5

OPERATIONS ON RATIONAL NUMBERS

5.1 INTRODUCTION

In this chapter, we shall learn about the operations of addition, subtraction, multiplication and division on rational numbers. We shall also learn about the properties of these operations on rational numbers.

5.2 ADDITION OF RATIONAL NUMBERS

In this section, we shall define the operation of addition of rational numbers. The addition of rational numbers is carried out in the same way as that of addition of fractions which you have learnt in earlier classes. If two rational numbers are to be added, we first express each one of them as rational numbers with positive denominator. For addition purpose, we divide the rational numbers into the following two categories:

5.2.1 RATIONAL NUMBERS WITH SAME DENOMINATORS

In order to add two rational numbers having the same denominator, we follow the following steps:

STEP I *Obtain the numerators of two given rational numbers and their common denominator.*

STEP II *Add the numerators obtained in step I*

STEP III *Write a rational number whose numerator is the sum obtained in step II, and whose denominator is the common denominator of the given rational numbers.*

It follows from the above steps that if $\frac{p}{q}$ and $\frac{r}{q}$ are two rational numbers with the same denominator, then

$$\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q}$$

ILLUSTRATIVE EXAMPLES

Example 1 Add $\frac{3}{5}$ and $\frac{13}{5}$.

Solution We have,

$$\frac{3}{5}+\frac{13}{5}=\frac{3+13}{5}=\frac{16}{5} \qquad [\because 3+13=16]$$

Example 2 Add $\frac{7}{9}$ and $\frac{-12}{9}$.

Solution We have,

$$\frac{7}{9}+\frac{-12}{9}=\frac{7+(-12)}{9}=\frac{-5}{9} \qquad [\because 7+(-12)=-5]$$

Example 3 Add $\frac{-5}{9}$ and $\frac{-17}{9}$.

Solution We have,

$$\frac{-5}{9}+\frac{-17}{9}=\frac{(-5)+(-17)}{9}=\frac{-22}{9} \qquad [\because (-5)+(-17)=-22]$$

Example 4 Add $\frac{4}{-11}$ and $\frac{7}{11}$.

Solution We first express $\frac{4}{-11}$ as a rational number with positive denominator.

We have, $\frac{4}{-11}=\frac{4\times(-1)}{(-11)\times(-1)}=\frac{-4}{11}$

Now, $\frac{4}{-11}+\frac{7}{11}=\frac{-4}{11}+\frac{7}{11}=\frac{(-4)+7}{11}=\frac{3}{11}$

5.2.2 NUMBERS WITH DISTINCT DENOMINATORS

To find the sum of two rational numbers which do not have the same denominator, we follow the following steps:

STEP I *Obtain the rational numbers and see whether their denominators are positive or not. If the denominator of one (or both) of the numbers is negative, re–write it so that the denominator becomes positive.*

STEP II *Obtain the denominators of the rational numbers in step I.*

STEP III *Find the LCM of the denominators obtained in step II.*

STEP IV *Express each one of the rational numbers in step I so that the LCM obtained in step III becomes their common denominator.*

STEP V *Write a rational number whose numerator is equal to the sum of the numerators of rational numbers obtained in step IV and denominators as the LCM obtained in step III.*

STEP VI *The rational number obtained in step V is the required sum.*

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Add $\frac{5}{12}$ and $\frac{3}{8}$.

Solution Clearly, denominators of the given numbers are positive.

The L.C.M of denominators 12 and 8 is 24.

Now, we express $\frac{5}{12}$ and $\frac{3}{8}$ into forms in which both of them have the same denominator 24.

We have,

$$\frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24} \text{ and, } \frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24}$$

$$\therefore \quad \frac{5}{12}+\frac{3}{8}=\frac{10}{24}+\frac{9}{24}=\frac{10+9}{24}=\frac{19}{24}$$

Example 2 Add $\frac{7}{9}$ and 4.

Solution We have, $4 = \frac{4}{1}$

Clearly, denominators of the two rational numbers are positive. We now re-write them so that they have a common denominator equal to the LCM of the denominators. LCM of 9 and 1 is 9.

We have, $\frac{4}{1} = \frac{4 \times 9}{1 \times 9} = \frac{36}{9}$

$$\therefore \quad \frac{7}{9} + 4 = \frac{7}{9} + \frac{4}{1} = \frac{7}{9} + \frac{36}{9} = \frac{7+36}{9} = \frac{43}{9}$$

Example 3 Add $\frac{3}{8}$ and $\frac{-5}{12}$

Solution The denominators of the given rational numbers are 8 and 12 respectively. The LCM of 8 and 12 is 24.

Now, we rewrite the given rational numbers into forms in which both of them have the same denominator.

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24} \text{ and, } \frac{-5}{12} = \frac{-5 \times 2}{12 \times 2} = \frac{-10}{24}$$

$$\therefore \quad \frac{3}{8} + \frac{-5}{12} = \frac{9}{24} + \frac{(-10)}{24} = \frac{9-10}{24} = \frac{-1}{24}$$

Example 4 Simplify: $\frac{8}{-15} + \frac{4}{-3}$

Solution We have,

$$\frac{8}{-15} + \frac{4}{-3} = \frac{-8}{15} + \frac{-4}{3} \quad \left[\because \frac{8}{-15} = \frac{8 \times (-1)}{(-15) \times (-1)} = \frac{-8}{15} \text{ and, } \frac{4}{-3} = \frac{4 \times (-1)}{(-3) \times (-1)} = \frac{-4}{3}\right]$$

LCM of 15 and 3 is 15.

