14

LINES AND ANGLES

14.1 INTRODUCTION

In Class VI, we have learnt about lines, line segments, rays and angles. We have also learnt about acute, obtuse and reflex angles. In this chapter, we shall learn about pair of angles and their properties.

14.2 PAIRS OF ANGLES

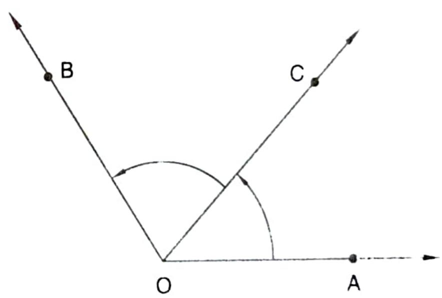
In geometry, we often come across pairs of angles which have been given specific names. In this section, we shall learn about such pair of angles.

ADJACENT ANGLES *Two angles in a plane are called adjacent angles, if*

(i) they have a common vertex,

(ii) they have a common arm, and

(iii) their other arms lie on the opposite sides of the common arm.



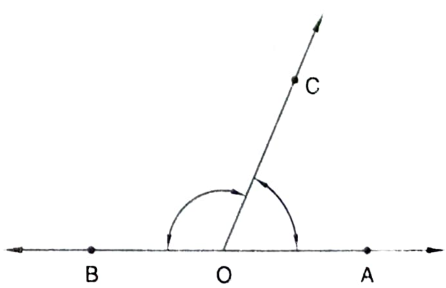
Adjacent angle

Fig. 1

In Fig. 1, $\angle AOC$ and $\angle BOC$ have the common vertex O. Also, they have a common arm OC and their other arms OA and OB lie on the opposite sides of the common arm OC. Therefore, $\angle AOC$ and $\angle BOC$ are adjacent angles.

Note that $\angle AOC$ and $\angle AOB$ are not adjacent angles, because their other arms OB and OC are not on the opposite side of the common arm OA.

LINEAR PAIR *Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.*



Linear pair

Fig. 2

In Fig. 2, OA and OB are two opposite rays and $\angle AOC$ and $\angle BOC$ are the adjacent angles. Therefore, $\angle AOC$ and $\angle BOC$ form a linear pair.

If you measure $\angle AOC$ and $\angle BOC$ with the help of the protractor, you will find the sum of their measures equal to $180°$.

Thus, the sum of the angles in a linear pair is $180°$.

VERTICALLY OPPOSITE ANGLES *Two angles formed by two intersecting lines having no common arm are called vertically opposite angles.*

In Fig. 3, two lines AB and CD are intersecting at a point O. We observe that with the intersection of these lines, four angles have been formed. Angles $\angle 1$ and $\angle 3$ form a pair of vertically opposite angles; while angles $\angle 2$ and $\angle 4$ form another pair of vertically opposite angles.

Clearly, Angles ∠1 and ∠2 form a linear pair.

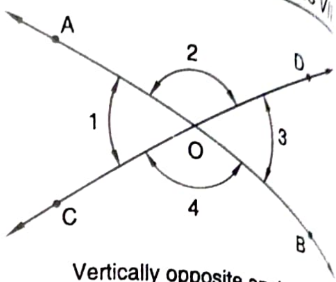
∴ $\angle 1 + \angle 2 = 180° \Rightarrow \angle 1 = 180° - \angle 2$ ...(i)

Also, ∠2 and ∠3 form a linear pair.

∴ $\angle 2 + \angle 3 = 180° \Rightarrow \angle 3 = 180° - \angle 2$ ...(ii)

From (i) and (ii), we get $\angle 1 = \angle 3$

Similarly, we can prove that $\angle 2 = \angle 4$.



Vertically opposite angles

Fig. 3

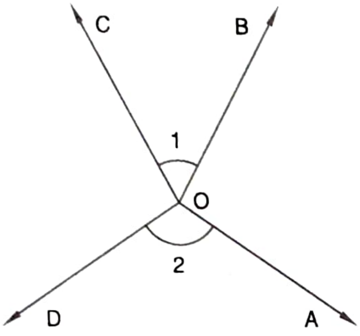


Fig. 4

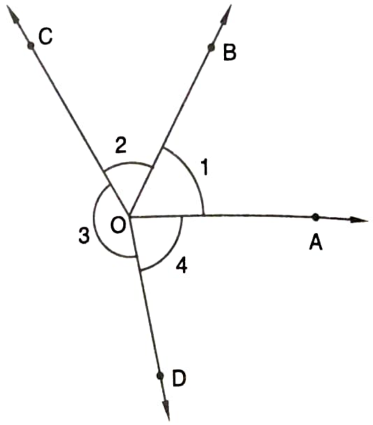


Fig. 5

Thus, if two lines intersect then vertically opposite angles are always equal.

In Fig. 4, ∠1 and ∠2 are not vertically opposite angles, because their arms do not form two pairs of opposite rays.

ANGLES AT A POINT *Angles formed by a number of rays having a common initial point are called angles at a point.*

In Fig. 5, rays *OA*, *OB*, *OC*, *OD* having a common initial point *O*, form ∠1, ∠2, ∠3, ∠4 at the point *O*.

If you find the measures of these angles, you will find that

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°.$$

Thus, the sum of the measures of all the angles at a point is 4 right angles or 360°.

COMPLEMENTARY ANGLES *If the sum of the measures of two angles is 90°, then the angles are called complementary angles and each is called a complement of the other.*

Angles of measures 35° and 55° are complementary angles. The angle of 35° is the complement of the angle of 55° and the angle of 55° is the complement of the angle of 35°.

The complement of an angle of measure 30° is the angle of 60°. And, the complement of the angle of measure 60° is the angle of 30°.

Observations:

(i) If two angles are complement of each other, then each is an acute angle. But any two acute angles need not be complementary. For example, angles of measure 30° and 50° are not complement of each other.

(ii) Two obtuse angles cannot be complement of each other.

(iii) Two right angles cannot be complement of each other.

SUPPLEMENTARY ANGLES *Two angles are said to be supplementary angles if the sum of their measures is 180°, and each of them is called a supplement of the other.*

Angles of 55° and 125° are supplementary angles.

The supplement of an angle of 130° is the angle of 50° and, the supplement of an angle of 50° is the angle of 130°.

Observations:

(i) Two acute angles cannot be supplement of each other.

(ii) Two right angles are always supplementary.

(iii) Two obtuse angles cannot be supplement of each other.

Note that the angles of a linear pair are supplementary. But supplementary angles need not form a linear pair. For example, in Fig. 6, $\angle ABC + \angle DEF = 120° + 60° = 180°$. So, they are supplementary angles but they do not form a linear pair as they are not adjacent angles.

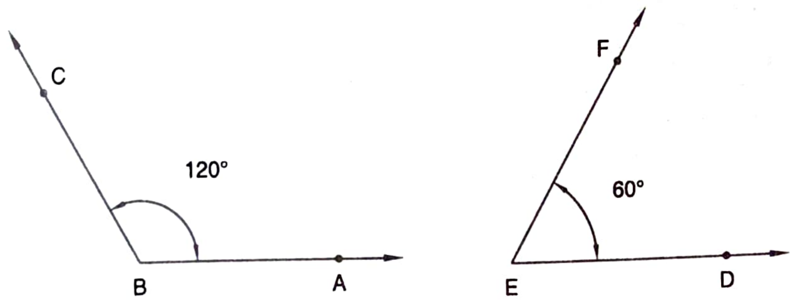


Fig. 6

ILLUSTRATIVE EXAMPLES

Example 1 Find the measure of an angle which is complement of itself.

Solution Let the measure of the angle be $x°$. Then, the measure of its complement is given to be $x°$.

Since the sum of the measures of an angle and its complement is 90°.

$\therefore \quad x° + x° = 90° \quad \Rightarrow \quad 2x° = 90° \quad \Rightarrow \quad x° = 45°$

Hence, the measure of the angle is 45°.

Example 2 Find the angle which is equal to its supplement.

Solution Let the measure of the angle be $x°$. Then,

Measure of its supplement $= x°$ [Given]

But,

Measure of an angle + Measure of its supplement = 180°

$\therefore \quad x° + x° = 180°$

$\Rightarrow \quad 2x° = 180°$

$\Rightarrow \quad x° = 90°$

Hence, the measure of the required angle is 90°.

Example 3 Two supplementary angles differ by 34°. Find the angles.

Solution Let one angle be $x°$. Then, the other angle is $(x + 34)°$.

Now, $x°$ and $(x + 34)°$ are supplementary angles.

$\therefore \quad x + (x + 34) = 180$

$\Rightarrow \quad 2x + 34 = 180$

$\Rightarrow \quad 2x = 180 - 34$

$\Rightarrow \quad 2x = 146 \quad \Rightarrow x = 73$

Hence, the measures of two angles are 73° and 73° + 34° = 107°.

Example 4 An angle is equal to five times its complement. Determine its measure.

Solution Let the measure of the given angle be x degrees. Then, its complement is $(90 -$

It is given that

Angle = $5 \times$ Complement of the angle.

$\therefore \quad x = 5(90 - x)$

$\Rightarrow \quad x = 450 - 5x$

$\Rightarrow \quad 6x = 450 \quad \Rightarrow x = 75$

Hence, the measure of the given angle is $75°$.

Example 5 In Fig. 7, OA and OB are opposite rays:

(i) If $x = 75$, what is the value of y? (ii) If $y = 110$, what is the value of x

Solution Since $\angle AOC$ and $\angle BOC$ form a linear pair.

$\therefore \quad \angle AOC + \angle BOC = 180°$

$\Rightarrow \quad x + y = 180° \quad$ ...(i)

(i) If $x = 75°$, then from (i), we have

$75° + y = 180° \Rightarrow y = 105°$.

(ii) If $y = 110°$ then from (i), we have

$x + 110° = 180° \Rightarrow x = 180° - 110° = 70°$.

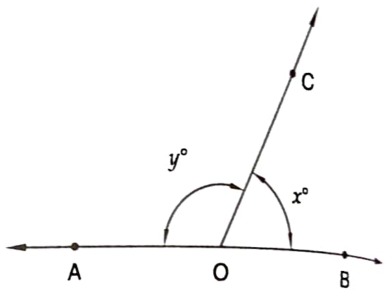


Fig. 7

Example 6 In Fig. 8, $\angle AOC$ and $\angle BOC$ form a linear pair. Determine the value of x.

Solution Since $\angle AOC$ and $\angle BOC$ form a linear pair.

$\therefore \quad \angle AOC + \angle BOC = 180°$

$\Rightarrow \quad 4x + 2x = 180$

$\Rightarrow \quad 6x = 180$

$\Rightarrow \quad x = \frac{180}{6} = 30$

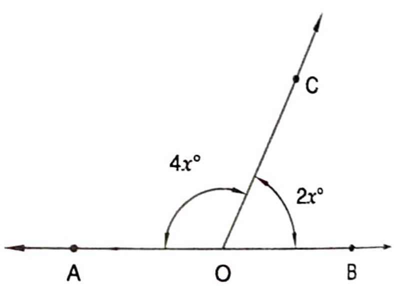


Fig. 8

Example 7 In Fig. 9, if ray OC stands on line AB such that $\angle AOC = \angle COB$, then show that $\angle AOC = 90°$.

Solution Since ray OC stands on line AB.

$\therefore \quad \angle AOC + \angle COB = 180° \quad$ [Linear pair]...(i)

But, $\angle AOC = \angle COB \quad$ [Given]

$\therefore \quad \angle AOC + \angle AOC = 180°$

$\Rightarrow \quad 2\angle AOC = 180° \Rightarrow \angle AOC = 90°$.

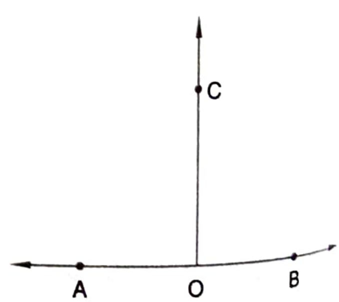


Fig. 9

Example 8 In Fig. 10, lines l_1 and l_2 intersect at O, forming angles as shown in the figure. If $a = 35$, find the values of b, c and d.

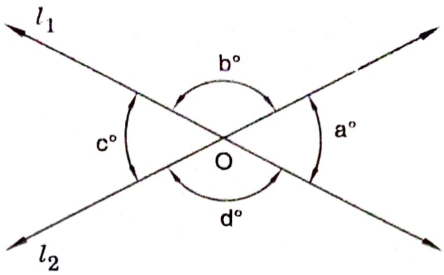


Fig. 10

Solution Since lines l_1 and l_2 intersect at O.

$\therefore \quad \angle a = \angle c$ [Vertically opposite angles]

$\Rightarrow \quad \angle c = 35°$ $[\because \angle a = 35°]$

Clearly, $\angle a + \angle b = 180°$

$\Rightarrow \quad 35° + \angle b = 180°$ $[\because \ \angle a$ and $\angle b$ are angles of a linear pair$]$

$\Rightarrow \quad \angle b = 180° - 35°$

$\Rightarrow \quad \angle b = 145°$

Since $\angle b$ and $\angle d$ are vertically opposite angles.

$\therefore \quad \angle d = \angle b \Rightarrow \angle d = 145°$ $[\because \angle b = 145°]$

Hence, $b = 145$, $c = 35$ and $d = 145$

Example 9 In Fig. 11, determine the value of y.

Solution Since $\angle COD$ and $\angle EOF$ are vertically opposite angles.

$\therefore \quad \angle COD = \angle EOF$

$\Rightarrow \quad \angle COD = 5y°$ $[\because \angle EOF = 5y°$ (Given)$]$

Now, OA and OB are opposite rays.

$\therefore \quad \angle AOD + \angle DOC + \angle COB = 180°$

$\Rightarrow \quad 2y° + 5y° + 5y° = 180°$

$\Rightarrow \quad 12y° = 180°$

$\Rightarrow \quad y = \dfrac{180}{12} = 15$

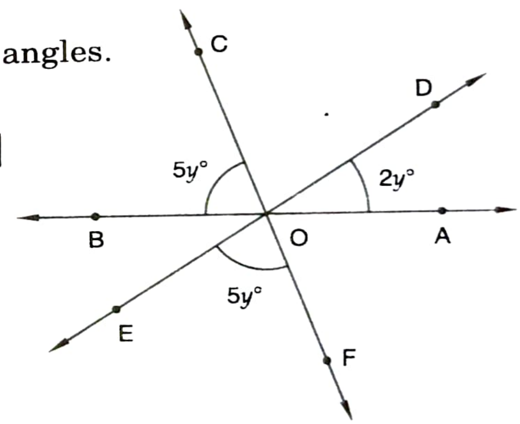


Fig. 11

Hence, $y = 15$.

Example 10 In Fig. 12, two straight lines PQ and RS intersect each other at O. If $\angle POT = 75°$, find the values of a, b and c.

Solution Since OR and OS are in the same line.

