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1

INTEGERS

1.1 INTRODUCTION

In class VI, we have learnt that the numbers

$$\ldots\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\ldots$$

are called integers. The numbers 1, 2, 3, 4, 5, 6, ..., i.e. natural numbers, are called positive integers and the numbers $-1, -2, -3, -4, -5, -6,$ ...... are called negative integers. The number 0 is simply an integer. It is neither positive nor negative. We have learnt about addition and subtraction of integers in class VI. In this chapter, we will study about multiplication and division of integers. We will also learn about various properties of these operations on integers.

1.2 MULTIPLICATION OF INTEGERS

In order to multiply integers, we use the following rules:

Rule 1 *The product of two integers of opposite signs is equal to the additive inverse of the product of their absolute values.*

Thus, to find the product of a positive and a negative integer, we find the product of their absolute values and assign minus sign to the product.

For example, (i) $7 \times (-4) = -(7 \times 4) = -28$ (ii) $(-8) \times 5 = -(8 \times 5) = -40$

Rule 2 *The product of two integers with like signs is equal to the product of their absolute values.*

Thus, to find the product of two integers, both positive or both negative, we find the product of their absolute values.

For example, (i) $7 \times 12 = 84$ (ii) $(-8) \times (-13) = 8 \times 13 = 104$

1.2.1 PROPERTIES OF MULTIPLICATION

In class VI, we have learnt properties of multiplication of whole numbers. All properties of multiplication of whole numbers also hold for integers. The multiplication of integers possesses the following properties:

Property 1 *(Closure property): The product of two integers is always an integer.*

That is, for any two integers a and b, $a \times b$ is an integer.

Verification: We have,

(i) $4 \times 3 = 12$, which is an integer. (ii) $3 \times (-5) = -15$, which is an integer.

(iii) $(-7) \times (-6) = 42$, which is an integer.

Property 2 *(Commutativity): For any two integers a and b, we have*

$$a \times b = b \times a$$

That is, multiplication of integers is commutative.

Verification: We have,

(i) $7 \times (-6) = -(7 \times 6) = -42$ and $(-6) \times 7 = -(6 \times 7) = -42$

$\therefore\ 7 \times (--6) = (-6) \times 7$

(ii) $(-5)\times(-9) = 5\times 9 = 45$ and $(-9)\times(-5) = 9\times 5 = 45$

$\therefore\ (-5)\times(-9) = (-9)\times(-5)$.

Property 3 *(Associativity): The multiplication of integers is associative, i.e., for any three integers a, b, c, we have*

$$a\times(b\times c) = (a\times b)\times c$$

Verification: We have,

(i) $(-3)\times\{4\times(-7)\} = (-3)\times(-28) = 3\times 28 = 84$

and, $\{(-3)\times 4\}\times(-7) = (-12)\times(-7) = 12\times 7 = 84$

$\therefore\ (-3)\times\{4\times(-7)\} = \{(-3)\times 4\}\times(-7)$

(ii) $(-2)\times\{(-3)\times(-5)\} = (-2)\times 15 = -(2\times 15) = -30$

and, $\{(-2)\times(-3)\}\times(-5) = 6\times(-5) = -(6\times 5) = -30$

$\therefore\ (-2)\times\{(-3)\times(-5)\} = \{(-2)\times(-3)\}\times(-5)$

Property 4 *(Distributivity of multiplication over addition): The multiplication of integers is distributive over their addition. That is, for any three integers a, b, c, we have*

(i) $a\times(b+c) = a\times b + a\times c$ *(ii)* $(b+c)\times a = b\times a + c\times a$

Verification: We have,

(i) $(-3)\times\{(-5)+2\} = (-3)\times(-3) = 3\times 3 = 9$

and, $(-3)\times(-5) + (-3)\times 2 = (3\times 5) - (3\times 2) = 15 - 6 = 9$

$\therefore\ (-3)\times\{(-5)+2\} = (-3)\times(-5) + (-3)\times 2$.

(ii) $(-4)\times\{(-2)+(-3)\} = (-4)\times(-5) = 4\times 5 = 20$

and, $(-4)\times(-2) + (-4)\times(-3) = (4\times 2) + (4\times 3) = 8 + 12 = 20$

$\therefore\ (-4)\times\{(-2)+(-3)\} = (-4)\times(-2) + (-4)\times(-3)$.

