 5} = \frac{12}{35}$ (ii) $\frac{7}{3} \times \frac{5}{2} = \frac{7 \times 5}{3 \times 2} = \frac{35}{6}$

(iii) $5 \times \frac{3}{7} = \frac{5}{1} \times \frac{3}{7} = \frac{5 \times 3}{1 \times 7} = \frac{15}{7}$ (iv) $\frac{5}{12} \times 9 = \frac{5}{12} \times \frac{9}{1} = \frac{5 \times \cancel{9}^{3}}{\cancel{12}_{4} \times 1} = \frac{5 \times 3}{4 \times 1} = \frac{15}{4}$

Sometimes for indicating multiplication of two fractions, we use the word 'of' as follows:

$$\frac{1}{2} \text{ of } 8 = \frac{1}{2} \times 8 = \frac{1}{2} \times \frac{8}{1} = \frac{1 \times 8}{2 \times 1} = \frac{8}{2} = 4$$

$$\frac{1}{5} \text{ of } 20 = \frac{1}{5} \times 20 = \frac{1}{5} \times \frac{20}{1} = \frac{1 \times 20}{5 \times 1} = \frac{20}{5} = 4$$

$$\frac{2}{5} \text{ of } 25 = \frac{2}{5} \times 25 = \frac{2}{5} \times \frac{25}{1} = \frac{2 \times 25}{5 \times 1} = \frac{50}{5} = 10$$

$$\frac{2}{3} \text{ of } \frac{5}{7} = \frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21} \text{ etc.}$$

ILLUSTRATIVE EXAMPLES

Example 1 Multiply:

(i) $\frac{2}{9}$ by $\frac{4}{5}$ (ii) $\frac{3}{5}$ by 12 (iii) $2\frac{1}{3}$ by $\frac{2}{5}$ (iv) $5\frac{3}{4}$ by $2\frac{3}{7}$

Solution We have,

(i) $\frac{2}{9}\times\frac{4}{5}=\frac{2\times4}{9\times5}=\frac{8}{45}$

(ii) $\frac{3}{5}\times12=\frac{3}{5}\times\frac{12}{1}=\frac{3\times12}{5\times1}=\frac{36}{5}=7\frac{1}{5}$

(iii) $2\frac{1}{3}\times\frac{2}{5}=\frac{7}{3}\times\frac{2}{5}=\frac{7\times2}{3\times5}=\frac{14}{15}$

(iv) $5\frac{3}{4}\times2\frac{3}{7}=\frac{23}{4}\times\frac{17}{7}=\frac{23\times17}{4\times7}=\frac{391}{28}=13\frac{27}{28}$

Example 2 Multiply and reduce to lowest form (if possible) :

(i) $\frac{2}{3}\times\frac{5}{4}$ (ii) $\frac{1}{3}\times\frac{15}{8}$ (iii) $\frac{4}{5}\times\frac{12}{7}$

Solution We have,

(i) $\frac{2}{3}\times\frac{5}{4}=\frac{{}^{1}\cancel{2}\times5}{3\times\cancel{4}^{2}}=\frac{1\times5}{3\times2}=\frac{5}{6}$

(ii) $\frac{1}{3}\times\frac{15}{8}=\frac{1\times\cancel{15}^{5}}{{}^{1}\cancel{3}\times8}=\frac{1\times5}{1\times8}=\frac{5}{8}$

(iii) $\frac{4}{5}\times\frac{12}{7}=\frac{4\times12}{5\times7}=\frac{48}{35}=1\frac{13}{35}$

Example 3 Simplify :

(i) $5\times\frac{3}{20}\times\frac{2}{15}$ (ii) $\frac{14}{25}\times\frac{35}{51}\times\frac{34}{49}$

Solution We have,

(i) $5\times\frac{3}{20}\times\frac{2}{15}=\frac{5}{1}\times\frac{3}{20}\times\frac{2}{15}=\frac{{}^{1}\cancel{5}\times{}^{1}\cancel{3}\times2}{1\times\cancel{20}_{4}\times\cancel{15}_{5}}=\frac{1\times1\times\cancel{2}^{1}}{1\times\cancel{4}_{2}\times5}=\frac{1}{2\times5}=\frac{1}{10}$

(ii) $\frac{14}{25}\times\frac{35}{51}\times\frac{34}{49}=\frac{{}^{2}\cancel{14}\times{}^{1}\cancel{35}\times\cancel{34}^{2}}{\cancel{25}_{5}\times\cancel{51}_{3}\times\cancel{49}_{7^{1}}}=\frac{2\times1\times2}{5\times3\times1}=\frac{4}{15}$

Example 4 Which is greater? $\frac{2}{7}$ of $\frac{3}{4}$ or, $\frac{3}{5}$ of $\frac{5}{8}$.

