15

PROPERTIES OF TRIANGLES

15.1 INTRODUCTION

In the previous chapter, we have learnt about lines, rays, line segments and angles. In this chapter, we shall study about a plane figure formed by three non-parallel line segments. Such a figure is called a triangle.

15.2 TRIANGLE

TRIANGLE *A plane figure formed by three non-parallel line segments is called a triangle.*

If A, B, C are three non-collinear points in the plane of the paper, then the figure made up by the three line segments AB, BC and CA is called a triangle with vertices A, B and C.

The triangle with vertices A, B and C is generally denoted by the symbol ΔABC.

Note that the triangle ABC consists of all the points on the line segments AB, BC and CA.

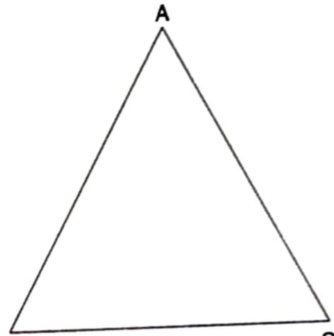


Fig. 1

SIDES *The three line segments AB, BC and CA, that form the triangle ABC, are called the sides of the triangle ABC.*

ANGLES *The three angles $\angle BAC$, $\angle ABC$ and $\angle ACB$ are called the angles of ΔABC.*

For the sake of convenience, we shall denote angles $\angle BAC$, $\angle ABC$ and $\angle ACB$ by $\angle A$, $\angle B$, $\angle C$ respectively.

ELEMENTS OR PARTS *The three sides AB, BC, CA and three angles $\angle A$, $\angle B$, $\angle C$ of a ΔABC are together called the six parts or elements of the ΔABC.*

In ΔABC, we observe that the sides AB and AC meet at vertex A and BC is the remaining side. So, we say that 'BC is the side opposite to vertex A and A is the vertex opposite to side BC'.

Similarly, CA is the side opposite to vertex B, and B is the vertex opposite to side CA.

Also, CA is the side opposite to $\angle B$, and $\angle B$ is the angle opposite to side CA.

Clearly, AB is the side opposite to vertex C, and C is the vertex opposite to side AB. We can also say that AB is the side opposite to $\angle C$, and $\angle C$ is the angle opposite to side AB.

15.3 INTERIOR AND EXTERIOR OF TRIANGLE

Consider a triangle ABC. We observe that all points in the plane of ΔABC are divided into following three parts:

(i) The points which lie inside the region enclosed by ΔABC.
(ii) The points which lie on the sides BC, CA and AB of ΔABC.
(iii) The points which lie outside the region enclosed by ΔABC.

So, we define the interior and exterior of a triangle as follows:

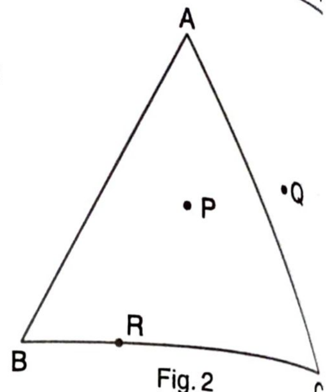


Fig. 2

INTERIOR *The part made up by all such points P which are enclosed by ΔABC is called the interior of ΔABC.*

EXTERIOR *The part made up by all such points Q which are not enclosed by ΔABC is called the exterior of ΔABC.*

Note that ΔABC is the boundary of its interior.

TRIANGULAR REGION *The interior of ΔABC together with the ΔABC itself, is called the triangular region ABC.*

The part of the plane which consists of all points such as R, i.e., points lying on the sides of the triangle, forms the triangle itself.

It may be noted that if we take any two points X and Y in the interior of ΔABC, then the line segment XY lies entirely in the interior of ΔABC.

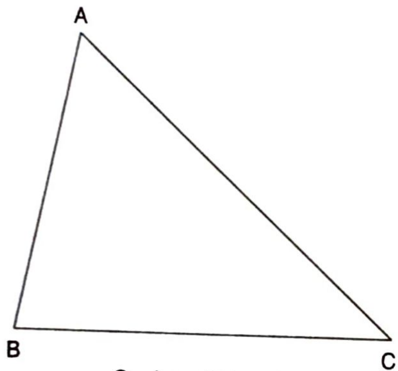
15.4 TYPES OF TRIANGLES

Triangles are named on the basis of the lengths of their sides and the measures of their angles as given below:

15.4.1 NAMING OF TRIANGLES BY CONSIDERING THE LENGTHS OF THEIR SIDES

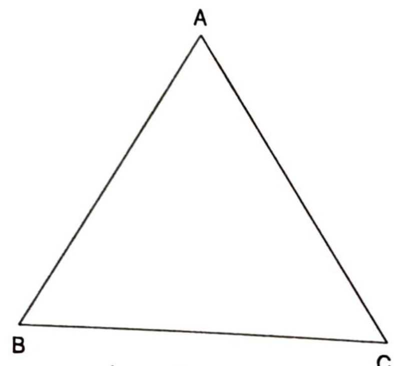
SCALENE TRIANGLE *A triangle whose no two sides are equal, is called a scalene triangle.*

In Fig. 3, ΔABC is a scalene triangle.



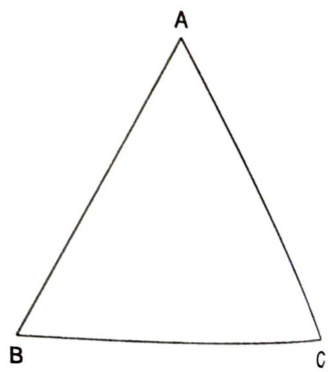
Scalene Triangle

Fig. 3



Isosceles Triangle

Fig. 4



Equilateral Triangle

Fig. 5

ISOSCELES TRIANGLE *A triangle whose two sides are equal is called an isosceles triangle.*

In Fig. 4, ΔABC is an isosceles triangle, where $AB = AC$.

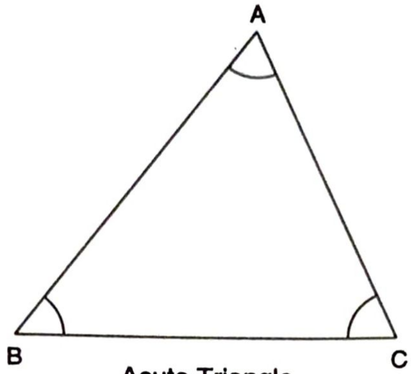
EQUILATERAL TRIANGLE *A triangle whose all sides are equal to one another, is called an equilateral triangle.*

In Fig. 5, ΔABC is an equilateral triangle, where $AB = BC = AC$

<u>NOTE:</u> *It should be noted that an equilateral triangle is isosceles triangle but the converse need not be true.*

15.4.2 NAMING TRIANGLES BY CONSIDERING THE MEASURES OF THEIR ANGLES

ACUTE TRIANGLE *A triangle whose all the angles are acute is called an acute-angled triangle or an acute triangle.*



Acute Triangle

Fig. 6

In Fig. 6, ΔABC is an acute-angled triangle where $\angle A, \angle B, \angle C$ are acute angles.

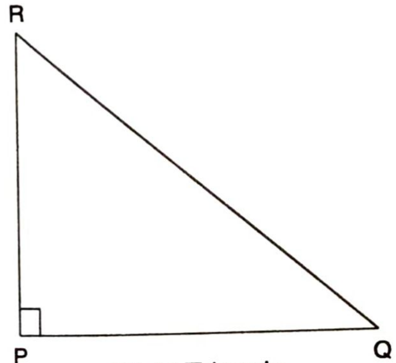
Note that an equilateral triangle is an acute angled triangle, because the measure of its each angle is 60°

RIGHT TRIANGLE *A triangle whose one angle is a right angle, is called a right-angled triangle or a right triangle.*

The side opposite to the right angle in a right-angled triangle is known as the hypotenuse of the triangle and the other two sides are called the legs of the triangle.

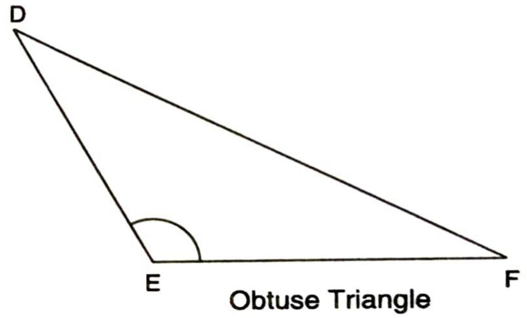
In Fig. 7, ΔPQR is a right-angled triangle, where $\angle P = 90°$.

Note that the remaining two angles of a right-angled triangle are acute.



Right Triangle

Fig. 7



Obtuse Triangle

Fig. 8

OBTUSE TRIANGLE *A triangle whose one angle is obtuse, is called an obtuse-angled triangle or an obtuse triangle.*

In Fig. 8, ΔDEF is an obtuse-angled triangle, where $\angle E$ is an obtuse angle.

Note that in a triangle only one angle can be obtuse and the remaining two angles are acute angles.

Let us perform the following experiments in the support of some facts mentioned in the above discussion.

Experiment 1 Draw some equilateral triangles and measure each one of the angles of each such triangle. You will find that the measure of each angle is 60°. Thus, the measure of each angle of an equilateral triangle is 60°.

Experiment 2 Draw an isosceles triangle and measure its angles. You will find that the angles opposite to equal sides are equal. Thus, an isosceles triangle has exactly two equal angles.

Experiment 3 Draw a scalene triangle, that is a triangle whose sides are of different lengths. Measure the angles of this triangle. You will find that the measures of the three angles are different. Thus, a scalene triangle has no two angles equal.

PERIMETER OF A TRIANGLE *The sum of the lengths of the sides of a triangle is called its perimeter.*

If the lengths of the sides of a triangle are a, b and c units, then

$$\text{Perimeter} = (a + b + c)$$

The perimeter of a triangle is generally denoted by $2s$, where s is the semi-perimeter of the triangle. Thus, $2s = a + b + c$

EXERCISE 15.1

1. Take three non-collinear points *A*, *B* and *C* on a page of your notebook. Join *AB*, *BC* and *CA*. What figure do you get? Name the triangle. Also, name
   (i) the side opposite to $\angle B$
   (ii) the angle opposite to side *AB*
   (iii) the vertex opposite to side *BC*
   (iv) the side opposite to vertex *B*.
2. Take three collinear points *A*, *B* and *C* on a page of your note book. Join *AB*, *BC* and *CA*. Is the figure a triangle? If not, why?
3. Distinguish between a triangle and its triangular region.
4. In Fig. 9, *D* is a point on side *BC* of a ΔABC. *AD* is joined. Name all the triangles that you can observe in the figure. How many are they?

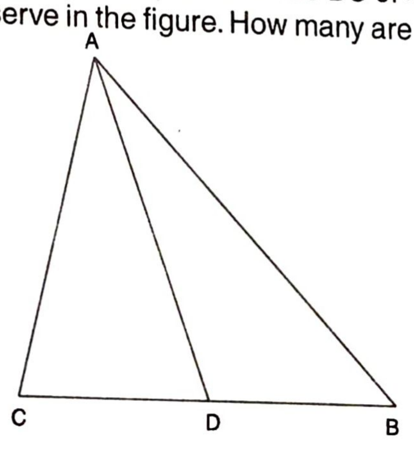


Fig. 9

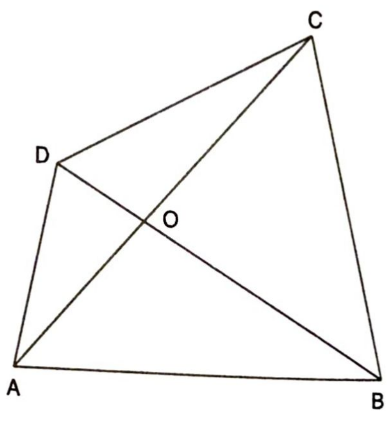


Fig. 10

5. In Fig. 10, *A*, *B*, *C* and *D* are four points, and no three points are collinear. *AC* and *BD* intersect at *O*. There are eight triangles that you can observe. Name all the triangles.
6. What is the difference between a triangle and triangular region?
7. Explain the following terms:
   (i) Triangle
   (ii) Parts or elements of a triangle
   (iii) Scalene triangle
   (iv) Isosceles triangle
   (v) Equilateral triangle
   (vi) Acute triangle
   (vii) Right triangle
   (viii) Obtuse triangle
   (ix) Interior of a triangle
   (x) Exterior of a triangle.
8. In Fig. 11, the length (in cm) of each side has been indicated along the side. State for each triangle whether it is scalene, isosceles or equilateral:

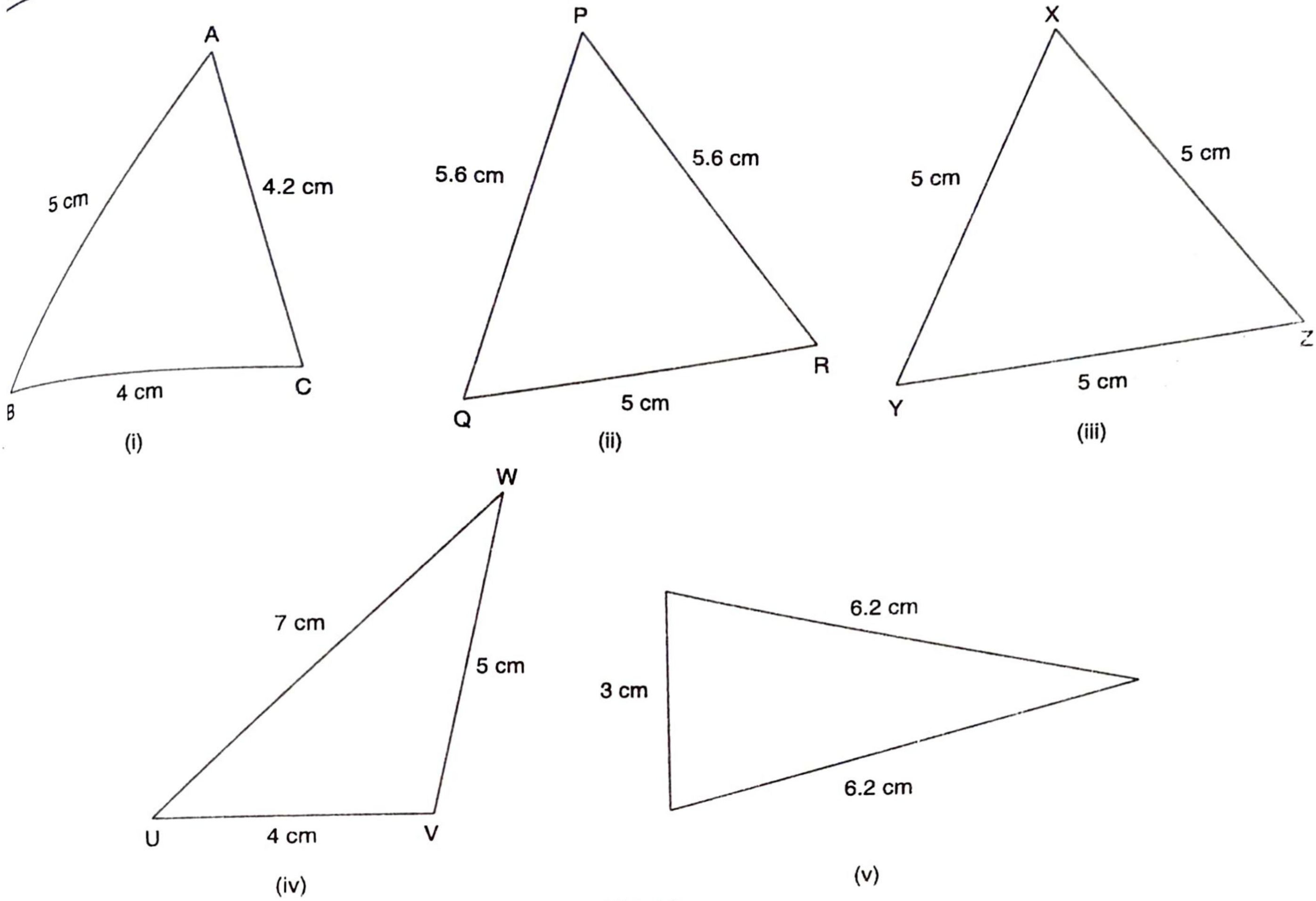


Fig. 11

9. In Fig. 12, there are five triangles. The measures of some of their angles have been indicated. State for each triangle whether it is acute, right or obtuse.