Remark 1 *A direct consequence of the distributivity of multiplication over addition is*

$$a\times(b-c) = a\times b - a\times c$$

Property 5 *(Existence of multiplicative identity): For every integer a, we have*

$$a\times 1 = a = 1\times a$$

The integer 1 is called the multiplicative identity for integers.

Property 6 *(Property of zero): For any integer, we have*

$$a\times 0 = 0 = 0\times a$$

Property 7 *For any integer a, we have*

$$a\times(-1) = -a = (-1)\times a$$

Remark *(i) We know that –a is additive inverse or opposite of a. Thus, to find the opposite or additive inverse or negative of an integer, we multiply the integer by –1.*

(ii) Since multiplication of integers is associative. Therefore, for any three integers a, b, c, we have

$$(a\times b)\times c = a\times(b\times c)$$

In what follows, we will write $a\times b\times c$ *for the equal products* $(a\times b)\times c$ *and* $a\times(b\times c)$.

(iii) Since multiplication of integers is both commutative and associative.

Therefore, in a product of three or more integers even if we rearrange the integers, the product will not change.

(iv) When the number of negative integers in a product is odd, the product is negative.

(v) When the number of negative integers in a product is even, the product is positive.

Property 8 *If a, b, c are integers, such that* $a > b$, *then*

(i) $a \times c > b \times c$, *if c is positive* *(ii)* $a \times c < b \times c$, *if c is negative.*

Property 9 *(i)* $(-a_1)\times(-a_2)\times(-a_3)\times\ldots\times(-a_n) = (a_1 \times a_2 \times a_3 \times \ldots \times a_n)$, *if n is even*

(ii) $(-a_1)\times(-a_2)\times(-a_3)\times\ldots\times(-a_n) = -(a_1 \times a_2 \times a_3 \times \ldots \times a_n)$, *if n is odd*

(iii) $\underbrace{(-1)\times(-1)\times(-1)\times\ldots\times(-1)}_{n-times} = 1$, *if n is even*

(iv) $\underbrace{(-1)\times(-1)\times(-1)\times\ldots\times(-1)}_{n-times} = -1$, *if n is odd*

ILLUSTRATIVE EXAMPLES

Example 1 Find each of the following products:

(i) $(-115)\times 8$ (ii) $9\times(-3)\times(-6)$ (iii) $(-12)\times(-13)\times(-5)$

Solution (i) We have,

$(-115)\times 8 = -(115\times 8) = -920$

(ii) We have,

$9\times(-3)\times(-6)$

$= \{9\times(-3)\}\times(-6)$

$= -(9\times 3)\times(-6)$

$= -27\times(-6) = 27\times 6 = 162$

(iii) We have,

$(-12)\times(-13)\times(-5)$

$= \{(-12)\times(-13)\}\times(-5)$

$= (12\times 13)\times(-5)$

$= 156\times(-5) = -(156\times 5) = -780$

Example 2 Evaluate each of the following products:

(i) $(-1)\times(-2)\times(-3)\times(-4)\times(-5)$ (ii) $(-3)\times(-6)\times(-9)\times(-12)$

(iii) $(-1)\times(-1)\times(-1)\times\ldots$ 50 *times* (iv) $(-1)\times(-1)\times(-1)\times\ldots$ 151 *times*

Solution (i) Since the number of negative integers in the product is odd. Therefore, their product is negative.

Thus, we have

$(-1)\times(-2)\times(-3)\times(-4)\times(-5)$

$= -(1\times 2\times 3\times 4\times 5)$

$= -(2\times 3\times 4\times 5)$ $[\because 1\times 2 = 2]$

$= -(6\times 4\times 5)$ $[\because 2\times 3 = 6]$

$= -(24\times 5) = -120$ $[\because 6\times 4 = 24]$

(ii) Since the number of negative integers in the given product is even. Therefore, their product is positive. Thus, we have

$(-3)\times(-6)\times(-9)\times(-12)$

$= (3\times 6\times 9\times 12)$

$= (18 \times 9 \times 12)$ $[\because 3 \times 6 = 18]$

$= (162 \times 12) = 1944$ $[\because 18 \times 9 = 162]$

(iii) The number of integers in the given product is even.

$\therefore$ $(-1) \times (-1) \times (-1) \times \ldots 50 \text{ times} = (1 \times 1 \times 1 \times \ldots 50 \text{ times}) = 1$

(iv) The number of integers in the given product is odd.

