8

LINEAR EQUATIONS IN ONE VARIABLE

8.1 INTRODUCTION

In this chapter, we shall study the meaning of an equation and also that of a linear equation. We shall also discuss the meaning of the solution of a linear equation and two methods of obtaining it. One by trial-and-error and another by a systematic method involving addition, subtraction, multiplication or division of some non-zero number on both sides of the equation. In the end of the chapter, we shall study formulation and solution of equations for some real-life problems.

8.2 EQUATIONS

In the earlier chapters, we have come across statements of the following type:

(i) $9+5=14$ (ii) $2\times(3+7)=2\times3+2\times7$ (iii) $(4+7)\times3=4\times3+7\times3$, etc.

All these statements involve the 'equals' symbol ' = '. Such a statement involving the 'equals' symbol ' = ' is called a statement of equality or simply an equality.

Clearly, none of the above statements involves a literal (variable). Let us now consider the following statements:

(i) 3 added to x is 8.
(ii) 5 subtracted from a number y is 12.
(iii) 7 less than a number x is 4.
(iv) Five times a number p is 32.
(v) A number x divided by 4 gives 3.
(vi) x multiplied by itself is 5 more than it.
(vii) The sum of a number x and twice the number y is 15.
(viii) 4 less from thrice a number m is 14.
(ix) The sum of the number x and its square is 20.

We can re-write the above statements as follows:

(i) $x+3=8$
(ii) $y-5=12$
(iii) $x-7=4$
(iv) $5p=32$
(v) $\frac{x}{4}=3$
(vi) $x^2=5+x$
(vii) $x+2y=15$
(viii) $3m-4=14$
(ix) $x+x^2=20.$

We observe that each one of the above statements is a statement of equality, involving one or more literals (variables). Such statements of equality are called equations as defined below.

EQUATION *A statement of equality which involves one or more literals (variables) is called an equation.*

Clearly, each of the equalities (i) to (ix) is an equation.

Every equation has two sides, namely, the left hand side (written as L.H.S.) and the right hand side (written as R.H.S.)

In the equation $x + 3 = 8$, $x + 3$ is L.H.S. and 8 is R.H.S, whereas in the equation $\frac{x}{4} = 3$, $\frac{x}{4}$ is L.H.S. and 3 is R.H.S. In the equation $x^2 = 5 + x$, x^2 is L.H.S. and $5 + x$ is R.H.S. The literal numbers involved in an equation are called variables or unknowns. Usually the variables are denoted by letters from the later part of the English alphabet, e.g. x, y, z, v, u, w etc.

An equation may contain any number of variables. The equation $x^2 - x = 5$ has only one variable whereas in equation $2x - 3y = 5$ there are two variables x and y.

8.3 LINEAR EQUATIONS

In the previous section, we have studied that an equation may involve any number of variables and the experiments or indices of the variables may be one or more than one. The nomenclature of the equations depends on the highest power of the variable(s) involved.

LINEAR EQUATION *An equation in which the highest power of the variables involved is 1, is called a linear equation.*

For example, equations $3x - 7 = 5$, $\frac{x}{4} + 5 = 3$, $3x - 2y = 7$ and $\frac{x}{2} + \frac{y}{3} = 4$ are linear equations.

The equations $2x^2 + x = 1$, $y + 5 = y^2$ and $x^3 = 8$ are not linear equations, because the highest power of the variable in each equation is greater than one.

In this chapter, we shall study linear equations in one variable only.

ILLUSTRATION Write the L.H.S. and the R.H.S. of each of the following equations:

(i) $x - 3 = 5$ (ii) $3x = 15 - 2x$

(iii) $3x = 21$ (iv) $3x - 4y = 9 + x$

Solution

Equation	L.H.S.	R.H.S.
(i) $x - 3 = 5$	$x - 3$	5
(ii) $3x = 15 - 2x$	$3x$	$15 - 2x$
(iii) $3x = 21$	$3x$	21
(iv) $3x - 4y = 9 + x$	$3x - 4y$	$9 + x$

8.3.1 SOLUTION OF AN EQUATION

Consider the linear equation

$$x - 10 = -7 \quad \text{...(i)}$$

L.H.S. of (i) is $x - 10$ and its R.H.S. is -7.

Let us now evaluate the L.H.S. and R.H.S. for some values of the variables x.

x	L.H.S.	R.H.S.
1	$1-10=-9$	-7
2	$2-10=-8$	-7
3	$3-10=-7$	-7

From the above table, we observe that the L.H.S. equals the R.H.S. only when we substitute 3 for x. For all other values of x, the two sides are not equal. In other words, the equation is satisfied by $x = 3$. Such a value of the variable is called the solution of the equation as defined below.

SOLUTION *A number, which when substituted for the variable in an equation, makes L.H.S. = R.H.S, is said to satisfy the equation and is called a solution or a root of the equation.*

ILLUSTRATION 1 Verify that $x = 3$ is the solution of the equation $2x - 3 = 3$.

Solution Putting $x = 3$ on L.H.S., we have

L.H.S. $= 2 \times 3 - 3 = 6 - 3 = 3$

And, R.H.S. = 3

Thus, for $x = 3$, we have L.H.S. = R.H.S.

Hence, $x = 3$ is the solution of the given equation.

ILLUSTRATION 2 Verify that $y = 9$ is the solution of the equation $\frac{y}{3} + 5 = 8$.

Solution Putting $y = 9$ in L.H.S. of the given equation, we obtain

L.H.S. $= \frac{9}{3} + 5 = 3 + 5 = 8$

And, R.H.S. = 8.

Thus, for $y = 9$, we have L.H.S. = R.H.S.

Hence, $y = 9$ is the solution of the given equation.

8.4 SOLVING LINEAR EQUATIONS

In the previous section, we have studied the meaning of the solution or root of an equation. Note that solving an equation means determining its roots. You will study in higher classes that a linear equation has only one root. Thus, solving a linear equation means finding its root.

In this section, we shall study three methods of solving a linear equation:

(i) By trial-and-error method. (ii) Systematic method (iii) Transposition method.

Let us discuss these methods one by one.

8.4.1 TRIAL-AND-ERROR METHOD

In this method, we often make a guess of the root of the equation. We find the values of L.H.S. and R.H.S. of the given equation for different values of the variable. The value of the variable for which L.H.S. = R.H.S. is the root of the equation.

Following examples illustrate the above method.

ILLUSTRATIVE EXAMPLES

Example 1 Solve the following equations by the trial-and-error method:

(i) $x + 7 = 10$ (ii) $x - 15 = 20$ (iii) $5x = 30$

(iv) $\frac{x}{8} = 9$ (v) $3x + 4 = 5x - 4$ (vi) $\frac{x}{3} + 8 = 11$

Solution Let us evaluate the L.H.S. and R.H.S. of each of the given equations for some values of x and continue to give new values till the L.H.S. becomes equal to the R.H.S.

(i) The given equation is $x + 7 = 10$. We have, L.H.S. $= x + 7$ and R.H.S. $= 10$

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
1	$1 + 7 = 8$	10	No
2	$2 + 7 = 9$	10	No
3	$3 + 7 = 10$	10	Yes

Clearly, L.H.S. = R.H.S. for $x = 3$.

Hence, $x = 3$ is the solution of given equation.

(ii) The given equation is $x - 15 = 20$, that is, 15 subtracted from x gives 20. So, we substitute values greater than 20. We have L.H.S. $= x - 15$, R.H.S. $=$ 20.

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
25	$25 - 15 = 10$	20	No
30	$30 - 15 = 15$	20	No
34	$34 - 15 = 19$	20	No
35	$35 - 15 = 20$	20	Yes

Clearly, L.H.S. = R.H.S. for $x = 35$.

Hence, $x = 35$ is the solution of the given equation.

(iii) The given equation is $5x = 30$. We have L.H.S. $= 5x$ and R.H.S. $= 30$.