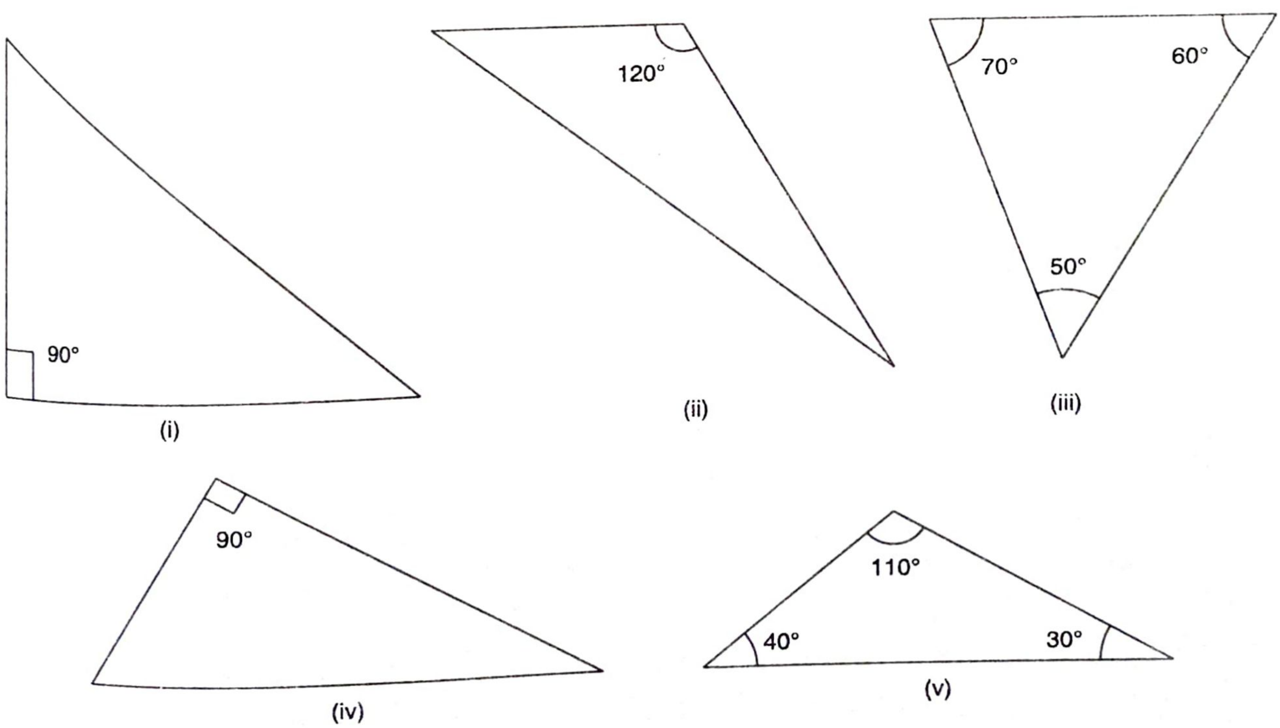


Fig. 12

10. Fill in the blanks with the correct word/symbol to make it a true statement:
    - (i) A triangle has .... sides.
    - (ii) A triangle has .... vertices.
    - (iii) A triangle has .... angles.
    - (iv) A triangle has .... parts.
    - (v) A triangle whose no two sides are equal is known as ....
    - (vi) A triangle whose two sides are equal is known as .....
    - (vii) A triangle whose all the sides are equal is known as ....
    - (viii) A triangle whose one angle is a right angle is known as ....
    - (ix) A triangle whose all the angles are of measure less than 90° is known as .....
    - (x) A triangle whose one angle is more than 90° is known as ....
11. In each of the following, state if the statement is true (T) or false (F):
    - (i) A triangle has three sides.
    - (ii) A triangle may have four vertices.
    - (iii) Any three line-segments make up a triangle.
    - (iv) The interior of a triangle includes its vertices.
    - (v) The triangular region includes the vertices of the corresponding triangle.
    - (vi) The vertices of a triangle are three collinear points.
    - (vii) An equilateral triangle is isosceles also.
    - (viii) Every right triangle is scalene.
    - (ix) Each acute triangle is equilateral.
    - (x) No isosceles triangle is obtuse.

ANSWERS

1. Triangle, ΔABC (i) AC (ii) $\angle ACB$ (iii) A (iv) AC
2. No, by definition of triangle
4. $\Delta ACD, \Delta ADB, \Delta ABC$; 3
5. $\Delta ABC, \Delta ABD, \Delta ABO, \Delta BCO, \Delta DCO, \Delta AOD, \Delta ACD, \Delta BCD$
8. (i) Scalene (ii) Isosceles (iii) Equilateral (iv) Scalene (v) Isosceles
9. (i) Right triangle (ii) Obtuse triangle (iii) Acute triangle (iv) Right triangle (v) Obtuse-triangle
10. (i) Three (ii) Three (iii) Three (iv) Six (v) Scalene (vi) Isosceles (vii) Equilateral (viii) Right triangle (ix) Acute triangle (x) Obtuse triangle
11. (i) T (ii) F (iii) F (iv) F (v) T (vi) F (vii) T (viii) F (ix) F (x) F

15.5 ANGLE SUM PROPERTY OF A TRIANGLE

In the previous sections, we have learnt about various types of triangles. The angles of a riangle possess an important property. In this section, we shall state and prove this roperty.

operty *The sum of the angles of a triangle is two right angles or 180°.*

Proof Let ABC be any triangle. Through A, draw a line XY parallel to the side BC as shown in Fig. 13. The angles are shown in Fig. 13.

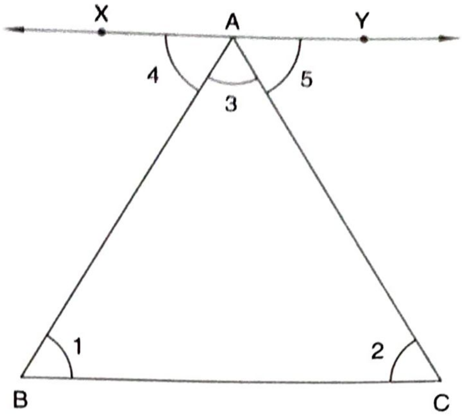


Fig. 13

Since $XY \parallel BC$ and the transversal AB cuts XY and BC at A and B respectively.

$\therefore \quad \angle 1 = \angle 4$ [$\because$ Alternate interior angles are equal]

Similarly, $XY \parallel BC$ and the transversal AC cuts XY and BC at A and C respectively.

$\therefore \quad \angle 2 = \angle 5$ [$\because$ Alternate interior angles are equal]

Also, $\angle 3 = \angle 3$

Adding the angles on the respective sides, we get

$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 3$

But, $\angle 4 + \angle 5 + \angle 3 = 180°$ [By linearity property]

$\therefore \quad \angle 1 + \angle 2 + \angle 3 = 180° = 2$ right angles.

Hence, the sum of the angles of a triangle is two right angles or 180°.

SOME IMPORTANT RESULTS *From the above property, we obtain the following useful results:*

(i) A triangle cannot have more than one right angle.

(ii) A triangle cannot have more than one obtuse angle i.e. if one angle of a triangle is obtuse, then the other two are acute.

(iii) In a right triangle, the other two angles are acute and their sum is 90°.

Following examples will illustrate the applications of the above property.

ILLUSTRATIVE EXAMPLES

Example 1 Two angles of a triangle are of measures 75° and 35°. Find the measure of the third angle.

Solution Let ABC be a triangle such that $\angle B = 75°$ and $\angle C = 35°$. Then, we have to find the measure of the third angle A.

Now, $\angle B = 75°$ and $\angle C = 35°$

$\Rightarrow \quad \angle B + \angle C = 75° + 35° = 110°$

By the angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + 110° = 180°$ [$\because \angle B + \angle C = 110°$]

$\Rightarrow \quad \angle A = 180° - 110°$

$\Rightarrow \quad \angle A = 70°.$

Example 2 One of the angles of a triangle has measure 80° and the other two angles are equal. Find these two angles.

Solution Let ABC be a triangle such that $\angle A = 80°$ and $\angle B = \angle C$.

By the angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 80° + \angle B + \angle B = 180°$ [$\because \angle A = 80°$ and $\angle C = \angle B$]

$\Rightarrow \quad 80° + 2\angle B = 180°$

$\Rightarrow \quad 2\angle B = 180° - 80°$

$\Rightarrow \quad 2\angle B = 100°$

$$\Rightarrow \quad \angle B = \left(\frac{100}{2}\right)^{\circ} = 50°$$

Hence, the measure of each of the remaining two angles is 50°.

Example 3 Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.

Solution Let the smallest angle of the given triangle be of $x°$. Then, the other two angles are of measures $2x°$ and $3x°$.

$\therefore \quad x + 2x + 3x = 180°$ [Angle sum property of a triangle]

$\Rightarrow \quad 6x = 180°$

$$\Rightarrow \quad x = \frac{180°}{6} = 30°$$

Hence, the angles of the triangle are 30°, 60° and 90°.

Example 4 Each of the two equal angles of a triangle is twice the third angle. Find the angles of the triangle.

Solution Let the measure of the smallest angle be $x°$. Then each of the other two angles has measure $2x°$.

Since the sum of the angles of a triangle is 180°.

$\therefore \quad x + 2x + 2x = 180$

$\Rightarrow \quad 5x = 180$

$$\Rightarrow \quad \frac{5x}{5} = \frac{180}{5} \quad \Rightarrow \quad x = 36$$

Thus, the angles of the triangle are 36°, 72°, 72°.

Example 5 If the angles of a triangle are in the ratio 2 : 3 : 4, determine three angles.

Solution Let measures of the angles of triangle be $2x°$, $3x°$ and $4x°$. Then,

$2x + 3x + 4x = 180°$ [Sum of the angles of a triangle is 180°]

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = 20°$

Hence, the angles of the triangle are 40°, 60° and 80°.

Example 6 The sum of two angles of a triangle is equal to its third angle. Determine the measure of the third angle.

Solution Let ABC be a triangle such that

$\angle A + \angle B = \angle C$ ...(i)

We know that

$\angle A + \angle B + \angle C = 180°$ ...(ii)

Putting $\angle A + \angle B = \angle C$ in (ii), we get

$\angle C + \angle C = 180°$

$\Rightarrow \quad 2\angle C = 180°$

$\Rightarrow \quad \angle C = 90°$

Thus, the third angle is of 90°.

Example 7 One of the acute angles of a right triangle is 58°. Find the other acute angle.

Solution Let the measure of the other acute angle be $x°$. Then, the angles of the triangle are 90°, 58° and $x°$.

Since the sum of the angles of a triangle is 180°.

$\therefore \quad 90° + 58° + x = 180°$

$\Rightarrow \quad 148° + x = 180°$

$\Rightarrow \quad x = 180° - 148°$

$\Rightarrow \quad x = 32°$

Hence, the measure of the other acute angle is 32°.

Example 8 In Fig. 14, ΔABC is right-angled at C, and $CD \perp AB$. Also, $\angle A = 65°$. Find
(i) $\angle ACD$ (ii) $\angle BCD$ (iii) $\angle CBD$.

Solution Since ΔABC is right-angled at C. Therefore, $\angle C = 90°$

Also, $\angle A = 65°$ [Given]

Now, sum of the angles of a triangle = 180°

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 65° + \angle B + 90° = 180°$

$\Rightarrow \quad \angle B = 180° - (65° + 90°)$

$\Rightarrow \quad \angle B = 180° - 155°$

$\Rightarrow \quad \angle B = 25°$

$\Rightarrow \quad \angle CBD = 25°$

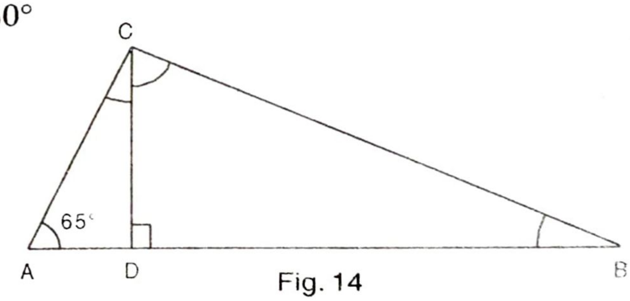


Fig. 14

Since $CD \perp AB$. Therefore, $\angle ADC = \angle CDB = 90°$

In ΔACD, we have

$\angle ACD + \angle CAD + \angle ADC = 180°$

$\Rightarrow \quad \angle ACD + 65° + 90° = 180°$

$\Rightarrow \quad \angle ACD + 155° = 180°$

$\Rightarrow \quad \angle ACD = 180° - 155° = 25°$

In ΔBCD, we have

$\angle BCD + \angle CBD + \angle BDC = 180°$

$\Rightarrow \quad \angle BCD + 25° + 90° = 180°$

$\Rightarrow \quad \angle BCD + 115° = 180°$

$\Rightarrow \quad \angle BCD = 180° - 115°$

$\Rightarrow \quad \angle BCD = 65°$

Hence, $\angle ACD = 25°$, $\angle BCD = 65°$ and $\angle CBD = 25°$.

Example 9 In Fig. 15, D, E are points on sides AB, AC of ΔABC such that $DE \parallel BC$. If $\angle B = 30°$ and $\angle A = 40°$, find $x°, y°, z°$.

Solution In ΔABC, we have

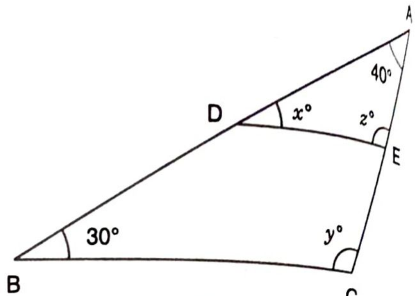


Fig. 15

$\angle A = 40°$ and $\angle B = 30°$

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 40° + 30° + \angle C = 180°$

$\Rightarrow \quad 70° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 70°$

$\Rightarrow \quad \angle C = 110°$

$\Rightarrow \quad y° = 110°$

Now, $DE \parallel BC$ and transversal AB cuts them at D and B respectively.

$\therefore \quad \angle B = \angle ADE$ [Corresponding angles]

$\Rightarrow \quad 30° = x° \quad \Rightarrow x° = 30°$

Again, $DE \parallel BC$ and transversal AC cuts them at E and C respectively.

$\therefore \quad \angle ACB = \angle AED$ [Corresponding angles]

$\Rightarrow \quad y° = z°$

$\Rightarrow \quad z° = 110°$ $[\because y° = 110°]$

Hence, $x° = 30°, y° = 110°$ and $z° = 110°$.

Example 10 The Fig. 16 has been obtained by using two triangles. Find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$.

Solution We know that the sum of the angles of a triangle is 180°.

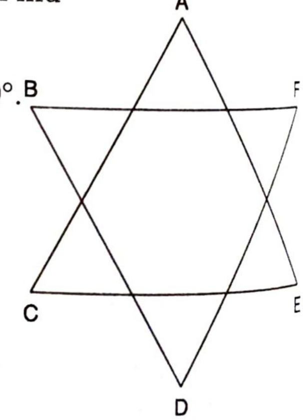


Fig. 16

$\therefore$ In ΔACE, we have $\angle A + \angle C + \angle E = 180°$ ...(i)

In ΔBDF, we have $\angle B + \angle D + \angle F = 180°$ ...(ii)

Adding the corresponding sides of (i) and (ii), we get

$(\angle A + \angle C + \angle E) + (\angle B + \angle D + \angle F)$

$\Rightarrow \quad 180° + 180°$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$

Example 11 Fig. 17 is made up of two triangles. Find $\angle DAB + \angle ABC + \angle BCD + \angle CDA$.