Rewriting $\frac{-4}{3}$ in the form in which it has denominator 15, we get

$$\frac{-4}{3} = \frac{-4 \times 5}{3 \times 5} = \frac{-20}{15}$$

$$\therefore \quad \frac{8}{-15} + \frac{4}{-3} = \frac{-8}{15} + \frac{-4}{3}$$

$$\Rightarrow \quad \frac{8}{-15} + \frac{4}{-3} = \frac{-8}{15} + \frac{-20}{15} \quad \left[\because \frac{4}{-3} = \frac{-20}{15}\right]$$

$$\Rightarrow \quad \frac{8}{-15} + \frac{4}{-3} = \frac{(-8)+(-20)}{15} = \frac{-28}{15}$$

Example 5 Simplify: $\frac{7}{-26} + \frac{16}{39}$

Solution We have,

$$\frac{7}{-26}+\frac{16}{39}=\frac{-7}{26}+\frac{16}{39} \qquad \left[\because \frac{7}{-26}=\frac{7\times(-1)}{(-26)\times(-1)}=\frac{-7}{26}\right]$$

LCM of 26 and 39 is 78.

Rewriting $\frac{-7}{26}$ and $\frac{16}{39}$ in forms having the same denominator 78, we get

$$\frac{7}{-26}=\frac{-7\times 3}{26\times 3}=\frac{-21}{78},\ \frac{16}{39}=\frac{16\times 2}{39\times 2}=\frac{32}{78}$$

$$\therefore \quad \frac{7}{-26}+\frac{16}{39}=\frac{-7}{26}+\frac{16}{39}=\frac{-21}{78}+\frac{32}{78}=\frac{11}{78}$$

<u>Remark:</u> *If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that q and s do not have a common factor other than 1, i.e., HCF of q and s is 1, then*

$$\frac{p}{q}+\frac{r}{s}=\frac{p\times s+r\times q}{q\times s}$$

For example, $\frac{5}{18}+\frac{3}{13}=\frac{5\times 13+3\times 18}{18\times 13}=\frac{65+54}{234}=\frac{119}{234}$

and, $\frac{-2}{11}+\frac{3}{14}=\frac{(-2)\times 14+(3\times 11)}{11\times 14}=\frac{-28+33}{154}=\frac{5}{154}$

EXERCISE 5.1

1. Add the following rational numbers:

 (i) $\frac{-5}{7}$ and $\frac{3}{7}$ (ii) $\frac{-15}{4}$ and $\frac{7}{4}$ (iii) $\frac{-8}{11}$ and $\frac{-4}{11}$ (iv) $\frac{6}{13}$ and $\frac{-9}{13}$

2. Add the following rational numbers:

 (i) $\frac{3}{4}$ and $\frac{-3}{5}$ (ii) -3 and $\frac{3}{5}$ (iii) $\frac{-7}{27}$ and $\frac{11}{18}$ (iv) $\frac{31}{-4}$ and $\frac{-5}{8}$

3. Simplify:

 (i) $\frac{8}{9}+\frac{-11}{6}$ (ii) $\frac{-5}{16}+\frac{7}{24}$ (iii) $\frac{1}{-12}+\frac{2}{-15}$ (iv) $\frac{-8}{19}+\frac{-4}{57}$

4. Add and express the sum as a mixed fraction:

 (i) $\frac{-12}{5}+\frac{43}{10}$ (ii) $\frac{24}{7}+\frac{-11}{4}$ (iii) $\frac{-31}{6}+\frac{-27}{8}$

ANSWERS

1. (i) $\frac{-2}{7}$ (ii) -2 (iii) $\frac{-12}{11}$ (iv) $\frac{-3}{13}$
2. (i) $\frac{3}{20}$ (ii) $\frac{-12}{5}$ (iii) $\frac{19}{54}$ (iv) $\frac{-67}{8}$
3. (i) $\frac{-17}{18}$ (ii) $\frac{-1}{48}$ (iii) $\frac{-13}{60}$ (iv) $\frac{-28}{57}$
4. (i) $1\frac{9}{10}$ (ii) $\frac{19}{28}$ (iii) $-8\frac{13}{24}$

5.3 SUBTRACTION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then subtracting $\frac{c}{d}$ from $\frac{a}{b}$ means adding additive inverse (negative) of $\frac{c}{d}$ to $\frac{a}{b}$. The subtraction of $\frac{c}{d}$ from $\frac{a}{b}$ is written as $\frac{a}{b}-\frac{c}{d}$.

Thus, we have

$$\frac{a}{b}-\frac{c}{d}=\frac{a}{b}+\left(\frac{-c}{d}\right) \qquad \left[\because \text{ Additive inverse of } \frac{c}{d} \text{ is } \frac{-c}{d}\right]$$

ILLUSTRATIVE EXAMPLES

Example 1 Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

Solution The additive inverse of $\frac{3}{4}$ is $\frac{-3}{4}$

$$\therefore \quad \frac{5}{6}-\frac{3}{4}=\frac{5}{6}+\frac{(-3)}{4}$$

$$\Rightarrow \quad \frac{5}{6}-\frac{3}{4}=\frac{5\times2}{6\times2}+\frac{(-3)\times3}{4\times3}=\frac{10}{12}+\frac{-9}{12}=\frac{10+(-9)}{12}=\frac{1}{12}$$

Example 2 Subtract $\frac{-3}{8}$ from $\frac{-5}{7}$.

Solution The additive inverse of $\frac{-3}{8}$ is $\frac{3}{8}$

$$\therefore \quad \frac{-5}{7}-\left(\frac{-3}{8}\right)=\frac{-5}{7}+\frac{3}{8} \qquad \left[\because -\left(\frac{-3}{8}\right)=\frac{3}{8}\right]$$

$$\Rightarrow \quad \frac{-5}{7}-\left(\frac{-3}{8}\right)=\frac{(-5)\times8+3\times7}{56}=\frac{-40+21}{56}=\frac{-19}{56}$$

Example 3 Subtract $\frac{-3}{5}$ from $\frac{9}{10}$.