Now,

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
1	$5 \times 1 = 5$	30	No
2	$5 \times 2 = 10$	30	No
3	$5 \times 3 = 15$	30	No
4	$5 \times 4 = 20$	30	No
5	$5 \times 5 = 25$	30	No
6	$5 \times 6 = 30$	30	Yes

Clearly, L.H.S. = R.H.S for $x = 6$.

Hence, $x = 6$ is the solution of the given equation.

(iv) The given equation is $\frac{x}{8} = 9$ that is, a number divided by 8 gives 9. This means that the number is a multiple of 8.

We have, L.H.S. = $\frac{x}{8}$ and R.H.S. = 9.

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
32	$\frac{32}{8} = 4$	9	No
40	$\frac{40}{8} = 5$	9	No
48	$\frac{48}{8} = 6$	9	No
64	$\frac{64}{8} = 8$	9	No
72	$\frac{72}{8} = 9$	9	Yes

Clearly, L.H.S. = R.H.S. for $x = 72$.

Hence, $x = 72$ is the solution of the given equation.

(v) The given equation is $3x + 4 = 5x - 4$.

We have, L.H.S. = $3x + 4$ and R.H.S. = $5x - 4$.

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
1	$3 \times 1 + 4 = 7$	$5 \times 1 - 4 = 1$	No
2	$3 \times 2 + 4 = 10$	$5 \times 2 - 4 = 6$	No
3	$3 \times 3 + 4 = 13$	$5 \times 3 - 4 = 11$	No
4	$3 \times 4 + 4 = 16$	$5 \times 4 - 4 = 16$	Yes

Clearly, L.H.S. = R.H.S. for $x = 4$.

Hence, $x = 4$ is the solution of the given equation.

(vi) The given equation is $\frac{x}{3} + 8 = 11$.

We have, L.H.S. = $\frac{x}{3} + 8$ and R.H.S. = 11.

Since R.H.S. is a natural number. So, $\frac{x}{3}$ must be a natural number. Thus, we give values of x which are multiples of 3.

x	L.H.S.	R.H.S.	Is L.H.S. = R.H.S.?
3	$\frac{3}{3} + 8 = 9$	11	No
6	$\frac{6}{3} + 8 = 10$	11	No
9	$\frac{9}{3} + 8 = 11$	11	Yes

Thus, L.H.S. = R.H.S. for $x = 9$.

Hence, $x = 9$ is the solution of the given equation.

EXERCISE 8.1

1. Verify by substitution that:

   (i) $x = 4$ is the root of $3x - 5 = 7$ (ii) $x = 3$ is the root of $5 + 3x = 14$

   (iii) $x = 2$ is the root of $3x - 2 = 8x - 12$ (iv) $x = 4$ is the root of $\frac{3x}{2} = 6$

   (v) $y = 2$ is the root of $y - 3 = 2y - 5$ (vi) $x = 8$ is the root of $\frac{1}{2}x + 7 = 11$

2. Solve each of the following equations by trial-and-error method:

   (i) $x + 3 = 12$ (ii) $x - 7 = 10$ (iii) $4x = 28$ (iv) $\frac{x}{2} + 7 = 11$

   (v) $2x + 4 = 3x$ (vi) $\frac{x}{4} = 12$ (vii) $\frac{15}{x} = 3$ (viii) $\frac{x}{18} = 20$

ANSWERS

2. (i) 9 (ii) 17 (iii) 7 (iv) 8

   (v) 4 (vi) 48 (vii) 5 (viii) 360

8.5 SYSTEMATIC METHOD

We have learnt about the trial-and-error method of solving linear equations in one variable. As we have seen that this method is time consuming and is not always direct.

In fact, it is a crude method. In the following discussion we shall study a better method of solving linear equations.

An equation can be compared with a weighing balance. The two sides of an equation are two pans and the equality symbol '=' tells us that the two pans are in balance as shown in Fig. 1

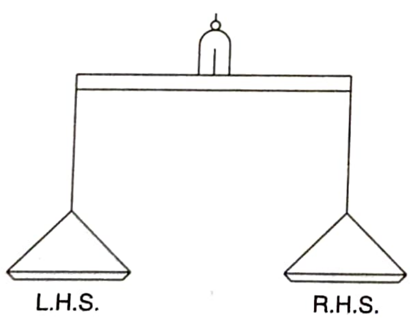

Fig.1

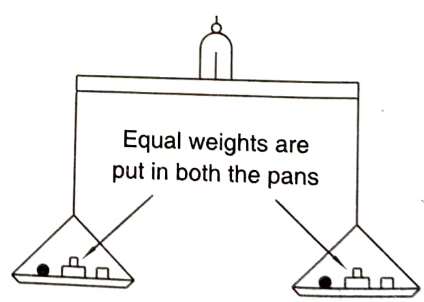

Fig. 2

All of us are familiar with the working of a balance. If equal weights are put in the two pans, we observe that the two pans remain in balance as shown in Fig. 2.

If we remove equal weights from both the pans, we find that the pans still remain in balance as shown in Fig. 3.

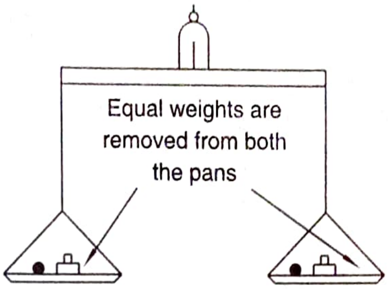

Fig. 3

Since multiplying a number by 4 (say) means adding it four times and dividing a number by 3 means subtracting the same number 3 times from it. Thus, the pans will still remain undisturbed if we multiply or divide the weights in two pans by the same quantity.

Similarly, in the case of an equation, we have the following rules:

Rule 1 *We can add the same number to both sides of the equation, i.e., if* $x + 5 = 7$, *then* $x + 5 + 2 = 7 + 2$.

Rule 2 *We can subtract the same number from both sides of the equation, i.e., if* $x + 5 = 7$, *then* $x + 5 - 2 = 7 - 2$.

Rule 3 *We can multiply both sides of the equation by the same non-zero number, i.e.,*

if $\frac{x}{3} = 4$, *then* $\frac{x}{3} \times 6 = 4 \times 6$. *Also,* $\frac{x}{3} \times 3 = 4 \times 3$.

Rule 4 *We can divide both sides of the equation by the same non-zero number, i.e.,*

if $3x = 10$, *then* $\frac{3x}{3} = \frac{10}{3}$. *Also,* $\frac{3x}{5} = \frac{10}{5}$.

Following examples will illustrate the applications of the above rules in solving linear equations.

ILLUSTRATIVE EXAMPLES

Type I EQUATIONS INVOLVING ADDITION

Example 1 Solve the equation $x - 3 = 5$ and check the result.

Solution We have, $x - 3 = 5$.

In order to solve this equation, we have to get x by itself on the L.H.S. To get x by itself on the L.H.S., we need to shift -3. This can be done by adding 3 to both sides of the given equation.

$x - 3 = 5$

$\Rightarrow \quad x - 3 + 3 = 5 + 3$ [Adding 3 to both sides]

$\Rightarrow \quad x + 0 = 8$ $[\because -3 + 3 = 0 \text{ and } 5 + 3 = 8]$

$\Rightarrow \quad x = 8$ $[\because x + 0 = x]$

So, $x = 8$ is the solution of the given equation.

Check Substituting $x = 8$ in the given equation, we get

L.H.S. $= 8 - 3 = 5$ and, R.H.S. $= 5$.

Thus, when $x = 8$, we have L.H.S. = R.H.S.

Example 2 Solve the equation $x - 7 = -2$ and check the result.

Solution We have, $x - 7 = -2$.

In order to solve this equation, we have to get x by itself on the L.H.S. To get x by itself on the L.H.S., we need to shift -7. This can be done by adding 7 to both sides of the given equation. Thus,

$x - 7 = -2$

$\Rightarrow x - 7 + 7 = -2 + 7$ [Adding 7 to both sides]

$\Rightarrow x + 0 = 5$ [$\because -7 + 7 = 0$ and $-2 + 7 = 5$]

$\Rightarrow x = 5$

Thus, $x = 5$ is the solution of the given equation.