$\therefore \quad \angle ROP + \angle POT + \angle TOS = 180°$

$\Rightarrow \quad 4b° + 75° + b° = 180°$

$\Rightarrow \quad 5b° + 75° = 180°$

$\Rightarrow \quad 5b° = 105°$

$\Rightarrow \quad b = 21°$

Since PQ and RS intersect at O. Therefore,

$\angle QOS = \angle POR$ [Vertically opposite angles]

$\Rightarrow \quad a = 4b$

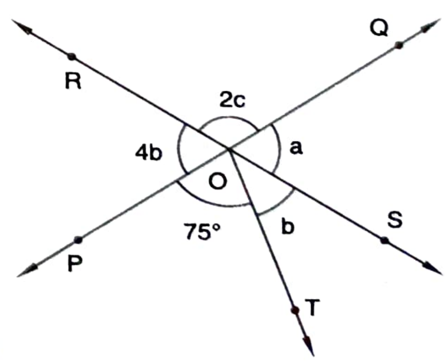


Fig. 12

$\Rightarrow \quad a = 4 \times 21° = 84°$ [∵ $b = 21°$]

Now, OR and OS are in the same line. Therefore,

$\angle ROQ + \angle QOS = 180°$ [Linear pair]

$\Rightarrow \quad 2c + a = 180°$

$\Rightarrow \quad 2c + 84° = 180°$ [∵ $a = 84°$]

$\Rightarrow \quad 2c = 96°$

$\Rightarrow \quad c = 48°$

Hence, $a = 84°$, $b = 21°$ and $c = 48°$.

EXERCISE 14.1

1. Write down each pair of adjacent angles shown in Fig. 13.

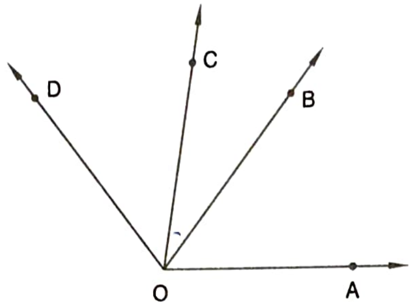


Fig. 13

2. In Fig. 14, name all the pairs of adjacent angles.

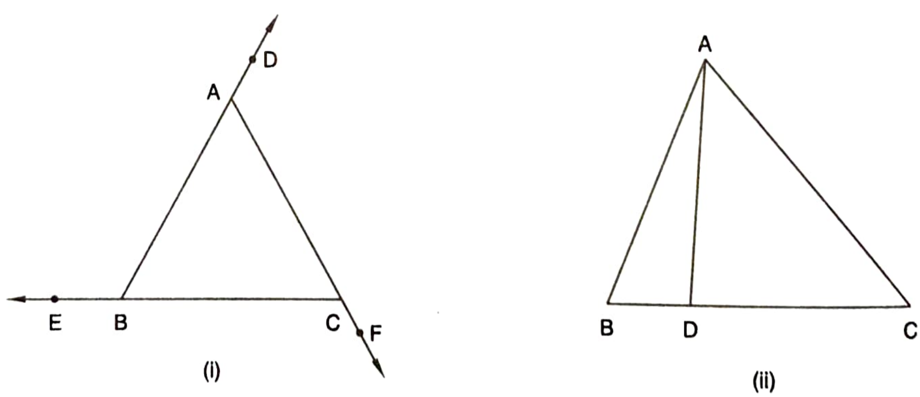


Fig. 14

3. In Fig. 15, write down: (i) each linear pair (ii) each pair of vertically opposite angles.

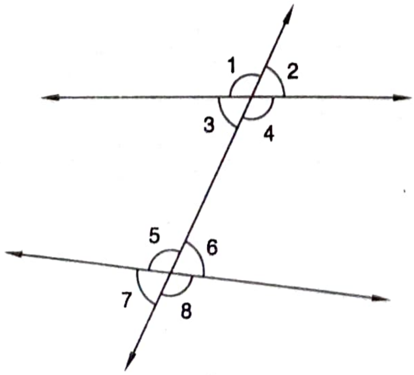


Fig. 15

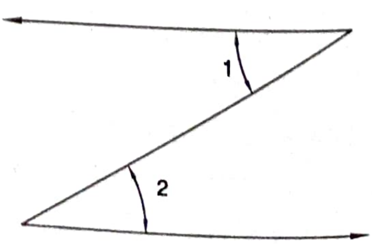


Fig. 16

4. Are the angles 1 and 2 given in Fig. 16 adjacent angles?
5. Find the complement of each of the following angles:
   (i) 35° (ii) 72° (iii) 45° (iv) 85°
6. Find the supplement of each of the following angles:
   (i) 70° (ii) 120° (iii) 135° (iv) 90°
7. Identify the complementary and supplementary pairs of angles from the following pairs:
   (i) 25°, 65° (ii) 120°, 60° (iii) 63°, 27° (iv) 100°, 80°
8. Can two angles be supplementary, if both of them be
   (i) obtuse? (ii) right? (iii) acute?
9. Name the four pairs of supplementary angles shown in Fig. 17.

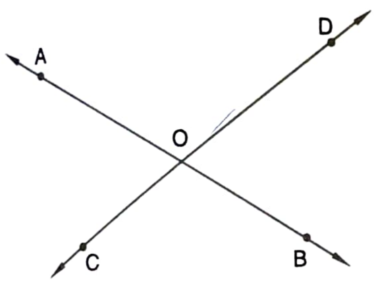


Fig. 17

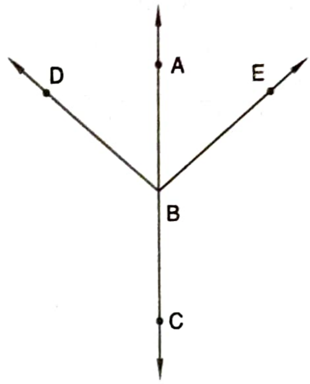


Fig. 18

10. In Fig. 18, A, B, C are collinear points and $\angle DBA = \angle EBA$.
    (i) Name two linear pairs (ii) Name two pairs of supplementary angles.
11. If two supplementary angles have equal measure, what is the measure of each angle?
12. If the complement of an angle is 28°, then find the supplement of the angle.
13. In Fig. 19, name each linear pair and each pair of vertically opposite angles:
14. In Fig. 20, OE is the bisector of $\angle BOD$. If $\angle 1 = 70°$, find the magnitudes of $\angle 2$, $\angle 3$ and $\angle 4$.

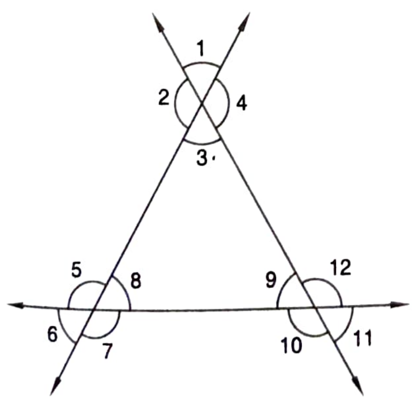


Fig. 19

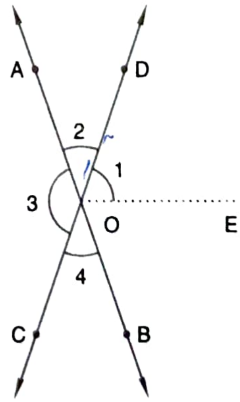


Fig. 20

15. One of the angles forming a linear pair is a right angle. What can you say about its other angle?
16. One of the angles forming a linear pair is an obtuse angle. What kind of angle is the other?
17. One of the angles forming a linear pair is an acute angle. What kind of angle is the other?

18. Can two acute angles form a linear pair?

19. If the supplement of an angle is 65°; then find its complement.

20. Find the value of x in each of the following figures.

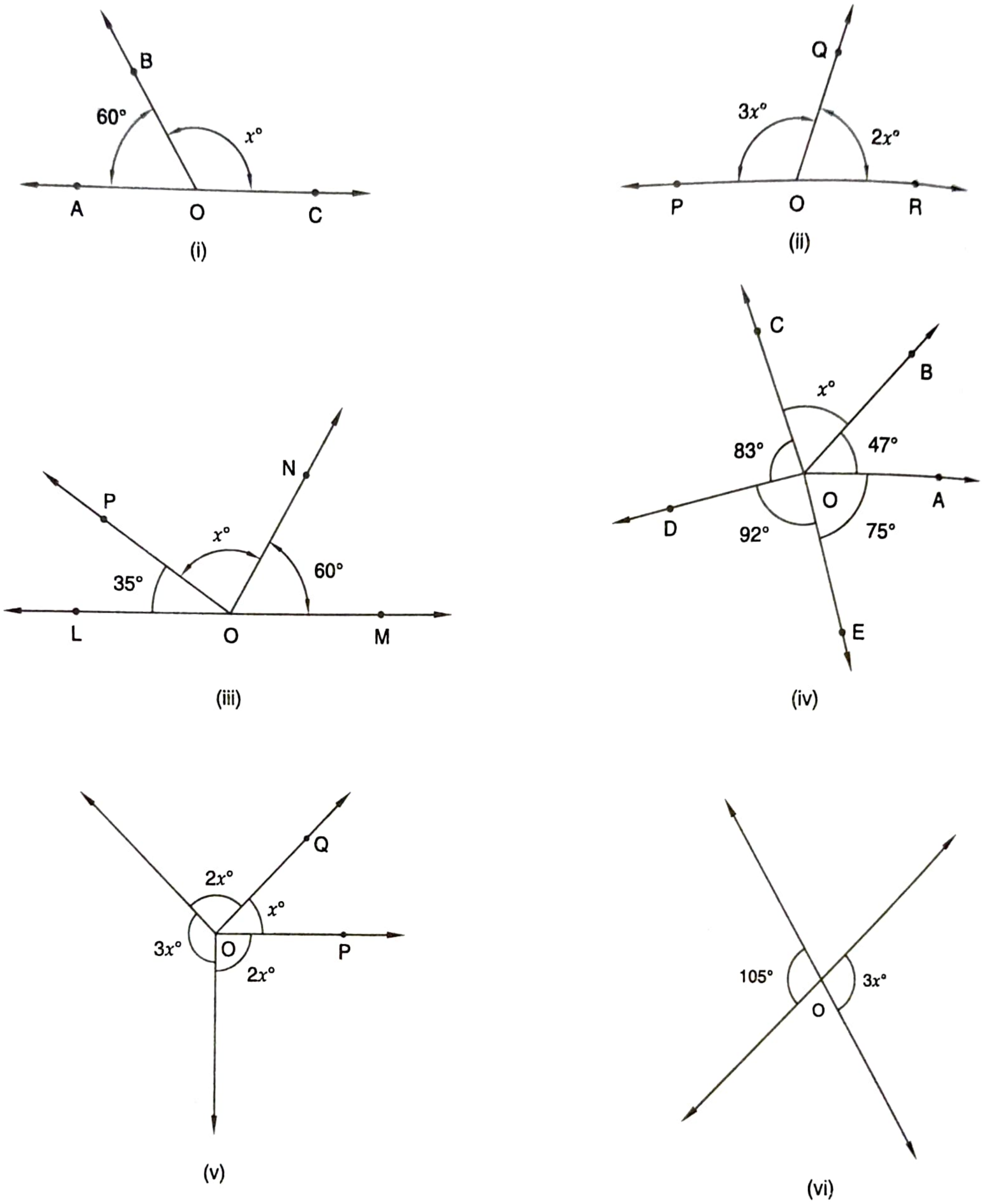


Fig. 21

21. In Fig. 22, it being given that $\angle 1 = 65°$, find all other angles.

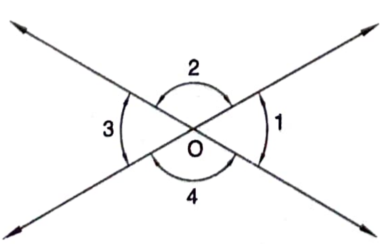


Fig. 22

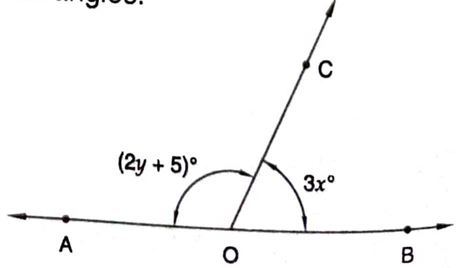


Fig. 23

22. In Fig. 23, OA and OB are opposite rays:
(i) If $x = 25°$, what is the value of y? (ii) If $y = 35°$, what is the value of x?
23. In Fig. 24, write all pairs of adjacent angles and all the linear pairs.

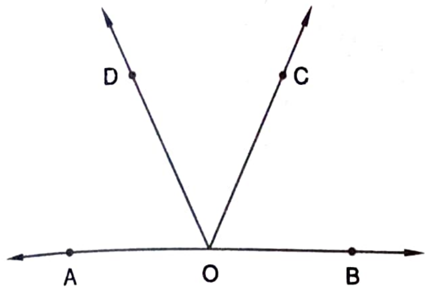


Fig. 24

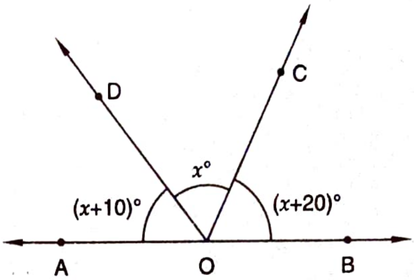


Fig. 25

24. In Fig. 25, find $\angle x$. Further find $\angle BOC$, $\angle COD$ and $\angle AOD$.
25. How many pairs of adjacent angles are formed when two lines intersect in a point?
26. How many pairs of adjacent angles, in all, can you name in Fig. 26?

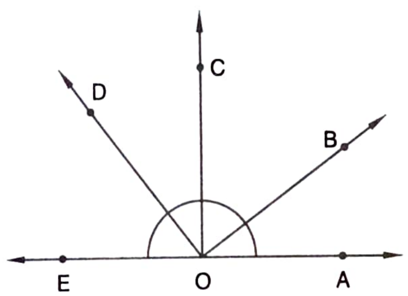


Fig. 26

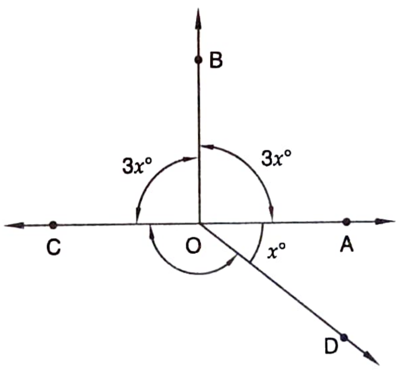


Fig. 27

27. In Fig. 27, determine the value of x.
28. In Fig. 28, AOC is a line, find x.

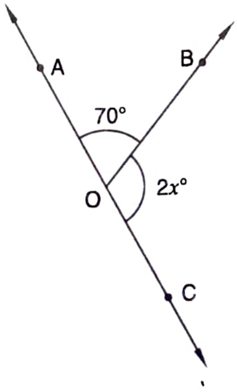


Fig. 28

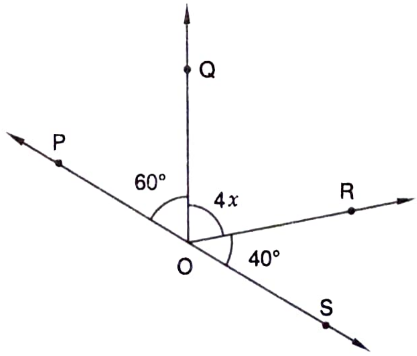


Fig. 29

29. In Fig. 29, POS is a line, find x.
30. In Fig. 30, lines l_1 and l_2 intersect at O, forming angles as shown in the figure. If $x = 45°$, find the values of y, z and u.