Solution The additive inverse of $\frac{-3}{5}=\frac{3}{5}$

$$\therefore \quad \frac{9}{10}-\left(\frac{-3}{5}\right)=\frac{9}{10}+\frac{3}{5}=\frac{9+3\times2}{10}=\frac{9+6}{10}=\frac{15}{10}$$

$$\Rightarrow \quad \frac{9}{10}-\left(\frac{-3}{5}\right)=\frac{3}{2}$$ [Dividing numerator and denominator by 5]

Example 4 The sum of two rational numbers is $\frac{-3}{5}$. If one of the number is $\frac{-9}{20}$, find the other.

Solution Sum of the numbers $= \frac{-3}{5}$, One number $= \frac{-9}{20}$

$\therefore$ The other number = Sum − One number

$$= \frac{-3}{5} - \left(\frac{-9}{20}\right)$$

$$= \frac{-3}{5} + \frac{9}{20} \qquad \left[\because -\left(\frac{-9}{20}\right) = \frac{9}{20}\right]$$

$$= \frac{(-3)\times 4 + 9\times 1}{20} = \frac{-12+9}{20} = \frac{-3}{20}$$

Example 5 What number should be added to $\frac{-5}{8}$ so as to get $\frac{5}{9}$?

Solution Sum of the given number and the required number $= \frac{5}{9}$

Given number $= \frac{-5}{8}$

$\therefore$ Required number = Sum − Given number

$$= \frac{5}{9} + \frac{5}{8} = \frac{5\times 8 + 5\times 9}{72} = \frac{40+45}{72} = \frac{85}{72}$$

Example 6 What should be subtracted from $\frac{-3}{4}$ so as to get $\frac{5}{6}$?

Solution Difference of the given number and the required number $= \frac{-5}{6}$

Given number $= \frac{-3}{4}$

$\therefore$ Required number $= \frac{-3}{4} - \frac{5}{6}$

$$= \frac{-3}{4} + \frac{-5}{6} = \frac{(-3)\times 3 + (-5)\times 2}{12} = \frac{(-9)+(-10)}{12} = \frac{-19}{12}$$

In order to simplify expressions involving the sum or difference of three or more rational numbers, we may use the following steps:

STEP I *Find the LCM of the denominator of all the numbers involved.*

STEP II *Write a rational number whose denominator is the LCM obtained in step I and numerator is computed as follows:*

Divide the LCM obtained in step I by the denominator of first rational number and get a quotient. Multiply the numerator of first rational number by this quoitent. Repeat this procedure for all rational numbers. Retain the given signs of addition and subtraction between the given rational numbers and get an expression involving integers. Simplify this expression to get an integer as the numerator.

STEP III *Reduce the rational number obtained in step II to the lowest form if it is not already so. This rational number so obtained is the required rational number.*

Following examples will illustrate the above procedure.

Example 7 Simplify :

(i) $\frac{-2}{3}+\frac{5}{9}-\frac{-7}{6}$ (ii) $\frac{5}{12}+\frac{-5}{18}-\frac{7}{24}$

Solution (i) We have,

$$\frac{-2}{3}+\frac{5}{9}-\frac{-7}{6}=\frac{-2}{3}+\frac{5}{9}+\frac{7}{6} \quad \left[\because -\left(\frac{-7}{6}\right)=\frac{7}{6}\right]$$

$$\Rightarrow \frac{-2}{3}+\frac{5}{9}-\frac{-7}{6}=\frac{(-2)\times 6+5\times 2+7\times 3}{18}=\frac{-12+10+21}{18}=\frac{-12+31}{18}=\frac{19}{18}$$

(ii) We have,

$$\frac{5}{12}+\frac{-5}{18}-\frac{7}{24}=\frac{5}{12}+\frac{-5}{18}+\frac{-7}{24}$$

$$\Rightarrow \frac{5}{12}+\frac{-5}{18}-\frac{7}{24}=\frac{5\times 6+(-5)\times 4+(-7)\times 3}{72} \quad [\because \text{LCM of 12, 18 and 24 is 72}]$$

$$\Rightarrow \frac{5}{12}+\frac{-5}{18}-\frac{7}{24}=\frac{30+(-20)+(-21)}{72}=\frac{30+(-41)}{72}=\frac{-11}{72}$$

EXERCISE 5.2

1. Subtract the first rational number from the second in each of the following:

   (i) $\frac{3}{8}, \frac{5}{8}$ (ii) $\frac{-7}{9}, \frac{4}{9}$ (iii) $\frac{-2}{11}, \frac{-9}{11}$ (iv) $\frac{11}{13}, \frac{-4}{13}$

2. Evaluate each of the following:

   (i) $\frac{2}{3}-\frac{3}{5}$ (ii) $-\frac{4}{7}-\frac{2}{-3}$ (iii) $\frac{4}{7}-\frac{-5}{-7}$ (iv) $-2-\frac{5}{9}$

3. The sum of the two numbers is $\frac{5}{9}$. If one of the numbers is $\frac{1}{3}$, find the other.

4. The sum of two numbers is $\frac{-1}{3}$. If one of the numbers is $\frac{-12}{3}$, find the other.

5. The sum of two numbers is $\frac{-4}{3}$. If one of the numbers is –5, find the other.

6. The sum of two rational numbers is –8. If one of the numbers is $\frac{-15}{7}$, find the other.

7. What should be added to $\frac{-7}{8}$ so as to get $\frac{5}{9}$?

8. What number should be added to $\frac{-5}{11}$ so as to get $\frac{26}{33}$?

9. What number should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

10. What number should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

11. What number should be subtracted from $\frac{3}{7}$ to get $\frac{5}{4}$?

12. What should be added to $\left(\frac{2}{3}+\frac{3}{5}\right)$ to get $\frac{-2}{15}$?

13. What should be added to $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{5}\right)$ to get 3?

14. What should be subtracted from $\left(\frac{3}{4}-\frac{2}{3}\right)$ to get $\frac{-1}{6}$?

15. Simplify:

(i) $\frac{-3}{2}+\frac{5}{4}-\frac{7}{4}$ (ii) $\frac{5}{3}-\frac{7}{6}+\frac{-2}{3}$ (iii) $\frac{5}{4}-\frac{7}{6}-\frac{-2}{3}$ (iv) $\frac{-2}{5}-\frac{-3}{10}-\frac{-4}{7}$