Check Substituting $x = 5$ in the given equation, we get

L.H.S. $= 5 - 7 = -2$ and R.H.S. $= -2$

Thus, when $x = 5$, we have L.H.S. = R.H.S.

Type II EQUATIONS INVOLVING SUBTRACTION

Example 3 Solve the equation $x + 7 = 5$ and check the result.

Solution In order to solve this equation, we have to obtain x by itself on the L.H.S. To get x by itself on L.H.S., we need to shift 7. This can be done by subtracting 7 from both the sides of the given equation. Thus,

$x + 7 = 5$

$\Rightarrow x + 7 - 7 = 5 - 7$ [Subtracting 7 from both the sides]

$\Rightarrow x + 0 = -2$ [$\because 7 - 7 = 0$ and $5 - 7 = -2$]

$\Rightarrow x = -2$ [$\because x + 0 = x$]

Thus, $x = -2$ is the solution of the given equation.

Check Substituting $x = -2$ in the given equation, we get

L.H.S. $= -2 + 7 = 5$ and R.H.S. $= 5$.

Thus, when $x = -2$, we have L.H.S. = R.H.S.

Example 4 Solve the equation $x + 4 = -2$ and check the result.

Solution In order to solve this equation, we have to obtain x by itself on L.H.S. To get x by itself on L.H.S., we need to shift 4. This can be done by subtracting 4 from both sides of the given equation.

Thus, $x + 4 = -2$

$\Rightarrow x + 4 - 4 = -2 - 4$ [Subtracting 4 from both sides]

$\Rightarrow x + 0 = -6$ [$\because 4 - 4 = 0$ and $-2 - 4 = -6$]

$\Rightarrow x = -6$ [$\because x + 0 = x$]

Thus, $x = -6$ is the solution of the given equation.

Check Substituting $x = -6$ in the given equation, we get

L.H.S. $= -6 + 4 = -2$ and R.H.S. $= -2$

Thus, when $x = -6$, we have L.H.S. = L.H.S.

Type III EQUATIONS INVOLVING MULTIPLICATION

Example 5 Solve the equation $\frac{y}{12} = 48$ and check the result.

Solution In order to solve this equation, we have to get y by itself on L.H.S. To get y by itself on L.H.S., we have to remove 12 from L.H.S. This can be done by multiplying both sides of the equation by 12 thus, we have

$$\frac{y}{12} = 48$$

$$\Rightarrow \frac{y}{12} \times 12 = 48 \times 12 \qquad \text{[Multiplying both sides by 12]}$$

$$\Rightarrow y = 576 \qquad \left[\because \frac{y}{12} \times 12 = y \text{ and } 48 \times 12 = 576\right]$$

Thus, $y = 576$ is the solution of the given equation.

Check Putting $y = 576$ in the given equation, we get

L.H.S. $= \frac{576}{12} = 48$ and R.H.S. $= 48$.

Thus, for $y = 576$, we have L.H.S. = R.H.S.

Type IV EQUATIONS INVOLVING DIVISION

Example 6 Solve the equation $15x = 21$ and verify the result.

Solution In order to solve this equation, we have to get x by itself on the L.H.S. For this, 15 has to be removed from the L.H.S. This can be done by dividing both sides of the equation by 15. Thus,

$$15x = 21$$

$$\Rightarrow \frac{15x}{15} = \frac{21}{15} \qquad \text{[Dividing both sides by 15]}$$

$$\Rightarrow x = \frac{7}{5} \qquad \left[\because \frac{15x}{15} = x \text{ and } \frac{21}{15} = \frac{3 \times 7}{3 \times 5} = \frac{7}{5}\right]$$

Thus, $x = \frac{7}{5}$ is the solution of the given equation.

Check Putting $x = \frac{7}{5}$ in the given equation, we get

L.H.S. $= 15 \times \frac{7}{5} = 3 \times 7 = 21$ and R.H.S. $= 21$.

Thus, for $x = \frac{7}{5}$, we have L.H.S. = R.H.S.

Example 7 Solve the equation $\frac{2}{3x} = 18$ and check the result.

Solution We have,

$$\frac{2}{3x} = 18$$

$$\Rightarrow \frac{2}{3x} \times \frac{3}{2} = 18 \times \frac{3}{2} \qquad \left[\text{Multiplying both sides by } \frac{3}{2}\right]$$

$$\Rightarrow \frac{2}{3} \times \frac{3}{2} \times \frac{1}{x} = 27$$

$$\Rightarrow \frac{1}{x} = 27 \Rightarrow x = \frac{1}{27}$$

Thus, $x = 27$ is the solution of the given equation.

Check Putting $x = 27$ in the given equation, we get

L.H.S. $= \frac{2}{3} \times 27 = 18$ and R.H.S. $= 18$.

Thus, for $x = 27$, we have L.H.S. = R.H.S.

Type V EQUATIONS SOLVABLE BY USING MORE THAN ONE RULE

In the previous examples, we have discussed those equations which can be solved by using any one of the rules given on page 8.7. Now, we shall discuss some examples of linear equations which can be solved by using two or more of the rules given on page 8.7.

Example 8 Solve the equation $3x + 2 = 11$ and check the result.

Solution We have,

$$3x + 2 = 11$$

$$\Rightarrow 3x + 2 - 2 = 11 - 2 \quad \text{[Subtracting 2 from both sides]}$$

$$\Rightarrow 3x + 0 = 9$$

$$\Rightarrow 3x = 9 \quad [\because 3x + 0 = 3x]$$

$$\Rightarrow \frac{3x}{3} = \frac{9}{3} \quad \text{[Dividing both sides by 3]}$$

$$\Rightarrow x = 3$$

Thus, $x = 3$ is the solution of the given equation.

Check Putting $x = 3$ in the given equation, we get

L.H.S. $= 3 \times 3 + 2 = 9 + 2 = 11 =$ R.H.S.

Thus, for $x = 3$, we have L.H.S. = R.H.S.

Example 9 Solve the equation $2x - \frac{1}{2} = 3$ and check the result.

Solution We have,

$$2x - \frac{1}{2} = 3$$

$$\Rightarrow 2x - \frac{1}{2} + \frac{1}{2} = 3 + \frac{1}{2} \quad \left[\text{Adding } \frac{1}{2} \text{ to both sides}\right]$$

$$\Rightarrow 2x + 0 = \frac{7}{2}$$

$$\Rightarrow 2x = \frac{7}{2} \quad [\because 2x + 0 = 2x]$$

$$\Rightarrow \frac{2x}{2} = \frac{7}{2} \times \frac{1}{2} \quad \text{[Dividing both sides by 2]}$$

$$\Rightarrow x = \frac{7}{4}$$

Thus, $x = \frac{7}{4}$ is the solution of the given equation.

Check Putting $x = \frac{7}{4}$ in the given equation, we get

L.H.S. $= 2 \times \frac{7}{4} - \frac{1}{2} = \frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$ and R.H.S. $= 3$

Thus, for $x = \frac{7}{4}$, we have L.H.S. = R.H.S.

Example 10 Solve the equation $3(x + 6) = 21$ and check the result.

Solution We have,

$3(x + 6) = 21$

$\Rightarrow \quad 3x + 18 = 21$ [On expanding the bracket]

$\Rightarrow \quad 3x + 18 - 18 = 21 - 18$ [Subtracting 18 from both sides]

$\Rightarrow \quad 3x + 0 = 3$

$\Rightarrow \quad 3x = 3$ $[\because \ 3x + 0 = 3x]$

$\Rightarrow \quad \frac{3x}{3} = \frac{3}{3}$ [Dividing both sides by 3]

$\Rightarrow \quad x = 1$

Thus, $x = 1$ is the solution of the given equation.