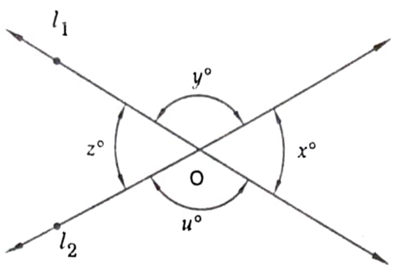


Fig. 30

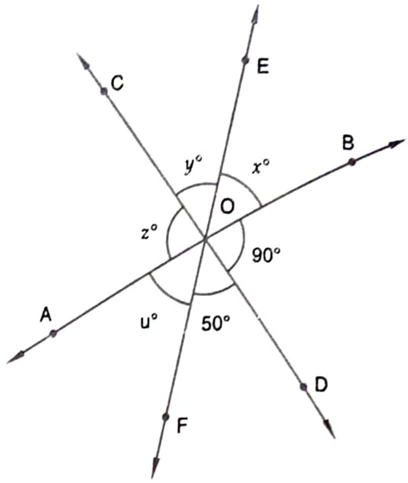


Fig. 31.

31. In Fig. 31, three coplanar lines intersect at a point O, forming angles as shown in the figure. Find the values of x, y, z and u.
32. In Fig. 32, find the values of x, y and z.

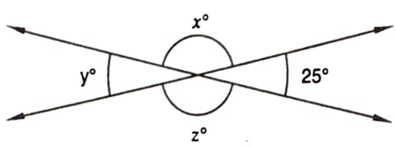


Fig. 32

ANSWERS

1. ∠AOB, ∠BOC ; ∠AOC, ∠COD ; ∠BOC, ∠COD ; ∠AOB, ∠BOD
2. (i) ∠DAC, ∠CAB ; ∠ACB, ∠FCB ; ∠ABC, ∠ABE
   (ii) ∠ADB, ∠ADC ; ∠BAD, ∠DAC
3. (i) ∠1, ∠2 ; ∠2, ∠4 ; ∠3, ∠4 ; ∠1, ∠3 ; ∠5, ∠6 ; ∠5, ∠7 ; ∠7, ∠8 ; ∠6, ∠8
   (ii) ∠1, ∠4 ; ∠2, ∠3 ; ∠5, ∠8 ; ∠6, ∠7
4. No   5. (i) 55°   (ii) 18°.   (iii) 45°   (iv) 5°
6. (i) 110°   (ii) 60°   (iii) 45°   (iv) 90°
7. Complementary   Supplementary
   (i), (iii)   (ii), (iv)
8. (i) No   (ii) Yes   (iii) No
9. ∠AOC, ∠COB ; ∠COB, ∠BOD ; ∠BOD, ∠DOA ; ∠DOA, ∠AOC
10. (i) ∠ABE, ∠EBC ; ∠ABD, ∠DBC (ii) ∠ABE, ∠EBC ; ∠ABD, ∠DBC
11. 90°   12. 152°
13. Linear pairs: ∠1, ∠2 ; ∠2, ∠3 ; ∠3, ∠4 ; ∠4, ∠1; ∠5, ∠6 ; ∠6, ∠7 ; ∠7, ∠8 ; ∠8, ∠5 ; ∠9, ∠10 ; ∠10, ∠11 ; ∠11, ∠12 ; ∠12, ∠9
    Pairs of vertically opposite angles: ∠1, ∠3 ; ∠2, ∠4 ; ∠5, ∠7; ∠6, ∠8°; ∠9, ∠11; ∠10, ∠12
14. 40°, 140°, 40°   15. 90°   16. Acute   17. Obtuse   18. No   19. Does not exist
20. (i) 120°   (ii) 36°   (iii) 85°   (iv) 63°   (v) 45°   (vi) 35°
21. ∠2 = 115°, ∠3 = 65°, ∠4 = 115°
22. (i) 50°   (ii) 35°
23. Adjacent angles: ∠AOD, ∠COD ; ∠BOC, ∠COD; ∠AOD, ∠BOD; ∠AOC, ∠BOC
    Linear Pairs: ∠AOD, ∠BOD ; ∠AOC, ∠BOC

24. $x = 50°$, $\angle BOC = 70°$, $\angle COD = 50°$, $\angle AOD = 60°$

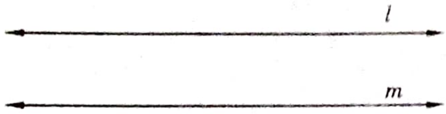
25. 4 26. 7 27. 30° 28. 55° 29. 20°

30. (i) $y = 135°$, $z = 45°$, $u = 135°$

31. $x = 40°$, $y = 50°$, $z = 90°$, $u = 40°$ 32. $x = 155°$, $y = 25°$, $z = 155°$

14.3 PARALLEL LINES

PARALLEL LINES *Two lines l and m in the same plane are said to be parallel lines if they do not intersect when produced indefinitely in either direction and we write $l \parallel m$ which is read as 'l is parallel to m'.*

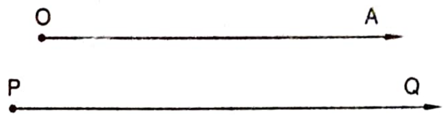


Parallel lines

Fig. 33

Clearly, when $l \parallel m$, we have $m \parallel l$.

PARALLEL RAYS *Two rays are parallel if the corresponding lines determined by them are parallel. In other words, two rays in the same plane are parallel if they do not intersect each other even if extended indefinitely beyond their initial points.*

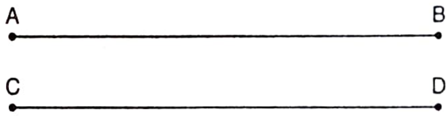


Parallel rays

Fig. 34

In Fig. 34, ray $OA \parallel$ ray PQ.

PARALLEL SEGMENTS *Two segments are parallel if the corresponding lines determined by them are parallel.*



Parallel segments

Fig. 35

In other words, two segments which are in the same plane and do not intersect each other even if extended indefinitely in both directions are said to be parallel.

In Fig. 35, segment $AB \parallel$ segment CD.

One segment and one ray are parallel if the corresponding lines determined by them are parallel.

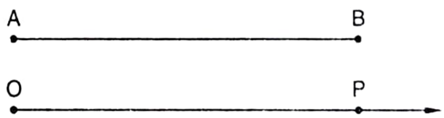


Fig. 36

In Fig. 36 segment $AB \parallel$ ray OP.

The opposite edges of this paper, opposite edges of a black board, the opposite edges of a ruler, railway lines etc. are all examples of parallel line segments.

<u>Remark</u> *It should be noted that if two lines in a plane are not parallel, then they intersect. Thus, two lines in a plane are either parallel or intersecting. However, two rays or two segments or one ray and one segment may neither be parallel nor intersecting. For example, in Fig. 37 (i) rays AB and CD are non-intersecting non-parallel rays, in Fig. 37 (ii), segments PQ and RS are non-parallel non intersecting segments and in Fig. 37 (iii), segment XY and ray UV are non-intersecting non-parallel.*

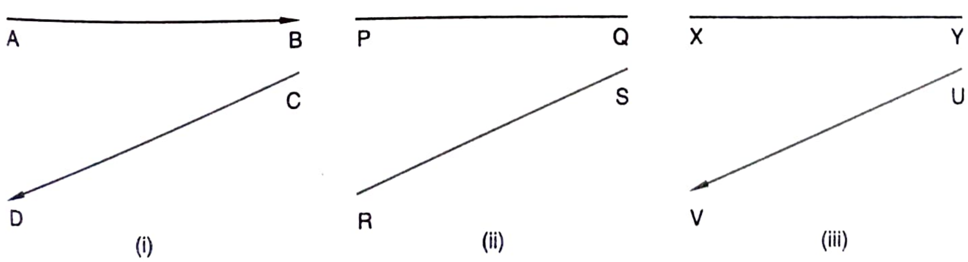


Fig. 37

14.3.1 TRANSVERSALS

TRANSVERSALS *A line intersecting two or more given lines in a plane at different points is called a transversal to the given lines.*

In Fig. 38 (i)-(vi), line l is the transversal to the given lines.

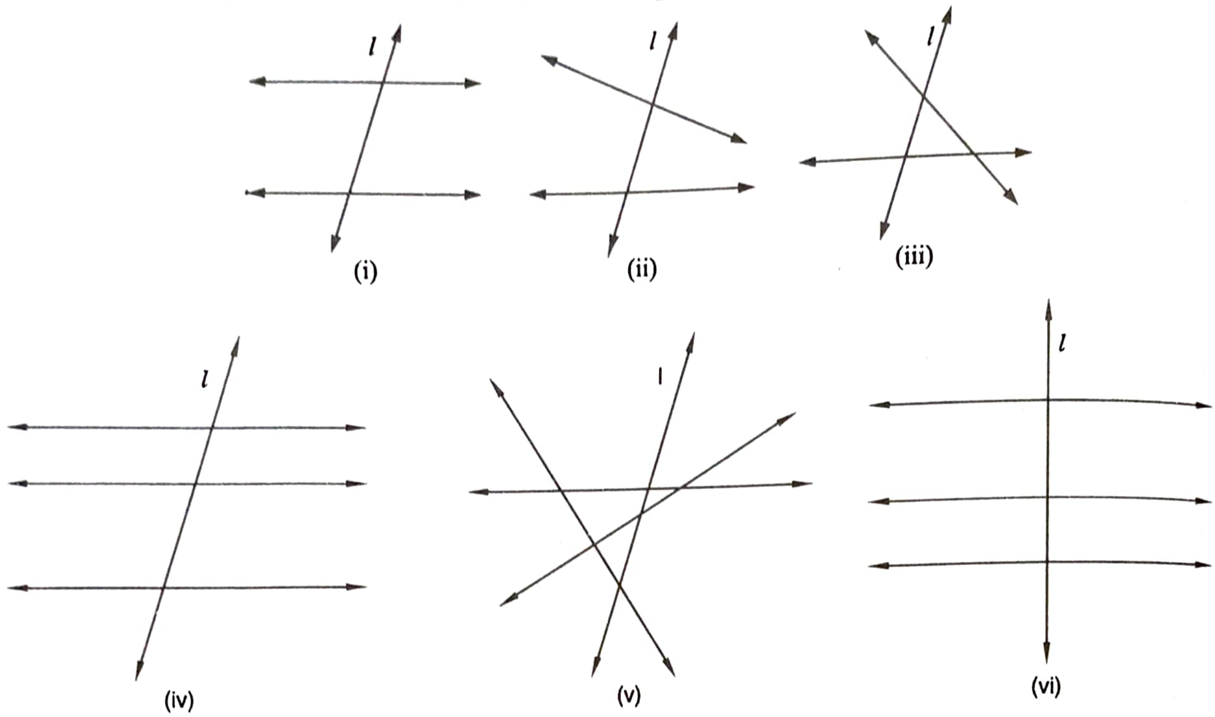


Fig. 38

Line m is not a transversal to the given lines shown in Fig. 39 (i) and (ii), because it does not intersect the lines at different points. In fact, it intersects two lines at the same point.

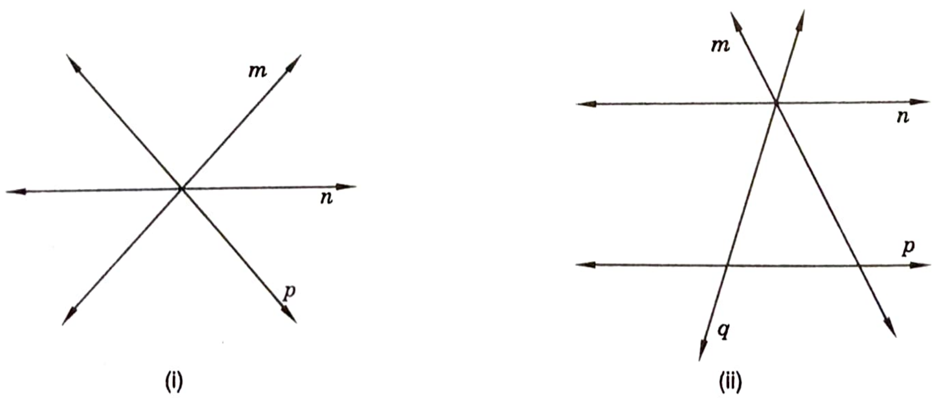


Fig. 39

It follows from the above definition that the point of intersection of the transversal and one of the given lines cannot be the point of intersection of the transversal and any other given line. Also, the given lines may or may not be parallel.

14.3.2 ANGLES MADE BY A TRANSVERSAL WITH TWO LINES

In this section, we shall learn about the angles made by a transversal with two given lines. Some of these angles can be paired together by virtue of the positions they occupy.

Let l and m be two lines and n be a transversal intersecting them at P and Q respectively as shown in Fig. 40.

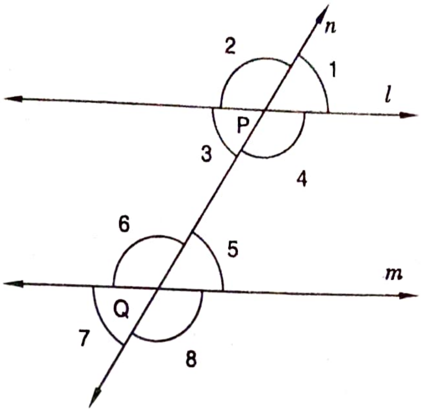


Fig. 40

learly, lines l, m and n make eight angles, four at P and the remaining four at Q. We label hem 1 to 8 for the sake of convenience and classify them in the following groups:

XTERIOR ANGLES *The angles whose arms do not include the line segment PQ are called xterior angles.*

n Fig. 41, angles 1, 2, 7 and 8 are exterior angles.

NTERIOR ANGLES *The angles whose arms include line segment PQ are called interior angles.*

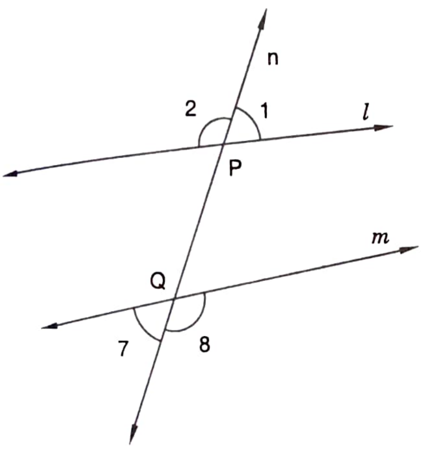


Fig. 41

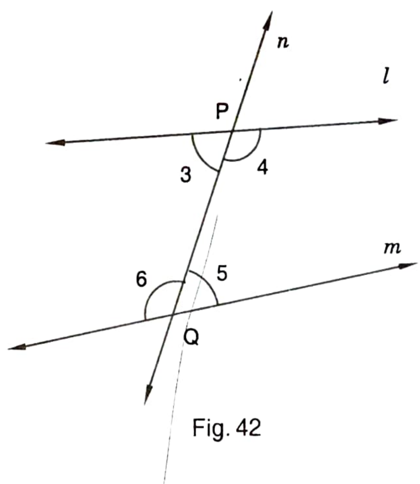


Fig. 42

n Fig. 42, angles 3, 4, 5 and 6 are interior angles.

ORRESPONDING ANGLES *A pair of angles in which one arm of both the angles is on the ame side of the transversal and their other arms are directed in the same sense is called a pair f corresponding angles.*

n Fig. 43, $\angle 1, \angle 5$; $\angle 2, \angle 6$; $\angle 3, \angle 7$ and $\angle 4, \angle 8$ are four pairs of corresponding angles.

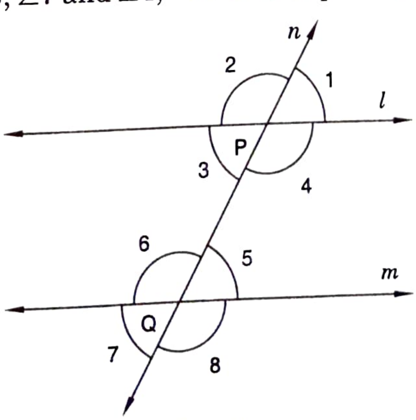


Fig. 43

We can also say that two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines.