16. Fill in the blanks:

(i) $\frac{-4}{13}-\frac{-3}{26}=\ldots.$ (ii) $\frac{-9}{14}+\ldots.=-1$ (iii) $\frac{-7}{9}+\ldots.=3$ (iv) $\ldots.+\frac{15}{23}=4$

ANSWERS

1. (i) $\frac{1}{4}$ (ii) $\frac{11}{9}$ (iii) $\frac{-7}{11}$ (iv) $\frac{-15}{13}$

2. (i) $\frac{1}{15}$ (ii) $\frac{2}{21}$ (iii) $\frac{-1}{7}$ (iv) $\frac{-23}{9}$ 3. $\frac{2}{9}$ 4. $\frac{11}{3}$

5. $\frac{11}{3}$ 6. $\frac{-41}{7}$ 7. $\frac{103}{72}$ 8. $\frac{41}{33}$ 9. $\frac{1}{21}$ 10. $\frac{-5}{2}$

11. $\frac{-23}{28}$ 12. $\frac{-7}{5}$ 13. $\frac{59}{30}$ 14. $\frac{1}{4}$

15. (i) -2 (ii) $\frac{-1}{6}$ (iii) $\frac{3}{4}$ (iv) $\frac{33}{70}$

16. (i) $\frac{-5}{26}$ (ii) $\frac{-5}{14}$ (iii) $\frac{34}{9}$ (iv) $\frac{77}{23}$

5.4 MULTIPLICATION OF RATIONAL NUMBERS

In earlier classes, we have learnt how to multiply two fractions. Recall that the product of two given fractions is a fraction whose numerator is the product of the numerators of the given fractions and whose denominator is the product of the denominators of the given fractions. In other words,

$$\text{Product of two given fractions} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

This rule is also true for the product of rational numbers.

$\therefore$ Product of two rational numbers $= \dfrac{\text{Product of their numerators}}{\text{Product of their denominators}}$

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

ILLUSTRATIVE EXAMPLES

Example 1 Multiply:

(i) $\frac{3}{4}$ by $\frac{5}{7}$ (ii) $\frac{3}{7}$ by $\left(\frac{-4}{5}\right)$ (iii) $\left(\frac{-5}{9}\right)$ by 4 (iv) $\left(\frac{-36}{7}\right)$ by $\left(-\frac{28}{9}\right)$

Solution We have,

(i) $\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$

(ii) $\frac{3}{7} \times \left(\frac{-4}{5}\right) = \frac{3 \times -4}{7 \times 5} = \frac{-12}{35}$

(iii) $\left(\frac{-5}{9}\right) \times 4 = \frac{-5}{9} \times \frac{4}{1} = \frac{(-5) \times 4}{9 \times 1} = \frac{-20}{9}$

(iv) $\left(\frac{-36}{7}\right) \times \left(-\frac{28}{9}\right) = \frac{-36}{7} \times \frac{-28}{9} = \frac{-\overset{-4}{\cancel{36}} \times -\overset{-4}{\cancel{28}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{9}}} = 16$

Example 2 Simplify:

(i) $\frac{-8}{7} \times \frac{14}{5}$ (ii) $\frac{13}{6} \times \frac{-18}{91}$ (iii) $\frac{-5}{9} \times \frac{72}{-125}$ (iv) $\frac{-22}{9} \times \frac{-51}{-88}$

Solution (i) We have,

$$\frac{-8}{7} \times \frac{14}{5} = \frac{-8 \times 14}{7 \times 5} = \frac{-8 \times 2}{1 \times 5} = \frac{-16}{5}$$

(ii) We have,

$$\frac{13}{6} \times \frac{-18}{91} = \frac{\overset{1}{\cancel{13}} \times (-\overset{-3}{\cancel{18}})}{\underset{1}{\cancel{6}} \times \underset{7}{\cancel{91}}} = \frac{1 \times -3}{1 \times 7} = \frac{-3}{7}$$

(iii) We have,

$$\frac{-5}{9} \times \frac{72}{-125} = \frac{(-\overset{-1}{\cancel{5}}) \times \overset{8}{\cancel{72}}}{\underset{1}{\cancel{9}} \times -\underset{-25}{\cancel{125}}} = \frac{-1 \times 8}{1 \times -25} = \frac{-8}{-25} = \frac{8}{25}$$

(iv) We have,

$$\frac{-22}{9} \times \frac{-51}{-88} = \frac{-22}{9} \times \frac{51}{88} \qquad \left[\because \frac{-51}{-88} = \frac{51}{88}\right]$$

$$\Rightarrow \frac{-22}{9} \times \frac{-51}{-88} = \frac{-\overset{-1}{\cancel{22}} \times \overset{17}{\cancel{51}}}{\underset{3}{\cancel{9}} \times \underset{4}{\cancel{88}}} = \frac{-1 \times 17}{3 \times 4} = \frac{-17}{12}$$

Example 3 Simplify:

(i) $\left(\frac{-16}{5}\times\frac{20}{8}\right)-\left(\frac{15}{5}\times\frac{-35}{5}\right)$ (ii) $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1\times3}{2\times4}\right)$