Check Substituting $x = 1$ in the given equation, we get

L.H.S. $= 3(1 + 6) = 3 \times 7 = 21$ and R.H.S. $= 21$

Thus, for $x = 1$, we have L.H.S. = R.H.S.

Example 11 Solve the equation $16(3x - 5) - 10(4x - 8) = 40$ and verify the result.

Solution We have,

$16(3x - 5) - 10(4x - 8) = 40$

$\Rightarrow \quad 16 \times 3x - 16 \times 5 - 10 \times 4x + 10 \times 8 = 40$ [On expanding the brackets]

$\Rightarrow \quad 48x - 80 - 40x + 80 = 40$

$\Rightarrow \quad 48x - 40x - 80 + 80 = 40$

$\Rightarrow \quad (48 - 40)x + 0 = 40$

$\Rightarrow \quad 8x + 0 = 40$

$\Rightarrow \quad 8x = 40$

$\Rightarrow \quad \frac{8x}{8} = \frac{40}{8}$ [Dividing both sides by 8]

$\Rightarrow \quad x = 5$

Thus, $x = 5$ is the solution of the given equation.

Check Substituting $x = 5$ in the given equation, we get L.H.S.

$= 16(3 \times 5 - 5) - 10(4 \times 5 - 8)$

$= 16(15 - 5) - 10(20 - 8)$

$= 16 \times 10 - 10 \times 12$

$= 160 - 120 = 40$

and, R.H.S. $= 40$

Thus, for $x = 5$, we have L.H.S. = R.H.S.

EXERCISE 8.2

Solve each of the following equations and check your answers:

1. $x - 3 = 5$
2. $x + 9 = 13$
3. $x - \frac{3}{5} = \frac{7}{5}$
4. $3x = 0$
5. $\frac{x}{2} = 0$
6. $x - \frac{1}{3} = \frac{2}{3}$
7. $x + \frac{1}{2} = \frac{7}{2}$
8. $10 - y = 6$
9. $7 + 4y = -5$
10. $\frac{4}{5} - x = \frac{3}{5}$
11. $2y - \frac{1}{2} = -\frac{1}{3}$
12. $14 = \frac{7x}{10} - 8$
13. $3(x + 2) = 15$
14. $\frac{x}{4} = \frac{7}{8}$
15. $\frac{1}{3} - 2x = 0$
16. $3(x + 6) = 24$
17. $3(x + 2) - 2(x - 1) = 7$
18. $8(2x - 5) - 6(3x - 7) = 1$
19. $6(1 - 4x) + 7(2 + 5x) = 53$
20. $5(2 - 3x) - 17(2x - 5) = 16$
21. $\frac{x - 3}{5} - 2 = -1$
22. $5(x - 2) + 3(x + 1) = 25$

ANSWERS

1. 8 2. 4 3. 2 4. 0 5. 0 6. 1
7. 3 8. 4 9. –3 10. $\frac{1}{5}$ 11. $\frac{1}{12}$ 12. $\frac{220}{7}$
13. 3 14. $\frac{7}{2}$ 15. $\frac{1}{6}$ 16. 2 17. –1 18. $\frac{1}{2}$
19. 3 20. $\frac{79}{49}$ 21. 8 22. 4

8.6 TRANSPOSITION METHOD

By transposing a term of an equation, we simply mean changing its sign and carrying it to the other side of the equation. Any term of an equation may be taken to the other side with its sign changed without affecting the equality. This process is called transposition. When we carry a term of an equation from L.H.S. to R.H.S or R.H.S to L.H.S., the plus sign of the term changes into minus sign on the other side and vice-versa.

The transposition method involves the following steps:

STEP I *Obtain the linear equation.*

STEP II *Identify the unknown quantity (variable).*

STEP III *Simplify the L.H.S. and R.H.S by removing grouping symbols (if any).*

STEP IV *Transfer all terms containing the variable on the L.H.S. and constant terms on the R.H.S. of the equation. Note that the signs of the terms will change in carrying them from L.H.S. to R.H.S and vice-versa.*

STEP V *Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.*

STEP VI *Solve the equation obtained in step V by using the rules (given on page 8.7).*

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Solve: $3(x-1) = 2x - 11$ and check the result.

Solution We have,

$$3(x-1) = 2x - 11$$

$\Rightarrow \quad 3x - 3 = 2x - 11$ [Expanding the bracket on L.H.S.]

$\Rightarrow \quad 3x - 2x = 3 - 11$ [Transposing $2x$ to L.H.S. and -3 to R.H.S.]

$\Rightarrow \quad x = -8$

Thus, $x = -8$ is the solution of the given equation.

Check Substituting $x = -8$ in the given equation, we get

L.H.S. $= 3(-8-1) = 3 \times (-9) = -27$ and, R.H.S. $= 2 \times (-8) - 11 = -16 - 11 = -27$

Thus, for $x = -8$, we have, L.H.S. = R.H.S.

Example 2 Solve: $3(x+3) - 2(x-1) = 5(x-5)$ and check the result.

Solution We have,

$$3(x+3) - 2(x-1) = 5(x-5)$$

$\Rightarrow \quad 3x + 9 - 2x + 2 = 5x - 25$ [Expanding brackets on both side]

$\Rightarrow \quad 3x - 2x + 9 + 2 = 5x - 25$

$\Rightarrow \quad x + 11 = 5x - 25$ [Simplifying L.H.S. and R.H.S. separately]

$\Rightarrow \quad x - 5x = -25 - 11$ [Transposing $5x$ on L.H.S. and 11 on R.H.S.]

$\Rightarrow \quad -4x = -36$

$\Rightarrow \quad \dfrac{-4x}{-4} = \dfrac{-36}{-4}$ [Dividing both sides by -4]

$\Rightarrow \quad x = 9$

Thus, $x = 9$ is the solution of the given equation.

Check Substituting $x = 9$ in the given equation, we get

L.H.S. $= 3(9+3) - 2(9-1) = 3 \times 12 - 2 \times 8 = 36 - 16 = 20$

and, R.H.S. $= 5(9-5) = 5 \times 4 = 20$

Thus, for $x = 9$, we have L.H.S. = R.H.S.

Example 3 Solve: $\dfrac{x}{2} - 1 = \dfrac{x}{3} + 4$

Solution We have,

$$\frac{x}{2} - 1 = \frac{x}{3} + 4$$

$\Rightarrow \quad \dfrac{x}{2} - \dfrac{x}{3} = 1 + 4$ [Transposing $\dfrac{x}{3}$ on L.H.S. and -1 on R.H.S.]

$\Rightarrow \quad \left(\dfrac{1}{2} - \dfrac{1}{3}\right)x = 5$

$\Rightarrow \quad \left(\dfrac{3-2}{6}\right)x = 5$

$\Rightarrow \quad \frac{1}{6}x = 5$

$\Rightarrow \quad 6 \times \frac{1}{6}x = 6 \times 5$ [Multiplying both sides by 6]

$\Rightarrow \quad x = 30$

Thus, $x = 30$ is the solution of the given equation.

Aliter We have,

$$\frac{x}{2} - 1 = \frac{x}{3} + 4$$

Multiplying each term by 6, the L.C.M. of 2 and 3, the given equation becomes

$$6 \times \frac{x}{2} - 6 = 6 \times \frac{x}{3} + 6 \times 4$$

$\Rightarrow \quad 3x - 6 = 2x + 24$

$\Rightarrow \quad 3x - 2x = 6 + 24$ [Transposing $2x$ on L.H.S. and -6 on R.H.S.]

$\Rightarrow \quad x = 30$

Check Substituting $x = 30$ in the given equation, we get

L.H.S. $= \frac{30}{2} - 1 = 15 - 1 = 14$ and, R.H.S. $= \frac{30}{3} + 4 = 10 + 4 = 14$

Thus, L.H.S. = R.H.S when $x = 30$

Example 4 Solve: $\frac{2x-1}{3} + 1 = \frac{x-2}{3} + 2$ and check the result.

