9

RATIO AND PROPORTION

9.1 INTRODUCTION

In Class VI, we have gone through the concept of Ratio and Proportion. In this chapter, we will have a brief revision of the same to strengthen our position in the applications of these concepts.

9.2 RATIO

In our daily life, we often come across many situations where we need to compare quantities in terms of their magnitudes/measurements. Generally, we compare two quantities either by finding the difference of their magnitudes or by finding the division of their magnitudes. When we want to see how much more (or less) one quantity is than the other, we find the difference of their magnitudes and such a comparison is known as the comparison by difference. If we want to see how many times more (or less) one quantity is than the other, we find the ratio (or division) of their magnitudes and such a comparison is known as the comparison by division. In fact, when we compare two quantities of the same kind by division, we say that we form a ratio of the two quantities. Suppose Heena's height is 150 cm and her brother Amir's height is 100 cm. If we wish to compare heights of Heena and Amir by difference, then we can say that Heena is 50 cm taller than her brother Amir.

By comparing their heights by division, we can also say that Heena is $\frac{3}{2}$ times taller than her brother Amir. Here we observe that the comparison by division makes better sense than the comparison by taking the difference. When we compare two quantities of the same kind by division, we say that we form a ratio of the two quantities. Thus, we can say that the ratio of the height of Heena to that of Amir is $\frac{150}{100}$. Usually, we use the symbol ':' to express a ratio. Therefore, the ratio of the height of Heena to that of Amir is written as 150 : 100 and it is read as '150 is to 100' or '150 to 100'.

It is evident from the above discussion that a ratio is a fraction that shows how many times a quantity is of another quantity of the same kind.

RATIO *The ratio of two quantities of the same kind and in the same units is a fraction that shows how many times the one quantity is of the other.*

Thus, the ratio of two quantities a and b $(b \neq 0)$ is $a \div b$ or, $\frac{a}{b}$ and is denoted by $a : b$.

In the ratio $a : b$, the quantities (numbers) a and b are called the terms of the ratio. The former 'a' is called the first term or antecedent and the later 'b' is known as the second term or consequent.

We know that a fraction does not change when its numerator and denominator are multiplied or divided by the same non-zero number. So, a ratio does not alter, if its first and second terms are multiplied or divided by the same non-zero number.

$\therefore$ $150:100 = 15:10$ [Dividing the first and second term by 10]

$150:100 = 300:200$ [Multiplying the first and second term by 2]

RATIO IN THE SIMPLEST FORM *A ratio a : b is said to be in the simplest form if its antecedent a and consequent b have no common factor other than 1.*

A ratio in the simplest form is also called the ratio in the lowest terms.

The ratio 80 : 32 is not in the simplest form, because 16 is a common factor of its antecedent and consequent. The simplest form of this ratio is 5 : 2 (dividing the first and second term by 16).

Usually, a ratio is expressed in the simplest form.

NOTE 1 *In a ratio, we compare two quantities. The comparison becomes meaningless if the quantities being compared are not of the same kind i.e. they are not measured in the same units. It is just meaningless to compare 20 bags with 200 crows. Therefore, to find the ratio of two quantities, they must be expressed in the same units.*

NOTE 2 *Since the ratio of two quantities of the same kind determines how many times one quantity is contained by the other. So, the ratio of any two quantities of the same kind is an abstract quantity. In other words, ratio has no unit or it is independent of the units used in the quantities compared.*

NOTE 3 *The order of the terms in a ratio a : b is very important. The ratio 3 : 2 is different from the ratio 2 : 3.*

Let us now illustrate these ideals through some examples.

ILLUSTRATIVE EXAMPLES

Example 1 Express the ratio 45 : 108 in its simplest form:

Solution In order to express the given ratio in its simplest form we divide its first and second terms by their HCF.

We have,

$$45 = 3 \times 3 \times 5 \text{ and } 108 = 2 \times 2 \times 3 \times 3 \times 3$$

So, HCF of 45 and 108 is $3 \times 3 = 9$.

$$\therefore \quad 45:108 = \frac{45}{108} = \frac{45 \div 9}{108 \div 9} = \frac{5}{12} = 5:12$$

Hence, 45 : 108 in its simplest form is 5 : 12.