Solution (i) We have,

$$\left(\frac{-16}{5}\times\frac{20}{8}\right)-\left(\frac{15}{5}\times\frac{-35}{5}\right)$$

$$=\left(\frac{-\overset{2}{\cancel{16}}\times\overset{4}{\cancel{20}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{8}}}\right)-\left(\frac{\overset{3}{\cancel{15}}\times-\overset{-7}{\cancel{35}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}}\right)$$

$$=\left(\frac{-2\times4}{1\times1}\right)-\left(\frac{3\times-7}{1\times1}\right)=\frac{-8}{1}-\frac{-21}{1}=-8-(-21)=-8+21=13$$

(ii) We have,

$$\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1\times3}{2\times4}\right)$$

$$=\frac{-3\times\overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}}\times5}+\frac{\overset{3}{\cancel{9}}\times-\overset{-2}{\cancel{10}}}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{3}}}-\frac{1\times3}{2\times4}$$

$$=\frac{-3\times2}{1\times5}+\frac{3\times-2}{1\times1}-\frac{3}{8}=\frac{-6}{5}+\frac{-6}{1}-\frac{3}{8}=\frac{-6}{5}+\frac{-6}{1}+\frac{-3}{8}$$

EXERCISE 5.3

1. Multiply:

(i) $\frac{7}{11}$ by $\frac{5}{4}$ (ii) $\frac{5}{7}$ by $\left(\frac{-3}{4}\right)$ (iii) $\frac{(-2)}{9}$ by $\frac{5}{11}$ (iv) $\frac{-3}{17}$ by $\frac{-5}{-4}$

2. Multiply:

(i) $\frac{-5}{17}$ by $\frac{51}{-60}$ (ii) $\frac{-6}{11}$ by $\frac{-55}{36}$ (iii) $\frac{-8}{25}$ by $\frac{-5}{16}$ (iv) $\frac{6}{7}$ by $\frac{-49}{36}$

3. Simplify each of the following and express the result as a rational number in standard form:

(i) $\frac{-16}{21}\times\frac{14}{5}$ (ii) $\frac{7}{6}\times\frac{-3}{28}$ (iii) $\frac{-19}{36}\times16$ (iv) $\frac{-13}{9}\times\frac{27}{-26}$

4. Simplify:

(i) $\left(-5\times\frac{2}{15}\right)-\left(-6\times\frac{2}{9}\right)$ (ii) $\left(\frac{-9}{4}\times\frac{5}{3}\right)+\left(\frac{13}{2}\times\frac{5}{6}\right)$

5. Simplify:

(i) $\left(\frac{13}{9}\times\frac{-15}{2}\right)+\left(\frac{7}{3}\times\frac{8}{5}\right)+\left(\frac{3}{5}\times\frac{1}{2}\right)$ (ii) $\left(\frac{3}{11}\times\frac{5}{6}\right)-\left(\frac{9}{12}\times\frac{4}{3}\right)+\left(\frac{5}{13}\times\frac{6}{15}\right)$

ANSWERS

1. (i) $\frac{35}{44}$ (ii) $\frac{-15}{28}$ (iii) $\frac{-10}{99}$ (iv) $\frac{-15}{68}$ 2. (i) $\frac{1}{4}$ (ii) $\frac{5}{6}$

(iii) $\frac{1}{10}$ (iv) $\frac{-7}{6}$ 3. (i) $\frac{-32}{15}$ (ii) $\frac{-1}{8}$ (iii) $\frac{-76}{9}$ (iv) $\frac{3}{2}$

4. (i) $\frac{2}{3}$ (ii) $\frac{5}{3}$ 5. (i) $\frac{-34}{5}$ (ii) $\frac{-177}{286}$

5.5 RECIPROCAL OF A NON-ZERO RATIONAL NUMBER

For every non-zero rational number $\frac{a}{b}$ *there exists a rational number* $\frac{b}{a}$ *such that*

$$\frac{a}{b}\times\frac{b}{a}=1=\frac{b}{a}\times\frac{a}{b}$$

The rational number $\frac{b}{a}$ *is called the multiplicative inverse or reciprocal of* $\frac{a}{b}$ *and is denoted by* $\left(\frac{a}{b}\right)^{-1}$.

The reciprocal of $\frac{5}{12}$ is $\frac{12}{5}$ i.e., $\left(\frac{5}{12}\right)^{-1}=\frac{12}{5}$

The reciprocal of $\frac{-3}{7}$ is $\frac{7}{-3}$ i.e., $\left(\frac{-3}{7}\right)^{-1}=\frac{7}{-3}$

The reciprocal of -5 is $\frac{1}{-5}$, since $-5\times\frac{1}{-5}=\frac{-5}{1}\times\frac{1}{-5}=\frac{-5\times1}{-5\times1}=1$

NOTE: *The reciprocal of 1 is 1 and the reciprocal of –1 is –1. 1 and –1 are the only rational numbers which are their own reciprocals. No other rational number is its own reciprocal.*

Remark: *We know that there is no rational number which when multiplied with 0, gives 1. Therefore, rational number 0 has no reciprocal or multiplicative inverse.*

ILLUSTRATION 1 Write the reciprocal of each of the following rational numbers:

(i) 7 (ii) –11 (iii) $\frac{2}{5}$ (iv) $\frac{-7}{15}$

Solution (i) Reciprocal of 7 is $\frac{1}{7}$ i.e., $7^{-1}=\frac{1}{7}$

(ii) Reciprocal of –11 is $\frac{1}{-11}$ i.e., $(-11)^{-1}=\frac{1}{-11}$

(iii) Reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$ i.e., $\left(\frac{2}{5}\right)^{-1}=\frac{2}{5}$