$\therefore$ $(-1) \times (-1) \times (-1) \times \ldots 151 \text{ times} = -(1 \times 1 \times 1 \times \ldots 151 \text{ times}) = -1$

Example 3 Find the value of

(i) $15625 \times (-2) + (-15625) \times 98$ (ii) $18946 \times 99 - (-18946)$

(iii) $1569 \times 887 - 569 \times 887$

Solution (i) $15625 \times (-2) + (-15625) \times 98$

$= (-15625) \times 2 + (-15625) \times 98$ $[\because 15625 \times (-2) = -(15625 \times 2) = (-15625 \times 2)]$

$= (-15625) \times (2 + 98)$ [Using: $a \times b + a \times c = a \times (b + c)$]

$= (-15625) \times 100$

$= -(15625 \times 100) = -1562500$

(ii) $18946 \times 99 - (-18946)$

$= 18946 \times 99 + 18946$

$= 18946 \times 99 + 18946 \times 1$ $[\because 18946 = 18946 \times 1]$

$= 18946 \times (99 + 1)$ [Using: $a \times b + a \times c = a \times (b + c)$]

$= 18946 \times 100 = 1894600$

(iii) $1569 \times 887 - 569 \times 887$

$= (1569 - 569) \times 887$ $[\because b \times a - c \times a = (b - c) \times a]$

$= 1000 \times 887 = 887000$

EXERCISE 1.1

1. Determine each of the following products:

   (i) 12×7 (ii) $(-15) \times 8$ (iii) $(-25) \times (-9)$ (iv) $125 \times (-8)$

2. Find each of the following products:

   (i) $3 \times (-8) \times 5$ (ii) $9 \times (-3) \times (-6)$ (iii) $(-2) \times 36 \times (-5)$

   (iv) $(-2) \times (-4) \times (-6) \times (-8)$

3. Find the value of:

   (i) $1487 \times 327 + (-487) \times 327$ (ii) $28945 \times 99 - (-28945)$

4. Complete the following multiplication table:

Second number

First number

×	–4	–3	–2	–1	0	1	2	3	4
–4	16	12	8	4	0	-4	-8	-12	-16
–3	12	9	6	3	0	-3	-6	-9	-12
–2	8	6	4	2	0	-2	-4	-6	-8
–1	4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0	0
1	-4	-3	-2	-1	0	1	2	3	4
2	-8	-6	-4	-2	0	2	4	6	8
3	-12	-9	-6	-3	0	3	6	9	12
4	-16	-12	-8	-4	0	4	8	12	16

Is the multiplication table symmetrical about the diagonal joining the upper left corner to the lower right corner?

5. Determine the integer whose product with '–1' is

   (i) 58 (ii) 0 (iii) –225

6. What will be the sign of the product if we multiply together

   (i) 8 negative integers and 1 positive integer?

   (ii) 21 negative integers and 3 positive integers?

   (iii) 199 negative integers and 10 positive integers?

7. State which is greater:

   (i) $(8+9)\times 10$ and $8+9\times 10$ (ii) $(8-9)\times 10$ and $8-9\times 10$

   (iii) $\{(-2)-5\}\times(-6)$ and $(-2)-5\times(-6)$.

8. (i) If $a\times(-1)=-30$, is the integer a positive or negative?

   (ii) If $a\times(-1)=30$, is the integer a positive or negative?

9. Verify the following:

   (i) $19\times\{7+(-3)\}=19\times 7+19\times(-3)$

   (ii) $(-23)\{(-5)+(+19)\}=(-23)\times(-5)+(-23)\times(+19)$

10. Which of the following statements are true?

   (i) The product of a positive and a negative integer is negative.

   (ii) The product of three negative integers is a negative integer.

   (iii) Of the two integers, if one is negative, then their product must be positive.

   (iv) For all non-zero integers a and b, $a\times b$ is always greater than either a or b.

   (v) The product of a negative and a positive integer may be zero.

   (vi) There does not exist an integer b such that for $a>1$, $a\times b=b\times a=b$.