Example 2 Shikha and Sumeet take ₹ 600 and ₹ 750 respectively from their parents as monthly pocket expenses. What is the ratio of their expenses in the simplest form?

Solution We have,

Required ratio = ₹ 600 : ₹ 750

$$= \frac{600}{750} = \frac{600 \div 150}{750 \div 150}$$ [Dividing the antecedent and consequent by the HCF of 600 and 750 which is 150]

$$= \frac{4}{5} = 4:5$$

Example 3 Divide 108 in two parts in the ratio 4 : 5.

Solution We have,

Sum of the terms of the ratio $= (4+5) = 9$

$\therefore$ First part $= \frac{4}{9} \times 108 = 4 \times 12 = 48$

Second part $= \frac{5}{9} \times 108 = 5 \times 12 = 60$

Aliter Since the given ratio is 4 : 5. So, let the two parts be $4x$ and $5x$. Thus, we have to divide 108 into two parts $4x$ *and* $5x$.

$\therefore$ $4x + 5x = 108$

$\Rightarrow$ $9x = 108$

$\Rightarrow$ $\frac{9x}{9} = \frac{108}{9}$ [Dividing both sides by 9]

$\Rightarrow$ $x = 12.$

$\therefore$ First part $= 4x = 4 \times 12 = 48$

Second part $= 5x = 5 \times 12 = 60.$

Example 4 Divide ₹ 1250 between Aman and Amit in the ratio 2 : 3.

Solution We have,

Sum of the terms of the ratio $= (2+3) = 5$

$\therefore$ Aman's share = ₹ $\left(\frac{2}{5} \times 1250\right)$ = ₹ (2×250) = ₹ 500

Amit's share = ₹ $\left(\frac{3}{5} \times 1250\right)$ = ₹ (3×250) = ₹ 750.

Example 5 Two numbers are in the ratio 4 : 5. If the sum of the numbers is 135, find the numbers.

Solution We have,

Sum of the terms of the ratio $= (4+5) = 9$

Sum of the numbers $= 135.$

$\therefore$ First number $= \frac{4}{9} \times 135 = 4 \times 15 = 60$

Second number $= \frac{5}{9} \times 135 = 5 \times 15 = 75.$

Hence, the numbers are 60 and 75.

Example 6 Divide ₹ 1500 among *A*, *B*, *C* in the ratio 3 : 5 : 2.

Solution We have,

Sum of the terms of the ratio $= (3+5+2) = 10$

$\therefore$ *A*'s share = ₹ $\left(\frac{3}{10} \times 1500\right)$ = ₹ 450

$$B\text{'s share} = ₹\left(\frac{5}{10}\times 1500\right) = ₹\,750$$

$$C\text{'s share} = ₹\left(\frac{2}{10}\times 1500\right) = ₹\,300$$

Example 7 Ratio of the number of male and female workers in a factory is 5 : 3. If there are 115 male workers, determine the number of female workers in the factory.

Solution We have, Male : Female = 5 : 3

So, let there be $5x$ male workers and $3x$ be female workers. It is given that there are 115 male workers.

$\therefore \quad 5x = 115$

$\Rightarrow \quad \frac{5x}{5} = \frac{115}{5}$ [Dividing both sides by 5]

$\Rightarrow \quad x = 23$

$\therefore$ Number of female workers $= 3x = 3\times 23 = 69$

Thus, there are 69 female workers in the factory.

Example 8 The ratio of copper and zinc in an alloy is 5 : 3. If the weight of the copper in the alloy is 30.5 grams, find the weight of zinc in the alloy.

Solution We have,

Weight of copper : Weight of zinc = 5 : 3

So, let the weight of copper in the alloy be $5x$ grams and, Weight of zinc in the alloy be $3x$ grams

But, the weight of the copper in the alloy is given to be 30.5 grams.

$\therefore \quad 5x = 30.5$

$\Rightarrow \quad \frac{5x}{5} = \frac{30.5}{5}$ [Dividing both sides by 5]

$\Rightarrow \quad x = 6.1$

$\therefore$ Weight of zinc in the alloy $= 3x$ grams $= 3\times 6.1$ grams $= 18.3$ grams

<u>Aliter</u> It is given that the alloy contains copper and zinc in the ratio 5 : 3. This means that if the weight of copper in the alloy is 5 grams, then the weight of zinc is 3 grams.