ALTERNATE INTERIOR ANGLES *A pair of angles in which one arm of each of the angle is on opposite sides of the transversal and whose other arms include segment PQ as shown in Fig. 44 is called a pair of alternate interior angles.*

In Fig. 44, ∠3 and ∠5, ∠4 and ∠6 form pairs of alternate interior angles.

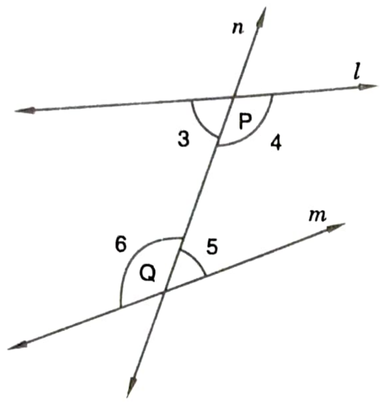


Fig. 44

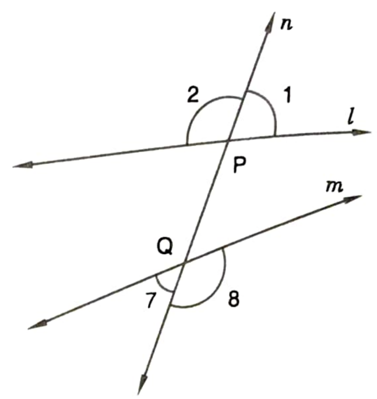


Fig. 45

ALTERNATE EXTERIOR ANGLES *A pair of angles in which one arm of each of the angles is on opposite sides of the transversal and whose other arms are directed in opposite direction and do not include segment PQ is called a pair of alternate exterior angles.*

In Fig. 45, ∠2 and ∠8, ∠1 and ∠7 form pairs of alternate exterior angles.

14.4 ANGLES MADE BY A TRANSVERSAL TO TWO PARALLEL LINES

In the previous section, we have learnt that when two lines are cut by a transversal, several pairs of angles are formed. These pairs of angles have special properties if the lines are parallel. So, we study them only for parallel lines and perform the following experiments to know their properties.

Experiment 1 Let us draw two parallel lines l and m. Also, draw a transversal intersecting lines l and m at the points P and Q respectively. Clearly, eight angles are formed by these lines. Let us label them 1 to 8. Measure all these angles with the help of a protractor.

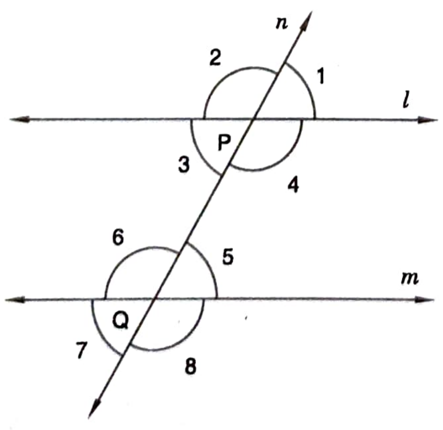


Fig. 46

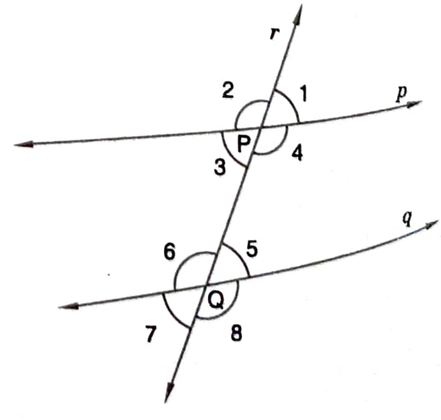


Fig. 47

You will find that:

(i) $\angle 3 = \angle 5, \angle 4 = \angle 6, \angle 2 = \angle 8$ and $\angle 1 = \angle 7$. That is, pairs of alternate interior and exterior angles are equal.

(ii) $\angle 1 = \angle 5, \angle 4 = \angle 8, \angle 2 = \angle 6$ and $\angle 3 = \angle 7$. That is, pairs of corresponding angles are equal.

(iii) $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$. That is, the sum of interior angles on the same side of the transversal is 180°.

We can repeat this experiment by drawing another pair of parallel lines and a transversal to it. We will obtain the same results.

From the above experiment, we observe the following properties of angles formed by a transversal to two parallel lines:

Property 1 *Pairs of alternate (interior or exterior) angles are equal.*

Property 2 *Pairs of corresponding angles are equal.*

Property 3 *The sum of the interior (or exterior) angles on the same side of the transversal is 180°. In other words, the interior (or exterior) angles on the same side of the transversal are supplementary.*

<u>Experiment 2</u> Let us draw two non-parallel lines p and q. Also, draw a transversal r intersecting lines p and q at P and Q respectively. Clearly, eight angles are formed. Let us label them 1 to 8. Measure all these angles with the help of a protractor.

You will find that:

(i) $\angle 3 \neq \angle 5, \angle 4 \neq \angle 6, \angle 2 \neq \angle 8$ and $\angle 1 \neq \angle 7$. That is, none of the pairs of alternate angles are equal.

(ii) $\angle 1 \neq \angle 5, \angle 4 \neq \angle 8, \angle 2 \neq \angle 6$ and $\angle 3 \neq \angle 7$. That is, none of the pairs of the corresponding angles are equal.

(iii) $\angle 3 + \angle 6 \neq 180°$ and $\angle 4 + \angle 5 \neq 180°$. That is, the sum of the interior angles on the same side of the transversal is not equal to 180°.

Thus, we observe that none of the above three properties which are true for parallel lines hold for non-parallel lines. Moreover, we conclude the following:

If a transversal cuts two lines such that any one of the following conditions holds:

(i) Pairs of alternate angles are equal.

(ii) Pairs of corresponding angles are equal.

(iii) The sum of the interior angles on the same side of the transversal is 180°.

Then, the two lines are parallel.

The following examples will illustrate the use of the above properties in solving problems.

ILLUSTRATIVE EXAMPLES

Example 1 In Fig. 48, line $l \parallel$ line m, n is transversal and $\angle 1 = 40°$. Find all the other angles marked in the figure.

Solution Since $\angle 1$ and $\angle 5$ are corresponding angles. Therefore,

$\angle 5 = \angle 1 \Rightarrow \angle 5 = 40°$ $[\because \angle 1 = 40° \text{ (given)}]$

$\angle 1$ and $\angle 2$ form a linear pair.

$\therefore \quad \angle 2 + \angle 1 = 180°$

$\Rightarrow \quad \angle 2 = 180° - \angle 1 = 180° - 40° = 140°$

Again, $\angle 5$ and $\angle 3$ are alternate interior angles. Therefore,

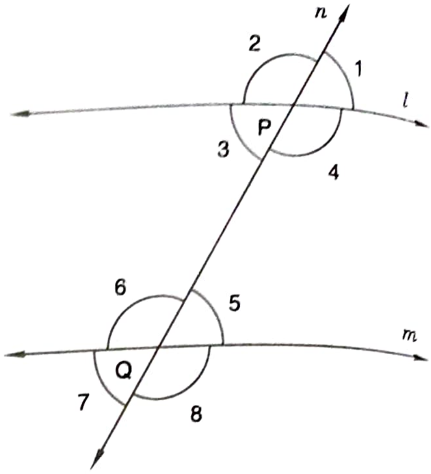


Fig. 48

$\angle 3 = \angle 5 \Rightarrow \angle 3 = 40° \quad [\because \angle 5 = 40°]$

Since $\angle 3$ and $\angle 4$ form a linear pair.

$\therefore \quad \angle 3 + \angle 4 = 180°$

$\Rightarrow \quad \angle 4 = 180° - \angle 3 = 180° - 40° = 140°$

Since $\angle 6$ and $\angle 4$ are alternate interior angles.

$\therefore \quad \angle 6 = \angle 4$

But, $\angle 4 = 140°$

$\therefore \quad \angle 6 = 140°$

Also, $\angle 3$ and $\angle 7$ are corresponding angles. Therefore,

$\angle 7 = \angle 3$

$\Rightarrow \quad \angle 7 = 40° \quad [\because \angle 3 = 40°]$

Since $\angle 7$ and $\angle 8$ form a linear pair

$\therefore \quad \angle 7 + \angle 8 = 180°$

$\Rightarrow \quad \angle 8 = 180° - \angle 7 = 180° - 40° = 140° \quad [\because \angle 7 = 40°]$

Hence, we have the following measurements:

$\angle 1 = 40°$, $\angle 2 = 140°$, $\angle 3 = 40°$, $\angle 4 = 140°$, $\angle 5 = 40°$, $\angle 6 = 140°$, $\angle 7 = 40°$ and $\angle 8 = 140°$

Example 2 In Fig. 49, $m \parallel n$ and $\angle 1 = 65°$. Find $\angle 5$ and $\angle 8$.

Solution We have,

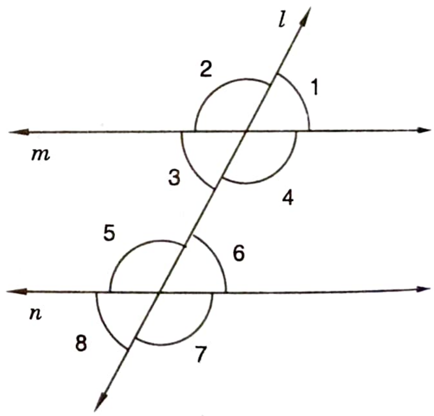


Fig. 49

$\angle 1 = \angle 3$ [Vertically opposite angles]

and, $\angle 3 = \angle 8$ [Corresponding angles]

$\therefore \quad \angle 1 = \angle 8$

$\Rightarrow \quad \angle 8 = 65° \quad [\because \angle 1 = 65° \text{ (given)}]$

Now, $\angle 5 + \angle 8 = 180°$ [Linear pair]

$\Rightarrow \quad \angle 5 + 65° = 180°$

$\Rightarrow \quad \angle 5 = 180° - 65° = 115°$

Thus, $\angle 5 = 115°$ and $\angle 8 = 65°$

Example 3 In Fig. 50, $m \parallel n$ and angles 1 and 2 are in the ratio 3 : 2. Determine all the angles from 1 to 8.

Solution It is given that $\angle 1 : \angle 2 = 3 : 2$.

So, let $\angle 1 = 3x°$ and $\angle 2 = 2x°$

But, $\angle 1$ and $\angle 2$ form a linear pair.

$\therefore \quad \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad 3x° + 2x° = 180°$

$\Rightarrow \quad 5x° = 180°$

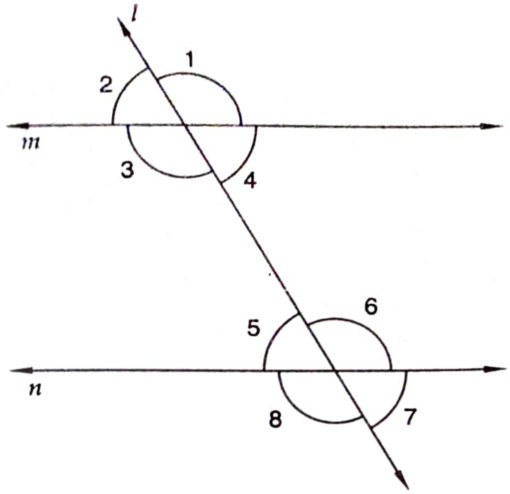


Fig. 50

$\Rightarrow \quad x = \dfrac{180}{5} = 36°$

$\therefore \quad \angle 1 = 3x° = (3 \times 36)° = 108°$

and, $\angle 2 = 2x° = (2 \times 36)° = 72°$

Now, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Vertically opposite angles]

$\therefore \quad \angle 4 = 72°$ and $\angle 3 = 108°$

Now, $\angle 6 = \angle 1$ and $\angle 4 = \angle 7$ [Corresponding angles]

[$\because \angle 2 = 72°$]

$\Rightarrow \quad \angle 6 = 72°$ and $\angle 7 = 108°$

Again, $\angle 5 = \angle 7$ and $\angle 8 = \angle 6$ [Vertically opposite angles]

$\therefore \quad \angle 5 = 108°$ and $\angle 8 = 72°$

Hence, $\angle 1 = 108°$, $\angle 2 = 72°$, $\angle 3 = 108°$, $\angle 4 = 72°$, $\angle 5 = 108°$, $\angle 6 = 72°$, $\angle 7 = 108°$ and $\angle 8 = 72°$.

Example 4 In Fig. 51, l, m and n are parallel lines intersected by a transversal p at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$. Give reasons.

Solution Since parallel lines l and n are intersected by the transversal p at X and Z respectively. Therefore,

[Corresponding angles]

$\angle 2 = \angle 4$

$\Rightarrow \quad \angle 2 = 180° - 50°$ [$\because \angle 4 + 50° = 180°$ (Linear pair)]

$\Rightarrow \quad \angle 2 = 130°$

Since $\angle 2$ and $\angle 3$ are vertically opposite angles.

$\therefore \quad \angle 3 = \angle 2$

$\Rightarrow \quad \angle 3 = 130°$ [$\because \angle 2 = 130°$]

Since parallel lines m and n are intersected by the transversal p at Y and Z respectively. Therefore,

$\angle 1 = \angle 3$ [Alternate interior angles]

$\Rightarrow \quad \angle 1 = 130°$ [$\because \angle 3 = 130°$]

Thus, $\angle 1 = \angle 2 = \angle 3 = 130°$

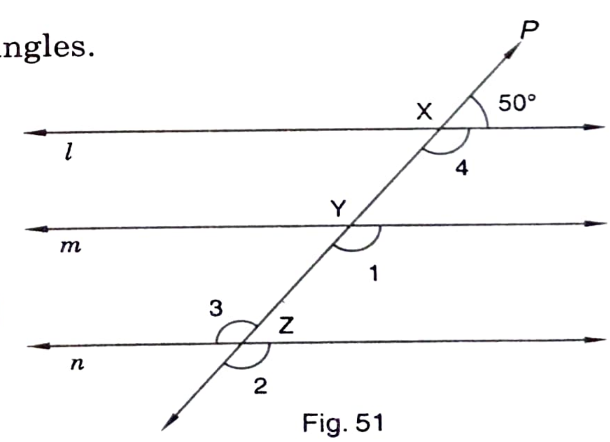


Fig. 51

Example 5 In Fig. 52, $AB \parallel CD$. Determine $\angle a$.

Solution Through O draw a line l parallel to both AB and CD.

Clearly, $\angle a = \angle 1 + \angle 2$ ...(i)

Now, $\angle 1 = 55°$ [Alternate $\angle s$]

and, $\angle 2 = 38°$ [Alternate $\angle s$]

$\therefore \quad \angle a = 55° + 38°$ [Using (i)]

$\Rightarrow \quad \angle a = 93°$

Thus, $\angle a = 93°$.

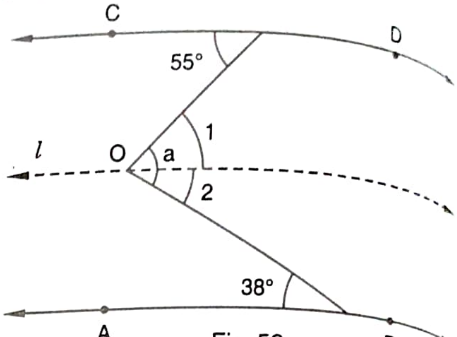


Fig. 52

Example 6 In Fig. 53, $AB \parallel CD$. Determine x.