Solution We have,

$\frac{2}{7}$ of $\frac{3}{4}=\frac{2}{7}\times\frac{3}{4}=\frac{\cancel{2}^{1}\times3}{7\times\cancel{4}_{2}}=\frac{1\times3}{7\times2}=\frac{3}{14}$ and, $\frac{3}{5}$ of $\frac{5}{8}=\frac{3}{5}\times\frac{5}{8}=\frac{3\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times8}=\frac{3\times1}{1\times8}=\frac{3}{8}$

In order to compare these fractions, we convert them into equivalent fractions having same denominator equal to the LCM of 14 and 8.

$$\begin{array}{r|rr} 2 & 14 & 8 \\ \hline & 7 & 4 \end{array}$$

LCM of 14 and 8 $= 2\times7\times4 = 56$

$\therefore \quad \frac{3}{14} = \frac{3\times4}{14\times4} = \frac{12}{56}$ and $\frac{3}{8} = \frac{3\times7}{8\times7} = \frac{21}{56}$

Clearly, $21 > 12$

$\therefore \quad \frac{21}{56} > \frac{12}{56} \Rightarrow \frac{3}{8} > \frac{3}{14}$

Hence, $\frac{3}{5}$ of $\frac{5}{8}$ is greater than $\frac{2}{7}$ of $\frac{3}{4}$.

Aliter We know that if the numerators of two fractions are same, then the fraction having smaller denominator is greater.

$\therefore \quad \frac{3}{8} > \frac{3}{14} \Rightarrow \frac{3}{5}$ of $\frac{5}{8}$ is greater than $\frac{2}{7}$ of $\frac{3}{4}$.

Example 5 Find:

(i) $\frac{3}{5}$ of a rupee (ii) $\frac{3}{4}$ of a year (iii) $\frac{2}{3}$ of a day

(iv) $\frac{5}{8}$ of a kilogram (v) $\frac{2}{3}$ of an hour (vi) $\frac{7}{25}$ of a litre

Solution (i) We have, 1 rupee = 100 paise

$\therefore \quad \frac{3}{5}$ of a rupee $= \frac{3}{5}$ of 100 paise

Now, $\frac{3}{5}$ of $100 = \frac{3}{5}\times100 = \frac{3}{5}\times\frac{100}{1} = \frac{3\times\cancel{100}^{20}}{\cancel{5}_{1}\times1} = \frac{3\times20}{1\times1} = 60$

$\therefore \quad \frac{3}{5}$ of a rupee = 60 paise

(ii) We have, 1 year = 12 months

$\therefore \quad \frac{3}{4}$ of a year $= \frac{3}{4}$ of 12 months

Now, $\frac{3}{4}$ of $12 = \frac{3}{4}\times12 = \frac{3\times\overset{3}{\cancel{12}}}{\underset{1}{\cancel{4}}\times1} = \frac{3\times3}{1\times1} = 9$

$\therefore \quad \frac{3}{4}$ of a year = 9 months

(iii) We have, 1 day = 24 hours

$\therefore \quad \frac{2}{3}$ of a day $= \frac{2}{3}$ of 24 hours

Now, $\frac{2}{3}$ of $24 = \frac{2}{3}\times24 = \frac{2}{3}\times\frac{24}{1} = \frac{2\times\overset{8}{\cancel{24}}}{\underset{1}{\cancel{3}}\times1} = \frac{2\times8}{1\times1} = 16$

$\therefore \quad \frac{2}{3}$ of a day = 16 hours

(iv) We have, 1 kilogram = 1000 grams

$\therefore \quad \frac{5}{8}$ of a kilogram $= \frac{5}{8}$ of 1000 grams $= \left(\frac{5}{8} \times 1000\right)$ grams

Now, $\frac{5}{8} \times 1000 = \frac{5}{8} \times \frac{1000}{1} = \frac{5 \times \overset{125}{\cancel{1000}}}{\underset{1}{\cancel{8}} \times 1} = \frac{5 \times 125}{1 \times 1} = 625$

$\therefore \quad \frac{5}{8}$ of a kilogram = 625 grams

(v) We have, 1 hour = 60 minutes

$\therefore \quad \frac{2}{3}$ of an hour $= \left(\frac{2}{3} \times 60\right)$ minutes

Now, $\frac{2}{3} \times 60 = \frac{2}{3} \times \frac{60}{1} = \frac{2 \times \overset{20}{\cancel{60}}}{\underset{1}{\cancel{3}} \times 1} = \frac{2 \times 20}{1 \times 1} = 40$

$\therefore \quad \frac{2}{3}$ of an hour = 40 minutes

(vi) We have, 1 litre = 1000 ml

$\therefore \quad \frac{7}{25}$ of a litre $= \left(\frac{7}{25} \times 1000\right)$ ml

Now, $\frac{7}{25} \times 1000 = \frac{7}{25} \times \frac{1000}{1} = \frac{7 \times \overset{40}{\cancel{1000}}}{\underset{1}{\cancel{25}} \times 1} = 7 \times 40 = 280$

$\therefore \quad \frac{7}{25}$ of a litre = 280 ml.