Solution We have,

$$\frac{2x-1}{3} + 1 = \frac{x-2}{3} + 2$$

$\Rightarrow \quad \frac{2x}{3} - \frac{1}{3} + 1 = \frac{x}{3} - \frac{2}{3} + 2$

$\Rightarrow \quad \frac{2x}{3} + \frac{3-1}{3} = \frac{x}{3} + \frac{6-2}{3}$

$\Rightarrow \quad \frac{2x}{3} + \frac{2}{3} = \frac{x}{3} + \frac{4}{3}$

$\Rightarrow \quad \frac{2x}{3} - \frac{x}{3} = \frac{4}{3} - \frac{2}{3}$ $\left[\text{Transposing } \frac{x}{3} \text{ on L.H.S. and } \frac{2}{3} \text{ on R.H.S.}\right]$

$\Rightarrow \quad \frac{x}{3} = \frac{2}{3}$

$\Rightarrow \quad \frac{x}{3} \times 3 = \frac{2}{3} \times 3$ [Multiplying both sides by 3]

$\Rightarrow \quad x = 2.$

Thus, $x = 2$ is the solution of the given equation.

Check Substituting $x = 2$ in the given equation, we get

$$\text{L.H.S.} = \frac{2x-1}{3} + 1 = \frac{2\times 2-1}{3} + 1 = \frac{3}{3} + 1 = 2$$

$$\text{R.H.S.} = \frac{2-2}{3} + 2 = \frac{0}{3} + 2 = 0 + 2 = 2$$

Thus, L.H.S. = R.H.S. when $x = 2$.

Example 5 *Solve:* $\dfrac{3x}{10} + \dfrac{2x}{5} = \dfrac{7x}{25} + \dfrac{29}{25}$

Solution We have,

$$\frac{3x}{10} + \frac{2x}{5} = \frac{7x}{25} + \frac{29}{25}$$

The denominators on two sides are 10, 5 and 25. Their L.C.M. is 50.
Multiplying both sides of the given equation by 50, we get

$$50\times\left(\frac{3x}{10} + \frac{2x}{5}\right) = 50\times\left(\frac{7x}{25} + \frac{29}{25}\right)$$

$$\Rightarrow \quad 50\times\frac{3x}{10} + 50\times\frac{2x}{5} = 50\times\frac{7x}{25} + 50\times\frac{29}{25}$$

$$\Rightarrow \quad 15x + 20x = 14x + 58$$

$$\Rightarrow \quad 35x = 14x + 58$$

$$\Rightarrow \quad 35x - 14x = 58 \qquad \text{[On transposing } 14x \text{ to L.H.S.]}$$

$$\Rightarrow \quad 21x = 58$$

$$\Rightarrow \quad \frac{21x}{21} = \frac{58}{21} \qquad \text{[Dividing both sides by 21]}$$

$$\Rightarrow \quad x = \frac{58}{21}$$

Thus, $x = \dfrac{58}{21}$ is the solution of the given equation.

Check Substituting $x = \dfrac{58}{21}$ in the given equation, we get

$$\text{L.H.S.} = \frac{3x}{10} + \frac{2x}{5}$$

$$= \frac{3}{10}\times\frac{58}{21} + \frac{2}{5}\times\frac{58}{21}$$

$$= \frac{29}{35} + \frac{116}{105} = \frac{29\times 3 + 116}{105} = \frac{87+116}{105} = \frac{203}{105} = \frac{29}{15}$$

and,

$$\text{R.H.S.} = \frac{7x}{25} + \frac{29}{25}$$

$$= \frac{7}{25} \times \frac{58}{21} + \frac{29}{25} = \frac{58}{75} + \frac{29}{25} = \frac{58 + 3 \times 29}{75} = \frac{145}{75} = \frac{29}{15}$$

Thus, for $x = \frac{58}{21}$, we have L.H.S. = R.H.S.

Example 6 Solve: $\frac{12}{7}(x-5) = 24 + 8x$

Solution We have, $\frac{12}{7}(x-5) = 24 + 8x$

Multiplying both sides by 7, we get

$$7 \times \frac{12}{7}(x-5) = 7 \times (24 + 8x)$$

$\Rightarrow$ $12(x-5) = 7(24 + 8x)$

$\Rightarrow$ $12x - 60 = 168 + 56x$

$\Rightarrow$ $12x - 56x = 168 + 60$ [Transposing $56x$ to L.H.S. and -60 to R.H.S]

$\Rightarrow$ $-44x = 228$

$\Rightarrow$ $\frac{-44x}{-44} = \frac{228}{-44}$ [Dividing both sides by -44]

$\Rightarrow$ $x = -\frac{57}{11}$

Thus, $x = -\frac{57}{11}$ is the solution of the given equation.

Check Substituting $x = -\frac{57}{11}$ in the given equation, we get

$$\text{L.H.S.} = \frac{12}{7}(x-5)$$

$$= \frac{12}{7} \times \left(\frac{-57}{11} - 5\right)$$

$$= \frac{12}{7} \times \left(\frac{-57 - 55}{11}\right) = \frac{12}{7} \times -\frac{112}{11} = 12 \times -\frac{16}{11} = -\frac{192}{11}$$

and,

$$\text{R.H.S.} = 24 + 8x$$

$$= 24 + 8 \times -\frac{57}{11} = 24 - \frac{456}{11} = \frac{24 \times 11 - 456}{11} = \frac{264 - 456}{11} = -\frac{192}{11}$$

Thus, L.H.S. = R.H.S. for $x = -\frac{57}{11}$

Example 7 Solve: $\dfrac{y-8}{3} = \dfrac{7-4y}{7}$

Solution We have, $\dfrac{y-8}{3} = \dfrac{7-4y}{7}$

The denominators on L.H.S. and R.H.S. are 3 and 7 respectively.

Multiplying both sides by the L.C.M. of 3 and 7, that is 21, we get

$$21\left(\frac{y-8}{3}\right) = 21\left(\frac{7-4y}{7}\right)$$

$\Rightarrow \quad 7(y-8) = 3(7-4y)$

$\Rightarrow \quad 7y - 56 = 21 - 12y$

$\Rightarrow \quad 7y + 12y = 21 + 56$ [Transposing $-12y$ to L.H.S. and -56 to R.H.S.]

$\Rightarrow \quad 19y = 77$

$\Rightarrow \quad y = \dfrac{77}{19}$

Thus, $y = \dfrac{77}{19}$ is the solution of the given equation.

Check Putting $y = \dfrac{77}{19}$ in the given equation, we get

$$\text{L.H.S.} = \frac{y-8}{3} = \frac{\frac{77}{19}-8}{3} = \frac{77-152}{19\times 3} = \frac{-75}{57} = \frac{-25}{19}$$

and, R.H.S. $= \dfrac{7-4y}{7}$

$$= \frac{7-4\times\frac{77}{19}}{7} = \frac{7-\frac{308}{19}}{7} = \frac{133-308}{7\times 19} = \frac{-175}{7\times 19} = \frac{-25}{19}$$

Thus, L.H.S. = R.H.S. for $y = \dfrac{77}{19}$

Example 8 Solve: $\dfrac{x-6}{4} - \dfrac{x-4}{6} = 1 - \dfrac{x}{10}$

Solution We have, $\dfrac{x-6}{4} - \dfrac{x-4}{6} = 1 - \dfrac{x}{10}$

Multiplying both sides by 60, the L.C.M. of 4, 6 and 10, we get

$$60\left(\frac{x-6}{4}\right) - 60\left(\frac{x-4}{6}\right) = 60\left(1-\frac{x}{10}\right)$$

$\Rightarrow \quad 15(x-6) - 10(x-4) = 60\left(1-\dfrac{x}{10}\right)$

$\Rightarrow \quad 15x - 90 - 10x + 40 = 60 - 6x$

$\Rightarrow \quad 15x - 10x - 90 + 40 = 60 - 6x$

$\Rightarrow \quad 5x - 50 = 60 - 6x$

$\Rightarrow \quad 5x + 6x = 60 + 50$ [Transposing $-6x$ to L.H.S. and -50 to R.H.S.