Solution We know that the sum of the angles of a triangle is 180°.

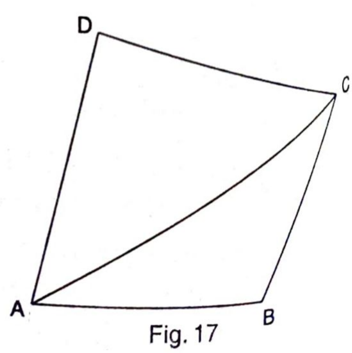


Fig. 17

$\therefore$ In ΔABC, we have

$\angle ABC + \angle BCA + \angle CAB = 180°$ ...(i)

In ΔACD, we have

$\angle DAC + \angle ACD + \angle CDA = 180°$ ...(ii)

Adding the corresponding sides of (i) and (ii), we get

$$(\angle ABC + \angle BCA + \angle CAB) + (\angle DAC + \angle ACD + \angle CDA) = 180^\circ + 180^\circ$$

$$\Rightarrow \quad \angle ABC + (\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) + \angle CDA = 360^\circ \quad \left[\begin{array}{l}\because \angle BCA + \angle ACD = \angle BCD \\ \text{and } \angle CAB + \angle DAC = \angle DAB\end{array}\right]$$

$$\Rightarrow \quad \angle ABC + \angle BCD + \angle DAB + \angle CDA = 360^\circ.$$

Example 12 In five cornered Fig. 18, AD, AC are joined. Find $\angle EAB + \angle ABC + \angle BCD + \angle CDE + \angle DEA$

Solution We know that the sum of the angles of a triangle is 180°.

$\therefore$ In ΔABC, we have

$$\angle ABC + \angle BAC + \angle BCA = 180^\circ \quad ...(i)$$

In ΔACD, we have

$$\angle ACD + \angle CAD + \angle CDA = 180^\circ \quad ...(ii)$$

In ΔADE, we have

$$\angle DAE + \angle ADE + \angle DEA = 180^\circ \quad ...(iii)$$

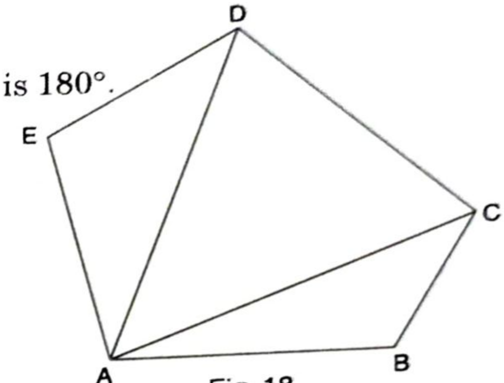


Fig. 18

Adding (i), (ii) and (iii), we get

$$(\angle ABC + \angle BAC + \angle BCA) + (\angle ACD + \angle CAD + \angle CDA) + (\angle DAE + \angle ADE + \angle DEA) = 180^\circ + 180^\circ + 180^\circ.$$

$$\Rightarrow \quad \angle ABC + (\angle BAC + \angle CAD + \angle DAE) + (\angle BCA + \angle ACD) + (\angle CDA + \angle ADE) + \angle DEA = 540^\circ$$

$$\Rightarrow \quad \angle ABC + \angle EAB + \angle BCD + \angle CDE + \angle DEA = 540^\circ. \quad \left[\begin{array}{l}\because \angle BAC + \angle CAD + \angle DAE = \angle EAB \\ \angle BCA + \angle ACD = \angle BCD \\ \text{and } \angle CDA + \angle ADE = \angle CDE\end{array}\right]$$

Example 13 The sides AB and AC of ΔABC are produced to P and Q respectively. The bisectors of exterior angles at B and C of ΔABC meet at O (Fig. 19). Prove that

$$\angle BOC = 90^\circ - \frac{1}{2}\angle A$$

Solution Since $\angle ABC$ and $\angle CBP$ form a linear pair.

$$\therefore \quad \angle ABC + \angle CBP = 180^\circ \quad \left[\begin{array}{l}\because BO \text{ is the bisector of} \\ \angle CBP \therefore \angle CBP = 2\angle 1\end{array}\right]$$

$$\Rightarrow \quad \angle B + 2\angle 1 = 180^\circ$$

$$\Rightarrow \quad 2\angle 1 = 180^\circ - \angle B$$

$$\Rightarrow \quad \angle 1 = 90^\circ - \frac{1}{2}\angle B \quad ...(i)$$

Again, $\angle ACB$ and $\angle QCB$ form a linear pair.

$$\therefore \quad \angle ACB + \angle QCB = 180^\circ$$

$$\Rightarrow \quad \angle C + 2\angle 2 = 180^\circ \quad \left[\begin{array}{l}\because OC \text{ is the bisector of } \angle QCB \\ \therefore \angle QCB = 2\angle 2\end{array}\right]$$

$$\Rightarrow \quad 2\angle 2 = 180^\circ - \angle C$$

$$\Rightarrow \quad \angle 2 = 90^\circ - \frac{1}{2}\angle C \quad ...(ii)$$

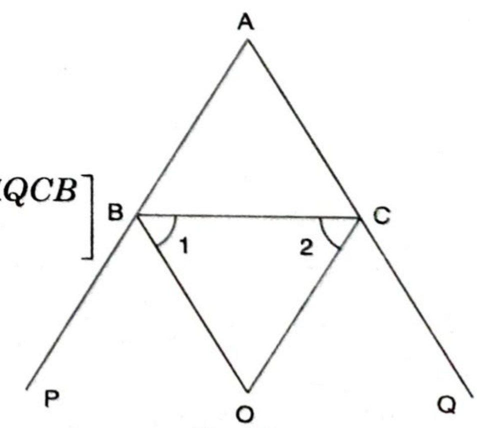


Fig. 19

Now, in ΔBOC, we have

$$\angle 1 + \angle 2 + \angle BOC = 180^\circ$$

$$\Rightarrow \quad 90^\circ - \frac{1}{2}\angle B + 90^\circ - \frac{1}{2}\angle C + \angle BOC = 180^\circ \qquad \text{[Using (i) and (ii)]}$$

$$\Rightarrow \quad 180^\circ - \frac{1}{2}(\angle B + \angle C) + \angle BOC = 180^\circ$$

$$\Rightarrow \quad \angle BOC = \frac{1}{2}(\angle B + \angle C)$$

$$\Rightarrow \quad \angle BOC = \frac{1}{2}(180^\circ - \angle A) \qquad \left[\begin{array}{l} \because \angle A + \angle B + \angle C = 180^\circ \\ \therefore \angle B + \angle C = 180^\circ - \angle A \end{array}\right]$$

$$\Rightarrow \quad \angle BOC = 90^\circ - \frac{1}{2}\angle A.$$

EXERCISE 15.2

1. Two angles of a triangle are of measures 105° and 30°. Find the measure of the third angle.
2. One of the angles of a triangle is 130°, and the other two angles are equal. What is the measure of each of these equal angles?
3. The three angles of a triangle are equal to one another. What is the measure of each of the angles?
4. If the angles of a triangle are in the ratio 1 : 2 : 3, determine three angles.
5. The angles of a triangle are $(x - 40)^\circ$, $(x - 20)^\circ$ and $\left(\frac{1}{2}x - 10\right)^\circ$. Find the value of x.
6. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°, find the three angles.
7. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30°. Determine all the angles of the triangle.
8. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.
9. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.
10. In each of the following, the measures of three angles are given. State in which cases, the angles can possibly be those of a triangle:
    (i) 63°, 37°, 80°
    (ii) 45°, 61°, 73°
    (iii) 59°, 72°, 61°
    (iv) 45°, 45°, 90°
    (v) 30°, 20°, 125°
11. The angles of a triangle are in the ratio 3 : 4 : 5. Find the smallest angle.
12. Two acute angles of a right triangle are equal. Find the two angles.
13. One angle of a triangle is greater than the sum of the other two. What can you say about the measure of this angle? What type of a triangle is this?

14. In the six cornered figure, (Fig. 20), AC, AD and AE are joined. Find $\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA$.

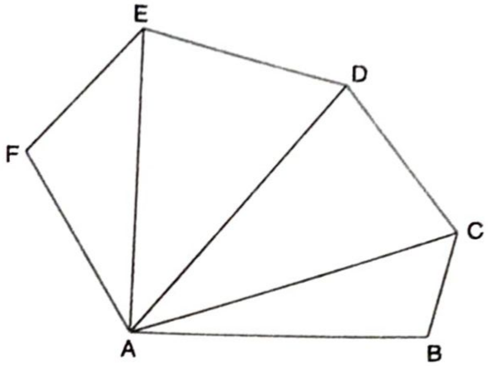


Fig. 20

15. Find x, y, z (whichever is required) in the Figures (Fig. 21) given below:

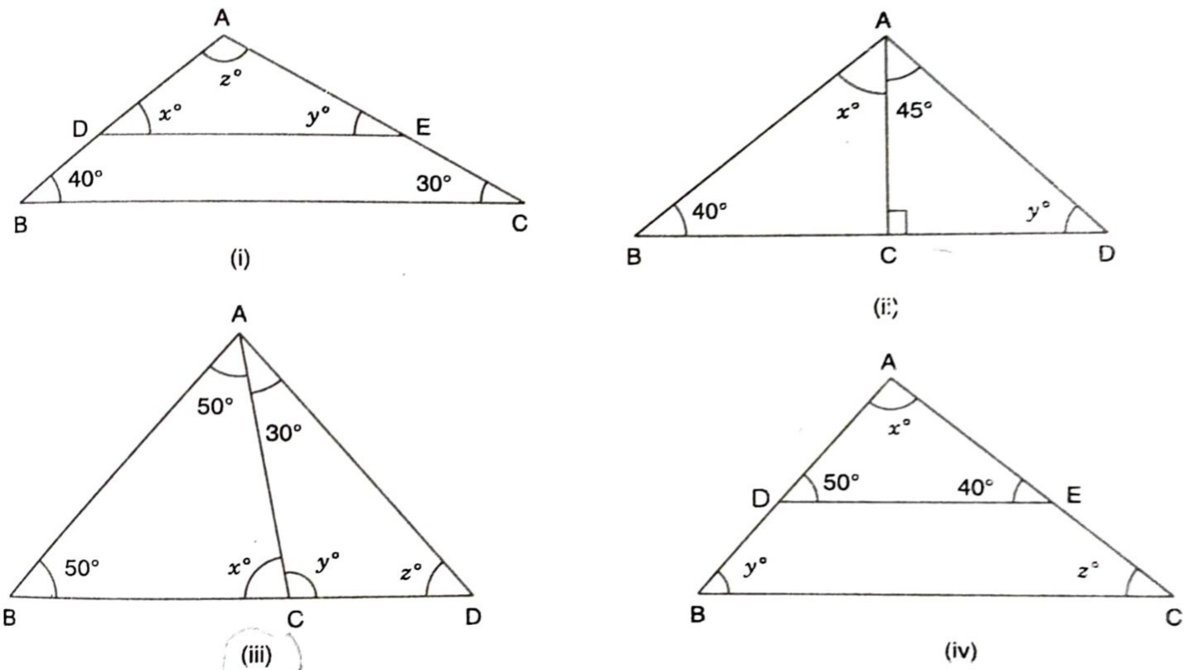


Fig. 21

16. If one angle of a triangle is 60° and the other two angles are in the ratio 1 : 2, find the angles.
17. If one angle of a triangle is 100° and the other two angles are in the ratio 2 : 3, find the angles.
18. In a ΔABC, if $3\angle A = 4\angle B = 6\angle C$, calculate the angles.
19. Is it possible to have a triangle, in which
   (i) two of the angles are right? (ii) two of the angles are obtuse?
   (iii) two of the angles are acute? (iv) each angle is less than 60°?
   (v) each angle is greater than 60°? (vi) each angle is equal to 60°?

   Give reasons in support of your answer in each case.
20. In ΔABC, $\angle A = 100°$, AD bisects $\angle A$ and $AD \perp BC$. Find $\angle B$.
21. In ΔABC, $\angle A = 50°$, $\angle B = 70°$ and bisector of $\angle C$ meets AB in D. Find the angles of the triangles ADC and BDC.
22. In ΔABC, $\angle A = 60°$, $\angle B = 80°$ and the bisectors of $\angle B$ and $\angle C$ meet at O. Find (i) $\angle C$ (ii) $\angle BOC$.

23. The bisectors of the acute angles of a right triangle meet at O. Find the angle at O between the two bisectors.

24. In ΔABC, $\angle A = 50^\circ$ and BC is produced to a point D. The bisectors of $\angle ABC$ and $\angle ACD$ meet at E. Find $\angle E$.

25. In ΔABC, $\angle B = 60^\circ$, $\angle C = 40^\circ$, $AL \perp BC$ and AD bisects $\angle A$ such that L and D lie on side BC. Find $\angle LAD$.

26. Line segments AB and CD intersect at O such that $AC \parallel DB$. If $\angle CAB = 35^\circ$ and $\angle CDB = 55^\circ$, find $\angle BOD$.

27. In Fig. 22, ΔABC is right angled at A. Q and R are points on line BC and P is a point such that $QP \parallel AC$ and $RP \parallel AB$. Find $\angle P$.

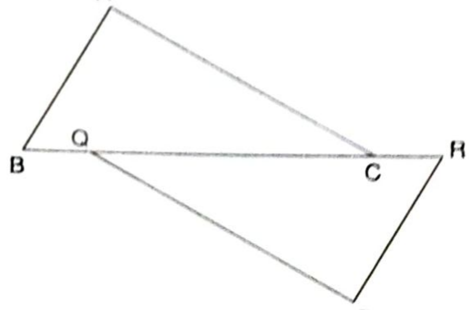


Fig. 22

ANSWERS

1. 45° 2. $25^\circ, 25^\circ$ 3. 60° 4. $30^\circ, 60^\circ, 90^\circ$

5. 100° 6. $50^\circ, 60^\circ, 70^\circ$ 7. $50^\circ, 50^\circ, 80^\circ$

10. (i), (iv) 11. 45° 12. $45^\circ, 45^\circ$ 13. More than 90°, Obtuse triangle

14. 720° 15. (i) $x = 40^\circ, y = 30^\circ, z = 110^\circ$ (ii) $x = 50^\circ, y = 45^\circ$
(iii) $x = 80^\circ, y = 100^\circ, z = 50^\circ$ (iv) $x = 90^\circ, y = 50^\circ, z = 40^\circ$

16. $40^\circ, 80^\circ$ 17. $32^\circ, 48^\circ$ 18. $80^\circ, 60^\circ, 40^\circ$

19. (i) No (ii) No (iii) Yes (iv) No (v) No (vi) Yes 20. 40°

21. For ΔADC, $\angle A = 50^\circ$, $\angle D = 100^\circ$, $\angle C = 30^\circ$, For ΔBDC, $\angle B = 70^\circ$, $\angle D = 80^\circ$, $\angle C = 30^\circ$

22. (i) 40° (ii) 120° 23. 135° 24. 25° 25. 10°

26. 90° 27. 90°

HINTS TO SELECTED PROBLEMS

6. Let the angles be x°, $(x+10)^\circ$ and $(x+20)^\circ$. Then,
$x + (x+10) + (x+20) = 180^\circ \Rightarrow 3x + 30 = 180 \Rightarrow 3x = 150 \Rightarrow x = 50$.
So, the angles are 50°, 60° and 70°.

7. Let the angles be x°, x° and $x + 30^\circ$. Then,
$x + x + (x+30) = 180 \Rightarrow 3x + 30 = 180 \Rightarrow x = 50$.

8. Let ABC be a triangle such that $\angle A = \angle B + \angle C$. Then,
$\angle A + \angle B + \angle C = 180^\circ \Rightarrow \angle A + \angle A = 180^\circ \Rightarrow \angle A = 90^\circ$

9. Let $\angle A < \angle B + \angle C$. Then $2\angle A < \angle A + \angle B + \angle C$

$\Rightarrow \quad 2\angle A < 180^\circ \Rightarrow \quad \angle A < 90^\circ$.

Similarly, $\angle B < 90^\circ$ and $\angle C < 90^\circ$.