Example 6 Sugar is sold at ₹ $17\frac{3}{4}$ per kg. Find the cost of $8\frac{1}{2}$ kg of a sugar.

Solution We have,

Cost of 1 kg of sugar $= ₹\, 17\frac{3}{4} = ₹\, \frac{71}{4}$

$\therefore$ Cost of $8\frac{1}{2}$ kg of sugar $= ₹ \left(\frac{71}{4} \times 8\frac{1}{2}\right)$

$= ₹ \left(\frac{71}{4} \times \frac{17}{2}\right) = ₹ \left(\frac{71 \times 17}{4 \times 2}\right) = ₹ \left(\frac{1207}{8}\right) = ₹\, 150\frac{7}{8}$

Hence, the cost of $8\frac{1}{2}$ kg of sugar is ₹ $150\frac{7}{8}$.

Example 7 A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol?

Solution In 1 litre petrol, car runs 16 km

$\therefore$ In $2\frac{3}{4}$ litres of petrol car will travel $= \left(2\frac{3}{4}\times 16\right)\text{km} = \left(\frac{11}{\cancel{4}_{1}}\times\frac{\cancel{16}^{4}}{1}\right)\text{km}$

$= (11\times 4)\text{ km} = 44\text{ km}$

Hence, car travels 44 km in $2\frac{3}{4}$ litres of petrol.

Example 8 Shikha has read $\frac{3}{4}$ of a book consisting of 288 pages. How many pages are still left?

Solution We have,

Total number of pages in the book = 288

Number of pages read by Shikha $= \frac{3}{4}$ of $288 = \frac{3}{4}\times 288$

$= \frac{3}{\cancel{4}_{1}}\times\frac{\cancel{288}^{72}}{1} = 3\times 72 = 216$

$\therefore$ Number of pages left $= (288 - 216) = 72$

Example 9 A rectangular park is $20\frac{3}{4}$ m long and $15\frac{1}{2}$ m wide. What is the area of the park?

Solution We have,

Length of the park $= 20\frac{3}{4}\text{ m} = \frac{83}{4}\text{ m}$, Width of the park $= 15\frac{1}{2}\text{ m} = \frac{31}{2}\text{ m}$

$\therefore$ Area of the park = Length × Width

$= \frac{83}{4}\times\frac{31}{2}\text{m}^2 = \frac{83\times 31}{4\times 2}\text{m}^2 = \frac{2573}{8}\text{m}^2 = 321\frac{5}{8}\text{m}^2$

Example 10 Find the area of a square field if its each side is $10\frac{3}{4}$ m long.

Solution We have,

Length of the square field $= 10\frac{3}{4}\text{ m} = \frac{43}{4}\text{ m}$.

Breadth of the square field $= 10\frac{3}{4}\text{ m} = \frac{43}{4}\text{ m}$.

$\therefore$ Area of the square field = Length × Breadth

$$= \frac{43}{4} \times \frac{43}{4}\ m^2 = \frac{43 \times 43}{4 \times 4}\ m^2 = \frac{1849}{16}\ m^2 = 115\frac{9}{16}\ m^2$$

Example 11 Priya spends $\frac{3}{5}$ of her income on household expenses and $\frac{1}{7}$ of her income on personal expenses. If her monthly income is ₹ 35000, find her monthly savings.

Solution Priya's total monthly income = ₹ 35000.

$$\text{Monthly expenditure} = \frac{3}{5} \text{ of } ₹\,35000 + \frac{1}{7} \text{ of } ₹\,35000$$

$$= ₹\left(\frac{3}{5} \times 35000\right) + ₹\left(\frac{1}{7} \times 35000\right)$$

$$= ₹\left(\frac{3}{5} \times \frac{35000}{1}\right) + ₹\left(\frac{1}{7} \times \frac{35000}{1}\right)$$

$$= ₹\left(\frac{3 \times \overset{7000}{\cancel{35000}}}{\underset{1}{\cancel{5} \times 1}}\right) + ₹\left(\frac{1 \times \overset{5000}{\cancel{35000}}}{\underset{1}{\cancel{7} \times 1}}\right)$$

$$= ₹\,(3 \times 7000) + ₹\,(1 \times 5000)$$

$$= ₹\,21000 + ₹\,5000 = ₹\,(21000 + 5000) = ₹\,26000$$

$\therefore$ Monthly savings $= ₹\,(35000 - 26000) = ₹\,9000$

Example 12 A carton contains 40 boxes of nails and each box weighs $3\frac{3}{4}$ kg. How much would a carton of nails weigh?