ANSWERS

1. (i) 84 (ii) –120 (iii) 225 (iv) –1000
2. (i) –120 (ii) 162 (iii) 360 (iv) 384
3. (i) 327000 (ii) 2894500

4. Yes

×	–4	–3	–2	–1	0	1	2	3	4
–4	16	12	8	4	0	–4	–8	–12	–16
–3	12	9	6	3	0	–3	–6	–9	–12
–2	8	6	4	2	0	–2	–4	–6	–8
–1	4	3	2	1	0	–1	–2	–3	–4
0	0	0	0	0	0	0	0	0	0
1	–4	–3	–2	–1	0	1	2	3	4
2	–8	–6	–4	–2	0	2	4	6	8
3	–12	–9	–6	–3	0	3	6	9	12
4	–16	–12	–8	–4	0	4	8	12	16

5. (i) –58 (ii) 0 (iii) 225

6. (i) positive (ii) negative (iii) negative

7. (i) $(8+9)\times 10$ (ii) $(8-9)\times 10$ (iii) $[(-2)-5]\times(-6)$

8. (i) positive (ii) negative 10. (i), (ii), (vi)

1.3 DIVISION OF INTEGERS

We know that division of whole numbers is an inverse process of multiplication. In this section, we shall extend the same idea to integers.

We know that dividing 20 by 5 means finding an integer which when multiplied with 5 gives us 20. Clearly, such an integer is 4. Therefore, we write

$$20 \div 5 = 4 \text{ or, } \frac{20}{5} = 4$$

Similarly, dividing 36 by –9 means, finding an integer which when multiplied with –9 gives 36. Obviously, such an integer is –4. Therefore, we write

$$36 \div (-9) = -4 \text{ or, } \frac{36}{-9} = -4$$

Dividing (–35) by (–7) means, what integer should be multiplied with (–7) to get (–35). Clearly, such an integer is 5. Therefore,

$$(-35) \div (-7) = 5 \text{ or, } \frac{-35}{-7} = 5$$

We have the following definitions:

Dividend The number to be divided is called dividend.

Divisor The number which divides is called the divisor.

Quotient The result of division is called the quotient.

It follows from the above discussion that when dividend is negative and divisor is negative, the quotient is positive. When the dividend is negative and divisor is positive, the quotient is negative.

Thus, we have the following rules for division of integers:

<u>Rule 1</u> *The quotient of two integers both positive or both negative is a positive integer equal to the quotient of the corresponding absolute values of the integers.*

Thus, for dividing two integers with like signs, we divide their values regardless of their sign and give plus sign to the quotient.

Rule 2 *The quotient of a positive and a negative integer is a negative integer and its absolute value is equal to the quotient of the corresponding absolute values of the integers.*

Thus, for dividing integers with unlike signs, we divide their values regardless of their sign and give minus sign to the quotient.

1.3.1 PROPERTIES OF DIVISION

Division of integers has the following properties:

(i) *If a and b are integers, then* $a \div b$ *is not necessarily an integer.*
For example, $15 \div 4$, $-14 \div 3$ are not integers.

(ii) *If a is an integer different from 0, then* $a \div a = 1$.

(iii) *For every integer a, we have* $a \div 1 = a$.

(iv) *If a is a non-zero integer, then* $0 \div a = 0$.

(v) *If a is an integer, then* $a \div 0$ *is not meaningful.*

(vi) *If a, b, c are non-zero integers, then* $(a \div b) \div c\ {}^{1}\ a \div (b \div c)$, *unless* $c = 1$.

(vii) *If a, b, c are integers, then*

(i) $a > b \Rightarrow a \div c > b \div c$, *if c is positive.*

(ii) $a > b \Rightarrow a \div c < b \div c$, *if c is negative.*

ILLUSTRATIVE EXAMPLES

Example 1 Divide:

(i) 84 by 7 (ii) –91 by 13 (iii) –98 by –14 (iv) 324 by –27

Solution (i) We have,

$$84 \div 7 = \frac{|84|}{|7|} = \frac{84}{7} = 12$$

(ii) We have,

$$-91 \div 13 = -\frac{|-91|}{|13|} = -\frac{91}{13} = -7$$

(iii) We have,

$$-98 \div (-14) = +\frac{|-98|}{|-14|} = \frac{98}{14} = 7$$

(iv) We have,

$$324 \div (-27) = -\frac{|324|}{|-27|} = -\frac{324}{27} = -12$$

Example 2 Find the quotient in each of the following:

(i) $(-1728) \div 12$ (ii) $(-15625) \div (-125)$ (iii) $30000 \div (-100)$

Solution (i) We have,

$$(-1728) \div 12 = -\frac{|-1728|}{|12|} = -\frac{1728}{12} = -144$$

(ii) We have,

$$(-15625) \div (-125) = \frac{|-15625|}{|-125|} = \frac{15625}{125} = 125$$

(iii) We have,

$$30000 \div (-100) = -\frac{|30000|}{|-100|} = -\frac{30000}{100} = -300$$

Example 3 Find the value of

(i) $[32+2\times17+(-6)]\div15$ (ii) $||-17|+17|\div||-25|-42|$

Solution (i) We have,

$[32+2\times17+(-6)]\div15$

$$= [32+34+(-6)]\div15 = (66-6)\div15 = 60\div15 = \frac{60}{15} = 4$$

(ii) We have,

$||-17|+17|\div||-25|-42|$

$$= |17+17|\div|25-42| = |34|\div|-17| = 34\div17 = \frac{34}{17} = 2$$

Example 4 Simplify: $\{36\div(-9)\}\div\{(-24)\div6\}$

Solution We have,

$\{36\div(-9)\}\div\{(-24)\div6\}$

$$= \left\{-\frac{|36|}{|-9|}\right\}\div\left\{-\frac{|24|}{|6|}\right\} = \left\{-\frac{36}{9}\right\}\div\left\{-\frac{24}{6}\right\} = (-4)\div(-4) = \frac{|-4|}{|-4|} = \frac{4}{4} = 1$$

EXERCISE 1.2

1. Divide:
   - (i) 102 by 17
   - (ii) −85 by 5
   - (iii) −161 by −23
   - (iv) 76 by −19
   - (v) 17654 by −17654
   - (vi) (−729) by (−27)
   - (vii) 21590 by −10
   - (viii) 0 by −135
2. Fill in the blanks:
   - (i) $296\div\ldots=-148$
   - (ii) $-88\div\ldots=11$
   - (iii) $84\div\ldots=12$
   - (iv) $\ldots\ldots\div-5=25$
   - (v) $\ldots\ldots\div156=-2$
   - (vi) $\ldots\ldots\div567=-1$
3. Which of the following statements are true?
   - (i) $0\div4=0$
   - (ii) $0\div(-7)=0$
   - (iii) $-15\div0=0$
   - (iv) $0\div0=0$
   - (v) $(-8)\div(-1)=-8$
   - (vi) $-8\div(-2)=4$

ANSWERS

1. (i) 6 (ii) −17 (iii) 7 (iv) −4 (v) −1 (vi) 27 (vii) −2159 (viii) 0
2. (i) −2 (ii) −8 (iii) 7 (iv) −125 (v) −312 (vi) −567
3. (i), (ii), (vi)

1.4 OPERATOR PRECEDENCE

In simplifying mathematical expressions consisting of the same type of operation, we perform one operation at a time generally starting from the left towards the right. If an expression has more than one fundamental operations, you cannot perform operations in the order they appear. Some operations have to be performed before the others. That is, each operation has its own precedence. Generally, the order in which we perform operations sequentially from left to right is: division, multiplication, addition, subtraction. This order is expressed in short as 'DMAS' where 'D' stands for division, 'M' for multiplication, 'A' for addition and, 'S' for subtraction.

We first do the divisions and multiplications starting from the left towards the right and then perform additions, subtractions left to right.

The following examples will illustrate the precedence of operations of addition, subtraction, multiplication and division.

ILLUSTRATIVE EXAMPLES

Example 1 Simplify: $24-4 \div 2\times3$

Solution We have,

$24-4 \div 2\times3$

$= 24-2\times3$ [Performing division $-4 \div 2 = -2$]

$= 24-6$ [Performing multiplication $2 \times 3 = 6$]

$= 18.$ [Performing subtraction]

Example 2 Simplify: $(-2\,0)+(-8) \div (-2)\times3$

Solution We have,

$(-20)+(-8) \div (-2)\times3$

$= (-20)+4\times3$

$= (-20)+12$ [Performing multiplication]

$= -8.$ [Performing subtraction]

Example 3 Simplify: $(-5)-(-48)\div(-16)+(-2)\times6$

Solution We have,

$(-5)-(-48) \div (-16)+(-2)\times6$

$= (-5)-3+(-2)\times6$ [Performing division]

$= (-5)-3+(-12)$ [Performing multiplication]

$= -5-3-12$

$= -20.$ [Performing addition]

EXERCISE 1.3

Find the value of

1. $36\div6+3$
2. $24+15\div3$
3. $120-20\div4$
4. $32-(3\times5)+4$
5. $3-(5-6\div3)$
6. $21-12\div3\times2$
7. $16+8\div4-2\times3$
8. $28-5\times6+2$
9. $(-20)\times(-1)+(-28)\div7$
10. $(-2)+(-8)\div(-4)$

11. $(-15)+4\div(5-3)$
12. $(-40)\times(-1)+(-28)\div 7$
13. $(-3)+(-8)\div(-4)-2\times(-2)$
14. $(-3)\times(-4)\div(-2)+(-1)$.