Solution Through O, draw a line l parallel to both AB and CD. Then,

$\angle 1 = 45°$ and, $\angle 2 = 30°$ [Alternate $\angle s$]

$\therefore \quad \angle BOC = \angle 1 + \angle 2$

$\Rightarrow \quad \angle BOC = 45° + 30° = 75°$

Clearly, x = reflex $\angle BOC$

$\therefore \quad x = 360 - \angle BOC$

$\Rightarrow \quad x = 360° - 75° = 285°$

Hence, $x = 285°$.

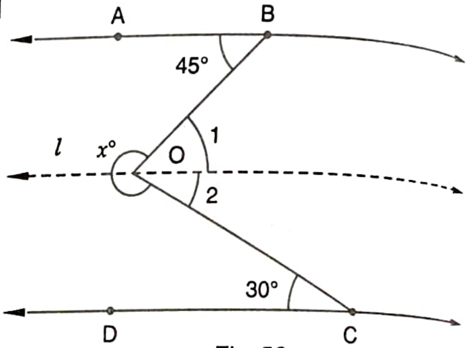


Fig. 53

Example 7 In Fig. 54, if $\angle 2 = 120°$ and $\angle 5 = 60°$, show that $m \parallel n$.

Solution We have,

$\angle 2 = 120°$ and $\angle 5 = 60°$

But, $\angle 2 = \angle 4$ [Vertically opposite angles]

$\therefore \quad \angle 4 = 120°$

$\Rightarrow \quad \angle 4 + \angle 5 = 120° + 60°$ [$\because \angle 5 = 60°$]

$\Rightarrow \quad \angle 4 + \angle 5 = 180°$

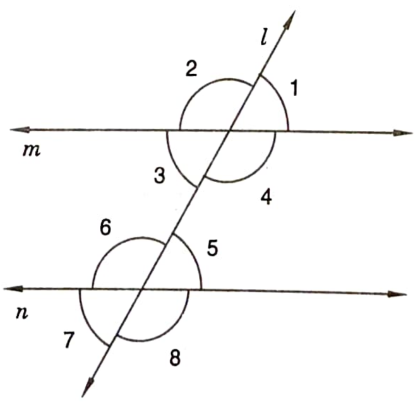


Fig. 54

$\Rightarrow$ $\angle 4$ and $\angle 5$ are supplementary angles.

$\Rightarrow$ Consecutive interior angles are supplementary.

$\Rightarrow$ $m \parallel n$.

Example 8 In Fig. 55, if $\angle 3 = 61°$ and $\angle 7 = 118°$. Is $m \parallel n$?

Solution We have, $\angle 3 = 61°$ and $\angle 7 = 118°$

Since $\angle 3$ and $\angle 4$ form a linear pair.

$\therefore \quad \angle 3 + \angle 4 = 180°$

$\Rightarrow \quad \angle 61° + \angle 4 = 180°$

$\Rightarrow \quad \angle 4 = 180° - 61° \Rightarrow \angle 4 = 119°$

Now, $\angle 4$ and $\angle 7$ are a pair of corresponding angles such that $\angle 4 = 119°$ and $\angle 7 = 118°$

i.e., $\angle 4 \neq \angle 7$.

So, m is not parallel to n.

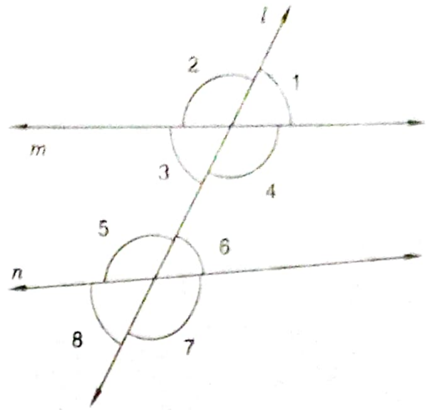


Fig. 55

Example 9 In Fig. 56, give reasons why $l_1 \parallel l_2$. Is $m_1 \parallel m_2$?

Solution Since lines l_1 and l_2 are intersected by a transversal m_2 such that the sum of two consecutive interior angles is 180° i.e. they are supplementary. Therefore, $l_1 \parallel l_2$

From Fig. 56, we observe that the lines m_1 and m_2 are intersected by transversal l_2 such that the alternate interior angles are equal. Therefore, $m_1 \parallel m_2$.

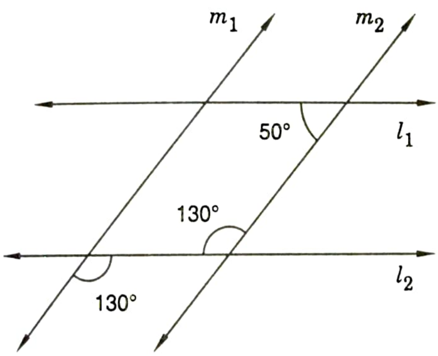


Fig. 56

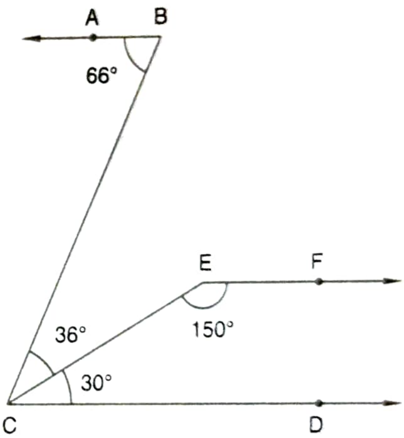


Fig. 57

Example 10 In Fig. 57, show that $AB \parallel EF$.

Solution We have,

$\angle BCD = \angle BCE + \angle ECD$

$\Rightarrow \quad \angle BCD = 36° + 30° = 66°$

$\therefore \quad \angle ABC = \angle BCD$

Thus, lines AB and CD are intersected by the line BC such that $\angle ABC = \angle BCD$ i.e., the alternate angles are equal. Therefore, $AB \parallel CD$ ...(i)

Now, $\angle ECD + \angle CEF = 30° + 150° = 180°$

This shows that the sum of the interior angles on the same side of the transversal CE is 180° i.e. they are supplementary.

$\therefore \quad EF \parallel CD$ ...(ii)

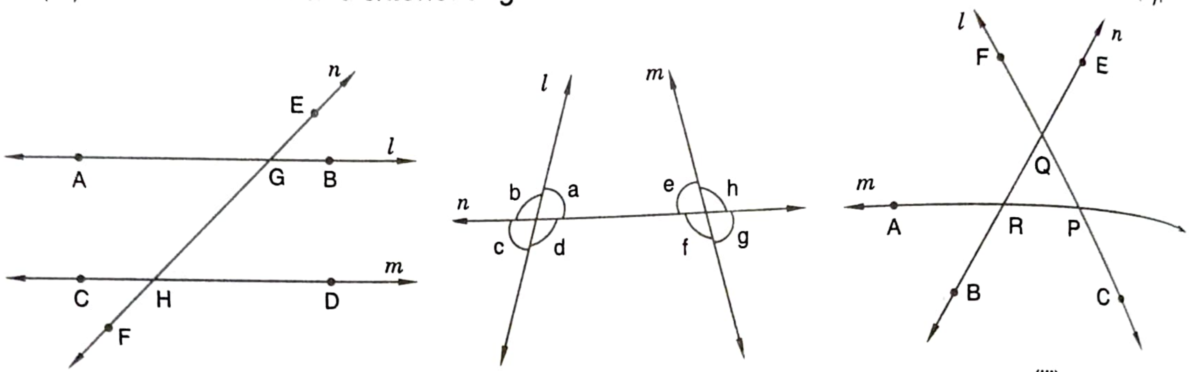
From (i) and (ii), we have $AB \parallel CD$ and $CD \parallel EF$

$\therefore \quad AB \parallel EF$

Hence, $AB \parallel EF$

EXERCISE 14.2

1. In Fig. 58, line n is a transversal to lines l and m. Identify the following:
   (i) Alternate and corresponding angles in Fig. 58 (i).
   (ii) Angles alternate to $\angle d$ and $\angle g$ and angles corresponding to angles $\angle f$ and $\angle h$ in Fig. 58 (ii).
   (iii) Angle alternate to $\angle PQR$, angle corresponding to $\angle RQF$ and angle alternate to $\angle PQE$ in Fig. 58 (iii).
   (iv) Pairs of interior and exterior angles on the same side of the transversal in Fig. 58 (ii).



(i) (ii) (iii)

Fig. 58

2. In Fig. 59, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively. If $\angle CMQ = 60°$, find all other angles in the figure.

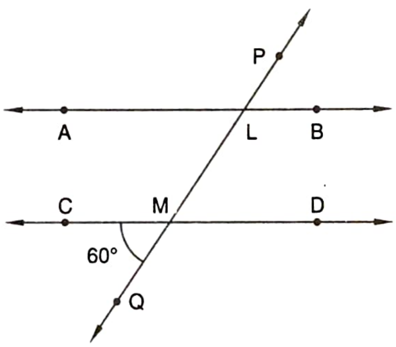


Fig. 59

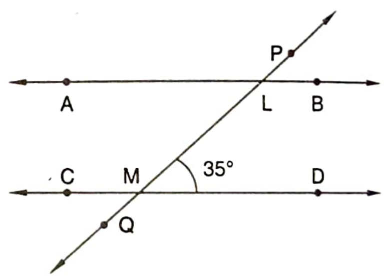


Fig. 60

3. In Fig. 60, AB and CD are parallel lines intersected by a transversal PQ at L and M respectively. If $\angle LMD = 35°$ find $\angle ALM$ and $\angle PLA$.

4. The line n is transversal to line l and m in Fig. 61. Identify the angle alternate to $\angle 13$, angle corresponding to $\angle 15$, and angle alternate to $\angle 15$.

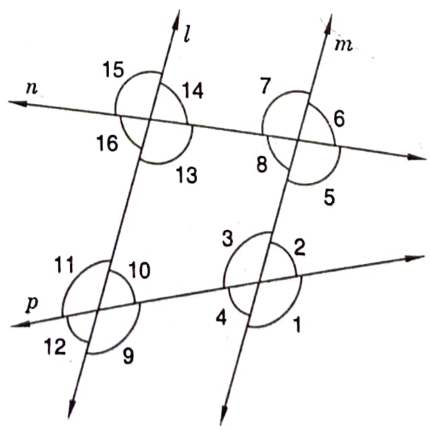


Fig. 61

5. In Fig. 62, line $l \parallel m$ and n is a transversal. If $\angle 1 = 40°$, find all the angles and check that all corresponding angles and alternate angles are equal.

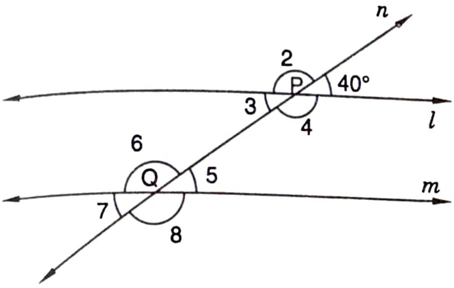


Fig. 62

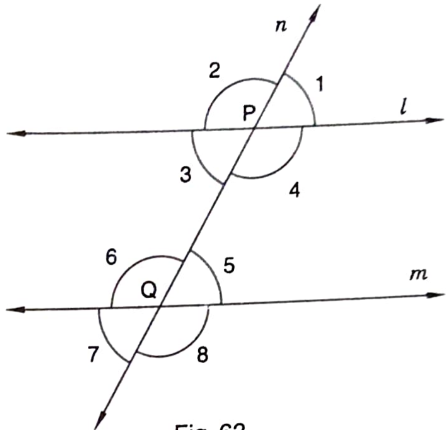


Fig. 63

6. In Fig. 63, line $l \parallel m$ and a transversal n cuts them at P and Q respectively. If $\angle 1 = 75°$, find all other angles.

7. In Fig. 64, $AB \parallel CD$ and a transversal PQ cuts them at L and M respectively. If $\angle QMD = 100°$, find all other angles.

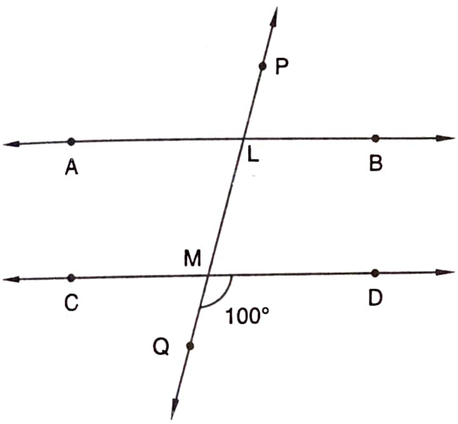


Fig. 64

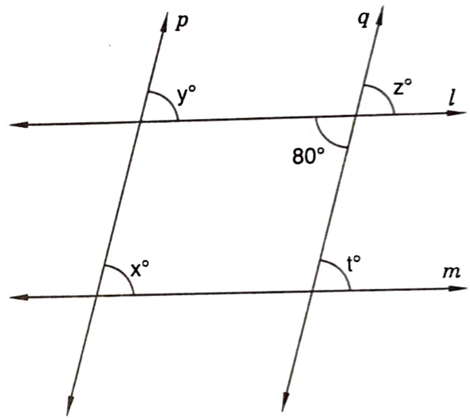


Fig. 65

8. In Fig. 65, $l \parallel m$ and $p \parallel q$. Find the values of x, y, z, t.

9. In Fig. 66, line $l \parallel m$, $\angle 1 = 120°$ and $\angle 2 = 100°$, find out $\angle 3$ and $\angle 4$.

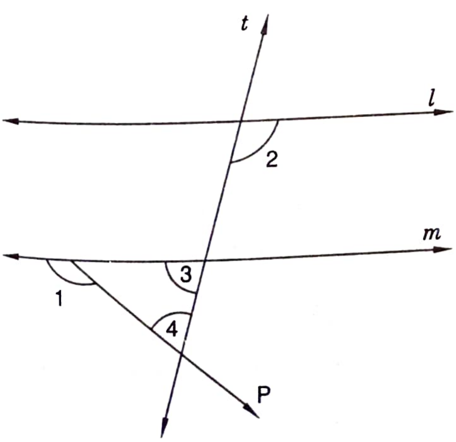


Fig. 66

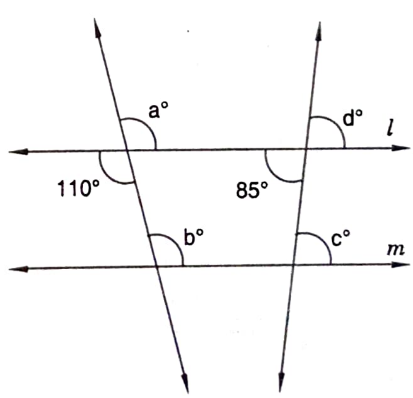


Fig. 67

10. In Fig. 67, line $l \parallel m$. Find the values of a, b, c, d. Give reasons.

11. In Fig. 68, $AB \parallel CD$ and $\angle 1$ and $\angle 2$ are in the ratio 3: 2. Determine all angles from 1 to 8.

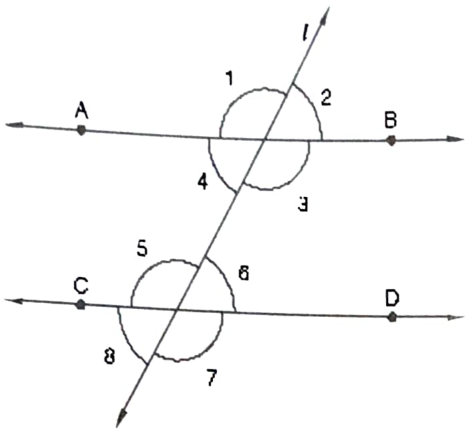


Fig. 68

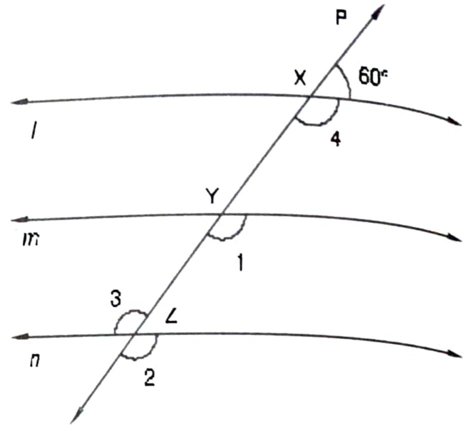


Fig. 69

12. In Fig.69, l, m and n are parallel lines intersected by transversal p at X, Y and Z respectively. Find $\angle 1$, $\angle 2$ and $\angle 3$.