Solution We have,

$$\text{Weight of 1 box} = 3\frac{3}{4}\ kg = \frac{15}{4}\ kg$$

$\therefore$ Weight of 40 boxes $= \left(\frac{15}{4} \times 40\right) kg = \left(\frac{15}{4} \times \frac{40}{1}\right) kg = \frac{15 \times \overset{10}{\cancel{40}}}{\underset{1}{\cancel{4} \times 1}}\ kg = 150\ kg$

Hence, weight of the carton is 150 kg.

EXERCISE 2.2

1. Multiply:

   (i) $\frac{7}{11}$ by $\frac{3}{5}$ (ii) $\frac{3}{5}$ by 25 (iii) $3\frac{4}{15}$ by 24 (iv) $3\frac{1}{8}$ by $\times 4\frac{10}{11}$

2. Find the product:

   (i) $\frac{4}{7} \times \frac{14}{25}$ (ii) $7\frac{1}{2} \times 2\frac{4}{15}$ (iii) $3\frac{6}{7} \times 4\frac{2}{3}$ (iv) $6\frac{11}{14} \times 3\frac{1}{2}$

3. Simplify:

(i) $\frac{12}{25} \times \frac{15}{28} \times \frac{35}{36}$ (ii) $\frac{10}{27} \times \frac{39}{56} \times \frac{28}{65}$ (iii) $2\frac{2}{17} \times 7\frac{2}{9} \times 1\frac{33}{52}$

4. Find:

(i) $\frac{1}{2}$ of $4\frac{2}{9}$ (ii) $\frac{5}{8}$ of $9\frac{2}{3}$ (iii) $\frac{2}{3}$ of $\frac{9}{16}$

5. Which is greater? $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

6. Find:

(i) $\frac{7}{11}$ of ₹ 330 (ii) $\frac{5}{9}$ of 108 metres (iii) $\frac{3}{7}$ of 42 litres

(iv) $\frac{1}{12}$ of an hour (v) $\frac{5}{6}$ of an year (vi) $\frac{3}{20}$ of a kg

(vii) $\frac{7}{20}$ of a litre (viii) $\frac{5}{6}$ of a day (ix) $\frac{2}{7}$ of a week

7. Shikha plants 5 saplings in a row in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

8. Ravish reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

9. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

10. Find the area of a rectangular park which is $41\frac{2}{3}$ m long and $18\frac{3}{5}$ m broad.

11. If milk is available at ₹ $17\frac{3}{4}$ per litre, find the cost of $7\frac{2}{5}$ litres of milk.

12. Sharda can walk $8\frac{1}{3}$ km in one hour. How much distance will she cover in $2\frac{2}{5}$ hours?

13. A sugar bag contains 30 kg of sugar. After consuming $\frac{2}{3}$ of it, how much sugar is left in the bag?

14. Each side of a square is $6\frac{2}{3}$ m long. Find its area.

15. There are 45 students in a class and $\frac{3}{5}$ of them are boys. How many girls are there in the class?

ANSWERS

1. (i) $\frac{21}{55}$ (ii) 15 (iii) $78\frac{2}{5}$ (iv) $15\frac{15}{44}$

2. (i) $\frac{8}{25}$ (ii) 17 (iii) 18 (iv) $23\frac{3}{4}$ 3. (i) $\frac{1}{4}$ (ii) $\frac{1}{9}$ (iii) 25

4. (i) $2\frac{1}{9}$ (ii) $6\frac{1}{24}$ (iii) $\frac{3}{8}$ 5. $\frac{1}{2}$ of $\frac{6}{7}$

6. (i) Rs 210 (ii) 60 metres (iii) 18 litres (iv) 5 minutes
   (v) 10 months (vi) 150 gms (vii) 350 ml
   (viii) 20 hours (ix) 2 days

7. 3 m 8. $\frac{11}{15}$ 9. $10\frac{1}{2}$ hours 10. 775 m^2 11. ₹ $131\frac{7}{20}$

12. 20 km 13. 10 kg 14. $44\frac{4}{9}$ m^2 15. 18

2.4 DIVISION OF FRACTIONS

RECIPROCAL OF A FRACTION *Two fractions are said to be the reciprocal or multiplicative inverse of each other, if their product is 1.*

For example,

(i) $\frac{3}{4}$ and $\frac{4}{3}$ are the reciprocals of each other, because $\frac{3}{4}\times\frac{4}{3}=1$.

(ii) The reciprocal of $\frac{1}{7}$ is $\frac{7}{1}$ i.e.; 7, because $\frac{1}{7}\times\frac{7}{1}=1$

(iii) The reciprocal of $\frac{1}{9}$ is 9, because $\frac{1}{9}\times 9=1$

(iv) The reciprocal of $2\frac{3}{5}$ i.e., $\frac{13}{5}$ is $\frac{5}{13}$, because $2\frac{3}{5}\times\frac{5}{13}=1$.

Reciprocal of 0 does not exist because division by zero is not possible.

Clearly, the reciprocal of a non-zero fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$.