ANSWERS

1. 9	2. 29	3. 115	4. 21	5. 0	6. 13
7. 12	8. 0	9. 16	10. 0	11. −13	12. 36
13. 3	14. −7				

1.5 USE OF BRACKETS

In the previous section, we have learnt about the precedence of fundamental operations of addition, subtraction, multiplication and division. According to it, the order in which operations are to be performed is first division, then multiplication after which addition and finally subtraction. But sometimes in complex expressions, we require a set of operations to be performed prior to the others. For example, if we want an addition to be performed before a division or a multiplication, then we need to use brackets. A bracket indicates that the operations within it are to be performed before the operations outside it. For example, the expression 24 ÷ 3 × 4, as per DMAS rule, is simplified by first dividing 24 by 3 and then multiplying by 4 as shown below:

$$24\div 3\times 4=(24\div 3)\times 4=8\times 4=32.$$

However, if we wish to multiply 3 and 4 first and then divide 24 by the resulting number, we write the expression as $24\div(3\times 4)$ and simplify it as

$$24\div(3\times 4)=24\div 12=2.$$

In complex expressions, sometimes, it is necessary to have brackets within brackets. Since same type of brackets one within another can be confusing, so different types of brackets are used. Most commonly used brackets are:

Brackets symbol	Name
( )	Parentheses or common brackets
{ }	Braces or Curly brackets
[ ]	Brackets or square brackets or box brackets
—	Vinculum

The left part of each bracket symbol indicates the start of the bracket and the right part indicates the end of the bracket. In writing mathematical expressions consisting of more than one brackets, parenthesis is used in the innermost part followed by braces and these two are covered by square brackets.

REMOVAL OF BRACKETS In order to simplify expressions involving more than one brackets, we use the following steps:

STEP I *See whether the given expression contains a vinculum or not. If a vinculum is present, then perform operations under it. Otherwise go to next step.*

STEP II *See the innermost bracket and perform operations within it.*

STEP III *Remove the innermost bracket by using following rules:*

Rule 1: *If a bracket is preceded by a plus sign, remove it by writing its terms as they are.*

Rule 2: *If a bracket is preceded by a minus sign, change positive signs within it to negative and vice-versa.*

Rule 3: *If there is no sign between a number and a grouping symbol, then it means multiplication.*

Rule 4: *If there is a number before some brackets then we multiply the number inside the brackets with the number outside the brackets.*

STEP IV *See the next innermost bracket and perform operations within it. Remove the second innermost bracket by using the rules given in step III. Continue this process till all the brackets are removed.*

Let us now illustrate these steps by means of some examples.

ILLUSTRATIVE EXAMPLES

Example 1 Simplify: $27-\left[5+\{28-(29-7)\}\right]$

Solution We have,

$27-\left[5+\{28-(29-7)\}\right]$

$= 27-\left[5+\{28-22\}\right]$ [Removing the innermost brackets]

$= 27-[5+6]$ [Removing braces]

$= 27-11 \quad = 16.$

Example 2 Simplify: $48-\left[18-\left\{16-(5-\overline{4-1})\right\}\right]$

Solution We have,

$48-\left[18-\left\{16-(5-\overline{4-1})\right\}\right]$

$= 48-\left[18-\{16-(5-3)\}\right]$ [Removing vinculum]

$= 48-\left[18-\{16-2\}\right]$ [Removing parentheses]

$= 48-[18-14]$ [Removing braces]

$= 48-4 \quad = 44.$

Example 3 Simplify: $222-\left[\frac{1}{3}\left\{42+(56-\overline{8+9})\right\}+108\right]$

Solution We have,

$222-\left[\frac{1}{3}\left\{42+(56-\overline{8+9})\right\}+108\right]$

$= 222-\left[\frac{1}{3}\{42+(56-17)\}+108\right]$ [Removing vinculum]

$= 222-\left[\frac{1}{3}\{42+39\}+108\right]$ [Removing parentheses]

$= 222-\left[\frac{81}{3}+108\right]$ [Removing braces]