(iv) Reciprocal of $\frac{-7}{15}$ is $\frac{15}{-7}$ i.e., $\left(\frac{-7}{15}\right)^{-1}=\frac{15}{-7}$

ILLUSTRATION 2 Find the reciprocal of : $\frac{-3}{8}\times\frac{-7}{13}$

Solution We have,

$$\frac{-3}{8}\times\frac{-7}{13}=\frac{(-3)\times(-7)}{8\times 13}=\frac{21}{104}$$

$\therefore$ The reciprocal of $\frac{-3}{8}\times\frac{-7}{13}$ = Reciprocal of $\frac{21}{104}=\frac{104}{21}$

5.6 DIVISION OF RATIONAL NUMBERS

In chapter 2, we have learnt how to divide a fraction by another fraction. Recall that division of fractions is the inverse of multiplication. In case of rational number also, division is the inverse of multiplication as defined below:

DIVISION *If x and y are two rational numbers such that $y \neq 0$, then the result of dividing x by y is the rational number obtained on multiplying x by the reciprocal of y.*

When x is divided by y, we write $x \div y$. Thus, $x \div y = x \times \frac{1}{y}$

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\left(\frac{c}{d}\right)^{-1}=\frac{a}{b}\times\frac{d}{c}$$

DIVIDEND *The number to be divided is called the dividend.*

DIVISOR *The number which divides the dividend is called the divisor.*

QUOTIENT *When dividend is divided by the divisor, the result of the division is called the quotient.*

If $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is the dividend, $\frac{c}{d}$ is the divisor and $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ is the quotient.

NOTE: *It should be noted that division by 0 is not defined.*

ILLUSTRATIVE EXAMPLES

Example 1 Divide: (i) $\frac{3}{5}$ by $\frac{4}{25}$ (ii) $\frac{8}{9}$ by $\frac{4}{3}$ (iii) $\frac{-16}{21}$ by $\frac{4}{3}$ (iv) $\frac{-8}{13}$ by $\frac{3}{-26}$

Solution We have,

(i) $$\frac{3}{5}\div\frac{4}{25}=\frac{3}{5}\times\frac{25}{4}=\frac{3\times \overset{5}{\cancel{25}}}{\underset{1}{\cancel{5}}\times 4}=\frac{15}{4}$$

(ii) $$\frac{-8}{9}\div\frac{4}{3}=\frac{-8}{9}\times\frac{3}{4}=\frac{-\overset{2}{\cancel{8}}\times\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}\times\underset{1}{\cancel{4}}}=\frac{-2\times 1}{3\times 1}=\frac{-2}{3}$$

(iii) $$\frac{-16}{21}\div\frac{-4}{3}=\frac{-16}{21}\times\frac{3}{-4}=\frac{-\overset{4}{\cancel{16}}\times\overset{1}{\cancel{3}}}{\underset{7}{\cancel{21}}\times(-\underset{1}{\cancel{4}})}=\frac{-4\times 1}{7\times -1}=\frac{-4}{-7}=\frac{4}{7}$$

(iv) $\frac{-8}{13} \div \frac{3}{-26} = \frac{-8}{13} \times \frac{-26}{3} = \frac{-8 \times -26}{13 \times 3} = \frac{8 \times 2}{1 \times 3} = \frac{16}{3}$

Example 2 The product of two rational numbers is $\frac{-28}{81}$. If one of the numbers is $\frac{14}{27}$, find the other.

Solution We have,

Product of two numbers $= \frac{-28}{81}$, One number $= \frac{14}{27}$

So, the other number is obtained by dividing the product by the given number.

$\therefore$ Other number

$$= \frac{-28}{81} \div \frac{14}{27} = \frac{-28}{81} \times \frac{27}{14} = \frac{-28 \times 27}{81 \times 14} = \frac{-(\overset{2}{\cancel{28}} \times \overset{1}{\cancel{27}})}{\underset{3}{\cancel{81}} \times \underset{1}{\cancel{14}}} = \frac{-(2 \times 1)}{3 \times 1} = \frac{-2}{3}$$

Example 3 By what number should we multiply $\frac{3}{-14}$, so that the product may be $\frac{5}{12}$.

Solution We have, Product of two numbers $= \frac{5}{12}$, One number $= \frac{3}{-14}$

$\therefore$ The other number

$$= \frac{5}{12} \div \frac{3}{-14} = \frac{5}{12} \times \frac{-14}{3} = \frac{5 \times (-14)}{12 \times 3} = \frac{-(5 \times \overset{7}{\cancel{14}})}{\underset{6}{\cancel{12}} \times 3} = \frac{-(5 \times 7)}{6 \times 3} = \frac{-35}{18}$$

EXERCISE 5.4

1. Divide:

(i) 1 by $\frac{1}{2}$ (ii) 5 by $\frac{-5}{7}$ (iii) $\frac{-3}{4}$ by $\frac{9}{-16}$ (iv) $\frac{-7}{8}$ by $\frac{-21}{16}$

(v) $\frac{7}{-4}$ by $\frac{63}{64}$ (vi) 0 by $\frac{-7}{5}$ (vii) $\frac{-3}{4}$ by -6 (viii) $\frac{2}{3}$ by $\frac{-7}{12}$

2. Find the value and express as a rational number in standard form:

(i) $\frac{2}{5} \div \frac{26}{15}$ (ii) $\frac{10}{3} \div \frac{-35}{12}$ (iii) $-6 \div \left(\frac{-8}{17}\right)$ (iv) $\frac{40}{98} \div (-20)$