$\Rightarrow \quad 11x = 110$

$\Rightarrow \quad \frac{11x}{11} = \frac{110}{11}$ [Dividing both sides by 11

$\Rightarrow \quad x = 10$

Thus, $x = 10$ is the solution of the given equation.

Example 9 Solve: $\frac{3}{4}(7x-1)-\left(2x-\frac{1-x}{2}\right) = x+\frac{3}{2}$

$\frac{2m-(m-1)}{2}$ $3-\frac{(m-2)}{3}$

Solution We have,

$$\frac{3}{4}(7x-1)-\left(2x-\frac{1-x}{2}\right) = x+\frac{3}{2}$$

$$\Rightarrow \quad \frac{3}{4}(7x-1)-\left(2x-\frac{1-x}{2}\right) = x+\frac{3}{2}$$

Multiplying both sides by 4, the L.C.M. of 4 and 2, we get

$$3(7x-1)-8x+4\left(\frac{1-x}{2}\right) = 4\left(x+\frac{3}{2}\right)$$

$$\Rightarrow \quad 3(7x-1)-8x+2(1-x) = 4\left(x+\frac{3}{2}\right)$$

$\Rightarrow \quad 21x-3-8x+2-2x = 4x+6$ [On expanding brackets]

$\Rightarrow \quad 21x-8x-2x-3+2 = 4x+6$

$\Rightarrow \quad 11x-1 = 4x+6$

$\Rightarrow \quad 11x-4x = 6+1$ [On transposing -1 to R.H.S. and $4x$ to L.H.S.

$\Rightarrow \quad 7x = 7$

$\Rightarrow \quad \frac{7x}{7} = \frac{7}{7}$ [Dividing both sides by 7]

$\Rightarrow \quad x = 1$

Thus, $x = 1$ is the solution of the given equation.

Example 10 Solve: $0.3x + 0.4 = 0.28x + 1.16$

Solution We have,

$0.3x + 0.4 = 0.28x + 1.16$

$\Rightarrow \quad 0.3x - 0.28x = 1.16 - 0.4$ [Transposing $0.28x$ to LHS and 0.4 to R.H.S.

$\Rightarrow \quad (0.3 - 0.28)x = 1.16 - 0.4$

$\Rightarrow \quad 0.02x = 0.76$

$\Rightarrow \quad \frac{0.02x}{0.02} = \frac{0.76}{0.02}$ [Dividing both sides by 0.02]

$\Rightarrow \quad x = \frac{76}{2}$

$\Rightarrow \quad x = 38.$

Thus, $x = 38$ is the solution of the given equation.

EXERCISE 8.3

Solve each of the following equations. Also, verify the result in each case.

1. $6x + 5 = 2x + 17$
2. $2(5x - 3) - 3(2x - 1) = 9$
3. $\frac{x}{2} = \frac{x}{3} + 1$
4. $\frac{x}{2} + \frac{3}{2} = \frac{2x}{5} - 1$
5. $\frac{3}{4}(x - 1) = x - 3$
6. $3(x - 3) = 5(2x + 1)$
7. $3x - 2(2x - 5) = 2(x + 3) - 8$
8. $x - \frac{x}{4} - \frac{1}{2} = 3 + \frac{x}{4}$
9. $\frac{6x - 2}{9} + \frac{3x + 5}{18} = \frac{1}{3}$
10. $m - \frac{m - 1}{2} = 1 - \frac{m - 2}{3}$
11. $\frac{(5x - 1)}{3} - \frac{(2x - 2)}{3} = 1$
12. $0.6x + \frac{4}{5} = 0.28x + 1.16$
13. $0.5x + \frac{x}{3} = 0.25x + 7$

ANSWERS

1. 3	2. 3	3. 6	4. −25	5. 9	6. −2
7. 4	8. 7	9. $\frac{1}{3}$	10. $\frac{7}{5}$	11. $\frac{2}{3}$	12. $\frac{9}{8}$
13. 12					

8.7 APPLICATIONS OF LINEAR EQUATIONS TO PRACTICAL PROBLEMS

In this section, we shall study formulation and solution of some practical problems. These problems involve relations among unkown quantities (variables) and known quantities (numbers) and are often stated in words. That is why we often refer to these problems as word problems. A word problem is first translated in the form of an equation containing unknown quantities (variables) and known quantities (numbers or constants) and then we solve it by using any one of the methods discussed in the earlier section. The procedure to translate a word problem in the form of an equation is known as the formulation of the problem. Thus, the process of solving a word problem consists of two parts, namely, formulation and solution.

Following steps should be followed to solve a word problem:

STEP I *Read the problem carefully and note what is given and what is required.*

STEP II *Denote the unknown quantity by some letters, say x, y, z, etc.*

STEP III *Translate the statements of the problem into mathematical statements.*

STEP IV *Using the condition (s) given in the problem, form the equation.*

STEP V *Solve the equation for the unknown.*

STEP VI *Check whether the solution satisfies the equation.*

Following examples will illustrate these steps.

ILLUSTRATIVE EXAMPLES

Example 1 5 added to a number gives 9. Find the number.

Formulation: Let the required number be x. Then, 5 added to x is equal to $x + 5$

It is given that when 5 is added to a number, we get 9. Thus, we obtain the following equation:

$$x + 5 = 9$$

Solution We have,

$$x + 5 = 9$$

$\Rightarrow \quad x = 9 - 5$ [On transposing 5 to R.H.S.]

$\Rightarrow \quad x = 4$

Hence, the required number is 4.

Example 2 If 7 is subtracted from five times a number, the result is 63. Find the number.

Formulation: Let the required number be x Then,

Five times $x = 5x$

When 7 is subtracted from five times x, we get $5x - 7$.

It is given that when 7 is subtracted from five times x, the result is 63. So, we obtain the following equation:

$$5x - 7 = 63$$

Solution We have,

$$5x - 7 = 63$$

$\Rightarrow \quad 5x = 63 + 7$ [On transposing – 7 to R.H.S]

$\Rightarrow \quad 5x = 70$

$\Rightarrow \quad \dfrac{5x}{5} = \dfrac{70}{5}$ [Dividing both sides by 5]

$\Rightarrow \quad x = 14.$

Hence, the required number is 14.

Example 3 What is the number which when multipled by 20 gives the product 60?

Formulation: Let the required number be x. Then,

Product of x and $20 = 20\,x$

It is given that when x is multiplied by 20, the product is 60.

Thus, we obtain the following equation:

$$20x = 60$$

Solution We have,

$$20x = 60$$

$\Rightarrow \quad \dfrac{20x}{20} = \dfrac{60}{20}$ [Dividing both sides by 20]

$\Rightarrow \quad x = 3$

Hence, the required number is 3.

Example 4 Find the number which when divided by 9 gives 4.

Formulation: Let the required number be x. Then,

$$x \text{ divided by } 9 = \frac{x}{9}$$

It is given that x divided by 9 is 4. So, we obtain the following equation:

$$\frac{x}{9} = 4$$

Solution We have,

$$\frac{x}{9} = 4$$

$$\Rightarrow \quad \frac{x}{9} \times 9 = 4 \times 9 \qquad \text{[Multiplying both sides by 9]}$$

$$\Rightarrow \quad x = 36$$

Hence, the required number is 36.

Example 5 The sum of two consecutive numbers is 53. find the numbers.

Formulation: Let one number be x. Then, the next consecutive number is $x+1$. It is given that the sum of two consecutive numbers is 53. So, we obtain the following equation:

$$x + (x+1) = 53$$

Solution We have,

$$x + (x+1) = 53$$

$$\Rightarrow \quad 2x + 1 = 53$$

$$\Rightarrow \quad 2x = 53 - 1 \qquad \text{[On transposing 1 on R.H.S.]}$$

$$\Rightarrow \quad 2x = 52$$

$$\Rightarrow \quad \frac{2x}{2} = \frac{52}{2} \qquad \text{[Dividing both sides by 2]}$$

$$\Rightarrow \quad x = 26$$

$\therefore$ One number = 26

Another number $= 26 + 1 = 27$.