13. In Fig. 70, if $l \parallel m \parallel n$ and $\angle 1 = 60°$, find $\angle 2$.

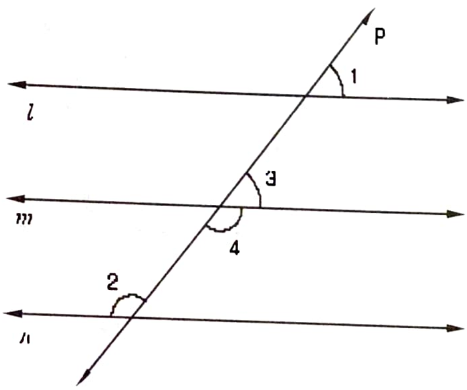


Fig. 70

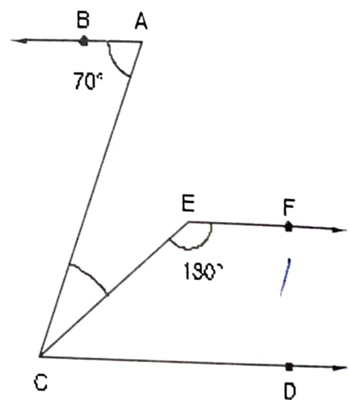


Fig. 71

14. In Fig. 71, if $AB \parallel CD$ and $CD \parallel EF$, find $\angle ACE$.

15. In Fig. 72, if $l \parallel m$, $n \parallel p$ and $\angle 1 = 85°$, find $\angle 2$.

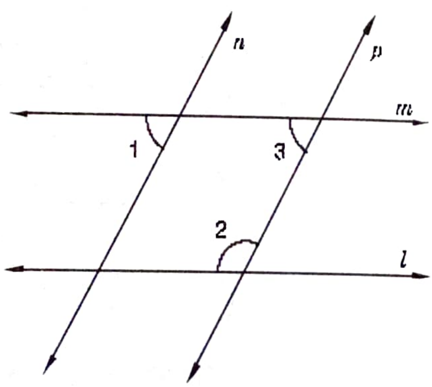


Fig. 72

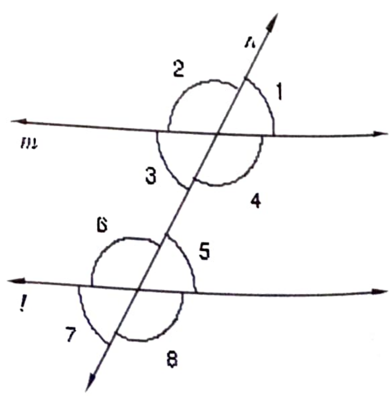


Fig. 73

16. In Fig. 73, a transversal n cuts two lines l and m. If $\angle 1 = 70°$ and $\angle 7 = 80°$, is $l \parallel m$?

17. In Fig. 74, a transversal n cuts two lines l and m such that $\angle 2 = 65^\circ$ and $\angle 8 = 65^\circ$. Are the lines parallel?

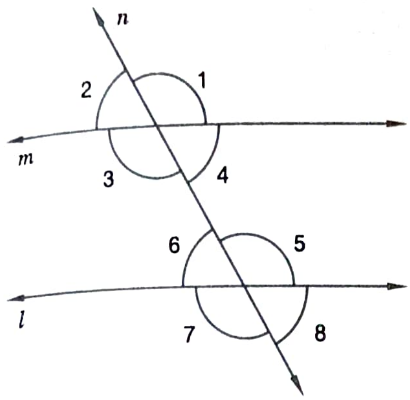


Fig. 74

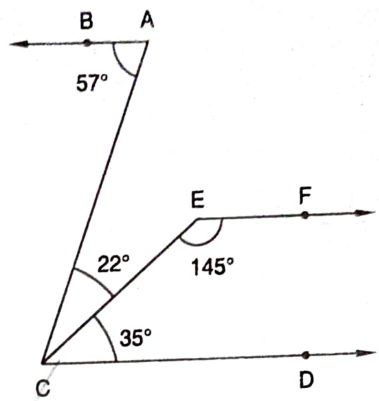


Fig. 75

18. In Fig. 75, show that $AB \parallel EF$.

19. In Fig. 76, $AB \parallel CD$. Find the values of x, y, z.

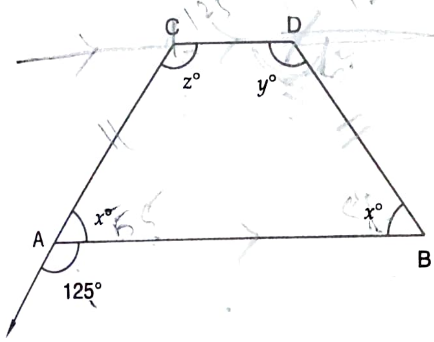


Fig. 76

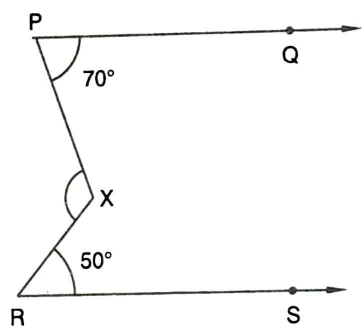


Fig. 77

20. In Fig. 77, find out $\angle PXR$, if $PQ \parallel RS$.

21. In Fig. 78, we have

(i) $\angle MLY = 2\angle LMQ$, find $\angle LMQ$.
(ii) $\angle XLM = (2x - 10)^\circ$ and $\angle LMQ = x + 30^\circ$, find x.
(iii) $\angle XLM = \angle PML$, find $\angle ALY$
(iv) $\angle ALY = (2x - 15)^\circ$, $\angle LMQ = (x + 40)^\circ$, find x

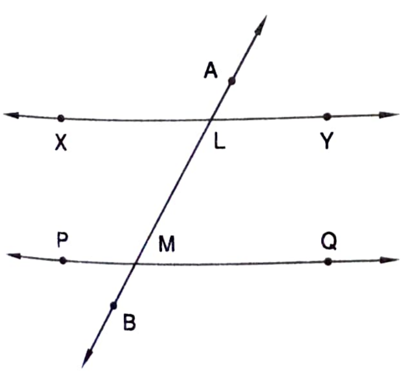


Fig. 78

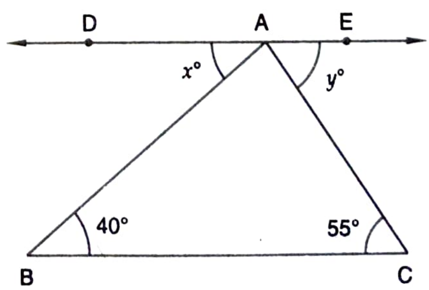


Fig. 79

22. In Fig. 79, $DE \parallel BC$. Find the values of x and y.

23. In Fig. 80, line $AC \parallel$ line DE and $\angle ABD = 32°$. Find out the angles x and y if $\angle E = 122°$.

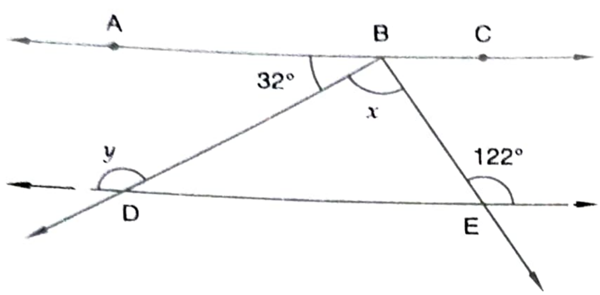


Fig. 80

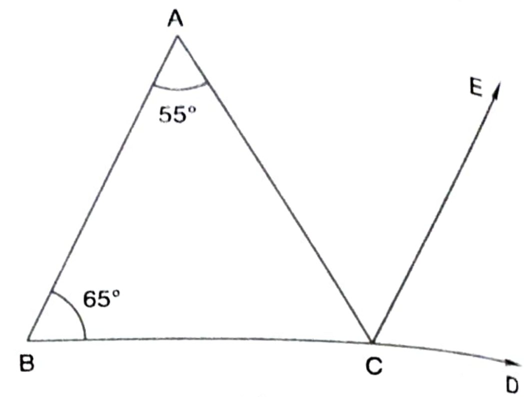


Fig. 81

24. In Fig. 81, side BC of ΔABC has been produced to D and $CE \parallel BA$. If $\angle ABC = 65°$, $\angle BAC = 55°$, find $\angle ACE$, $\angle ECD$ and $\angle ACD$.

25. In Fig. 82, line $CA \perp AB \parallel$ line CR and line $PR \parallel$ line BD. Find $\angle x$, $\angle y$ and $\angle z$.

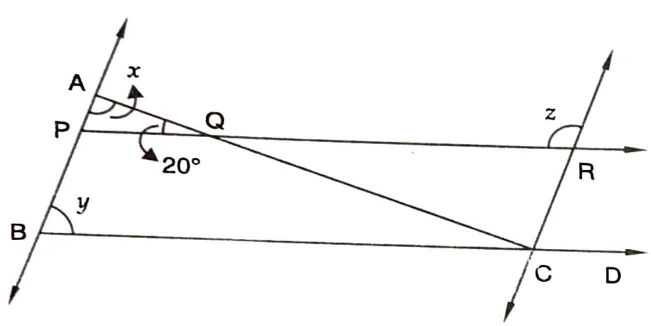


Fig. 82

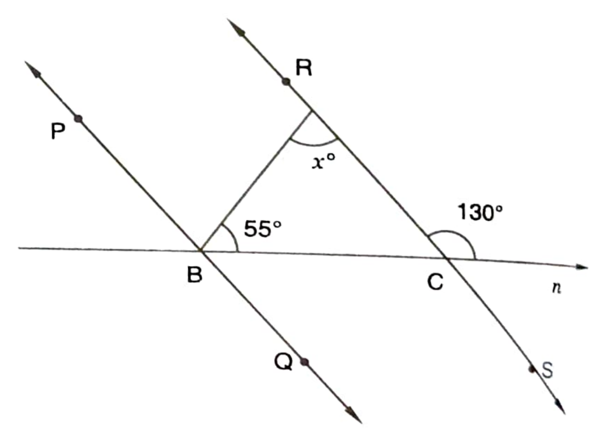


Fig. 83

26. In Fig. 83, $PQ \parallel RS$. Find the value of x.

27. In Fig. 84, $AB \parallel CD$ and $AE \parallel CF$; $\angle FCG = 90°$ and $\angle BAC = 120°$. Find the values of x, y and z.

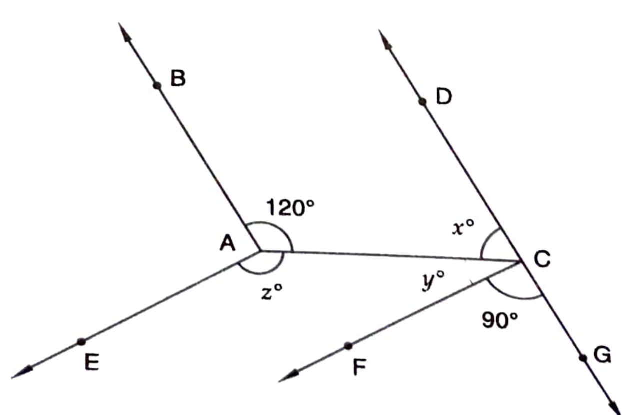


Fig. 84

28. In Fig. 85, $AB \parallel CD$ and $AC \parallel BD$. Find the values of x, y, z.

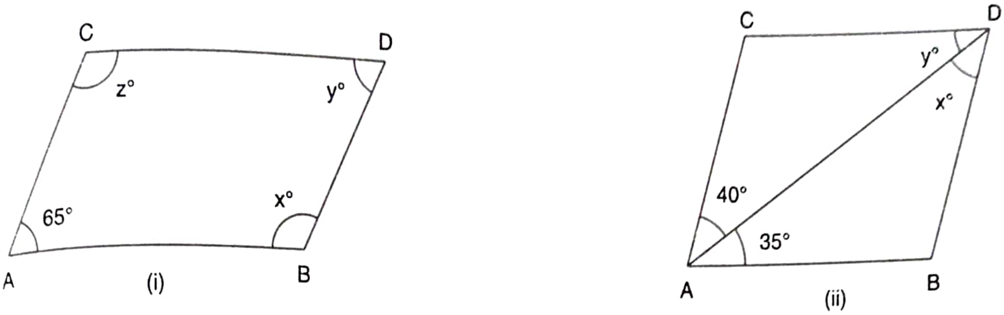


Fig. 85

29. In Fig. 86, state which lines are parallel and why?

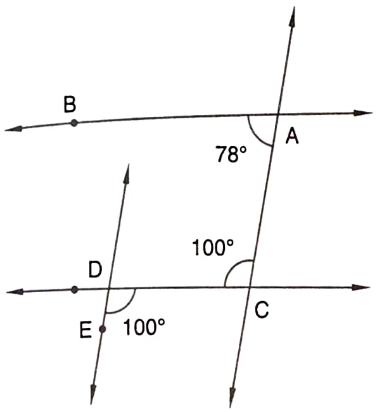


Fig. 86

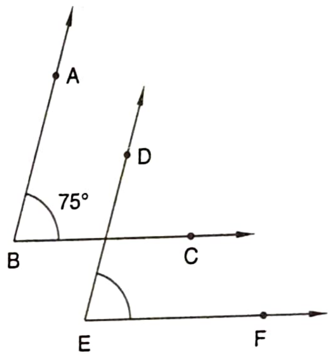


Fig. 87

30. In Fig. 87, the corresponding arms of $\angle ABC$ and $\angle DEF$ are parallel. If $\angle ABC = 75°$, find $\angle DEF$.