$= 222-[27+108] = 222-135 = 87$

Example 4 Simplify: $39-\left[23-\left\{29-(17-\overline{9-3})\right\}\right]$

Solution We have,

$39-\left[23-\left\{29-(17-\overline{9-3})\right\}\right]$

$= 39-\left[23-\{29-(17-6)\}\right]$ [Removing vinculum]

$= 39-\left[23-\{29-11\}\right]$ [Removing parentheses]

$= 39-\left[23-18\right]$ [Removing braces]

$= 39-5 \quad = 34$

Example 5 Simplify:

(i) $118-\left[121\div(11\times11)-(-4)-\{3-\overline{9-2}\}\right]$

Solution (i) We have,

$118-\left[121\div(11\times11)-(-4)-\{3-\overline{9-2}\}\right]$

$= 118-\left[121\div(11\times11)-(-4)-\{3-7\}\right]$

$=118\left[121\div121-(-4)-\{3-7\}\right]$

$= 118-\left[1-(-4)-(-4)\right]$ [Performing division]

$= 118-\left[1+4+4\right] = 118-9 = 109$

EXERCISE 1.4

Simplify each of the following:

1. $3-(5-6\div3)$
2. $-25+14\div(5-3)$
3. $25-\frac{1}{2}\{5+4-(3+2-\overline{1+3})\}$
4. $27-\left[38-\{46-(15-\overline{13-2})\}\right]$
5. $36-\left[18-\{14-(15-4\div2\times2)\}\right]$
6. $45-\left[38-\{60\div3-(6-9\div3)\div3\}\right]$
7. $23-\left[23-\{23-(23-\overline{23-23})\}\right]$
8. $2550-\left[510-\{270-(90-\overline{80+70})\}\right]$
9. $4+\frac{1}{5}\left[\{-10\times(25-\overline{13-3})\}\div(-5)\right]$
10. $22-\frac{1}{4}\{-5-(-48)\div(-16)\}$
11. $63-\left[(-3)\{-2-\overline{8-3}\}\right]\div\left[3\{5+(-2)(-1)\}\right]$
12. $\left[29-(-2)\{6-(7-3)\}\right]\div\left[3\times\{5+(-3)\times(-2)\}\right]$
13. Using brackets, write a mathematical expression for each of the following:
    (i) Nine multiplied by the sum of two and five.
    (ii) Twelve divided by the sum of one and three.
    (iii) Twenty divided by the difference of seven and two.
    (iv) Eight subtracted from the product of two and three.
    (v) Forty divided by one more than the sum of nine and ten.
    (vi) Two multiplied by one less than the difference of nineteen and six.

ANSWERS

1. 0 2. −18 3. 21 4. 31 5. 21 6. 26

7. 0 8. 2370 9. 10 10. 24 11. 62 12. 1

13. (i) $9(2+5)$ (ii) $12\div(1+3)$ (iii) $20\div(7-2)$

(iv) $2\times3-8$ (v) $40\div\{1+(9+10)\}$ (vi) $2\times\{(19-6)-1\}$

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. $(-1)\times(-1)\times(-1)\times(-1)\times$......500 times =

   (a) -1 (b) 1 (c) 500 (d) -500

2. $(-1)+(-1)+(-1)+(-1)+$......500 times =

   (a) 500 (b) 1 (c) -1 (d) -500

3. The additive inverse of -7 is

   (a) -7 (b) $-\frac{1}{7}$ (c) 7 (d) $\frac{1}{7}$

4. The modulus of an integer x is 9, then

   (a) $x=9$ only (b) $x=-9$ only (c) $x=\pm 9$ (d) None of these

5. By how much does 5 exceed -4?

   (a) 1 (b) -1 (c) 9 (d) -9

6. By how much less than -3 is -7?

   (a) 4 (b) -4 (c) 10 (d) -10

7. The sum of two integers is 24. If one of them is -19, then the other is

   (a) 43 (b) -43 (c) 5 (d) -5

8. What must be subtracted from -6 to obtain -14?

   (a) 8 (b) 20 (c) -20 (d) -8

9. What should be divided by 6 to get -18?

   (a) -3 (b) 3 (c) -108 (d) 108

10. Which of the following is correct?

    (a) $-12>-9$ (b) $-12<-9$ (c) $(-12)+9>0$ (d) $(-12)\times 9>0$

11. The sum of two integers is -8. If one of the integers is 12, then the other is

    (a) 20 (b) 4 (c) -4 (d) -20

12. On subtracting -14 from -18, we get

    (a) 4 (b) -4 (c) -32 (d) 32

13. $(-35)\times 2+(-35)\times 8=$

    (a) -350 (b) -70 (c) -280 (d) 350

14. If $x \div 29 = 0$, then x =

    (a) 29 (b) -29 (c) 0 (d) None of these

15. If $x = (-10) + (-10) + \ldots 15$ times and $y = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$, then $x - y =$