Now,

If the weight of copper in the alloy is 5 gm, then the weight of zinc = 3 gm.

$\therefore$ If the weight of copper is 1 gm, then the weight of the zinc $= \frac{3}{5}$ grams

If the weight of copper is 30.5 gm, then the weight of zinc

$$= \left(\frac{3}{5}\times 30.5\right)\text{gm} = 18.3 \text{ gm}$$

Example 9 If $x : y = 2 : 3$, find the value of $(3x + 2y) : (2x + 5y)$.

Solution We have,

$$x : y = 2 : 3 \Rightarrow \frac{x}{y} = \frac{2}{3}$$

Now,

$$(3x + 2y) : (2x + 5y)$$

$$= \frac{3x + 2y}{2x + 5y}$$

$$= \frac{\frac{3x + 2y}{y}}{\frac{2x + 5y}{y}} \qquad \text{[Dividing numerator and denominator by } y\text{]}$$

$$= \frac{3\left(\frac{x}{y}\right) + 2}{2\left(\frac{x}{y}\right) + 5} = \frac{3 \times \frac{2}{3} + 2}{2 \times \frac{2}{3} + 5} \qquad \left[\because \frac{x}{y} = \frac{2}{3}\right]$$

$$= \frac{2 + 2}{\frac{4}{3} + 5} = \frac{4}{\frac{19}{3}} = \frac{12}{19} = 12 : 19$$

Example 10 If $(4x + 5) : (3x + 11) = 13 : 17$, find the value of x.

Solution We have,

$$(4x + 5) : (3x + 11) = 13 : 17$$

$$\Rightarrow \frac{4x + 5}{3x + 11} = \frac{13}{17}$$

$$\Rightarrow 17(4x + 5) = 13(3x + 11)$$

$$\Rightarrow 68x + 85 = 39x + 143$$

$$\Rightarrow 68x - 39x = 143 - 85$$

$$\Rightarrow 29x = 58$$

$$\Rightarrow x = \frac{58}{29} = 2$$

Example 11 What must be added to each term of the ratio 2 : 5 so that it may become equal to 5 : 6?

Solution Let the number to be added be x. Then,

$$(2 + x) : (5 + x) = 5 : 6$$

$$\Rightarrow \frac{2 + x}{5 + x} = \frac{5}{6}$$

$\Rightarrow \quad 6(2+x) = 5(5+x)$

$\Rightarrow \quad 12 + 6x = 25 + 5x$

$\Rightarrow \quad 6x - 5x = 25 - 12$

$\Rightarrow \quad x = 13$

Hence, the required number is 13.

Example 12 A bag contains ₹ 187 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio 3 : 4 : 5. Find the number of each type of coins.

Solution Let the number of 1 ₹, 50 paise and 10 paise coins be $3x$, $4x$ and $5x$ respectively. Then,

$$3x + 4x \times \frac{50}{100} + 5x \times \frac{10}{100} = 187$$

$$\Rightarrow \quad 3x + 2x + \frac{x}{2} = 187$$

$$\Rightarrow \quad \frac{11}{2}x = 187 \Rightarrow x = 34$$

$$\therefore \quad 3x = 3 \times 34 = 102, 4x = 4 \times 34 = 136 \text{ and } 5x = 5 \times 34 = 170$$

Hence, the number of 1 ₹, 50 paise and 10 paise coins are 102, 136 and 170 respectively.