DIVISION OF FRACTIONS *The division of a fraction $\frac{a}{b}$ by a non-zero fraction $\frac{c}{d}$ is defined as the product of $\frac{a}{b}$ with the multiplicative inverse or reciprocal of $\frac{c}{d}$.*

i.e., $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$

For example,

(i) $\frac{3}{5}\div\frac{5}{9}=\frac{3}{5}\times\frac{9}{5}=\frac{3\times 9}{5\times 5}=\frac{27}{25}$ (ii) $\frac{2}{3}\div 8=\frac{2}{3}\times\frac{1}{8}=\frac{\overset{1}{\cancel{2}}\times 1}{3\times\underset{4}{\cancel{8}}}=\frac{1}{12}$

(iii) $4 \div \frac{6}{7} = \frac{4}{1} \div \frac{6}{7} = \frac{4}{1} \times \frac{7}{6} = \frac{\overset{2}{\cancel{4}} \times 7}{1 \times \underset{3}{\cancel{6}}} = \frac{14}{3} = 4\frac{2}{3}$

(iv) $4\frac{2}{3} \div 3\frac{1}{2} = \frac{14}{3} \div \frac{7}{2} = \frac{14}{3} \times \frac{2}{7} = \frac{\overset{2}{\cancel{14}} \times 2}{3 \times \underset{1}{\cancel{7}}} = \frac{4}{3}$

ILLUSTRATIVE EXAMPLES

Example 1 Divide:

(i) $\frac{5}{9}$ by $\frac{2}{3}$ (ii) 28 by $\frac{7}{4}$ (iii) 36 by $6\frac{2}{3}$

Solution We have,

(i) $\frac{5}{9} \div \frac{2}{3} = \frac{5}{9} \times \frac{3}{2} = \frac{5 \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times 2} = \frac{5 \times 1}{3 \times 2} = \frac{5}{6}$

(ii) $28 \div \frac{7}{4} = \frac{28}{1} \div \frac{7}{4} = \frac{28}{1} \times \frac{4}{7} = \frac{\overset{4}{\cancel{28}} \times 4}{1 \times \underset{1}{\cancel{7}}} = \frac{4 \times 4}{1 \times 1} = \frac{16}{1}$

(iii) $36 \div 6\frac{2}{3} = 36 \div \frac{20}{3} = \frac{36}{1} \div \frac{20}{3} = \frac{\overset{9}{\cancel{36}}}{1} \times \frac{3}{\underset{5}{\cancel{20}}} = \frac{27}{5} = 5\frac{2}{5}$

Example 2 Simplify:

(i) $\frac{4}{9} \div \frac{2}{3}$ (ii) $3\frac{3}{7} \div \frac{8}{21}$ (iii) $15\frac{3}{7} \div 1\frac{23}{49}$

Solution We have,

(i) $\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{\overset{2}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{2}}} = \frac{2 \times 1}{3 \times 1} = \frac{2}{3}$

(ii) $3\frac{3}{7} \div \frac{8}{21} = \frac{24}{7} \div \frac{8}{21} = \frac{24}{7} \times \frac{21}{8} = \frac{\overset{3}{\cancel{24}} \times \overset{3}{\cancel{21}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{8}}} = \frac{3 \times 3}{1 \times 1} = 9$

(iii) $15\frac{3}{7} \div 1\frac{23}{49} = \frac{108}{7} \div \frac{72}{49} = \frac{108}{7} \times \frac{49}{72} = \frac{\overset{3}{\cancel{108}} \times \overset{7}{\cancel{49}}}{\underset{1}{\cancel{7}} \times \underset{2}{\cancel{72}}} = \frac{3 \times 7}{1 \times 2} = \frac{21}{2}$

Example 3 Simplify: $\left(\frac{16}{5} \div \frac{8}{20}\right) + \left(\frac{15}{5} \div \frac{3}{35}\right)$

Solution We have,

$$\left(\frac{16}{5}\div\frac{8}{20}\right)+\left(\frac{15}{5}\div\frac{3}{35}\right)=\left(\frac{16}{5}\times\frac{20}{8}\right)+\left(\frac{15}{5}\times\frac{35}{3}\right)$$

$$=\frac{\overset{2}{\cancel{16}}\times\overset{4}{\cancel{20}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{8}}}+\frac{\overset{5}{\cancel{15}}\times\overset{7}{\cancel{35}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{3}}}=\frac{8}{1}+\frac{35}{1}=\frac{43}{1}=43$$

Example 4 The cost of $5\frac{2}{5}$ kg of sugar is ₹ $101\frac{1}{4}$, find its cost per kg.