3. The product of two rational numbers is 15. If one of the numbers is –10, find the other.

4. The product of two rational numbers is $\frac{-8}{9}$. If one of the numbers is $\frac{-4}{15}$, find the other.

5. By what number should we multiply $\frac{-1}{6}$ so that the product may be $\frac{-23}{9}$?

6. By what number should we multiply $\frac{-15}{28}$ so that the product may be $\frac{-5}{7}$?

7. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

8. By what number should $\frac{-3}{4}$ be multiplied in order to produce $\frac{2}{3}$?

9. Find $(x+y) \div (x-y)$, if

(i) $x = \frac{2}{3}$, $y = \frac{3}{2}$ (ii) $x = \frac{2}{5}$, $y = \frac{1}{2}$ (iii) $x = \frac{5}{4}$, $y = \frac{-1}{3}$

10. The cost of $7\frac{2}{3}$ metres of rope is ₹ $12\frac{3}{4}$. Find its cost per metre.

11. The cost of $2\frac{1}{3}$ metres of cloth is ₹ $75\frac{1}{4}$. Find the cost of cloth per metre.

12. By what number should $\frac{-33}{16}$ be divided to get $\frac{-11}{4}$?

13. Divide the sum of $\frac{-13}{5}$ and $\frac{12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{-1}{2}$.

14. Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.

15. If 24 trousers of equal size can be prepared in 54 metres of cloth, what length of cloth is required for each trouser?

ANSWERS

1. (i) 2 (ii) –7 (iii) $\frac{4}{3}$ (iv) $\frac{2}{3}$ (v) $\frac{-16}{9}$ (vi) 0 (vii) $\frac{1}{8}$ (viii) $\frac{-8}{7}$

2. (i) $\frac{3}{13}$ (ii) $\frac{-8}{7}$ (iii) $\frac{51}{4}$ (iv) $-\frac{1}{49}$

3. (i) $\frac{-3}{2}$

4. $\frac{10}{3}$

5. $\frac{46}{3}$

6. $\frac{4}{3}$

7. -39

8. $\frac{-8}{9}$

9. (i) $\frac{-13}{5}$ (ii) -9 (iii) $\frac{11}{19}$

10. ₹ $1\frac{61}{92}$

11. ₹ $32\frac{1}{4}$

12. $\frac{3}{4}$

13. $\frac{-2}{5}$

14. $\frac{97}{33}$

15. $\frac{9}{4}$ metres.

5.7 INSERTION OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

In class VI, we have learnt about integers and properties of various operations on them. We have learnt that between two non–consecutive integers m and n there are $(m-n-1)$ integers. However, there is no integer between two consecutive integers. For example, between – 9 and 4 there are $4-(-9)-1 = 12$ integers, namely, – 8 ,–7,– 6 ,– 5 , – 4 , – 3, – 2 ,– 1, 0, 1, 2 and 3 but there is no integer between 4 and 5. Thus, we find that between two given integers there may or may not lie any integer.

In this section, we shall see that between any two rational numbers infinitely many rational numbers can be inserted. This property of rational numbers is known as the dense property.

Let us now try to find some rational numbers lying between two given rational numbers, say between $\frac{-2}{5}$ and $\frac{3}{5}$.

The four rational numbers $\frac{-1}{5}, \frac{0}{5}, \frac{1}{5}$ and $\frac{2}{5}$ lie between $\frac{-2}{5}$ and $\frac{3}{5}$.

If required, we can insert more rational numbers between $\frac{-2}{5}$ and $\frac{3}{5}$ by the technique as discussed below.

The rational numbers $\frac{-2}{5}$ and $\frac{3}{5}$ can also be written as $\frac{-20}{50}$ and $\frac{30}{50}$ respectively.

Clearly, $\frac{-19}{50}, \frac{-18}{50}, \ldots \frac{0}{50}, \frac{1}{50}, \ldots, \frac{29}{50}$ are rational numbers between $\frac{-2}{5}$ and $\frac{3}{5}$.

The total number of these rational numbers is same as the number of integers between -20 and 30, i.e., $30-(-20)-1 = 49$.

Similarly, by re-writing $\frac{-2}{5}$ and $\frac{3}{5}$ as $\frac{-200}{500}$ and $\frac{300}{500}$, we can insert $300-(-200)-1 = 499$ rational numbers between $\frac{-2}{5}$ and $\frac{3}{5}$.

Continuing in this manner, we can insert as many rational numbers between $\frac{-2}{5}$ and $\frac{3}{5}$ as we wish.

ILLUSTRATIVE EXAMPLES

Example 1 Insert 10 rational numbers between $\frac{-3}{11}$ a nd $\frac{8}{11}$.

Solution We know that

$$-3 < -2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8$$

$$\therefore \quad \frac{-3}{11} < \frac{-2}{11} < \frac{-1}{11} < \frac{0}{11} < \frac{1}{11} < \frac{2}{11} < \frac{3}{11} < \frac{4}{11} < \frac{5}{11} < \frac{6}{11} < \frac{7}{11} < \frac{8}{11}$$

Hence, 10 rational numbers between $\frac{-3}{11}$ and $\frac{8}{11}$ are :

$$\frac{-2}{11}, \frac{-1}{11}, \frac{0}{11}, \frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{5}{11}, \frac{6}{11} \text{ and } \frac{7}{11}$$

Example 2 Insert 100 rational numbers between $\frac{-3}{13}$ and $\frac{9}{13}$.