EXERCISE 9.1

1. If $x : y = 3 : 5$, find the ratio $3x + 4y : 8x + 5y$.
2. If $x : y = 8 : 9$, find the ratio $(7x - 4y) : 3x + 2y$.
3. If two numbers are in the ratio 6 : 13 and their l.c.m. is 312, find the numbers.
4. Two numbers are in the ratio 3 : 5. If 8 is added to each number, the ratio becomes 2 : 3. Find the numbers.
5. What should be added to each term of the ratio 7 : 13 so that the ratio becomes 2 : 3
6. Three numbers are in the ratio 2 : 3 : 5 and the sum of these numbers is 800. Find the numbers.
7. The ages of two persons are in the ratio 5 : 7. Eighteen years ago their ages were in the ratio 8 : 13. Find their present ages.
8. Two numbers are in the ratio 7 : 11. If 7 is added to each of the numbers, the ratio becomes 2 : 3. Find the numbers.
9. Two numbers are in the ratio 2 : 7. If the sum of the numbers is 810, find the numbers.
10. Divide ₹ 1350 between Ravish and Shikha in the ratio 2 : 3.
11. Divide ₹ 2000 among *P, Q, R* in the ratio 2 : 3 : 5.
12. The boys and the girls in a school are in the ratio 7 : 4. If total strength of the school be 550, find the number of boys and girls.
13. The ratio of monthly income to the savings of a family is 7 : 2. If the savings be of ₹ 500, find the income and expenditure.
14. The sides of a triangle are in the ratio 1 : 2 : 3. If the perimeter is 36 cm, find its sides.

15. A sum of ₹ 5500 is to be divided between Raman and Aman in the ratio 2 : 3. How much will each get ?

16. The ratio of zinc and copper in an alloy is 7 : 9. If the weight of the copper in the alloy is 11.7 kg, find the weight of the zinc in the alloy.

17. In the ratio 7 : 8, if the consequent is 40, what is the antecedent?

18. Divide ₹ 351 into two parts such that one may be to the other as 2 : 7.

19. Find the ratio of the price of pencil to that of ball pen, if pencils cost ₹ 16 per score and ball pens cost ₹ 8.40 per dozen.

20. In a class, one out of every six students fails. If there are 42 students in the class, how many pass?

ANSWERS

1. 29 : 49	2. 10 : 21	3. 24, 52	4. 24, 40
5. 5	6. 160, 240, 400	7. 50, 70 yrs	8. 49, 77
9. 180, 630	10. ₹ 540, ₹ 810	11. ₹ 400, ₹ 600, ₹ 1000	
12. Boys 350, Girls 200	13. Income = ₹ 1750, Expenditure = ₹ 1250		
14. 6 cm, 12 cm, 18 cm	15. Raman : ₹ 2200 Aman : ₹ 3300		16. 9.1 kg
17. 35	18. ₹ 78, ₹ 273	19. 8 : 7	20. 35

HINTS TO SELECTED PROBLEMS

6. Let the numbers be $2x$, $3x$, $5x$. Then, $2x + 3x + 5x = 800 \Rightarrow x = 80$

7. Let the present ages be $5x$ and $7x$ years. Then, $\dfrac{5x - 18}{7x - 18} = \dfrac{8}{13} \Rightarrow x = 10$

8. Let the numbers be $7x$ and $11x$. Then, $\dfrac{7x + 7}{11x + 7} = \dfrac{2}{3}$

17. Let the antecedent and consequent be $7x$ and $8x$ respectively. Then, consequent $= 40 \Rightarrow 8x = 40 \Rightarrow x = 5$. Therefore, antecedent $= 7x = 7 \times 5 = 35$.

20. Ratio of the number of pass students to the number of fail students = 5 : 1.

9.3 COMPARISON OF RATIOS

In order to compare two given ratios, we follow the following steps:

STEP I *Obtain the given ratios.*

STEP II *Express each one of them in the form of a fraction in the simplest form.*

STEP III *Find the L.C.M. of the denominators of the fractions obtained in step II.*

STEP IV *Obtain first fraction and its denominator. Divide the L.C.M. obtained in step III by the denominator to get a number x (say).*

Now, multiply the numerator and denominator of the fraction by x. Apply the same procedure to the other fraction.

Now, the denominators of all fractions will be same.

STEP V *Compare the numerators of the fractions obtained in step IV. The fraction having larger numerator will be larger than the other.*

Following example will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Compare the ratios: 5 : 12 and 3 : 8

Solution Writing the given ratios as fractions, we have

$$5:12=\frac{5}{12} \text{ and } 3:8=\frac{3}{8}$$

Now, L.C.M of 12 and 8 is 24.