Solution We have,

$$\text{Cost of } 5\frac{2}{5} \text{ kg of sugar} = ₹\ 101\ \frac{1}{4}$$

$$\Rightarrow \text{Cost of } \frac{27}{5} \text{ kg of sugar} = ₹\ \frac{405}{4}$$

$$\Rightarrow \text{Cost of 1 kg of sugar}$$

$$=₹\left(\frac{405}{4}\div\frac{27}{5}\right)=₹\left(\frac{405}{4}\times\frac{5}{27}\right)$$

$$=₹\left(\frac{\overset{15}{\cancel{405}}\times 5}{4\times\underset{1}{\cancel{27}}}\right)=₹\ \frac{75}{4}=₹\ 18\frac{3}{4}$$

Hence, the cost of 1 kg of sugar is ₹ $18\ \frac{3}{4}$.

Example 5 The product of two numbers is $20\frac{5}{7}$. If one of the numbers is $6\ \frac{2}{3}$, find the other.

Solution We have,

$$\text{Product of two numbers} = 20\frac{5}{7}=\frac{145}{7}$$

$$\text{One of the numbers} = 6\frac{2}{3}=\frac{20}{3}$$

$\therefore$ The other number = (Product of the numbers $\div$ One of the numbers)

$$=\frac{145}{7}\div\frac{20}{3}$$

$$=\frac{145}{7}\times\frac{3}{20}=\frac{\overset{29}{\cancel{145}}\times 3}{7\times\underset{4}{\cancel{20}}}=\frac{29\times 3}{7\times 4}=\frac{87}{28}=3\frac{3}{28}$$

Hence, the other number is $3\ \frac{3}{28}$.

Example 6 By what number should $5\frac{5}{6}$ be multiplied to get $3\frac{1}{3}$?

Solution We have,

$$\text{Product of two numbers} = 3\frac{1}{3} = \frac{10}{3}$$

$$\text{One of the numbers} = 5\frac{5}{6} = \frac{35}{6}$$

$\therefore$ The other number = Product of the numbers $\div$ One of the numbers

$$\Rightarrow \text{The other number} = \frac{10}{3} \div \frac{35}{6} = \frac{10}{3} \times \frac{6}{35} = \frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \times \underset{7}{\cancel{35}}} = \frac{2 \times 2}{1 \times 7} = \frac{4}{7}$$

Hence, required number is $\frac{4}{7}$.

Example 7 If the cost of a notebook is ₹ $8\frac{3}{4}$, how many notebooks can be purchased for ₹ $131\frac{1}{4}$?

Solution We have,

$$\text{Cost of one note book} = ₹\, 8\frac{3}{4} = ₹\, \frac{35}{4}$$

$$\text{Total amount} = ₹\, 131\frac{1}{4} = ₹\, \frac{525}{4}$$

$\therefore$ $$\text{Number of notebooks} = \frac{\text{Total amount}}{\text{Cost of one notebook}}$$

$$= \frac{525}{4} \div \frac{35}{4} = \frac{525}{4} \times \frac{4}{35} = \frac{\overset{15}{\cancel{525}} \times \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{35}}} = 15$$

Example 8 A bucket contains $24\frac{3}{4}$ litres of water. How many $\frac{3}{4}$ litre jugs can be filled from the bucket to get it emptied?

Solution We have,

Volume of water in the bucket = $24\frac{3}{4}$ litres = $\frac{99}{4}$ litres

Capacity of jug = $\frac{3}{4}$ litre

$\therefore$ Number of jugs that can be filled to get the bucket emptied

$$= \frac{99}{4} \div \frac{3}{4} = \frac{99}{4} \times \frac{4}{3} = \frac{\overset{33}{\cancel{99}} \times \overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}}} = 33$$

Hence, 33 jugs of $\frac{3}{4}$ litre can be filled to get the bucket emptied.

EXERCISE 2.3

1. Find the reciprocal of each of the following fractions and classify them as proper, improper and whole numbers:

(i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$ (v) $\frac{12}{7}$ (vi) $\frac{1}{8}$

2. Divide:

   (i) $\frac{3}{8}$ by $\frac{5}{9}$ (ii) $3\frac{1}{4}$ by $\frac{2}{3}$ (iii) $\frac{7}{8}$ by $4\frac{1}{2}$ (iv) $6\frac{1}{4}$ by $2\frac{3}{5}$

3. Divide:

   (i) $\frac{3}{8}$ by 4 (ii) $\frac{9}{16}$ by 6 (iii) 9 by $\frac{3}{16}$ (iv) 10 by $\frac{100}{3}$

4. Simplify:

   (i) $\frac{3}{10} \div \frac{10}{3}$ (ii) $4\frac{3}{5} \div \frac{4}{5}$ (iii) $5\frac{4}{7} \div 1\frac{3}{10}$ (iv) $4 \div 2\frac{2}{5}$

5. A wire of length $12\frac{1}{2}$ m is cut into 10 pieces of equal length. Find the length of each piece.

6. The length of a rectangular plot of area $65\frac{1}{3}$ m^2 is $12\frac{1}{4}$ m. What is the width of the plot?

7. By what number should $6\frac{2}{9}$ be multiplied to get $4\frac{4}{9}$?

8. The product of two numbers is $25\frac{5}{6}$. If one of the numbers is $6\frac{2}{3}$, find the other.

9. The cost of $6\frac{1}{4}$ kg of apples is ₹ 400. At what rate per kg are the apples being sold?

10. By selling oranges at the rate of ₹ $5\frac{1}{4}$ per orange, a fruit-seller gets ₹ 630. How many dozens of oranges does he sell?