15.6 EXTERIOR AND INTERIOR OPPOSITE ANGLES OF A TRIANGLE

EXTERIOR ANGLE *If the side BC of a triangle ABC is produced to form ray BX, then $\angle ACX$ is called an exterior angle of ΔABC at C.*

INTERIOR OPPOSITE ANGLES *With respect to exterior angle $\angle ACX$ of ΔABC at C, the angles $\angle BAC$ and $\angle ABC$ are called interior opposite angles or remote interior angles.*

INTERIOR ADJACENT ANGLES *With respect to exterior $\angle ACX$ of ΔABC at C, $\angle ACB$ is called interior adjacent angle.*

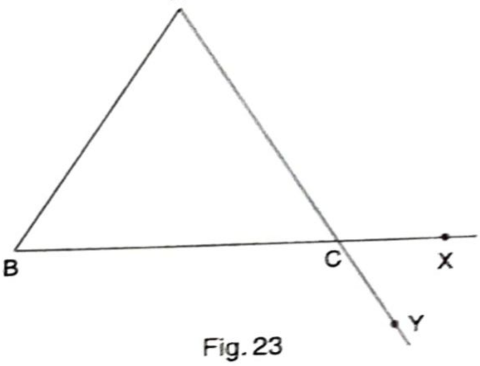


Fig. 23

If we produce side AC to form ray AY, then $\angle BCY$ is also an exterior angle of ΔABC at C. Clearly, $\angle ACX$ and $\angle BCY$ are vertically opposite angles.

$\therefore \quad \angle ACX = \angle BCY$.

Thus, at each vertex of a triangle, we have a pair of exterior angles of the triangle which are equal to each other.

With respect to exterior $\angle BCY$ of ΔABC at C, $\angle ACB$ is the interior adjacent angle and $\angle A$ and $\angle B$ are the two interior opposite angles, same as for exterior $\angle ACX$.

An exterior angle of a triangle is closely related to the interior opposite angles as given below:

THEOREM *(Exterior Angle Property of a Triangle): If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.*

Proof Let ABC be a triangle such that its side BC is produced to form ray BX. Then, $\angle ACX = \angle 4$ is the exterior angle of ΔABC at C and $\angle 1$ and $\angle 2$ are two interior opposite angles.

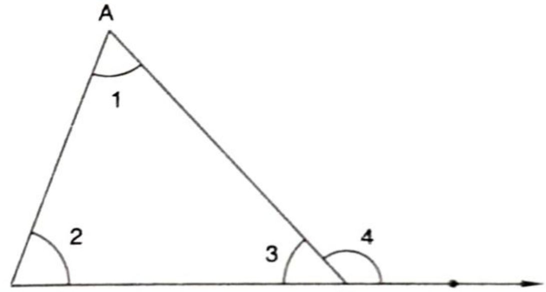


Fig. 24

We have to prove that

$\angle 4 = \angle 1 + \angle 2$

The sum of the angles of a triangle is 180°.

$\therefore \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ$ ...(i)

Also, $\quad \angle 3 + \angle 4 = 180^\circ$ ...(ii)

[$\because$ $\angle 3$ and $\angle 4$ form a linear pair]

From (i) and (ii), we get

$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle 1 + \angle 2 = \angle 4$ [Subtracting $\angle 3$ from both sides]

Hence, $\angle 4 = \angle 1 + \angle 2$.

<u>An Important Result:</u> *In Fig. 24, we have*

$\angle ACX = \angle A + \angle B \quad \Rightarrow \quad \angle ACX > \angle A$ *and* $\angle ACX > \angle B$.

Thus, in a triangle an exterior angle is greater than either of the interior opposite angles.

<u>Remark</u> *An exterior angle and the interior adjacent angle form a linear pair.*

Let us now discuss some problems to discuss the applications of the above property. Following examples will illustrate the same.

ILLUSTRATIVE EXAMPLES

Example 1 In Fig. 25, two of the angles are indicated. What is the measure of $\angle ACD$?

Solution Clearly, $\angle ACD$ is the exterior angle of ΔABC at C and an exterior angle is equal to the sum of the two interior opposite angles.

$\therefore \quad \angle ACD = \angle A + \angle B$

$\Rightarrow \quad \angle ACD = 60° + 70° = 130°$

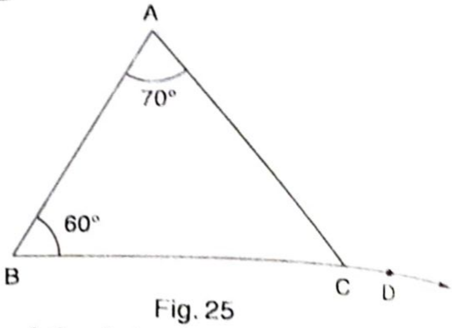


Fig. 25

Example 2 An exterior angle of a triangle is 110°, and one of the interior opposite angles is 30°. Find the other two angles of the triangle.

Solution Let ABC be a triangle whose side BC is produced to D to form an exterior angle $\angle ACD$ such that $\angle ACD = 110°$. (Fig. 26).

Let $\angle B = 30°$. By exterior angle theorem, we have

$\angle ACD = \angle B + \angle A$

$\Rightarrow \quad 110° = 30° + \angle A$

$\Rightarrow \quad \angle A = 110° - 30° = 80°$

Now, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 80° + 30° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - (80° + 30°) = 70°$

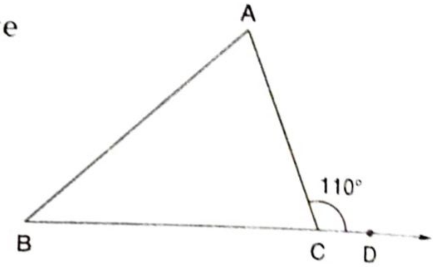


Fig. 26

Hence, the other two angles of the triangle are 80° and 70°.

Example 3 One of the exterior angles of a triangle is 80° and the interior opposite angles are in the ratio 3 : 5. Find the angles of the triangle.

Solution Let $\angle ACX$ be the exterior angle of ΔABC at C such that $\angle ACX = 80°$. Clearly, $\angle A$ and $\angle B$ are the interior opposite angles (Fig. 27).

It is given that $\angle A : \angle B = 3 : 5$. So, let $\angle A = 3x°$ and $\angle B = 5x°$.

$\angle ACX = \angle A + \angle B$

$\Rightarrow \quad 80° = 3x + 5x$ [By exterior angle property]

$\Rightarrow \quad 8x = 80°$

$\Rightarrow \quad \frac{8x}{8} = \frac{80°}{8}$ [Dividing both sides by 8]

$\Rightarrow \quad x = 10°$

$\therefore \quad \angle A = 3x° = 30°$ and $\angle B = 5x° = 50°$

Now, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 30° + 50° + \angle C = 180°$

$\Rightarrow \quad 80° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 80° = 100°$

Hence, $\angle A = 30°$, $\angle B = 50°$ and $\angle C = 100°$.

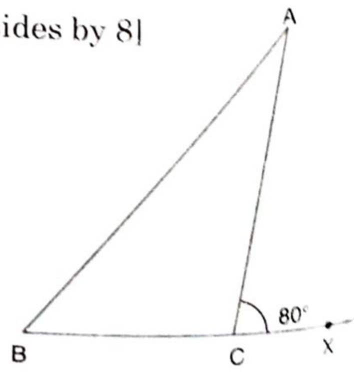


Fig. 27

Example 4 In Fig. 28, the measures of some of the angles are indicated. Find the values of x° and y°.

Solution In ΔABC, exterior $\angle CBX$ at B and adjacent interior $\angle CBA$ form a linear pair.

$\therefore \quad \angle CBX + \angle CBA = 180^\circ$

$\Rightarrow \quad 70^\circ + y^\circ = 180^\circ$

$\Rightarrow \quad y^\circ = 180^\circ - 70^\circ = 110^\circ$

$\therefore \quad y^\circ = 110^\circ$

Again, $\angle BCY$ is exterior of ΔABC at C, and $\angle CAB$ and $\angle CBA$ are interior opposite angles.

$\therefore \quad \angle BCY = \angle CAB + \angle CBA$

$\Rightarrow \quad x^\circ = 40^\circ + y^\circ$

$\Rightarrow \quad x^\circ = 40^\circ + 110^\circ \qquad [\because y^\circ = 110^\circ]$

$\Rightarrow \quad x^\circ = 150^\circ$

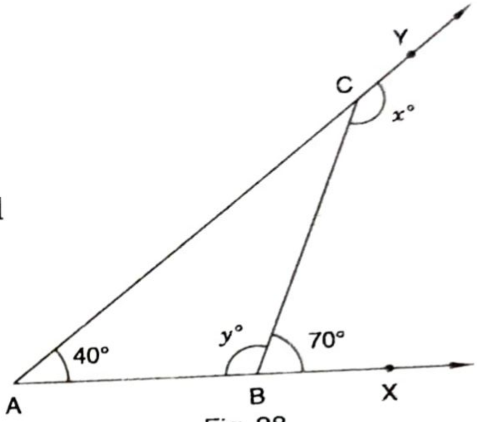


Fig. 28

Hence, we have $x^\circ = 150^\circ$ and $y^\circ = 110^\circ$.

Example 5 In Fig. 29, find $\angle ABD$. Also, if $\angle C = 3\angle ABC$, find $\angle ABC$.

Solution In ΔABD, exterior $\angle BAC$ at A and adjacent interior $\angle BAD$ form a linear pair.

$\therefore \quad \angle BAD + \angle BAC = 180^\circ$

$\Rightarrow \quad \angle BAD + 100^\circ = 180^\circ$

$\Rightarrow \quad \angle BAD = 180^\circ - 100^\circ$

$\Rightarrow \quad \angle BAD = 80^\circ$

In ΔABD, the sum of the angles is 180°.

$\therefore \quad \angle ABD + \angle BAD + \angle ADB = 180^\circ$

$\Rightarrow \quad \angle ABD + 80^\circ + 70^\circ = 180^\circ$

$\Rightarrow \quad \angle ABD + 150^\circ = 180^\circ$

$\Rightarrow \quad \angle ABD = 180^\circ - 150^\circ$

$\Rightarrow \quad \angle ABD = 30^\circ$

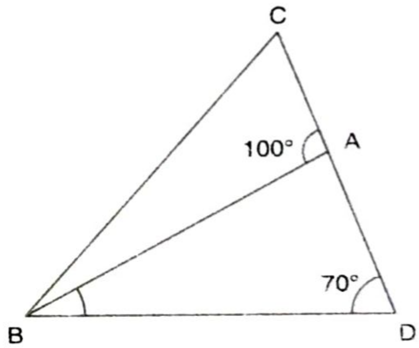


Fig. 29

Again, in ΔABC, we have

$\angle ABC + \angle C + \angle BAC = 180^\circ$

$\Rightarrow \quad \angle ABC + 3\angle ABC + 100^\circ = 180^\circ \qquad [\because C = 3\angle ABC \text{ (Given)}]$

$\Rightarrow \quad 4\angle ABC + 100^\circ = 180^\circ$

$\Rightarrow \quad 4\angle ABC = 180^\circ - 100^\circ$

$\Rightarrow \quad 4\angle ABC = 80^\circ$

$$\Rightarrow \quad \angle ABC = \frac{80^\circ}{4} = 20^\circ$$

Example 6 In Fig. 30, find: (i) $\angle ACD$ (ii) $\angle AED$

Solution (i) In ΔABC, $\angle ACD$ is the exterior angle at C and $\angle CBA$ and $\angle CAB$ are interior opposite angles.

$\therefore \quad \angle ACD = \angle CBA + \angle CAB$

$\Rightarrow \angle ACD = 40° + 15°$

$\Rightarrow \angle ACD = 55°$.

(ii) In ΔCDE, $\angle AED$ is the exterior angle at E and $\angle ECD$ and $\angle CDE$ are interior opposite angles.

$\therefore \angle AED = \angle ECD + \angle CDE$

$\Rightarrow \angle AED = \angle ACD + \angle CDE \quad [\because \angle ECD = \angle ACD]$

$\Rightarrow \angle AED = 55° + 52° = 107°$

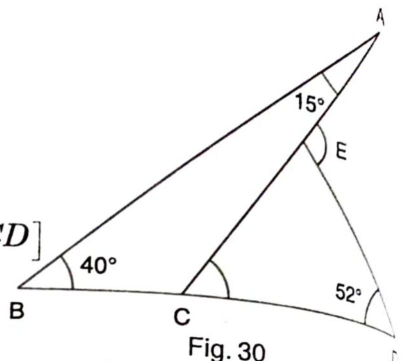


Fig. 30

Example 7 In Fig. 31, the sides BC, CA and AB of a ΔABC, are produced in order, forming exterior angles $\angle ACD$, $\angle BAE$ and $\angle CBF$. Show that $\angle ACD + \angle BAE + \angle CBF = 360°$

Solution By using exterior angle theorem, we have

$\angle ACD = \angle 1 + \angle 2$, $\angle BAE = \angle 2 + \angle 3$,

and, $\angle CBF = \angle 1 + \angle 3$.

Adding these, we get

$\angle ACD + \angle BAE + \angle CBF$

$= (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 1 + \angle 3)$

$= 2(\angle 1 + \angle 2 + \angle 3)$

$= 2 \times 180° \quad [\because \angle 1 + \angle 2 + \angle 3 = 180°]$

$= 360°$

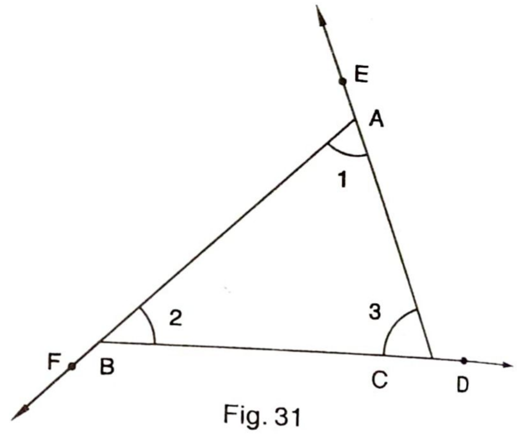


Fig. 31

Example 8 The side BC of a ΔABC is produced on both sides. Show that the sum of the exterior angles so formed is greater than $\angle A$ by two right angles (Fig. 32).

Solution By exterior angle theorem, we have

$\angle 4 = \angle 1 + \angle 3$ and, $\angle 5 = \angle 1 + \angle 2$

Adding these two, we get

$\angle 4 + \angle 5 = (\angle 1 + \angle 3) + (\angle 1 + \angle 2)$

$\Rightarrow \angle 4 + \angle 5 = \angle 1 + (\angle 1 + \angle 2 + \angle 3)$

$\Rightarrow \angle 4 + \angle 5 = \angle 1 + 180° \quad [\because \angle 1 + \angle 2 + \angle 3 = 180°]$

$\Rightarrow \angle 4 + \angle 5 = \angle A + 2 \times 90°$

$\Rightarrow \angle 4 + \angle 5 - \angle A = 2 \times 90°$

$\Rightarrow \angle 4 + \angle 5$ exceeds $\angle A$ by two right angles.

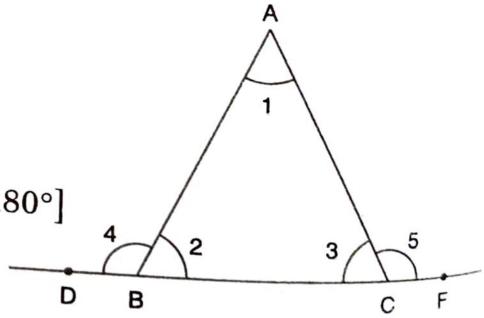


Fig. 32

Example 9 Sides BC, CA and BA of a triangle ABC are produced to D, Q, P respectively as shown in Fig.33. If $\angle ACD = 100°$ and $\angle QAP = 35°$, find all the angles of the triangle.

Solution Since $\angle QAP$ and $\angle BAC$ are vertically opposite angles.