Making the denominator of each fraction equal to 24, we have

$$\frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24} \text{ and } \frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24}$$

Clearly, 10 > 9

$$\therefore \quad \frac{10}{24}>\frac{9}{24}\Rightarrow\frac{5}{12}>\frac{3}{8}$$

Example 2 Compare the ratios: 7 : 6 and 24 : 9

Solution Writing the given ratios as fractions, we have

$$7:6=\frac{7}{6} \text{ and } 4:9=\frac{4}{9}$$

Now, L.C.M. of 6 and 9 is 18.

Making the denominator of each fraction equal to 18, we have

$$\frac{7}{6}=\frac{7\times3}{6\times3}=\frac{21}{18} \text{ and } \frac{4}{9}=\frac{4\times2}{9\times2}=\frac{8}{18}$$

Clearly, 21 > 8 .

$$\therefore \quad \frac{21}{18}>\frac{8}{18}\Rightarrow\frac{7}{6}>\frac{4}{9}$$

9.4 EQUIVALENT RATIOS

EQUIVALENT RATIO *A ratio obtained by multiplying or dividing the numerator and denominator of a given ratio by the same number is called an equivalent ratio.*

Consider the ratio 6 : 4.

We have,

$$\frac{6}{4}=\frac{6\times2}{4\times2}=\frac{12}{8},\ \frac{6}{4}=\frac{6\times3}{4\times3}=\frac{18}{12},\ \frac{6}{4}=\frac{6\times4}{4\times4}=\frac{24}{16} \text{ and so on.}$$

Also, $\frac{6}{4}=\frac{6\div2}{4\div2}=\frac{3}{2}$

Thus, $\frac{12}{8},\frac{18}{12},\frac{24}{16},\frac{3}{2}$, etc. are ratios equivalent to the ratio $\frac{6}{4}$.

If $a:b$ and $c:d$ are two equivalent ratios, we write $\frac{a}{b}=\frac{c}{d}$.

ILLUSTRATIVE EXAMPLES

Example 1 Find two equivalent ratios of 6 : 15.

Solution We have,

$$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

$\therefore$ 2 : 5 is an equivalent ratio of 6 : 15

Also, $\frac{6}{15} = \frac{6 \times 2}{15 \times 2} = \frac{12}{30}$

So, 12 : 30 is an equivalent ratio of 6 : 15.

Hence, 2 : 5 and 12 : 30 are equivalent ratios of 6 : 15.

Example 2 Fill in the following blanks:

$$\frac{14}{21} = \frac{\square}{3} = \frac{6}{\square}$$

Solution In order to find the first missing number, we consider the denominators, 21 and 3.

We have,

$$21 \div 3 = 7$$

So, we divide the numerator and denominator of $\frac{14}{21}$ by 7 to get

$$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$$

Hence, first missing number is 2. Consequently, the second ratio is $\frac{2}{3}$.

To find the second missing number, we consider $\frac{2}{3} = \frac{6}{\square}$

We have $6 \div 2 = 3$. So, we multiply the numerator and denominator of $\frac{2}{3}$ by 3

to get $\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$.

Hence, the second missing number is 9.

Consequently, the third ratio is $\frac{6}{9}$

Aliter Let $\frac{14}{21} = \frac{x}{3} = \frac{6}{y}$. Then,

$$\frac{14}{21} = \frac{x}{3} \Rightarrow 14 \times 3 = 21 \times x \Rightarrow x = \frac{14 \times 3}{21} = 2$$

and, $\frac{14}{21} = \frac{6}{y} \Rightarrow 14 \times y = 21 \times 6 \Rightarrow y = \frac{\overset{3}{\cancel{21}} \times \overset{3}{\cancel{6}}}{\underset{\cancel{2}1}{\cancel{14}}} = 9$

EXERCISE 9.2

1. Which ratio is larger in the following pairs?
   (i) 3 : 4 or 9 : 16 (ii) 15 : 16 or 24 : 25 (iii) 4 : 7 or 5 : 8
   (iv) 9 : 20 or 8 : 13 (v) 1 : 2 or 13 : 27
2. Give two equivalent ratios of 6 : 8.
3. Fill in the following blanks: $\frac{12}{20} = \frac{\square}{5} = \frac{9}{\square}$

ANSWERS

1. (i) 3 : 4 (ii) 24 : 25 (iii) 5 : 8 (iv) 8 : 13 (v) 1 : 2
2. 3 : 4, 12 : 16 3. 3, 15

9.5 PROPORTION

PROPORTION An equality of two ratios is called a proportion.