11. In mid-day meal scheme $\frac{3}{10}$ litre of milk is given to each student of a primary school. If 30 litres of milk is distributed every day in the school, how many students are there in the school?

12. In a charity show ₹ 6496 were collected by selling some tickets. If the price of each ticket was ₹ $50\frac{3}{4}$, how many tickets were sold?

ANSWERS

1. (i) $\frac{7}{3}$, improper (ii) $\frac{8}{5}$, improper (iii) $\frac{7}{9}$, proper (iv) $\frac{5}{6}$, proper
   (v) $\frac{7}{12}$, proper (vi) 8, whole number

2. (i) $\frac{27}{40}$ (ii) $4\frac{7}{8}$ (iii) $\frac{7}{36}$ (iv) $2\frac{21}{52}$

3. (i) $\frac{3}{32}$ (ii) $\frac{3}{32}$ (iii) 48 (iv) $\frac{3}{10}$

4. (i) $\frac{9}{100}$ (ii) $5\frac{3}{4}$ (iii) $4\frac{2}{7}$ (iv) $1\frac{2}{3}$

5. $1\frac{1}{4}$ m 6. $5\frac{1}{3}$ m 7. $\frac{5}{7}$ 8. $3\frac{7}{8}$ 9. ₹ 64 10. 10 dozens
11. 100 students 12. 128

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. If a fraction $\frac{a}{b}$ is a lowest terms, then HCF of a and b is

   (a) a (b) b (c) 1 (d) ab

2. The fraction $\frac{84}{98}$ in its lowest terms is

   (a) $\frac{42}{49}$ (b) $\frac{12}{14}$ (c) $\frac{6}{7}$ (d) $\frac{3}{7}$

3. Which of the following is a vulgar fraction?

   (a) $\frac{7}{10}$ (b) $\frac{13}{1000}$ (c) $2\frac{9}{10}$ (d) $\frac{7}{9}$

4. Which of the following fraction is an irreducible (or in its lowest terms)?

   (a) $\frac{91}{104}$ (b) $\frac{105}{112}$ (c) $\frac{51}{85}$ (d) $\frac{43}{83}$

5. Which of the following is a proper fraction?

   (a) $\frac{13}{17}$ (b) $\frac{17}{13}$ (c) $\frac{12}{5}$ (d) $1\frac{3}{4}$

6. The reciprocal of the fraction $2\frac{3}{5}$ is

   (a) $2\frac{5}{3}$ (b) $\frac{13}{5}$ (c) $\frac{5}{13}$ (d) $2\frac{2}{5}$

7. $4\frac{1}{3}-2\frac{1}{3}=$

   (a) $2\frac{1}{3}$ (b) 2 (c) $3\frac{1}{3}$ (d) $\frac{1}{2}$

8. $2\frac{3}{5}\div\frac{5}{7}=$

   (a) $\frac{13}{7}$ (b) $\frac{13}{25}$ (c) $\frac{91}{25}$ (d) $\frac{25}{91}$

9. By what number should $1\frac{3}{4}$ be divided to get $2\frac{1}{2}$?

   (a) $\frac{3}{7}$ (b) $1\frac{2}{5}$ (c) $\frac{7}{10}$ (d) $1\frac{3}{7}$

10. By what number $4\frac{3}{5}$ be multiplied to get $2\frac{3}{7}$?

    (a) $\frac{391}{35}$ (b) $\frac{85}{91}$ (c) $\frac{91}{85}$ (d) None of these

11. $\left(5\frac{1}{4}-3\frac{1}{3}\right)=$

(a) $\frac{12}{23}$ (b) 2 (c) $1\frac{11}{12}$ (d) $\frac{11}{12}$

12. The fraction equivalent to $1\frac{2}{3}$ is

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{10}{6}$ (d) $\frac{6}{10}$

13. By what number $9\frac{4}{5}$ be multiplied to get 42?

(a) $\frac{30}{7}$ (b) $\frac{7}{30}$ (c) $4\frac{1}{7}$ (d) $4\frac{3}{7}$

14. Which of the following statements is true?

(a) $\frac{7}{12}<\frac{4}{21}$ (b) $\frac{7}{12}=\frac{4}{21}$ (c) $\frac{7}{12}>\frac{4}{21}$ (d) None of these

15. Which one of the following is the correct statement?

(a) $\frac{3}{4}<\frac{2}{3}<\frac{12}{15}$ (b) $\frac{2}{3}<\frac{3}{4}<\frac{12}{15}$ (c) $\frac{2}{3}<\frac{12}{15}<\frac{3}{4}$ (d) $\frac{12}{15}<\frac{2}{3}<\frac{3}{4}$