$\therefore \angle BAC = \angle QAP$

$\Rightarrow \angle BAC = 35° \quad [\because \angle QAP = 35°]$

By exterior angle theorem, we have

$\angle ACD = \angle BAC + \angle CBA$

$\Rightarrow \quad 100° = 35° + \angle CBA$

$\Rightarrow \quad \angle CBA = 100° - 35°$

$\Rightarrow \quad \angle CBA = 65°$

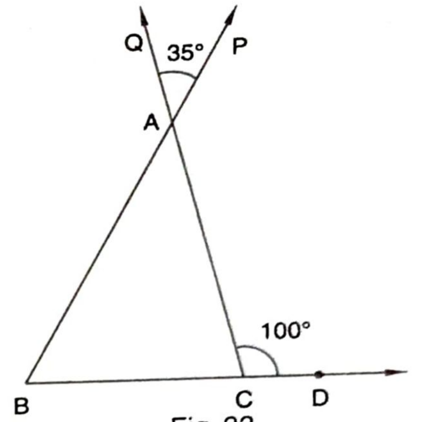


Fig. 33

Since $\angle ACB$ and $\angle ACD$ form linear pairs.

$\therefore \quad \angle ACB + \angle ACD = 180°$

$\Rightarrow \quad \angle ACB + 100° = 180°$

$\Rightarrow \quad \angle ACB = 180° - 100° = 80°$

Hence, the angles of the ΔABC are $\angle A = 35°$, $B = 65°$ and $\angle C = 80°$.

Example 10 In Fig. 34, the side BC of ΔABC is produced to form ray BD. Ray CE is drawn parallel to BA. Show directly, without using the angle sum property of a triangle that $\angle ACD = \angle A + \angle B$ and deduce that $\angle A + \angle B + \angle C = 180°$.

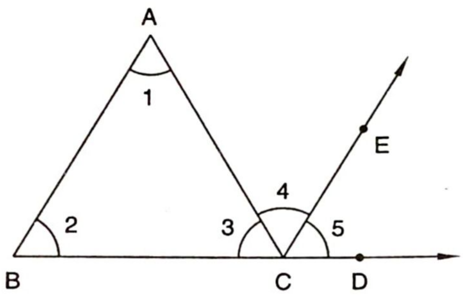


Fig. 34

Solution Since $AB \parallel CE$ and transversal AC cuts them at A and C respectively.

$\therefore \quad \angle 1 = \angle 4$ [Alternate interior angles] ...(i)

Again, $AB \parallel CE$ and transversal BD cuts them at B and C

$\therefore \quad \angle 2 = \angle 5$ [Corresponding angles] ...(ii)

Adding (i) and (ii), we get

$\angle 1 + \angle 2 = \angle 4 + \angle 5$

$\Rightarrow \quad \angle A + \angle B = \angle ACD$ $[\because \angle 4 + \angle 5 = \angle ACD]$

This proves the first part.

$\angle A + \angle B = \angle ACD$

$\Rightarrow \quad \angle A + \angle B + \angle C = \angle ACD + \angle C$ [Adding $\angle C$ on both sides]

$\Rightarrow \quad \angle A + \angle B + \angle C = 180°$ $\left[\begin{array}{l}\because \angle ACD \text{ and } \angle C \text{ form a linear pair} \\ \therefore \angle ACD + \angle C = 180°\end{array}\right]$

EXERCISE 15.3

1. In Fig. 35, $\angle CBX$ is an exterior angle of ΔABC at B. Name
   (i) the interior adjacent angle (ii) the interior opposite angles to exterior $\angle CBX$.
   Also, name the interior opposite angles to an exterior angle at A.

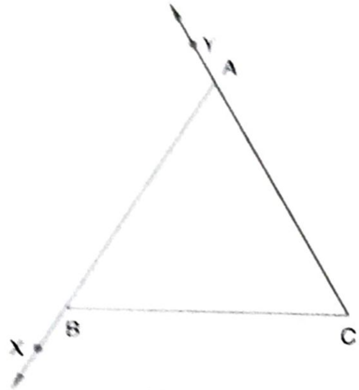


Fig. 35

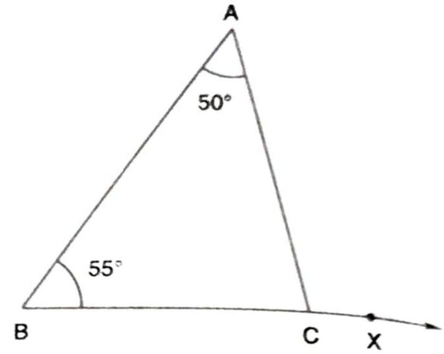


Fig. 36

2. In Fig. 36, two of the angles are indicated. What are the measures of $\angle ACX$ and $\angle ACB$?
3. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angles is 55°. Find all the angles of the triangle.
4. One of the exterior angles of a triangle is 80°, and the interior opposite angles are equal to each other. What is the measure of each of these two angles?
5. The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136°. Find all the angles of the triangle.
6. In Fig. 37, the sides BC, CA and BA of a ΔABC have been produced to D, E and F respectively. If $\angle ACD = 105°$ and $\angle EAF = 45°$; find all the angles of the ΔABC.

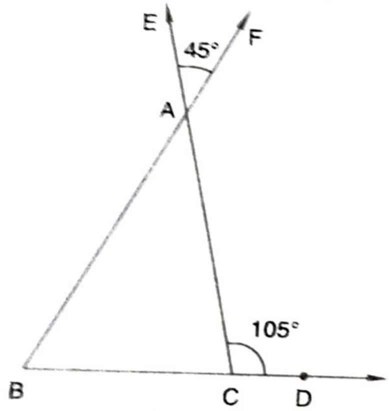


Fig. 37

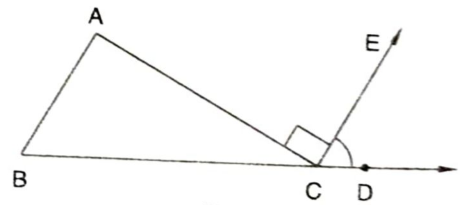


Fig. 38

7. In Fig. 38, $AC \perp CE$ and $\angle A : \angle B : \angle C = 3 : 2 : 1$, find the value of $\angle ECD$.
8. A student when asked to measure two exterior angles of ΔABC observed that the exterior angles at A and B are of 103° and 74° respectively. Is this possible? Why or why not?
9. In Fig. 39, AD and CF are respectively perpendiculars to sides BC and AB of ΔABC. If $\angle FCD = 50°$, find $\angle BAD$.

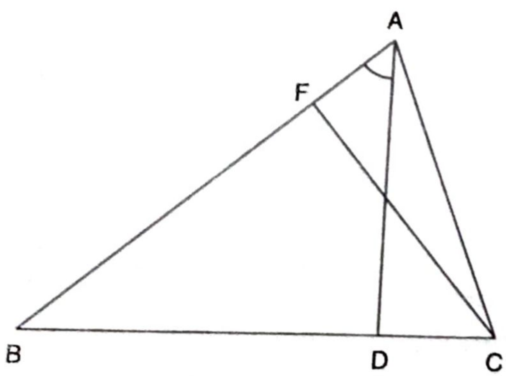


Fig. 39

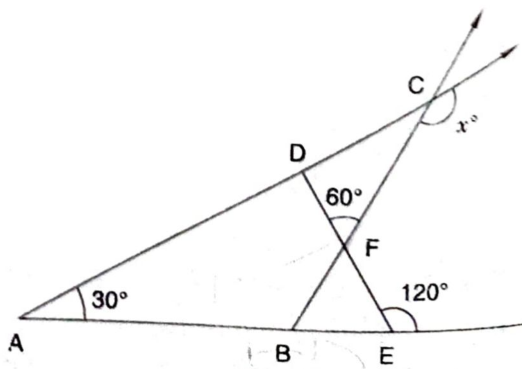


Fig. 40

10. In Fig. 40, measures of some angles are indicated. Find the value of x.

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and $DE \perp BC$, intersecting AC at F. If $\angle AFE = 130°$, find
(i) $\angle BDE$ (ii) $\angle BCA$ (iii) $\angle ABC$

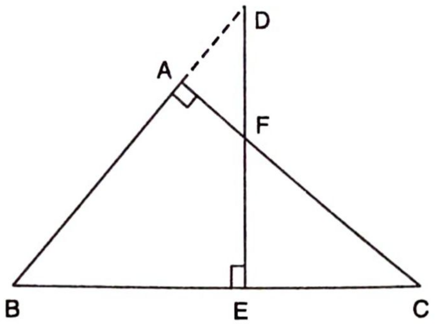


Fig. 41

12. ABC is a triangle in which $\angle B = \angle C$ and ray AX bisects the exterior angle DAC. If $\angle DAX = 70°$, find $\angle ACB$.
13. The side BC of ΔABC is produced to a point D. The bisector of $\angle A$ meets side BC in L. If $\angle ABC = 30°$ and $\angle ACD = 115°$, find $\angle ALC$.
14. D is a point on the side BC of ΔABC. A line PDQ, through D, meets side AC in P and AB produced at Q. If $\angle A = 80°$, $\angle ABC = 60°$ and $\angle PDC = 15°$, find (i) $\angle AQD$ (ii) $\angle APD$.
15. Explain the concept of interior and exterior angles and in each of the figures given below, find x and y (Fig. 42).

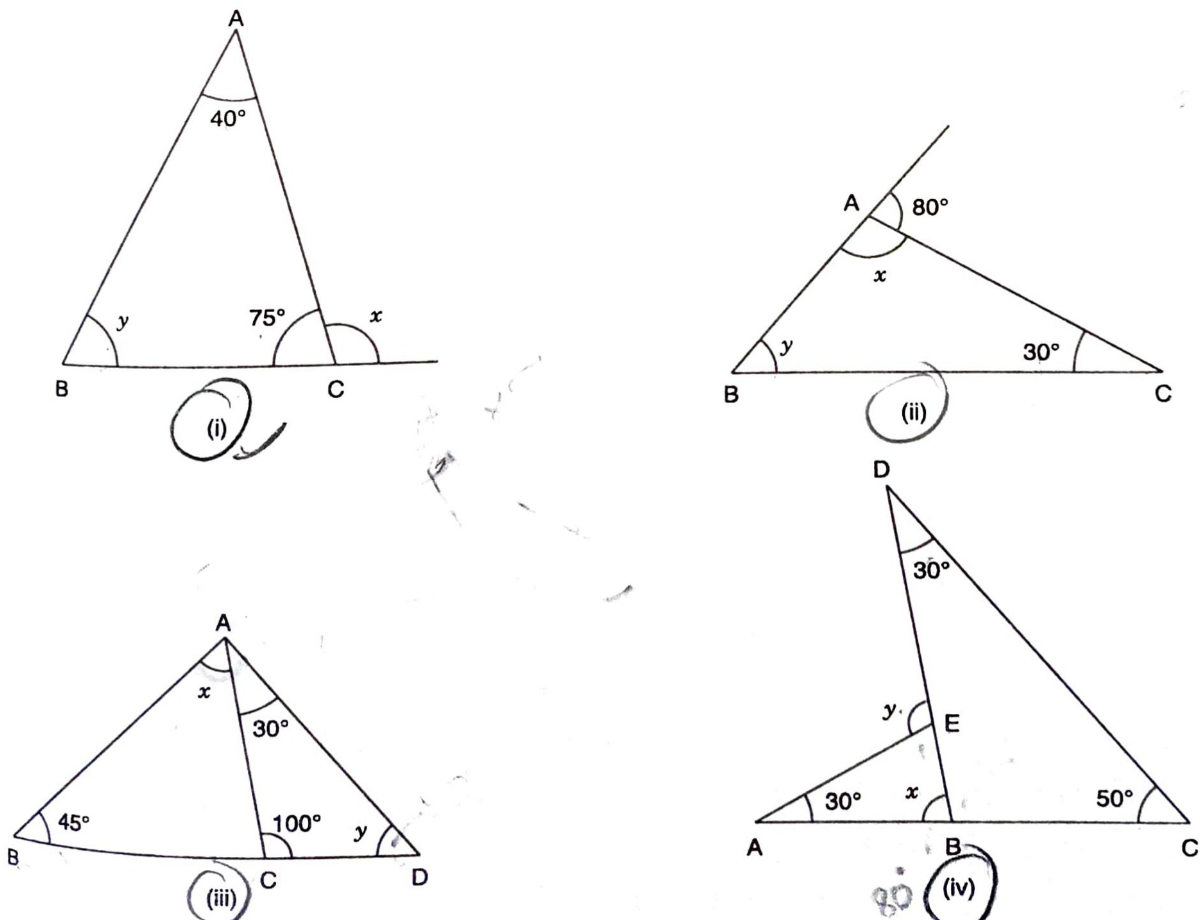


Fig. 42

16. Compute the value of x in each of the following figures:

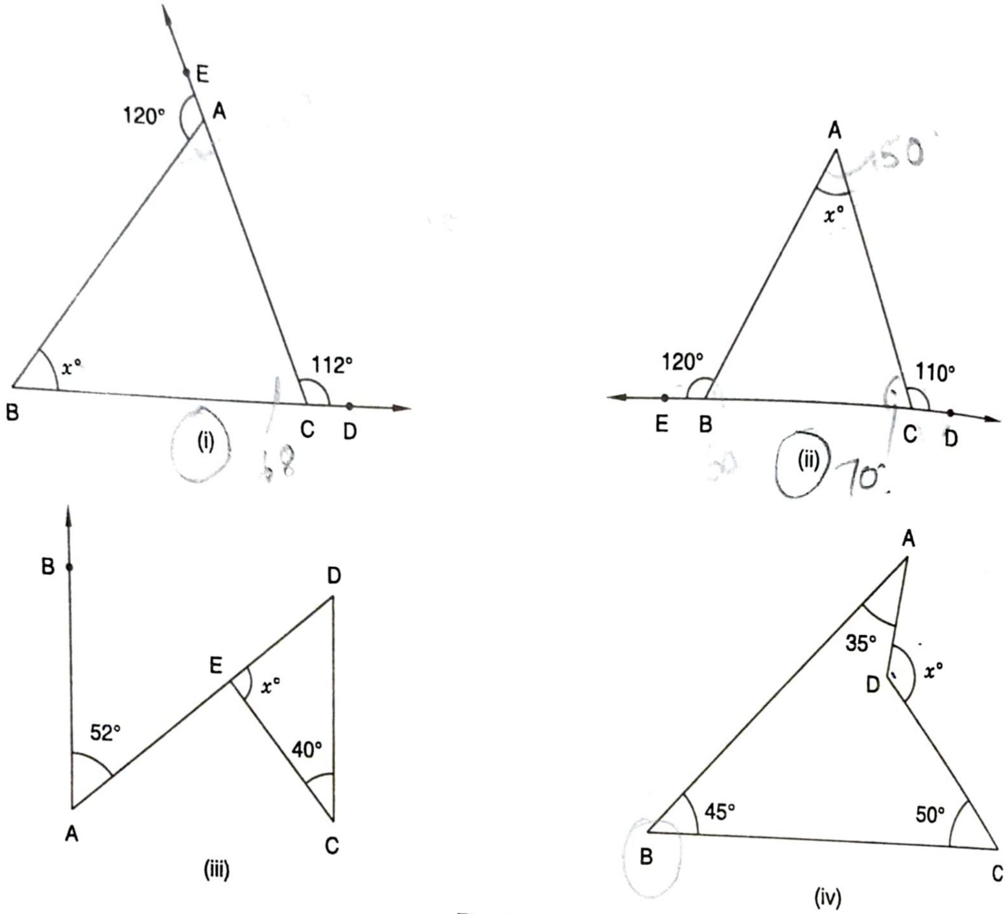


Fig. 43

ANSWERS

1. (i) $\angle ABC$ (ii) $\angle BAC, \angle ACB; \angle ABC, \angle ACB$
2. $\angle ACX = 105°, \angle ACB = 75°$
3. 55°, 40°, 85°
4. 40° each
5. 60°, 76°, 44°
6. $\angle A = 45°$; $\angle C = 75°$; $\angle B = 60°$
7. 60°
8. No, since sum of interior angles A and $B = 183° > 180°$
9. 50°
10. 150°
11. (i) 40° (ii) 40° (iii) 50°
12. 70°
13. $72\frac{1}{2}°$
14. (i) 45° (ii) 55°
15. (i) 105°, 65° (ii) 100°, 50° (iii) 55°, 50° (iv) 80°, 110°
16. 52°, 50°, 88°, 130°

15.7 TRIANGLE INEQUALITY PROPERTY

Consider a triangle ABC as shown in Fig. 44. It has three sides BC, CA and AB. Let us denote the sides opposite the vertices A, B, C by a, b, c respectively. That is,

$$a = BC, b = CA \text{ and } c = AB$$

The sides of a triangle satisfy an important property as stated below:

Property *The sum of any two sides of a triangle is greater than the third side. That is, in a triangle ABC, we have*

$$b + c > a, c + a > b \text{ and } a + b > c$$

This important property of a triangle is known as Triangle inequality.