Example 6 The sum of two consecutive even numbers is 86. Find the numbers.

Formulation: Let one of the even numbers be x. Then, the next consecutive even number $= x + 2$.

Now,

Sum of two consecutive even numbers = 86

$$\Rightarrow \quad x + x + 2 = 86$$

$$\Rightarrow \quad 2x + 2 = 86$$

Solution We have,

$$2x + 2 = 86$$

$$\Rightarrow \quad 2x = 86 - 2 \qquad \text{[On transposing 2 on R.H.S.]}$$

$$\Rightarrow \quad 2x = 84$$

$$\Rightarrow \quad \frac{2x}{2} = \frac{84}{2} \qquad \text{[Dividing both sides by 2]}$$

$$\Rightarrow \quad x = 42.$$

Thus,

One of the even numbers = 42

The other consecutive even number $= x + 2 = 42 + 2 = 44$.

Example 7 The sum of two consecutive odd numbers is 68. Find the numbers.

Formulation: Let one of the odd numbers be x. Then,

The next consecutive odd numbers $= x + 2$.

Now,

Sum of two consecutive odd numbers = 68

$\Rightarrow \quad x + (x + 2) = 68$

$\Rightarrow \quad 2x + 2 = 68$

Solution We have,

$2x + 2 = 68$

$\Rightarrow \quad 2x = 68 - 2$ [On transposing 2 on R.H.S.]

$\Rightarrow \quad 2x = 66$

$\Rightarrow \quad \frac{2x}{2} = \frac{66}{2}$ [Dividing both sides by 2]

$\Rightarrow \quad x = 33.$

Thus, one odd number = 33.

Other odd number $= x + 2 = 33 + 2 = 35.$

Example 8 Find two numbers such that one of them exceeds the other by 9 and their sum is 81.

Formulation: Let the smaller number be x. Then,

Another number $= x + 9$

Now, sum of the numbers = 81

$\Rightarrow \quad x + x + 9 = 81$

$\Rightarrow \quad 2x + 9 = 81$

Solution We have, $2x + 9 = 81$

$\Rightarrow \quad 2x = 81 - 9$ [On transposing 9 on R.H.S.]

$\Rightarrow \quad 2x = 72$

$\Rightarrow \quad \frac{2x}{2} = \frac{72}{2}$

$\Rightarrow \quad x = 36$ [Dividing both sides by 2]

$\therefore$ One number = 36

Another number $= 36 + 9 = 45.$

Example 9 Find a number which when multiplied by 5 is increased by 80.

Formulation: Let the required number be x. Then x multiplied by $5 = 5x$.

It is given that when the number is multiplied by 5, the new number is 80 more than the number x. So, we obtain

$$5x = x + 80$$

Solution We have,

$5x = x + 80$

$\Rightarrow \quad 5x - x = 80$ [On transposing x on L.H.S.]

$\Rightarrow \quad 4x = 80$

$\Rightarrow \quad \frac{4x}{4} = \frac{80}{4}$ [Dividing both sides by 4]

$\Rightarrow \quad x = 20$

Thus, the required number is 20.

Example 10 The sum of ages of father and his son is 75 years. If the age of the son is 25 years, find the age of the father.

Formulation: Let the age of the father be x years.

The age of son = 25 years.

$\therefore$ Sum of the two ages $= (x + 25)$ years.

But the sum of the ages is given as 75 years.

$\therefore \quad x + 25 = 75$

Solution We have,

$x + 25 = 75$

$\Rightarrow \quad x = 75 - 25$ [On transposing 25 on R.H.S.]

$\Rightarrow \quad x = 50$

Thus, the age of the father is 50 years.

Example 11 Rahim's father is three times as old as Rahim. If sum of their ages is 56 years, find their ages.

Formulation: Let Rahim's age be x years. Then,

Rahim's father's age = $3x$ years.

$\therefore$ Sum of their ages $= x + 3x$.

But the sum of their ages is given as 56 years. Therefore, we have the following equation.

$$x + 3x = 56$$

Solution We have, $x + 3x = 56$

$\Rightarrow \quad 4x = 56$

$\Rightarrow \quad \frac{4x}{4} = \frac{56}{4}$ [Dividing both sides by 4]

$\Rightarrow \quad x = 14$

Thus, Rahim's age = 14 years

Rahim's father's age = $3x$ year $= 3 \times 14$ years = 42 years.

Example 12 Mona's father is thrice as old as Mona. After 12 years he will be just twice his daughter. Find their present ages.

Formulation: Let Mona's present age be x years. Then,

Her father's present age = $3x$ years.

Mona's age after 12 years $= (x + 12)$ years.

Mona's father's age after 12 years $= (3x + 12)$ years.

It is given that after 12 years Mona's father will be just twice his daughter. Therefore, we obtain the following equation:

$$3x + 12 = 2(x + 12)$$

Solution We have,

$$3x + 12 = 2(x + 12)$$

$\Rightarrow$ $3x + 12 = 2x + 24$ [Expanding bracket on R.H.S.]

$\Rightarrow$ $3x - 2x = 24 - 12$ [Transposing $2x$ on L.H.S. and 12 on R.H.S]

$\Rightarrow$ $x = 12.$

Thus, Mona's present age = 12 years.

Her father's present age $= (3 \times 12)$ years = 36 years.

Example 13 A sum of ₹ 8400 is made up of 50, 20, 10 and 5 rupee notes. The number of 10 rupee notes is five times the number of 5 rupee notes, four times the number of 20 rupee notes and ten times the number of 50 rupee notes. What is the number of notes in each denominator?

Formulation: Let the number of 10 rupee notes be x. Then.

Number of 50 rupee notes $= \frac{x}{10}$.

Number of 20 rupee notes $= \frac{x}{4}$

Number of 5 rupee notes $= \frac{x}{5}$

Now,

Value of x ten rupee notes $=$ ₹ $(10 \times x) =$ ₹ $10x$

Value of $\frac{x}{10}$ fifty rupee notes $=$ ₹ $\left(50 \times \frac{x}{10}\right) =$ ₹ $5x$

Value of $\frac{x}{4}$ twenty rupee notes $=$ ₹ $\left(20 \times \frac{x}{4}\right) =$ ₹ $5x$

Value of $\frac{x}{5}$ five rupee notes $=$ ₹ $\left(5 \times \frac{x}{5}\right) =$ ₹ x

$\therefore$ Total value of all notes $=$ ₹ $(10x + 5x + 5x + x)$

But the total amount is given as ₹ 8400. So, we obtain the following equation:

$$10x + 5x + 5x + x = 8400$$

Solution We have,

$$10x + 5x + 5x + x = 8400$$

$\Rightarrow$ $21x = 8400$

$\Rightarrow$ $\frac{21x}{21} = \frac{8400}{21}$

$\Rightarrow$ $x = 400$

Hence,

Number of ten rupee notes $= 400$

Number of fifty rupee notes $= \frac{x}{10} = \frac{400}{10} = 40$

Number of twenty rupee notes $= \frac{x}{4} = \frac{400}{4} = 100$

Number of five rupee notes $= \frac{x}{5} = \frac{400}{5} = 80.$

Example 14 Ravish owns a plot of rectangular shape. He has fenced it with a wire of length 750 m. The length of the plot exceeds the breadth by 5 m. Find the length and breadth of the plot.

Formulation: Let the breadth of the plot be x metres. Then,

Length $= (x + 5)$ metres.