16. Which of the following fractions lies between $\frac{2}{3}$ and $\frac{5}{7}$?

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) None of these

17. Which one of the following is true?

(a) $\frac{1}{2}<\frac{9}{13}<\frac{3}{4}<\frac{12}{17}$ (b) $\frac{3}{4}<\frac{9}{13}<\frac{1}{2}<\frac{12}{17}$

(c) $\frac{1}{2}<\frac{3}{4}<\frac{9}{13}<\frac{12}{17}$ (d) $\frac{1}{2}<\frac{9}{13}<\frac{12}{17}<\frac{3}{4}$

18. The smallest of the fractions $\frac{2}{3},\frac{4}{7},\frac{8}{11}$ and $\frac{5}{9}$ is

(a) $\frac{2}{3}$ (b) $\frac{4}{7}$ (c) $\frac{8}{11}$ (d) $\frac{5}{9}$

19. $9\times\left(-\frac{1}{3}\right)\times(-3)\times\left(-\frac{1}{9}\right)=$

(a) 1 (b) -1 (c) -3 (d) 3

20. Which of the following is correct?

(a) $\frac{2}{3}<\frac{3}{5}<\frac{11}{15}$ (b) $\frac{3}{5}<\frac{2}{3}<\frac{11}{15}$ (c) $\frac{11}{15}<\frac{3}{5}<\frac{2}{3}$ (d) $\frac{3}{5}<\frac{11}{15}<\frac{2}{3}$

21. Which is the smallest of the following fractions?

(a) $\frac{4}{9}$ (b) $\frac{2}{5}$ (c) $\frac{3}{7}$ (d) $\frac{1}{4}$

22. The difference between the greatest and the least fractions out of $\frac{6}{7}, \frac{7}{8}, \frac{8}{9}$ and $\frac{9}{10}$ is

(a) $\frac{3}{70}$ (b) $\frac{1}{56}$ (c) $\frac{1}{40}$ (d) $\frac{1}{72}$

23. Which of the following fractions is greater than $\frac{3}{4}$ and less than $\frac{5}{6}$?

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{9}{10}$

24. Which of the following fractions is more than one-third?

(a) $\frac{23}{70}$ (b) $\frac{205}{819}$ (c) $\frac{26}{75}$ (d) $\frac{118}{335}$

ANSWERS

1. (c)	2. (c)	3. (d)	4. (d)	5. (a)	6. (c)	7. (b)
8. (c)	9. (c)	10. (d)	11. (c)	12. (c)	13. (a)	14. (c)
15. (b)	16. (d)	17. (d)	18. (d)	19. (b)	20. (b)	21. (d)
22. (a)	23. (c)	24. (c)				

THINGS TO REMEMBER

1. A fraction is a number representing a part of a whole.
2. A fraction can be expressed in the form $\frac{a}{b}$, where a, b are whole numbers and $b \neq 0$.
3. In a fraction $\frac{a}{b}$, we call 'a' as numerator and 'b' as denominator.
4. A fraction whose numerator is less than the denominator is called a proper fraction.
5. A fraction whose numerator is more than or equal to the denominator is called an improper fraction.
6. A combination of a whole number and a proper fraction is called a mixed fraction.
7. To get a fraction equivalent to a given fraction, we multiply (or divide) its numerator and denominator by the same non-zero number.
8. Fractions having the same denominators are called like fractions. Otherwise, they are called unlike fractions.
9. A fraction is said to be in its lowest terms if its numerator and denominator have no common factor other than 1.
10. To compare fractions, we use the following steps:
    - Step I Find the LCM of the denomiantors of the given fractions.
    - Step II Convert each fraction to its equivalent fraction with denominator equal to the LCM obtained in step I.
    - Step III Arrange the fractions in ascending or descending order by arranging numerators in ascending or descending order.
11. To convert unlike fractions into like fractions, we use the following steps:
    - Step I Find the LCM of the denomiantors of the given fractions.
    - Step II Convert each of the given fractions into an equivalent fraction having denominator equal to the LCM obtained in step I.
12. To add (or subtract) fractions, we may use the following steps:
    - Step I Obtain the fractions and their denominators.
    - Step II Find the LCM of the denominators.
    - Step III Convert each fraction into an equivalent fraction having its denominator equal to the LCM obtained in step II.
    - Step IV Add (or subtract) like fractions obtained in Step III.
13. Product of two fraction $= \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$
14. Two fractions are said to be reciprocal of each other, if their product is 1. The reciprocal of a non-zero fraction $\frac{a}{b}$ is equal to $\frac{b}{a}$.
15. The division of a fraction $\frac{a}{b}$ by a non-zero fraction $\frac{c}{d}$ is the product of $\frac{a}{b}$ with the reciprocal of $\frac{c}{d}$.