In order to verify the above property, let us perform the following experiment.

Experiment Draw any three triangles T_1, T_2 and T_3. Label each one as ΔABC. Measure, in each case, the three sides $a = BC$, $b = CA$ and $c = AB$.

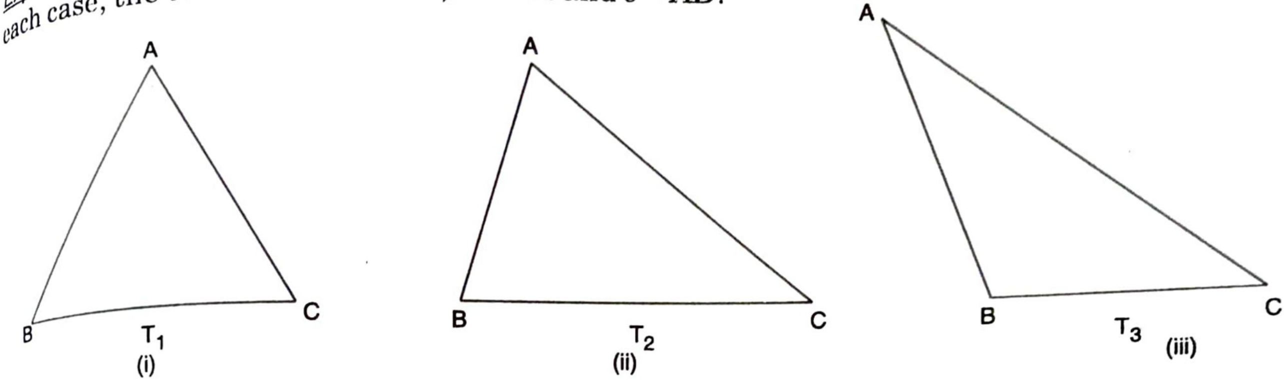


Fig. 44

Tabulate your observations as follows:

Triangle	Side of ΔABC			Computations					
	a	b	c	$b+c$	$b+c-a$	$c+a$	$c+a-b$	$a+b$	$a+b-c$
T_1									
T_2									
T_3									

From the above table you will find that:

(i) each value of $b + c - a$ is positive; (ii) each value of $c + a - b$ is positive ;

(iii) each value of $a + b - c$ is positive.

Now,

$b + c - a$ is positive $\Rightarrow$ $b + c - a > 0$ $\Rightarrow$ $b + c > a$

$c + a - b$ is positive $\Rightarrow$ $c + a - b > 0$ $\Rightarrow$ $c + a > b$

$a + b - c$ is positive $\Rightarrow$ $a + b - c > 0$ $\Rightarrow$ $a + b > c$

Thus, we have verified the following geometrical truth:

The sum of any two sides of a triangle is greater than the third side.

Remark 1 *From the above discussion, we find that three line segments whose lengths are equal to three given numbers, form the sides of a triangle if the sum of the lengths of every pair of two of these is greater than the length of the third.*

Remark 2 *In a triangle, the angle opposite the largest side is the largest.*

ILLUSTRATIVE EXAMPLES

Example 1 In each of the following there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:

(i) 2, 3, 4 (ii) 4, 5, 3 (iii) 2.5, 1.5, 4

Solution

(i) We have,

$2 + 3 > 4$, $2 + 4 > 3$ and $3 + 4 > 2$

That is, the sum of any two of the given numbers is greater than the third number.

So, 2 cm, 3 cm and 4 cm can be the lengths of the sides of a triangle.

(ii) We have,

$4 + 5 > 3, 4 + 3 > 5$ and $5 + 3 > 4$

That is, the sum of any two of the given numbers is greater than the third number.

So, 4 cm, 5 cm and 3 cm can be the lengths of the sides of a triangle.

(iii) We have,

$2.5 + 1.5 \not> 4$.

So, the given numbers cannot be the lengths of the sides of a triangle.

Example 2 In Fig. 45, $\Delta ABC, AB = 3cm, BC = 4cm$ and $AC = 5cm$. Name the smallest and the largest angles of the triangle.

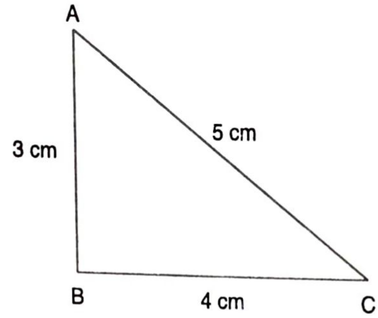


Fig. 45

Solution As the largest angle is always opposite to the largest side. Therefore, $\angle B$ is the largest angle and smallest angle is opposite to smallest side.

$\therefore$ $\angle C$ is the smallest angle.

EXERCISE 15.4

1. In each of the following, there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:
   (i) 5, 7, 9 (ii) 2, 10, 15 (iii) 3, 4, 5 (iv) 2, 5, 7 (v) 5, 8, 20
2. In Fig. 46, P is the point on the side BC. Complete each of the following statements using symbol '=', '>' or '<' so as to make it true:

   (i) $AP ... AB + BP$ (ii) $AP .... AC + PC$ (iii) $AP .... \frac{1}{2}(AB + AC + BC)$

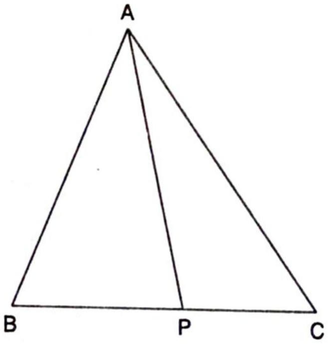


Fig. 46

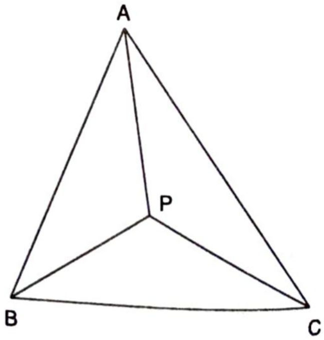


Fig. 47

3. P is a point in the interior of ΔABC as shown in Fig. 47. State which of the following statements are true (T) or false (F):
   (i) $AP + PB < AB$ (ii) $AP + PC > AC$ (iii) $BP + PC = BC$

4. O is a point in the exterior of ΔABC. What symbol '>', '<' or '=' will you use to complete the statement $OA + OB .. AB$? Write two other similar statements and show that

$$OA + OB + OC > \frac{1}{2}(AB + BC + CA)$$

5. In ΔABC, $\angle A = 100°$, $\angle B = 30°$, $\angle C = 50°$. Name the smallest and the largest sides of the triangle.

ANSWERS

1. (i), (iii) 2. (i) < (ii) < (iii) < 3. (i) F (ii) T (iii) F 4. >
5. AC, BC

15.8 PYTHAGORAS THEOREM

Pythagoras, the famous Greek philosopher, was born in about 580 B.C. in Samos, Ionia and died in about 520 B.C. in Metapontum, Lucania. He gave a wonderful relation between the lengths of the sides of a right triangle which is generally known as Pythagoras theorem. Although this theorem was known to the Babylonians 1000 years earlier. But, Pythagoras is believed to have been the first to discover a proof of this theorem. And, for this reason, the theorem bears his name.

STATEMENT *In a right triangle, the square of the hypotenuse equals the sum of the squares of its remaining two sides.*

Thus, if ΔABC is a right triangle right angled at C, so that AB is the hypotenuse and, AC and BC are the sides of the right angle, then

$$(AB)^2 = (BC)^2 + (CA)^2$$

i.e., $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

or, $c^2 = a^2 + b^2$, where $a = BC$, $b = CA$ and $c = AB$.

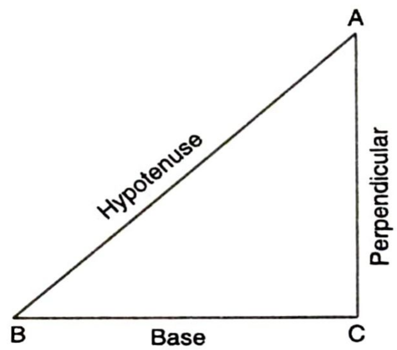


Fig. 48

Verification: In order to verify the above stated theorem, we may perform the following experiment.

Experiment Draw any three right triangles say T_1, T_2 and T_3. Label each as ΔABC with $\angle C$ as the right angle.

In each case, measure the sides a, b and the hypotenuse c of the triangle. Compute a^2, b^2 and c^2 and tabulate the results as follows:

Right Triangle	Measurements			Square of a, b			Square of c	Difference $c^2 - (a^2 + b^2)$
	a	b	c	a^2	b^2	$a^2 + b^2$	c^2	
T_1								
T_2								
T_3								

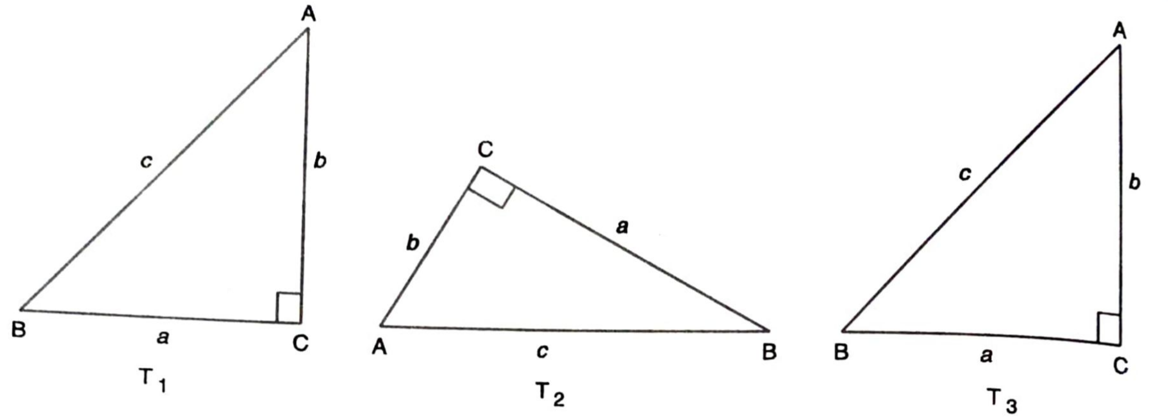


Fig. 49

You will observe that in each case the difference $c^2 - (a^2 + b^2)$ is 0.

$\therefore \quad c^2 = a^2 + b^2$

COROLLARY 1 If $\Delta\, ABC$ is a right triangle right angled at C such that $AB = c$, $BC = a$ and $AC = b$, then by Pythagoras theorem, we have

$$c^2 = a^2 + b^2$$

$$\Rightarrow \quad c^2 > a^2 \text{ and } c^2 > b^2$$

$$\Rightarrow \quad c > a \text{ and } c > b$$

Thus, in a right triangle, the hypotenuse is greater than each one of the remaining two sides of the triangle.

In other words, in a right triangle, the hypotenuse is the longest side.

COROLLARY 2 Let P be a point and l be a line such that the point P is outside line l. Let $PN \perp l$ and let Q be any point on l, other than the foot of the perpendicular drawn from P i.e., N. Join PQ.

Now, $\Delta\, PNQ$ is a right triangle, right-angled at N. So, PQ is the hypotenuse and PN is a side of this right triangle.

$\therefore \quad PN < FQ$ [Using Corrollary 1]

This is true for all points Q, other than N on line l. Hence, we conclude that:

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

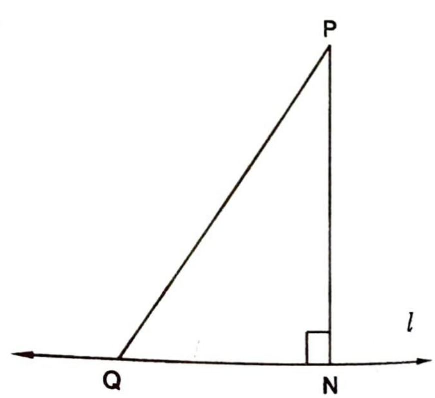


Fig. 50

15.8.1 CONVERSE OF PYTHAGORAS THEOREM

We have learnt that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Now, a natural question arises: if in a triangle the square of one side is equal to the sum of the squares of the remaining two sides, whether the triangle is a right triangle or not. The answer is in affirmative. In other words, the converse of the Pythagoras theorem is also true.

STATEMENT If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle with the angle opposite the first side as right angle.

or

If in a ΔABC, we have

$$AB^2 = BC^2 + CA^2$$

then the triangle is a right triangle, right angled at C.

In order to verify the above statement, let us consider the following experiments.

Experiment

(i) Let $a = 3$ cm, $b = 4$ cm and $c = 5$ cm. Then,

$$3^2 + 4^2 = 25 = 5^2$$

$$\Rightarrow \quad a^2 + b^2 = c^2$$

Now, draw ΔABC such that $AB = 5$ cm, $BC = 3$ cm and $CA = 4$ cm and measure $\angle ACB$. You will find that $\angle ACB = 90°$.

$\therefore \quad \Delta ABC$ is a right triangle, right-angled at C.

(ii) Let $a = 12$ cm, $b = 5$ cm and $c = 13$ cm. Then,

$$12^2 + 5^2 = 169 = 13^2$$

$$\Rightarrow \quad a^2 + b^2 = c^2$$

Now, draw a ΔABC such that $BC = 12$ cm, $CA = 5$ cm and $AB = 13$ cm and measure $\angle ACB$.

You will find that $\angle ACB = 90°$. So, the triangle ABC is a right triangle, right-angled at C.

PYTHAGOREAN TRIPLETS *Three positive numbers a, b, c in this order, are said to form a Pythagorean triplet, if* $c^2 = a^2 + b^2$

For example, each one of the triplets (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37) is a pythagorean triplet.

ILLUSTRATIVE EXAMPLES

Example 1 The hypotenuse of a right triangle is 13 cm long. If one of the remaining two sides is of length 5 cm, find the length of another side.

Solution Let ABC be a right triangle, right angled at C

Let $AB = 13$ cm and $BC = 5$ cm.

Then, by Pythagoras theorem, we have

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow \quad 13^2 = 5^2 + AC^2$$

$$\Rightarrow \quad 169 = 25 + AC^2$$

$$\Rightarrow \quad AC^2 = 169 - 25$$

$$\Rightarrow \quad AC^2 = 144$$

$$\Rightarrow \quad AC = \sqrt{144} \text{ cm} = 12 \text{ cm}.$$

Hence, the length of another side is 12 cm.

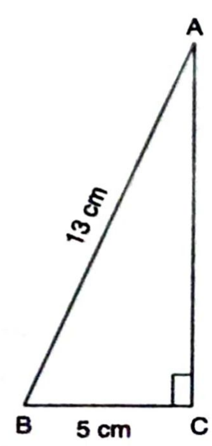


Fig. 51

Example 2 The sides of certain triangles are given below. Determine which of them are right triangles:

(i) $a = 6$ cm, $b = 8$ cm and $c = 10$ cm (ii) $a = 5$ cm, $b = 8$ cm and $c = 11$ cm.

Solution (i) Here the larger side is $c = 10$ cm.

We have: $a^2 + b^2 = 6^2 + 8^2 = 36 + 64 = 100 = c^2$.

So, the triangle with the given sides is a right triangle.