ANSWERS

1. (i) Alternate angles: $\angle BGH$ and $\angle CHG$; $\angle AGH$ and $\angle DHG$
   Corresponding angles: $\angle EGB$ and $\angle GHD$; $\angle EGA$ and $\angle GHC$; $\angle BGH$ and $\angle DHF$; $\angle AGH$ and $\angle CHF$.
   (ii) $\angle e, \angle b, \angle c, \angle a$ (iii) $\angle QRA, \angle BRA, \angle BRA$
   (iv) Interior angles: $\angle d, \angle f$; $\angle a, \angle e$; Exterior angles: $\angle c, \angle g$; $\angle b, \angle h$.
2. $\angle QMD = 120°, \angle PLB = 60°, \angle ALM = 60°, \angle MLB = 120°, \angle DML = 60°, \angle CML = 120°, \angle ALP = 120°$
3. $\angle ALM = 35°, \angle PLA = 145°$ 4. $\angle 7, \angle 7, \angle 5$
5. $\angle 2 = 140°, \angle 3 = 40°, \angle 4 = 140°, \angle 5 = 40°, \angle 6 = 140°, \angle 7 = 40°, \angle 8 = 140°$
6. $\angle 2 = 105°$; $\angle 3 = 75°, \angle 4 = 105°, \angle 5 = 75°, \angle 6 = 105°, \angle 7 = 75°$ and $\angle 8 = 105°$
7. $\angle PLB = 80°, \angle PLA = 100°, \angle ALM = 80°, \angle BLM = 100°, \angle LMD = 80°, \angle LMC = 100°, \angle CMQ = 80°$
8. $x = 80°, y = 80°, z = 80°, t = 80°$ 9. $\angle 3 = 80°, \angle 4 = 40°$
10. $a = 110°, b = 110°, c = 85°, d = 85°$
11. $\angle 1 = 108°, \angle 2 = 72°, \angle 3 = 108°, \angle 4 = 72°, \angle 5 = 108°, \angle 6 = 72°, \angle 7 = 108°, \angle 8 = 72°$
12. $\angle 1 = 120°, \angle 2 = 120°, \angle 3 = 120°$ 13. 120° 14. 20° 15. 95°
16. No 17. Yes 19. $x = 55°, y = 125°, z = 125°$ 20. 120°
21. (i) 60° (ii) 40° (iii) 90° (iv) 55° 22. $x = 40°, y = 55°$
23. $x = 90°, y = 148°$ 24. $\angle ACE = 55°, \angle ECD = 65°, \angle ACD = 120°$
25. $x = 90°, y = 70°, z = 110°$ 26. 75° 27. $x = 60°, y = 30°, z = 150°$

28. (i) $x = 115°, y = 65°, z = 115°$ (ii) $x = 40°, y = 35°$

29. $AC \parallel DE$, because alternate angles are equal 30. 75°

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. The sum of an angle and one third of its supplementary angle is 90°. The measure of the angle is
   (a) 135° (b) 120° (c) 60° (d) 45°
2. If angles of a linear pair are equal, then the measure of each angle is
   (a) 30° (b) 45° (c) 60° (d) 90°
3. Two complemntary angles are in the ratio 2 : 3. The measure of the larger angle is
   (a) 60° (b) 54° (c) 66° (d) 48°
4. An angle is thrice its supplement. The measure of the angle is
   (a) 120° (b) 105° (c) 135° (d) 150°
5. In Fig. 88 *PR* is a straight line and $\angle PQS : \angle SQR = 7 : 5$. The measure of $\angle SQR$ is
   (a) 60° (b) $62\frac{1}{2}^{\circ}$ (c) $67\frac{1}{2}^{\circ}$ (d) 75°

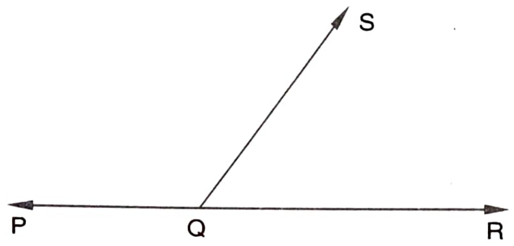


Fig. 88

6. The sum of an angle and half of its complementary angle is 75°. The measure of the angle is
   (a) 40° (b) 50° (c) 60° (d) 80°
7. $\angle A$ is an obtuse angle. The measure of $\angle A$ and twice its supplementary differ by 30°. Then, $\angle A$ can be
   (a) 150° (b) 110° (c) 140° (d) 120°
8. An angle is double of its supplement. The measure of the angle is
   (a) 60° (b) 120° (c) 40° (d) 80°
9. The measure of an angle which is its own complement is
   (a) 30° (b) 60° (c) 90° (d) 45°
10. Two supplementary angles are in the ratio 3 : 2. The smaller angle measures
    (a) 108° (b) 81° (c) 72° (d) 68°
11. In Fig. 89, the value of *x* is
    (a) 75 (b) 65 (c) 45 (d) 55

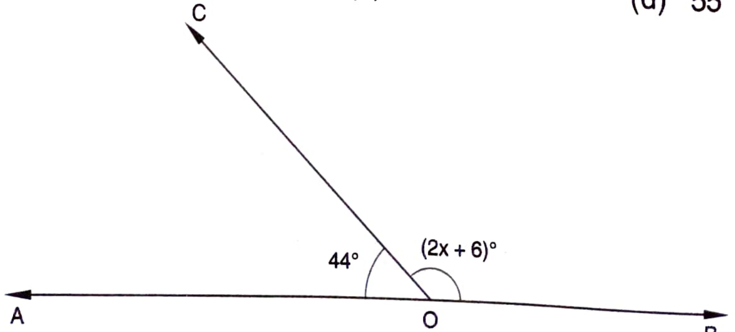


Fig. 89

12. In Fig. 90, AOB is a straight line and the ray OC stands on it. The value of x is
(a) 16 (b) 26 (c) 36 (d) 46

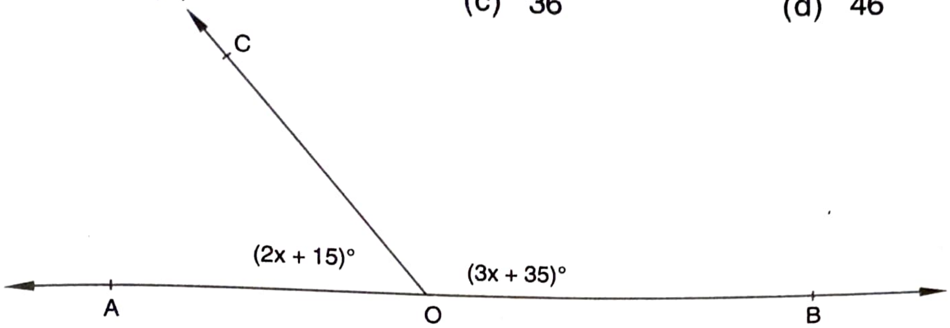


Fig. 90

13. In Fig. 91, AOB is a straight line and $4x = 5y$. The value of x is
(a) 100 (b) 105 (c) 110 (d) 115

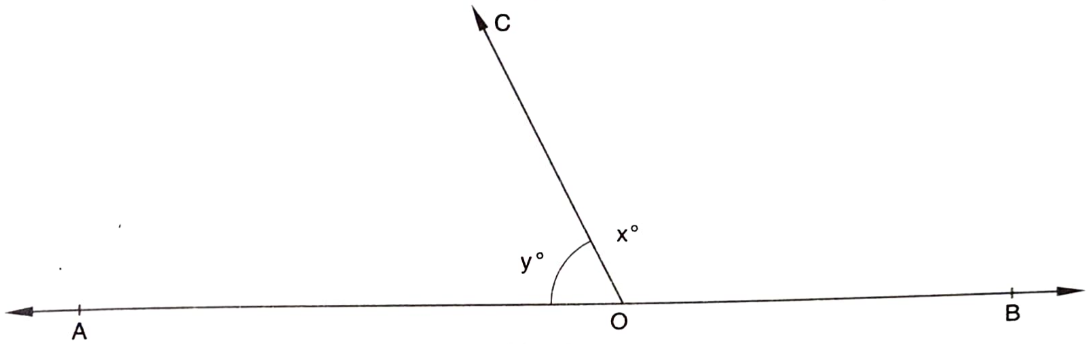


Fig. 91

14. In Fig. 92, AOB is a straight line such that $\angle AOC = (3x + 10)°$, $\angle COD = 50°$ and $\angle BOD = (x - 8)°$. The value of x is
(a) 32 (b) 36 (c) 42 (d) 52

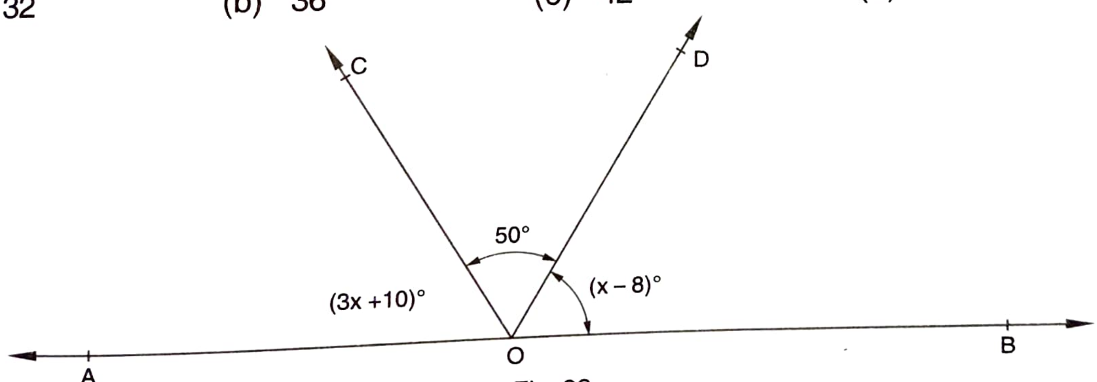


Fig. 92

15. In Fig. 93, if AOC is a straight line, then $x =$
(a) 42° (b) 52° (c) 142° (d) 38°

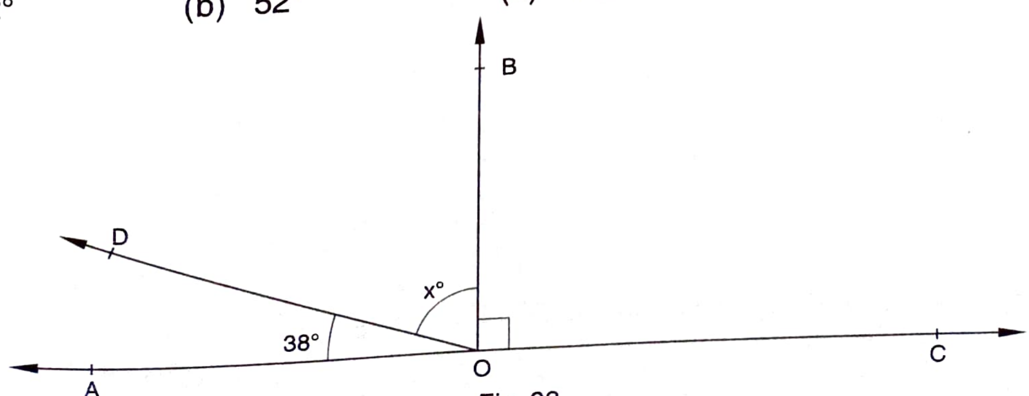


Fig. 93

16. In Fig. 94, if $\angle AOC$ is a straight line, then the value of x is
(a) 15 (b) 18 (c) 20 (d) 16

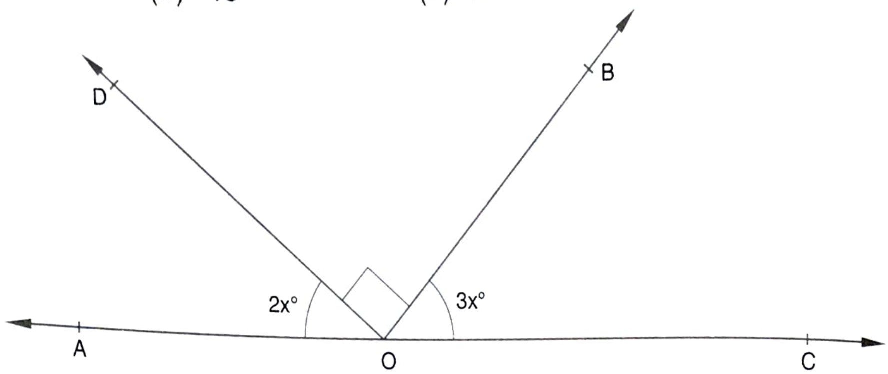


Fig. 94

17. In Fig. 95, if AB, CD and EF are straight lines, then $x =$
(a) 5 (b) 10 (c) 20 (d) 30

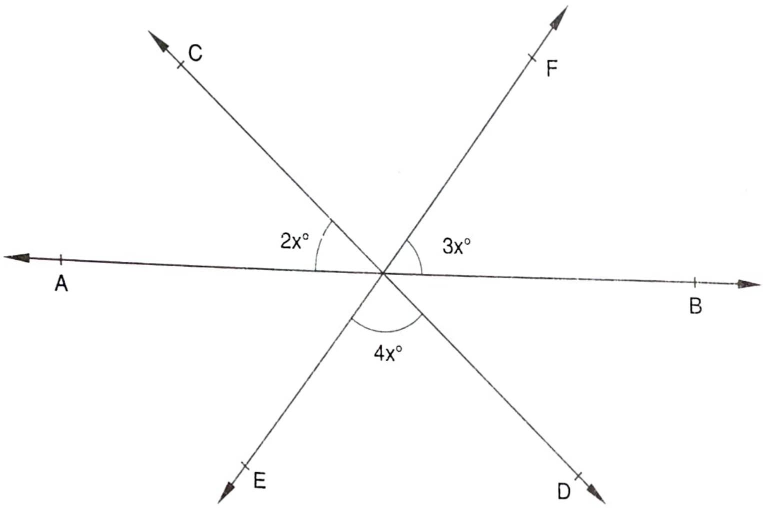


Fig. 95

18. In Fig. 96, if AB, CD and EF are straight lines, then $x + y + z =$
(a) 180 (b) 203 (c) 213 (d) 134

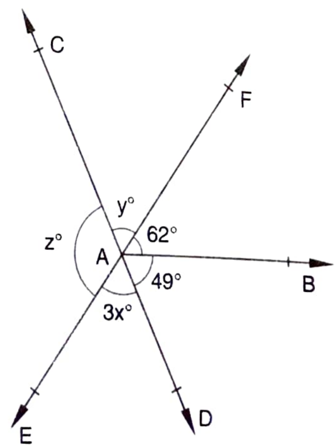


Fig. 96

19. In Fig. 97, if AB is parallel to CD, then the value of $\angle BPE$ is
(a) 106° (b) 76° (c) 74° (d) 84°

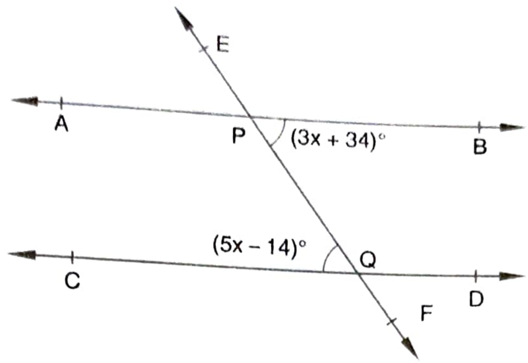


Fig. 97

20. In Fig. 98, if *AB* is parallel to *CD* and *EF* is a transversal, then $x =$

(a) 19 (b) 29 (c) 39 (d) 49

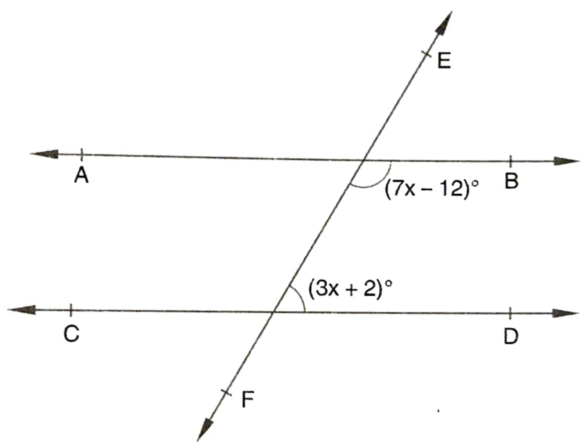


Fig. 98

21. In Fig. 99, $AB \parallel CD$ and *EF* is a transversal intersecting *AB* and *CD* at *P* and *Q* respectively. The measure of $\angle DPQ$ is