Solution We have,

$$\frac{-3}{13} = \frac{-3\times 10}{13\times 10} = \frac{-30}{130} \text{ and, } \frac{9}{13} = \frac{9\times 10}{13\times 10} = \frac{90}{130}$$

We know that

$$-30 < -29 < -28 < \ldots\ldots -1 < 0 < 1 < 2 < 3 < \ldots\ldots < 70$$

$$\Rightarrow \frac{-30}{130} < \frac{-29}{130} < \frac{-28}{130} < \ldots < \frac{-1}{130} < \frac{0}{130} < \frac{1}{130} < \frac{2}{130} < \ldots < \frac{70}{130}$$

Hence, 100 rational numbers between $\frac{-3}{13} = \frac{-30}{130}$ and $\frac{9}{13} = \frac{90}{130}$ are:

$\frac{-29}{130}, \frac{-28}{130}, \ldots, \frac{-1}{130}, \frac{0}{130}, \frac{1}{130}, \ldots, \frac{70}{130}$

EXERCISE 5.5

1. Find six rational numbers between $\frac{-4}{8}$ and $\frac{3}{8}$.
2. Find 10 rational numbers between $\frac{7}{13}$ and $\frac{-4}{13}$.
3. State true or false:
   (i) Between any two distinct integers there is always an integer.
   (ii) Between any two distinct rational numbers there is always a rational number.
   (iii) Between any two distinct rational numbers there are infinitely many rational numbers.

ANSWERS

1. $\frac{-3}{8}, \frac{-2}{8}, \frac{-1}{8}, \frac{0}{8}, \frac{1}{8}, \frac{2}{8}$
2. $\frac{-3}{13}, \frac{-2}{13}, \frac{-1}{13}, \frac{0}{13}, \frac{1}{13}, \frac{2}{13}, \frac{3}{13}, \frac{4}{13}, \frac{5}{13}, \frac{6}{13}$

11. (i) F (ii) T (iii) T

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. What should be added to $\frac{-7}{9}$ to get 2 ?
   (a) $\frac{11}{9}$ (b) $\frac{-11}{9}$ (c) $\frac{-25}{9}$ (d) $\frac{25}{9}$
2. What should be subtracted from $\frac{-2}{3}$ to get $\frac{4}{5}$?
   (a) $\frac{22}{15}$ (b) $\frac{-22}{15}$ (c) $\frac{15}{22}$ (d) $\frac{-15}{22}$
3. Reciprocal of $\frac{-3}{4}$ is
   (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{-4}{3}$ (d) None of these
4. The multiplicative inverse of $\frac{4}{-5}$ is
   (a) $-\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{5}{-4}$ (d) $\frac{-5}{-4}$
5. $1 \div \frac{-5}{7} =$
   (a) $\frac{2}{7}$ (b) $\frac{5}{7}$ (c) $-\frac{2}{7}$ (d) $\frac{-7}{5}$

6. $\frac{-5}{13} + ? = -1$

(a) $\frac{8}{13}$ (b) $\frac{-8}{13}$ (c) $\frac{-18}{13}$ (d) $\frac{18}{13}$

7. $0 \div \frac{3}{5} =$

(a) 0 (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

8. $-2\frac{3}{7} + 4 = ?$

(a) $\frac{-11}{7}$ (b) $\frac{11}{7}$ (c) $\frac{-45}{7}$ (d) $\frac{45}{7}$

9. If the product of two non-zero rational numbers is 1, then they are

(a) additive inverse of each other (b) multiplicative inverse of each other
(c) reciprocal of each other (d) both (b) and (c)

10. The product $3\frac{1}{7} \times 1\frac{5}{6} \times 1\frac{2}{5} \times 1\frac{1}{11}$ is equal to

(a) $5\frac{8}{5}$ (b) $5\frac{4}{5}$ (c) $8\frac{4}{5}$ (d) $7\frac{4}{5}$

11. $\frac{-7}{13} - \left(\frac{-8}{15}\right) =$

(a) $-\frac{239}{195}$ (b) $\frac{29}{195}$ (c) $\frac{-29}{195}$ (d) None of these

12. $1 \div \frac{1}{3} =$

(a) $\frac{1}{3}$ (b) 3 (c) $1\frac{1}{3}$ (d) $3\frac{1}{3}$

13. $(-2) \div \left(-\frac{5}{3}\right) =$

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$ (c) $\frac{6}{5}$ (d) $\frac{-6}{5}$

14. If $\frac{x}{2} + \frac{1}{3} = 1$, then $x =$

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{3}{4}$ (d) $\frac{-4}{3}$

15. $\frac{5}{4} - \frac{7}{6} - \frac{-2}{3} =$

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{-7}{12}$ (d) $\frac{7}{12}$

ANSWSERS

1. (d)	2. (b)	3. (c)	4. (c)	5. (d)	6. (b)	7. (a)
8. (b)	9. (d)	10. (c)	11. (d)	12. (b)	13. (c)	14. (b)
15. (a)						

THINGS TO REMEMBER

1. *For any two rational numbers* $\frac{p}{q}$ *and* $\frac{r}{q}$, *we define :*

$$\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q}$$

2. *For any two rational numbers* $\frac{p}{q}$ *and* $\frac{r}{s}$ *to find* $\frac{p}{q}+\frac{r}{s}$ *first we convert* $\frac{p}{q}$ *and* $\frac{r}{s}$ *to equivalent rational numbers having denominator equal to the LCM of q and s and then they are added.*

3. *For any two rational numbers* $\frac{p}{q}$ *and* $\frac{r}{s}$, *we have* $\frac{p}{q}-\frac{r}{s}=\frac{p}{q}+\left(\text{negative of } \frac{r}{s}\right)$

4. *For any two rational numbers* $\frac{p}{q}$ *and* $\frac{r}{s}$, *we have*

$$\frac{p}{q}\times\frac{r}{s}=\frac{p\times r}{q\times s}$$

5. *The reciprocal of a non-zero rational number* $\frac{p}{q}$ *is* $\frac{q}{p}$ *and we write* $\left(\frac{p}{q}\right)^{-1}=\frac{q}{p}$.

6. *For any two rational numbers* $\frac{p}{q}$ *and* $\frac{r}{s}(\neq 0)$, *we have*

$$\frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}$$