(ii) Here, the larger side is $c = 11$ cm.

Clearly, $a^2 + b^2 = 25 + 64 = 89 \neq c^2$.

So, the triangle with the given sides is not a right triangle.

Example 3 A ladder is placed in such a way that its foot is at a distance of 5 m from a wall and its top reaches a window 12 m above the ground. Determine the length of the ladder.

Solution Let AB be the ladder and B be the window.

Then, $BC = 12$ m and $AC = 5$ m.

Since ΔABC is a right triangle, right-angled at C.

$\therefore \quad AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AB^2 = 5^2 + 12^2 = 169$

$\Rightarrow \quad AB = \sqrt{169}$ m $= 13$ m

Hence, the length of the ladder is 13 m.

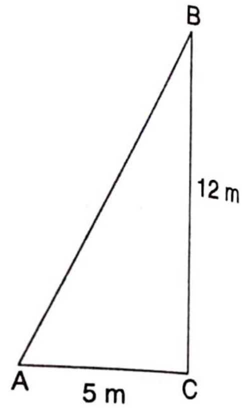


Fig. 52

Example 4 A ladder 25 m long reaches a window of a building 20 m above the ground. Determine the distance of the foot of the ladder from the building.

Solution Suppose that AB is the ladder, B is the window and CB is the building. Then, triangle ABC is a right triangle, with right angle at C

$\therefore \quad AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 25^2 = AC^2 + 20^2$

$\Rightarrow \quad AC^2 = 625 - 400 = 225$

$\Rightarrow \quad AC = \sqrt{225}$ m $= 15$ m.

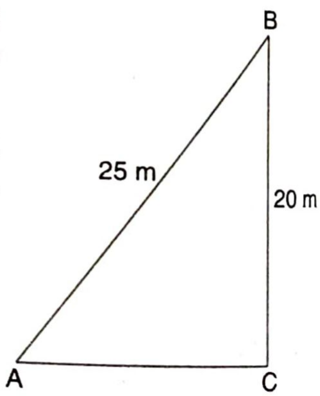


Fig. 53

Hence, the foot of the ladder is at a distance of 15 m from the building.

Example 5 A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 12 m high. Find the width of the street.

Solution Let AB be the street and C be the foot of the ladder. Let D and E be the windows at heights of 9 m and 12 m respectively from the ground. Then, CD and CE are the two positions of the ladder. Clearly, $AD = 9$ m, $BE = 12$ m, $CD = CE = 15$ m.

In right ΔACD, right-angled at A we have

$$CD^2 = AC^2 + AD^2$$

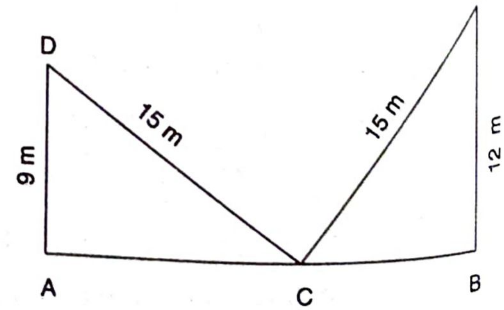


Fig. 54

$\Rightarrow \quad 15^2 = AC^2 + 9^2$

$\Rightarrow \quad AC^2 = 225 - 81 = 144$

$\Rightarrow \quad AC = \sqrt{144}$ m $= 12$ m

In right $\Delta\, BCE$, right angled at B, we have

$CE^2 = BC^2 + BE^2$

$\Rightarrow \quad 15^2 = BC^2 + 12^2$

$\Rightarrow \quad BC^2 = 225 - 144 = 81$

$\Rightarrow \quad BC = \sqrt{81}$ m $= 9$ m

Hence, width of the street $= AB = AC + CB = (12 + 9)$ m $= 21$ m.

Example 6 A man goes 10 m due east and then 24 m due north. Find the distance from the starting point.

Solution Let the initial position of the man be O and his final position be B. Since the man goes 10 m due east and then 24 m due north. Therefore, $\Delta\, OAB$ is a right triangle, right angled at A such that $OA = 10$ m and $AB = 24$ m.

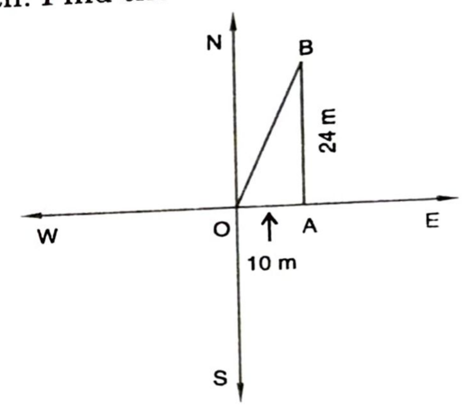


Fig. 55

By Pythagoras theorem, we have

$OB^2 = OA^2 + AB^2$

$\Rightarrow \quad OB^2 = 10^2 + 24^2$

$\Rightarrow \quad OB^2 = 100 + 576 = 676$

$\Rightarrow \quad OB = \sqrt{676}$ m $= 26$ m.

Hence, the man is at a distance of 26 m from the starting point.

Example 7 ABC is an isosceles right triangle, right-angled at C. Prove that: $AB^2 = 2AC^2$.

Solution Since $\Delta\, ABC$ is a right triangle, right-angled at C

$\therefore \quad AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AB^2 = AC^2 + AC^2 \qquad [\because BC = AC]$

$\Rightarrow \quad AB^2 = 2AC^2$.

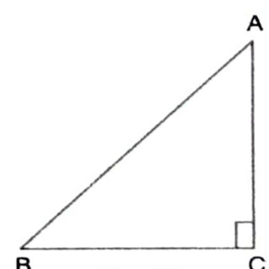


Fig. 56

Example 8 In a ΔABC, $AD \perp BC$ and $AD^2 = BD \times CD$. Prove that $\Delta\, ABC$ is a right triangle.

Solution In right triangles ADB and ADC, we have

$AB^2 = AD^2 + BD^2 \qquad$ ...(i)

and, $AC^2 = AD^2 + DC^2 \qquad$ ...(ii)

Adding (i) and (ii), we get

$AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$AB^2 + AC^2 = 2BD \times CD + BD^2 + CD^2$

$[\because AD^2 = BD \times CD \text{ (given)}]$

$AB^2 + AC^2 = (BD + CD)^2 = BC^2$

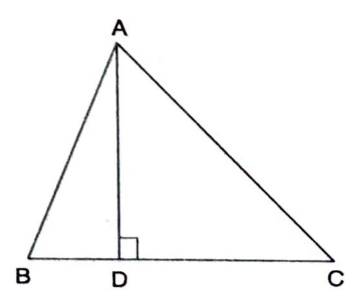


Fig. 57

Thus, in ΔABC, we have

$$AB^2 + AC^2 = BC^2 .$$

Hence, ΔABC is a right triangle, right-angled at A.

Example 9 A tree broke at a point but did not separate. Its top touched the ground at a distance of 6 dm from its base. If the point where it broke be at a height 2.5 dm from the ground, what was the total height of the tree before it broke?

Solution Let ACB represent the tree before it broke at the point C, and let the top A touch the ground at A' after it broke. Then, $\Delta\ A'BC$ is a right triangle, right angled at B such that

$A'B = 6$ dm, $BC = 2.5$ dm

Using Pythagoras theorem in $\Delta\ A'BC$, we have

$$(A'C)^2 = (A'B)^2 + (BC)^2$$

$$\Rightarrow \quad (A'C)^2 = 6^2 + (2.5)^2$$

$$\Rightarrow \quad (A'C)^2 = 36 + 6.25$$

$$\Rightarrow \quad (A'C)^2 = 42.5$$

$$\Rightarrow \quad (A'C)^2 = (6.5)^2$$

$$\Rightarrow \quad A'C = 6.5$$

$$\Rightarrow \quad AC = A'C = 6.5 \text{ dm} \qquad [\because AC = A'C]$$

$$\therefore \quad AB = AC + BC = (6.5 + 2.5) \text{ dm} = 9 \text{ dm}$$

Hence, the height of the tree before it broke was 9 dm.

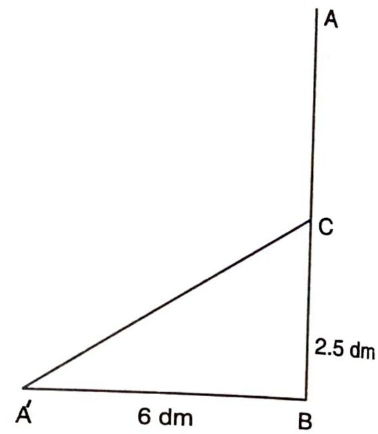


Fig. 58

EXERCISE 15.5

1. State Pythagoras theorem and its converse.
2. In right $\Delta\ ABC$, the lengths of the legs are given. Find the length of the hypotenuse.
   (i) $a = 6$ cm , $b = 8$ cm (ii) $a = 8$ cm , $b = 15$ cm
   (iii) $a = 3$ cm , $b = 4$ cm (iv) $a = 2$ cm , $b = 1.5$ cm
3. The hypotenuse of a triangle is 2.5 cm. If one of the sides is 1.5 cm, find the length of the other side.
4. A ladder 3.7 m long is placed against a wall in such a way that the foot of the ladder is 1.2 m away from the wall. Find the height of the wall to which the ladder reaches.
5. If the sides of a triangle are 3 cm, 4 cm and 6 cm long, determine whether the triangle is right-angled triangle.
6. The sides of certain triangles are given below. Determine which of them are right triangles.
   (i) $a = 7$ cm, $b = 24$ cm and $c = 25$ cm (ii) $a = 9$ cm, $b = 16$ cm and $c = 18$ cm
7. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
   [Hint: Find the hypotenuse of a right triangle having the sides $(11 - 6)$ m $= 5$ m and 12 m]
8. A man goes 15 m due west and then 8 m due north. How far is he from the starting point?

9. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its top reach?
10. A ladder 50 dm long when set against the wall of a house just reaches a window at a height of 48 dm. How far is the lower end of the ladder from the base of the wall?
11. The two legs of a right triangle are equal and the square of the hypotenuse is 50. Find the length of each leg.
12. Verify that the following numbers represent Pythagorean triplet:
    (i) 12, 35, 37 (ii) 7, 24, 25 (iii) 27, 36, 45 (iv) 15, 36, 39
13. In a ΔABC, $\angle ABC = 100°$, $\angle BAC = 35°$ and $BD \perp AC$ meets side AC in D. If $BD = 2$ cm, find $\angle C$ and length DC.
14. In a ΔABC, AD is the altitude from A such that $AD = 12$ cm, $BD = 9$ cm and $DC = 16$ cm. Examine if ΔABC is right angled at A.
15. Draw a triangle ABC, with $AC = 4$ cm, $BC = 3$ cm and $\angle C = 105°$. Measure AB. Is $(AB)^2 = (AC)^2 + (BC)^2$? If not, which one of the following is true: $(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?
16. Draw a triangle ABC, with $AC = 4$ cm, $BC = 3$ cm and $\angle C = 80°$. Measure AB. Is $(AB)^2 = (AC)^2 + (BC)^2$? If not, which one of the following is true: $(AB)^2 > (AC)^2 + (BC)^2$ or $(AB)^2 < (AC)^2 + (BC)^2$?

ANSWERS

2. (i) 10 cm (ii) 17 cm (iii) 5 cm (iv) 2.5 cm
3. 2 cm
4. 3.5 cm
5. No
6. (i) Yes (ii) No
7. 13 m
8. 17 m
9. 6 m
10. 14 dm
11. 5 units
12. (i) Yes (ii) Yes (iii) Yes (iv) Yes
13. 45°, 2 cm
14. Yes
15. No, $AB^2 > (AC^2 + BC^2)$
16. No, $AB^2 < AC^2 + BC^2$

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. If the measures of the angles of a triangle are $(2x)°$, $(3x - 5)°$ and $(4x - 13)°$. Then the value of x is
    (a) 22 (b) 18 (c) 20 (d) 30
2. The angles of a triangle are in the ratio 2 : 3 : 7. The measure of the largest angle is
    (a) 84° (b) 91° (c) 105° (d) 98°
3. In a ΔABC, if $2\angle A = 3\angle B = 6\angle C$, then the measure of the smallest angle is
    (a) 90° (b) 60° (c) 40° (d) 30°
4. In a ΔABC, if $\angle A + \angle B = 150°$ and $\angle B + \angle C = 75°$, then $\angle B =$
    (a) 35° (b) 45° (c) 55° (d) 25°
5. In a ΔABC, if $\angle A - \angle B = 33°$ and $\angle B - \angle C = 18°$, then $\angle B =$
    (a) 35° (b) 45° (c) 56° (d) 55°
6. If the measures of the angles of a triangle are $(2x - 5)°$, $\left(3x - \frac{1}{2}\right)°$ and $\left(30 - \frac{x}{2}\right)°$, then $x =$

(a) $\frac{311}{9}$ (b) $\frac{309}{11}$ (c) $\frac{310}{9}$ (d) $\frac{301}{9}$

7. In Fig. 59, the value of x is
   (a) 84 (b) 74 (c) 94 (d) 57

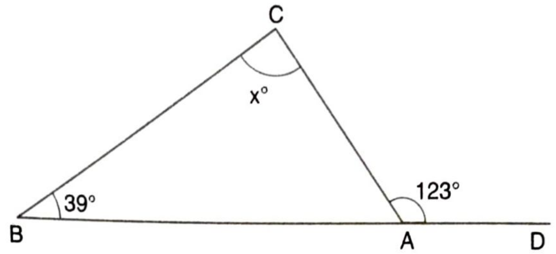


Fig. 59

8. In Fig. 60, the values of x and y are
   (a) $x = 20,\ y = 130$ (b) $x = 40,\ y = 140$
   (c) $x = 20,\ y = 140$ (d) $x = 15,\ y = 140$

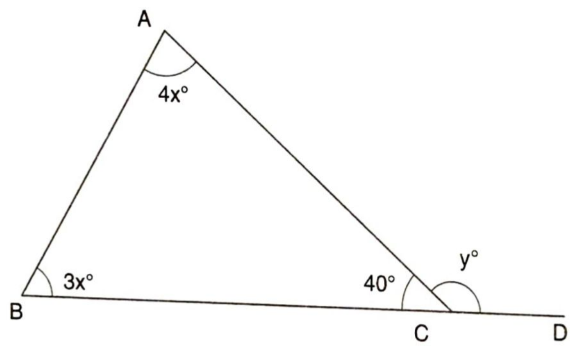


Fig. 60

9. In Fig. 61, the value of x is
   (a) 72° (b) 50 (c) 58 (d) 48

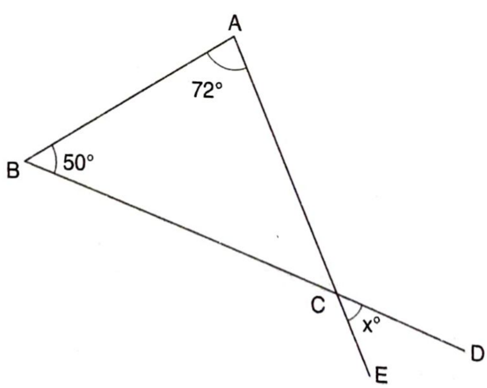


Fig. 61

10. In Fig. 62, if $AB \parallel DE$, then the value of x is
    (a) 25 (b) 35 (c) 40 (d) 45

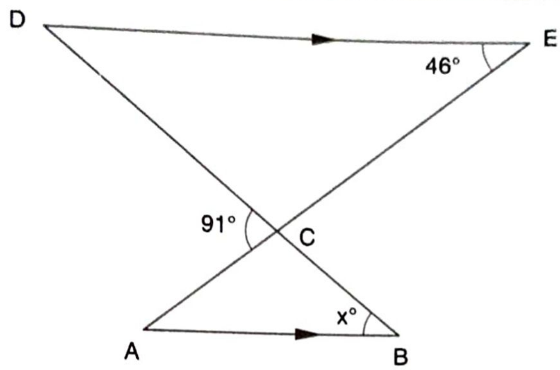


Fig. 62

11. In Fig. 63, if $AB \parallel CD$, the value of x is

(a) 25 (b) 35 (c) 15 (d) 20

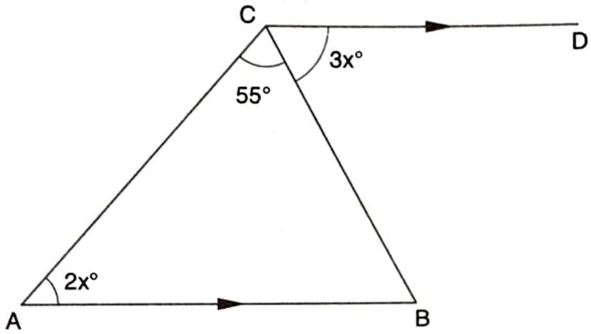


Fig. 63

12. In Fig. 64, if $AB \parallel CD$, the values of x and y are

(a) $x = 21$, $y = 28$ (b) $x = 21$, $y = 38$ (c) $x = 38$, $y = 21$ (d) $x = 22$, $y = 38$