(a) 100° (b) 80° (c) 110° (d) 70°

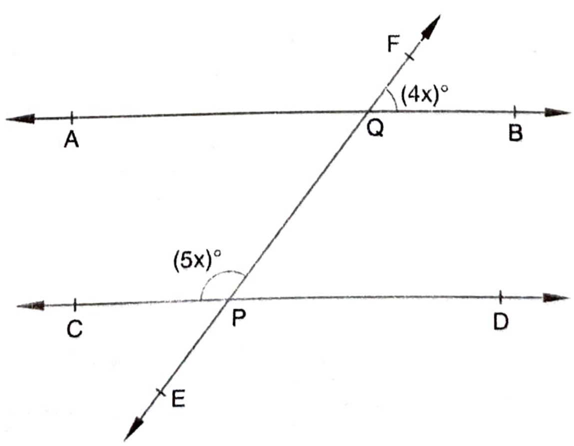


Fig. 99

22. In Fig. 100, $AB \parallel CD$ and EF is a transversal intersecting AB and CD at P and Q respectively. The measure of $\angle DQP$ is

(a) 65 (b) 25 (c) 115 (d) 105

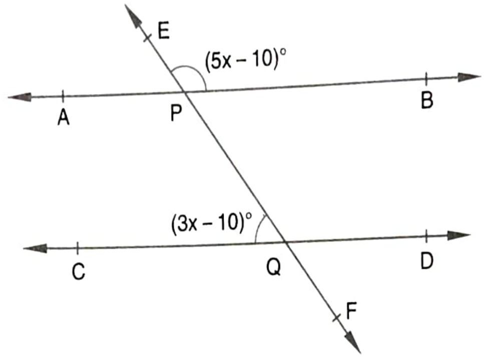


Fig. 100

23. In Fig. 101, $AB \parallel CD$ and EF is a transversal. The value of $y - x$ is

(a) 30 (b) 35 (c) 95 (d) 25

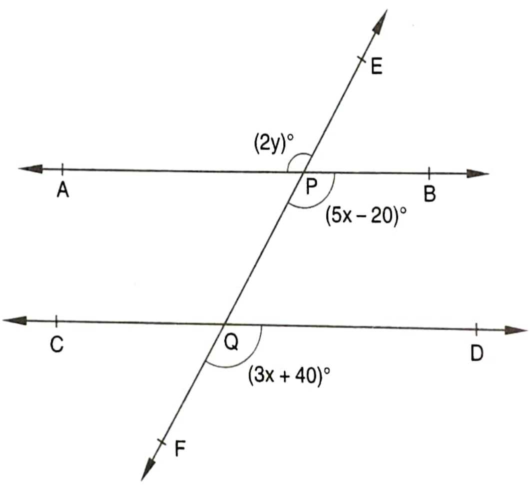


Fig. 101

24. In Fig. 102, $AB \parallel CD \parallel EF$, $\angle ABG = 110°$, $\angle GCD = 100°$ and $\angle BGC = x°$. The value of x is

(a) 35 (b) 50 (c) 30 (d) 40

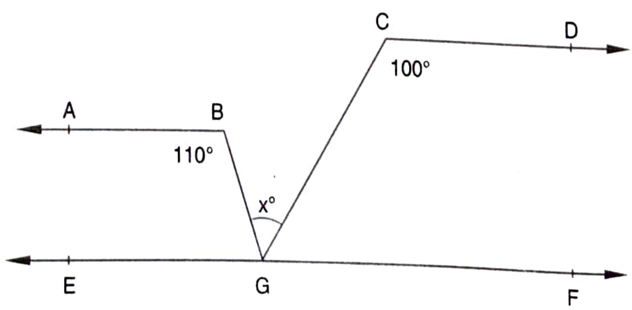


Fig. 102

25. In Fig. 103, $PQ \parallel RS$ and $\angle PAB = 60^\circ$ and $\angle ACS = 100^\circ$. Then, $\angle BAC =$

(a) 40° (b) 60° (c) 80° (d) 50°

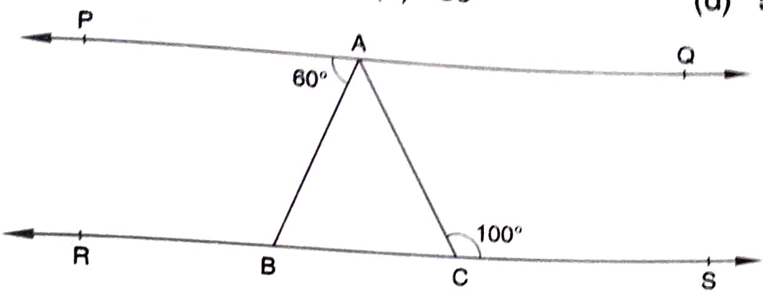


Fig. 103

26. In Fig. 104, $AB \parallel CD$, $\angle OAB = 150^\circ$ and $\angle OCD = 120^\circ$. Then, $\angle AOC =$

(a) 80° (b) 90° (c) 70° (d) 100°

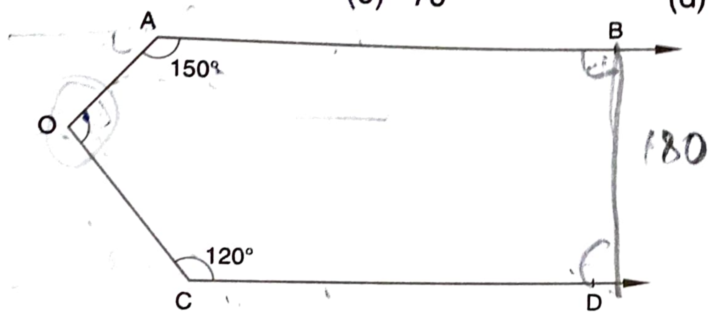


Fig. 104

27. In Fig. 105, if AOB and COD are straight lines. Then, $x + y =$

(a) 120 (b) 140 (c) 100 (d) 160

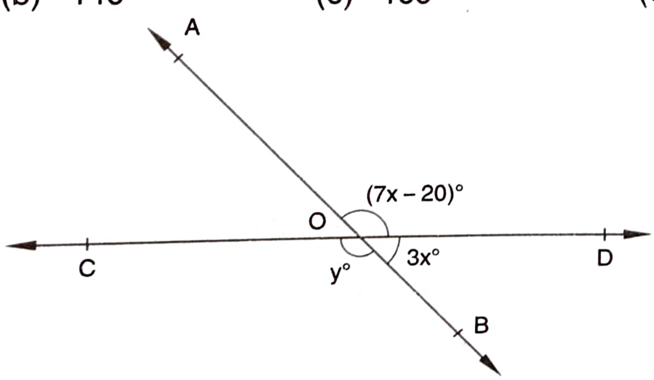


Fig. 105

28. In Fig. 106, the value of x is

(a) 22 (b) 20 (c) 21 (d) 24

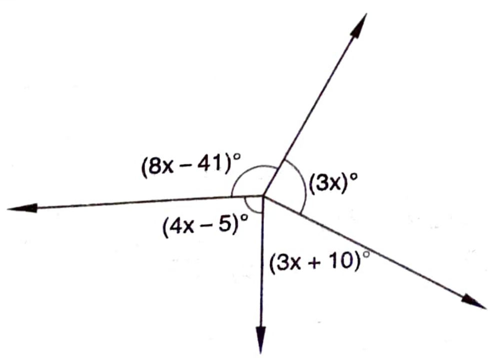


Fig. 106

29. In Fig. 107, if *AOB* and *COD* are straight lines, then

(a) $x = 29,\ y = 100$ (b) $x = 110,\ y = 29$

(c) $x = 29,\ y = 110$ (d) $x = 39,\ y = 110$

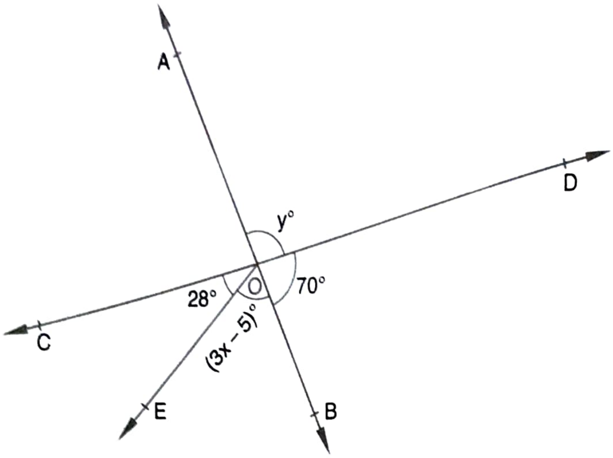


Fig. 107

30. In Fig. 108, if $AB \parallel CD$ then the value of x is

(a) 87 (b) 93 (c) 147 (d) 141

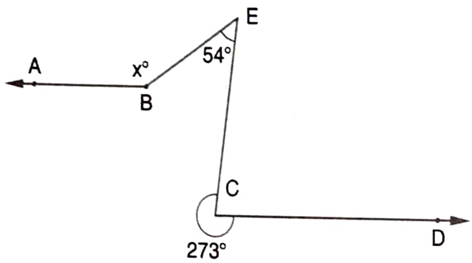


Fig. 108

31. In Fig. 109, if $AB \parallel CD$ then the value of x is

(a) 34 (b) 124 (c) 24 (d) 158

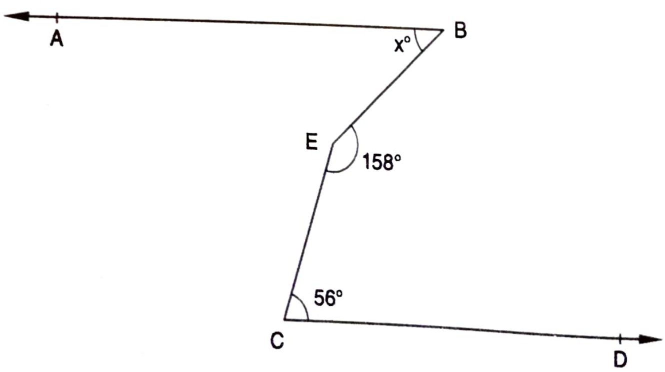


Fig. 109

32. In Fig. 110, if $AB \parallel CD$. The value of x is

(a) 122 (b) 238 (c) 58 (d) 119

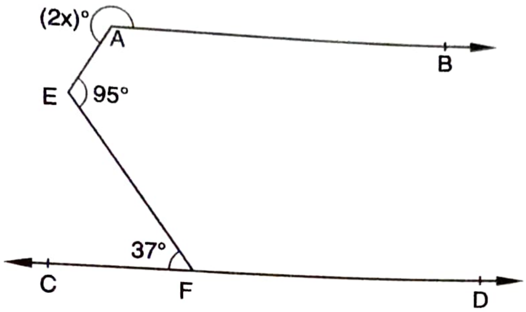


Fig. 110

33. In Fig. 111, if $AB \parallel CD$ then $x =$

(a) 154 (b) 139 (c) 144 (d) 164

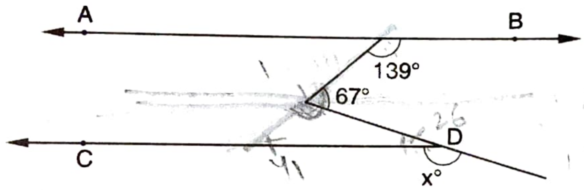


Fig. 111

34. In Fig. 112, if $AB \parallel CD$, then $x =$

(a) 32 (b) 42 (c) 52 (d) 31

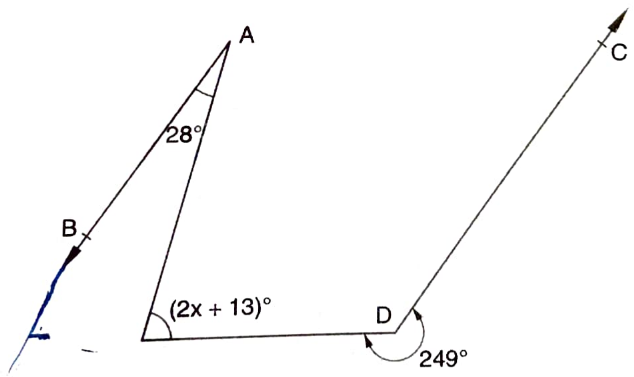


Fig. 112

35. In Fig. 113 if $AC \parallel DF$ and $AB \parallel CE$, then

(a) $x = 145, \; y = 223$ (b) $x = 223, \; y = 145$

(c) $x = 135, \; y = 233$ (d) $x = 233, \; y = 135$

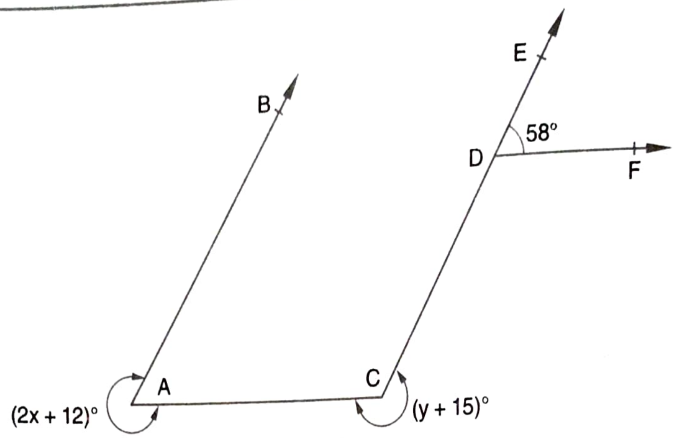


Fig. 113

ANSWSERS

1. (d)	2. (d)	3. (b)	4. (c)	5. (d)	6. (c)	7. (b)
8. (b)	9. (d)	10. (c)	11. (b)	12. (b)	13. (a)	14. (a)
15. (b)	16. (b)	17. (c)	18. (b)	19. (c)	20. (a)	21. (b)
22. (c)	23. (b)	24. (c)	25. (a)	26. (b)	27. (b)	28. (a)
29. (c)	30. (c)	31. (a)	32. (d)	33. (a)	34. (b)	35. (a)

THINGS TO REMEMBER

1. *A line which intersects two or more given lines at distinct points is called a transversal to the given lines.*
2. *Lines in a plane are parallel if they do not intersect when produced indefinitely in either direction.*
3. *The distance between two intersecting lines is zero.*
4. *The distance between two parallel lines is the same everywhere and is equal to the perpendicular distance between them.*
5. *If two parallel lines are intersected by a transversal then*
   - *(i) pairs of alternate (interior or exterior) angles are equal.*
   - *(ii) pairs of corresponding angles are equal.*
   - *(iii) interior angles on the same side of the transversal are supplementary.*
6. *If two non-parallel lines are intersected by transversal then none of (i), (ii) and (iii) hold true in 5.*
7. *If two lines are intersected by a transversal, then they are parallel if any one of the following is true:*
   - *(i) The angles of a pair of corresponding angles are equal.*
   - *(ii) The angles of a pair of alternate interior angles are equal.*
   - *(iii) The angles of a pair of interior angles on the same side of the transversal are supplementary.*