(a) 118 (b) -118 (c) -182 (d) 182

16. If $a = (-1) \times (-1) \times (-1) \ldots 100$ times and $b = (-1) \times (-1) \times (-1) \ldots 95$ times, then $a + b =$

(a) -1 (b) -2 (c) 0 (d) 1

17. $||3-12|-4| =$

(a) -5 (b) 5 (c) 7 (d) -7

18. If the difference of an integer a and (-9) is 5, then

(a) 4 (b) 5 (c) -4 (d) -9

19. The sum of two integers is 10. If one of them is negative, then other has to be

(a) negative (b) positive
(c) may be positive or negative (d) None of these

20. If $x = (-1) \times (-1) \times (-1) \times (-1) \times \ldots 25$ times, $y = (-3) \times (-3) \times (-3)$, then $xy =$

(a) -27 (b) 27 (c) 26 (d) -26

ANSWERS

1. (b)	2. (d)	3. (c)	4. (c)	5. (c)	6. (a)	7. (a)
8. (a)	9. (c)	10. (b)	11. (d)	12. (b)	13. (a)	14. (c)
15. (b)	16. (c)	17. (b)	18. (c)	19. (b)	20. (b)	

THINGS TO REMEMBER

1. *The numbers. . . . , –4, –3, –1, 0, 1, 2, 3, 4,. . . . etc. are integers.*
2. *1, 2, 3, 4, 5. . . . are positive integers and –1,–2, –3,. . . are negative integers.*
3. *0 is an integer which is neither positive nor negative.*
4. *On an integer number line, all numbers to the right of 0 are positive integers and all numbers to the left of 0 are negative integers.*
5. *0 is less than every positive integer and greater than every negative integer.*
6. *Every positive integer is greater than every negative integer.*
7. *Two integers that are at the same distance from 0, but on opposite sides of it are called opposite numbers.*
8. *The greater the number, the lesser is its opposite.*
9. *The sum of an integer and its opposite is zero.*
10. *The absolute value of an integer is the numerical value of the integer without regard to its sign. The absolute value of an integer a is denoted by $|a|$ and is given by*

$$|a| = \begin{cases} a, \text{ if } a \text{ is positive or } 0 \\ -a, \text{ if } a \text{ is negative} \end{cases}$$

11. *The sum of two integers of the same sign is an integer of the same sign whose absolute value is equal to the sum of the absolute values of the given integers.*
12. *The sum of two integers of opposite signs is an integer whose absolute value is the difference of the absolute values of addend and whose sign is the sign of the addend having greater absolute value.*
13. *To subtract an integer b from another integer a, we change the sign of b and add it to a.*
    Thus, $a - b = a + (-b)$
14. *All properties of operations on whole numbers are satisfied by these operations on integers.*
15. *If a and b are two integers, then $(a - b)$ is also an integer.*
16. *–a and a are negative or additive inverses of each other.*
17. *To find the product of two integers, we multiply their absolute values and give the result a plus sign if both the numbers have the same sign or a minus sign otherwise.*
18. *To find the quotient of one integer divided by another non-zero integer, we divide their absolute values and give the result a plus sign if both the numbers have the same sign or a minus sign otherwise.*
19. *All the properties applicable to whole numbers are applicable to integers in addition, the subtraction operation has the closure property.*
20. *Any integer when multiplied or divided by 1 gives itself and when multiplied or divided by –1 gives its opposite.*
21. *When expression has different types of operations, some operations have to be performed before the others. That is, each operation has its own precedence. The order in which operations are performed is division, multiplication, addition and finally subtraction (DMAS).*
22. *Brackets are used in an expression when we want a set of operations to be performed before the others.*
23. *While simplifying an expression containing brackets, the operations within the innermost set of brackets are performed first and then those brackets are removed followed by the ones immediately after them till all the brackets are removed.*
24. *While simplifying arithmetic expressions involving various brackets and operations, we use BODMAS rule.*