$\Rightarrow$ Perimeter of the plot $= 2\,(\text{length} + \text{breadth})$

$$= 2(x + 5 + x)$$

$$= 2(2x + 5)$$

$$= (4x + 10)$$

It is given that Ravish has fenced the plot with a wire of length 750 metres. This means that the perimeter of the plot is 750 metres. Thus, we obtain the following equation:

$$4x + 10 = 750.$$

Solution We have, $4x + 10 = 750$

$\Rightarrow \quad 4x = 750 - 10$ [Transposing 10 to R.H.S.]

$\Rightarrow \quad 4x = 740$

$\Rightarrow \quad \frac{4x}{4} = \frac{740}{4}$ [Dividing both sides by 4]

$\Rightarrow \quad x = 185$

Thus, length of the plot $= (x + 5)$ metres $= (185 + 5)$ metres $= 190$ metres and, breadth of the plot $= 185$ metres.

Example 15 Sara's mother is three times as old as Sara and four times as old as Sara's sister, Ann. Ann is three years younger than Sara. How old are Sara, Ann and their mother?

Solution Let Sara's age be x years.

It is given that Ann is three years younger than Sara.

$\therefore$ Ann's age $= (x - 3)$ years

It is given that Sara's mother is three times as old as Sara.

$\therefore$ Sara's mother's age $= 3x$ years ... (i)

Also, Sara's mother is four times as old as Ann

$\therefore$ Sara's mother's age $= 4\,(x - 3)$ years ... (ii)

From (i) and (ii), we have

$$3x = 4\,(x - 3)$$

$\Rightarrow \quad 3x = 4x - 12$

$\Rightarrow \quad 3x-4x=-12$ [Transposing $4x$ to LHS]

$\Rightarrow \quad -x=-12$ [Dividing both sides by -1]

$\Rightarrow \quad x=12$

$\therefore$ Sara's age = 12years

Ann's age = (12 - 3) years = 9years

Sara's mother's age = $3x = 3\times 12 = 36$years.

EXERCISE 8.4

1. If 5 is subtracted from three times a number, the result is 16. Find the number.
2. Find the number which when multiplied by 7 is increased by 78.
3. Find three consecutive natural numbers such that the sum of the first and second is 15 more than the third.
4. The difference between two numbers is 7. Six times the smaller plus the larger is 77. Find the numbers.
5. A man says, "I am thinking of a number. When I divide it by 3 and then add 5, my answer is twice the number I thought of". Find the number.
6. If a number is tripled and the result is increased by 5, we get 50. Find the number.
7. Shikha is 3 years younger to her brother Ravish. If the sum of their ages is 37 years, what are their present ages?
8. Mrs. Jain is 27 years older than her daughter Nilu. After 8 years she will be twice as old as Nilu. Find their present ages.
9. A man is 4 times as old as his son. After 16 years, he will be only twice as old as his son. Find the their present ages.
10. The difference in age between a girl and her younger sister is 4 years. The younger sister in turn is 4 years older than her brother. The sum of the ages of the younger sister and her brother is 16. How old are the three children?
11. One day, during their vacation at a beach resort, Shella found twice as many sea shells as Anita and Anita found 5 shells more than sandy. Together sandy and Shella found 16 sea shells. How many did each of them find?
12. Andy has twice as many marbles as Pandy, and Sandy has half as many has Andy and Pandy put together. If Andy has 75 marbles more than Sandy. How many does each of them have?
13. A bag contains 25 paise and 50 paise coins whose total value is Rs 30. If the number of 25 paise coins is four times that of 50 paise coins, find the number of each type of coins.
14. The length of a rectangular field is twice its breadth. If the perimeter of the field is 228 metres, find the dimensions of the field.
15. There are only 25 paise coins in a purse. The value of money in the purse is Rs 17.50. Find the number of coins in the purse.
16. In a hostel mess, 50 kg rice are consumed everyday. If each student gets 400 gm of rice per day, find the number of students who take meals in the hostel mess.

ANSWERS

1. 7 2. 13 3. 16, 17, 18 4. 10, 17 5. 3 6. 15

7. Shikha: 17 years, Ravish 20 years 8. Nilu: 19 years, Mrs. Jain: 46 years

9. Son: 8 years, Man 32 years 10. Brother: 6 years, Younger sister: 10 years, Girl: 14 years.
11. Sandy: 2, Anita: 7, Shella: 14 12. Pandy: 150 Marbles, Andy: 300 Marbles Sandy: 225 Marbles,
13. 50 paise coins : 20, 25 paise coins : 80 14. Length = 76 metres, Breadth = 38 metres
15. 70 16. 125

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. The zero of $3x + 2$ is
   (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{-3}{2}$
2. If $2x - \frac{3}{2} = 5x + \frac{3}{4}$, then $x =$
   (a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$
3. If $\frac{x}{2} - 4 = \frac{x}{3} - 1$, then $x =$
   (a) 3 (b) 6 (c) 18 (d) 2
4. If $\frac{x+2}{x-2} = \frac{2}{3}$
   (a) -10 (b) 10 (c) $\frac{4}{3}$ (d) $-\frac{4}{3}$
5. If $\frac{x}{6} + \frac{x}{4} = \frac{x}{2} + \frac{3}{4}$, then $x =$
   (a) 9 (b) 6 (c) -9 (d) 4
6. If $2x + \frac{5}{3} = \frac{1}{4}x + 4$, then $x =$
   (a) 3 (b) 4 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
7. If $\frac{x}{2} - \frac{x}{3} = 5$, then $x =$
   (a) 8 (b) 16 (c) 24 (d) 30
8. If $\frac{x-2}{3} = \frac{2x-1}{3} - 1$, then $x =$
   (a) 2 (b) 4 (c) 6 (d) 8
9. The sum of two consecutive whole numbers is 43. The smaller number is
   (a) 21 (b) 22 (c) 23 (d) 24
10. The sum of two consecutive odd numbers is 36. The larger number is
    (a) 17 (b) 15 (c) 19 (d) 21
11. Twice a number when increased by 7 gives 25. The number is
    (a) 7 (b) 9 (c) 10 (d) 8
12. The length of a rectangle is three times its width and its perimeter 56 m. The length is
    (a) 7 m (b) 14 m (c) 21 m (d) 28 m
13. Two-thirds of a number is greater than one-third of the number by 5. The number is
    (a) 10 (b) 5 (c) 15 (d) 12

14. If the sum of a number and its two-fifth is 70. Thenumber is
 (a) 70 (b) 50 (c) 60 (d) 90

15. $\frac{2}{3}$ of a number is less than the original number by 20. The number is
 (a) 30 (b) 40 (c) 50 (d) 60

16. A number is as much greater than 31 as it is less than 81. The number is
 (a) 46 (b) 56 (c) 66 (d) 76

17. Two complementary angles differ by 20°. The smaller angle is
 (a) 55° (b) 25° (c) 65° (d) 35°

18. Two supplementary angles differ by 40°. The measure of the larger angle is
 (a) 70° (b) 80° (c) 110° (d) 100°

19. The sum of three consecutive odd numbers is 81. The middle number is
 (a) 25 (b) 27 (c) 31 (d) 29

20. If $2\ (2n + 5) = 3\ (3n - 10)$, then $n =$
 (a) 5 (b) 3 (c) 7 (d) 8

ANSWSERS

1. (c)	2. (b)	3. (c)	4. (a)	5. (c)	6. (d)	7. (d)
8. (a)	9. (a)	10. (c)	11. (b)	12. (c)	13. (c)	14. (b)
15. (d)	16. (b)	17. (d)	18. (c)	19. (b)	20. (d)	

THINGS TO REMEMBER

1. *A statement of equality involving one or more variables (literals) is called an equation.*
2. *An equation involving only one literal number (variable) with the highest power one is called a linear equation in one variable.*
3. *While solving an equation we can*
   (i) *add the same number to both sides of the equation;*
   (ii) *subtract the same number from both sides of the equation;*
   (iii) *multiply both sides of the equation by the same non-zero number;*
   (iv) *divide both sides of the equation by the same non-zero number.*
4. *In an equation, we can drop a term from one side and put it on the other side with the opposite sign. This process is known as transposition.*