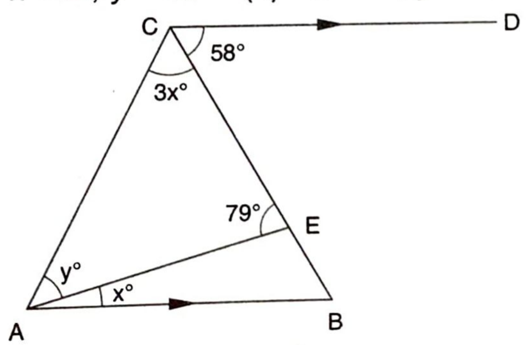


Fig. 64

13. In Fig. 65, if $AB \parallel CE$, then the values of x and y are

(a) $x = 26$, $y = 144$ (b) $x = 36$, $y = 108$ (c) $x = 154$, $y = 36$ (d) $x = 144$, $y = 26$

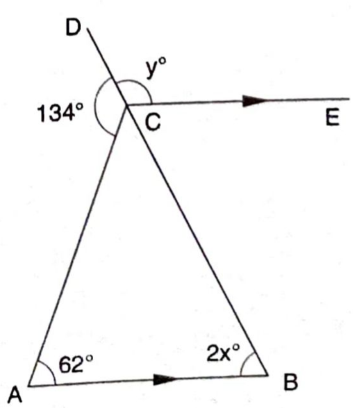


Fig. 65

14. In Fig. 66, if $AF \parallel DE$, then $x =$

(a) 37 (b) 57 (c) 47 (d) 67

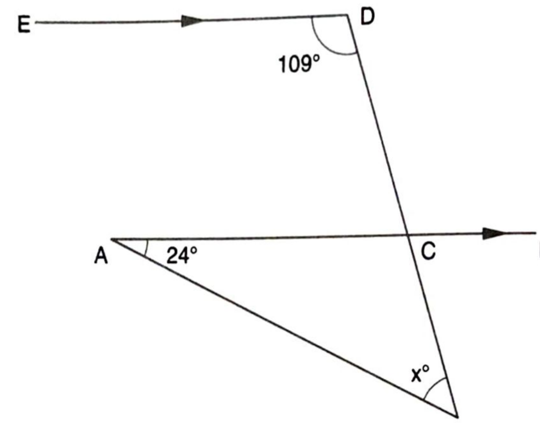


Fig. 66

15. In Fig. 67, the values of x and y are

(a) $x = 130,\ y = 120$ (b) $x = 120,\ y = 130$ (c) $x = 120,\ y = 120$ (d) $x = 130,\ y = 130$

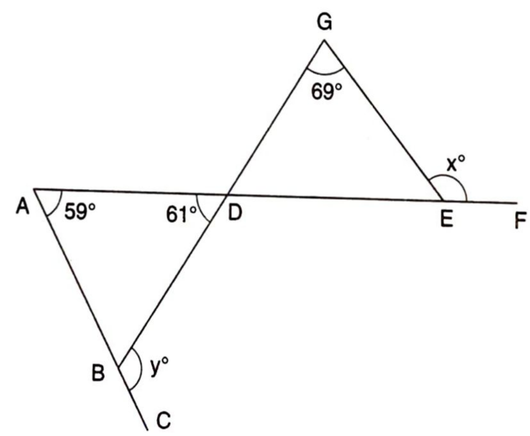


Fig. 67

16. In Fig. 68, the values of x and y are

(a) $x = 120,\ y = 150$ (b) $x = 110,\ y = 160$ (c) $x = 150,\ y = 120$ (d) $x = 110,\ y = 160$

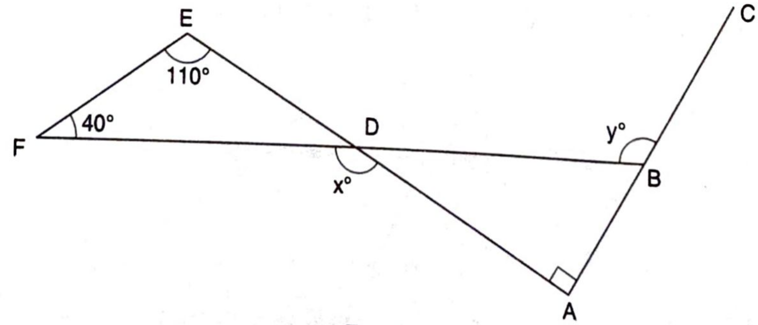


Fig. 68

17. In Fig. 69, if $AB \parallel CD$, then the values of x and y are

(a) $x = 106,\ y = 307$ (b) $x = 307,\ y = 106$ (c) $x = 107,\ y = 306$ (d) $x = 105, y = 308$

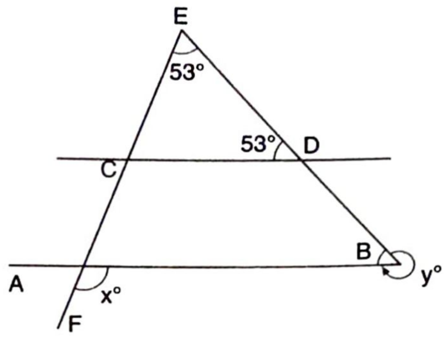


Fig. 69

18. In Fig. 70, if $AB \parallel CD$, then the values of x and y are
(a) $x = 24, y = 48$ (b) $x = 34, y = 68$ (c) $x = 24, y = 68$ (d) $x = 34, y = 48$

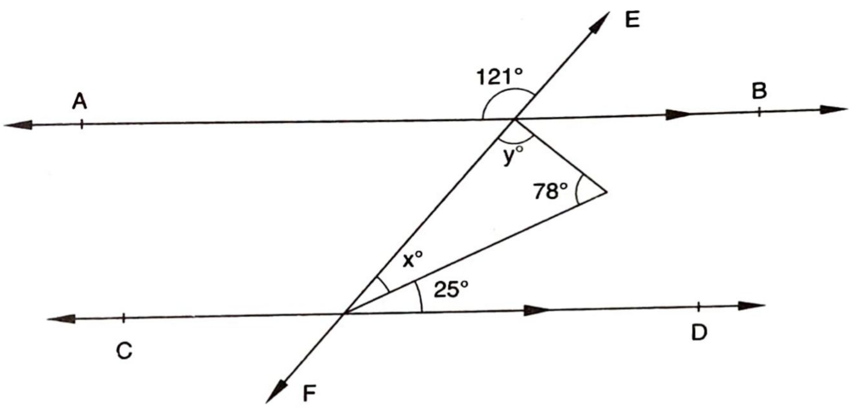


Fig. 70

19. In Fig. 71, if $AB \parallel CD$, then the values of x, y and z are
(a) $x = 56, y = 47, z = 77$ (b) $x = 47, y = 56, z = 77$
(c) $x = 77, y = 56, z = 47$ (d) $x = 56, y = 77, z = 47$

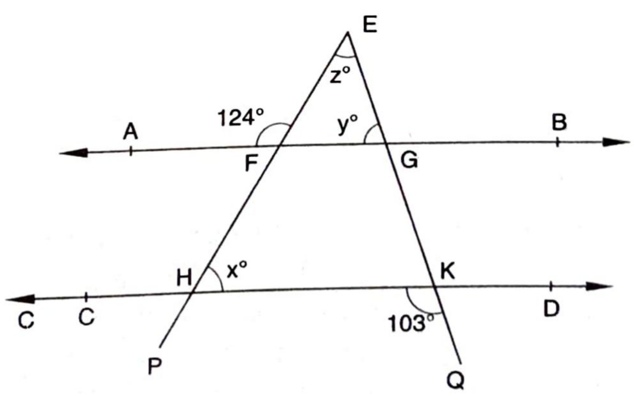


Fig. 71

20. In Fig. 72, if $AB \parallel CD$ and $AE \parallel BD$, then the value of x is
(a) 38 (b) 48 (c) 58 (d) 68

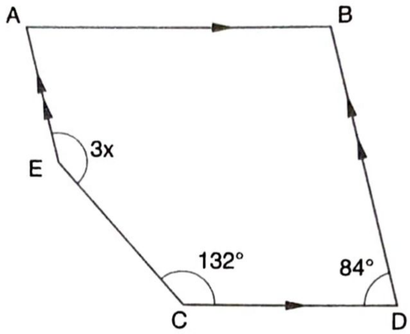


Fig. 72

21. If the exterior angles of a triangle are $(2x+10)°$, $(3x-5)°$ and $(2x+40)°$, then $x=$
    (a) 25 (b) 35 (c) 45 (d) 55
22. In Fig. 73, the value of x is
    (a) 20 (b) 30 (c) 40 (d) 25

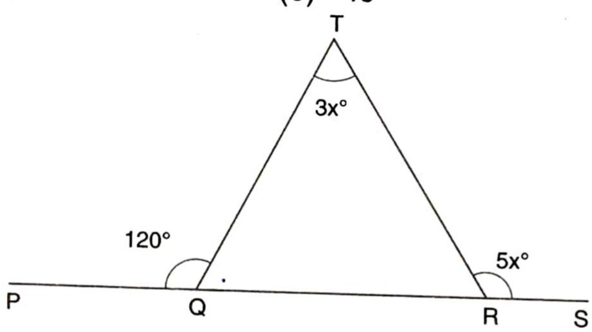


Fig. 73

23. In Fig. 74, if $AB \parallel CD$, $\angle CAB = 49°$, $\angle CBD = 27°$ and $\angle BDC = 112°$, then the values of x and y are
    (a) $x = 41$, $y = 90$ (b) $x = 41$, $y = 63$ (c) $x = 63$, $y = 41$ (d) $x = 90$, $y = 41$

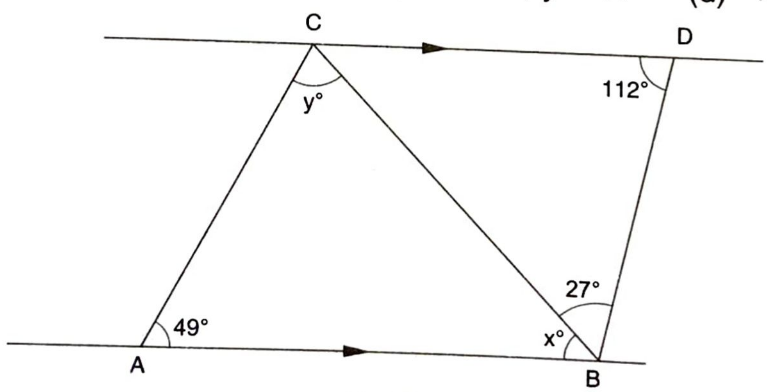


Fig. 74

24. Which of the following is the set of measures of the sides of a triangle?
    (a) 8 cm, 4 cm, 20 cm (b) 9 cm, 17 cm, 25 cm
    (c) 11 cm, 16 cm, 28 cm (d) None of these
25. In which of the following cases, a right triangle cannot be constructed?
    (a) 12 cm, 5 cm, 13 cm (b) 8 cm, 6 cm, 10 cm
    (c) 5 cm, 9 cm, 11 cm (d) None of these
26. Which of the following is/are not Pythagorean triplet (s)?
    (a) 3, 4, 5 (b) 8, 15, 17 (c) 7, 24, 25 (d) 13, 26, 29
27. In a right triangle, one of the acute angles is four times the other. Its measure is
    (a) 68° (b) 84° (c) 80° (d) 72°

28. In which of the following cases can a right triangle ABC be constructed?
    (a) $AB = 5$ cm, $BC = 7$ cm, $AC = 10$ cm (b) $AB = 7$ cm, $BC = 8$ cm, $AC = 12$ cm
    (c) $AB = 8$ cm, $BC = 17$ cm, $AC = 15$ cm (d) None of these
29. ΔABC is a right triangle right angled at A. If $AB = 24$ cm and $AC = 7$ cm, then $BC =$
    (a) 31 cm (b) 17 cm (c) 25 cm (d) 28 cm
30. If ΔABC is an isosceles right-triangle right angled at C such that $AC = 5$ cm. Then, $AB =$
    (a) 2.5 cm (b) $5\sqrt{2}$ cm (c) 10 cm (d) 5 cm
31. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, the distance between their tops is
    (a) 13 m (b) 14 m (c) 15 m (d) 12.8 m
32. A ladder is placed in such a way that its foot is 15 m away from the wall and its top reaches a window 20 m above the ground. The length of the ladder is
    (a) 35 m (b) 25 m (c) 18 m (d) 17.5 m
33. The hypotenuse of a right triangle is 26 cm long. If one of the remaining two sides is 10 cm long, the length of the other side is
    (a) 25 cm (b) 23 cm (c) 24 cm (d) 22 cm
34. A 15 m long ladder is placed against a wall in such away that the foot of the ladder is 9 m away from the wall. Up to what height does the ladder reach the wall?
    (a) 13 m (b) 10 m (c) 8 m (d) 12 m

ANSWSERS

1. (a)	2. (c)	3. (d)	4. (b)	5. (d)	6. (a)	7. (a)
8. (c)	9. (c)	10. (d)	11. (a)	12. (b)	13. (b)	14. (c)
15. (a)	16. (c)	17. (a)	18. (b)	19. (d)	20. (b)	21. (c)
22. (b)	23. (a)	24. (b)	25. (c)	26. (d)	27. (d)	28. (c)
29. (c)	30. (b)	31. (a)	32. (b)	33. (c)	34. (d)	

THINGS TO REMEMBER

1. A triangle is a figure made up by three line segments joining, in pairs, three non-collinear points. That is, if A, B, C are three non-collinear points, the figure formed by three line segments AB, BC and CA is called a triangle with vertices A, B, C.
2. The three line segments forming a triangle are called the sides of the triangle.
3. The three sides and three angles of a triangle are together called the six parts or elements of the triangle.
4. A triangle whose two sides are equal, is called an isosceles triangle.
5. A triangle whose all sides are equal, is called an equilateral triangle.
6. A triangle whose no two sides are equal, is called a scalene triangle.
7. A triangle whose all the angles are acute is called an acute triangle.
8. A triangle whose one of the angles is a right angle is called a right triangle.
9. A triangle whose one of the angles is an obtuse angle is called an obtuse triangle.
10. The interior of a triangle is made up of all such points P of the plane, as are enclosed by the triangle.
11. The exterior of a triangle is that part of the plane which consists of those points Q, which are neither on the triangle nor in its interior.
12. The interior of a triangle together with the triangle itself is called the triangular region.
13. The sum of the angles of a triangle is two right angles or 180°.
14. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the interior opposite angles.
15. In any triangle, an exterior angle is greater than either of the interior opposite angles.
16. The sum of any two sides of a triangle is greater than the third side.
17. In a right triangle, if a, b are the lengths of the sides and c that of the hypotenuse, then $c^2 = a^2 + b^2$.
18. If the sides of a triangle are of lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is right-angled and the side of length c is the hypotenuse.
19. Three positive numbers a, b, c in this order are said to form a pythagorean triplet, if $c^2 = a^2 + b^2$. Triplets (3, 4, 5) (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37) are some pythagorean triplets.