Consider the two ratios 6 : 18 and 8 : 24. We find that

$$6 : 18 = 1 : 3 \text{ and } 8 : 24 = 1 : 3.$$

$$\therefore \quad 6 : 18 = 8 : 24$$

Thus, 6 : 18 = 8 : 24 is a proportion.

Similarly, 40 : 70 = 200 : 350, 180 : 135 = 4 : 3 etc. are proportions.

Four numbers a, b, c, d are said to be in proportion, if the ratio of the first two is equal to the ratio of the last two, i.e., $a : b = c : d$.

If four numbers a, b, c, d are in proportion, then we write

$$a : b :: c : d$$

which is read as 'a is to b as c is to d' or 'a to b as c to d'. Here a, b, c and d are the first, second, third and fourth terms of the proportion. The first and fourth terms of a proportion are called extreme terms or extremes. The second and third terms are called the middle terms or means.

We observe that 2, 6, 18 and 54 are in proportion, because 2 : 6 = 18 : 54. Clearly, 2 and 54 are extremes whereas 6 and 18 are means.

Also, 40, 70, 200 and 350 are in proportion, because 40 : 70 = 200 : 350.

Consider the numbers 40, 70, 200, 350. We find that 40 : 70 = 200 : 350. So, the given numbers are in proportion. Clearly, 40 and 350 are extreme terms and 70 and 200 are middle terms. We find that

Product of extreme terms = $40 \times 350 = 14000$, Product of middle terms = $70 \times 200 = 14000$

$\therefore$ Product of extreme terms = Product of middle terms.

Thus, we observe that if four numbers are in proportion, then the product of the extreme terms is equal to the product of the middle terms.

In other words, $a : b = c : d$ if and only if $ad = bc$

If $ad \neq bc$, then a, b, c, d are not in proportion.

CONTINUED PROPORTION *Three numbers a, b, c are said to be in continued proportion if a, b, b, c, are in proportion.*

Thus, if a, b, c are in continued proportion, then

a, b, b, c are in proportion, i.e., $a : b :: b : c$

Product of extreme terms = Product of mean terms

$\Rightarrow$ $a \times c = b \times b$

$\Rightarrow$ $ac = b^2$

$\Rightarrow$ $b^2 = ac.$

$\Rightarrow$

MEAN PROPORTIONAL *If a, b, c are in continued proportion, then b is called the mean proportional between a and c.*

Clearly, if b is the mean proportional between a and c, then $b^2 = ac$.

The above concepts are illustrated with the help of the following examples.

ILLUSTRATIVE EXAMPLES

Example 1 Are 30, 40, 45, 60 in proportion?

Solution We have,

$$30:40 = \frac{30}{40} = \frac{3}{4} \quad \text{and,} \quad 45:60 = \frac{45}{60} = \frac{3}{4}$$

$\therefore$ $30:40 = 45:60$

Hence, 30, 40, 45, 60 are in proportion.

Aliter We have,

Product of extremes $= 30 \times 60 = 1800$

Product of means $= 40 \times 45 = 1800$

$\therefore$ Product of extremes = Product of means.

Hence, 30, 40, 45, 60 are in proportion.

Example 2 Are 36, 49, 6, 7 in proportion?

Solution We have,

Product of extremes $= 36 \times 7 = 252$

Product of means $= 49 \times 6 = 294$

Clearly, Product of extremes $\neq$ Product of means

Hence, 36, 49, 6, 7 are not in proportion.

Example 3 Are 4, 12, 36 in continued proportion?

Solution We know that three numbers a, b, c are in continued proportion, if a, b, b, c are in proportion.

Therefore, 4, 12, 36 will be in continued proportion if 4, 12, 12, 36 are in proportion.

We have,

Product of extremes $= 4 \times 36 = 144$

Product of means $= 12 \times 12 = 144$

$\therefore$ Product of extremes = Product of means

$\Rightarrow$ 4, 12, 12, 36 are in proportion

$\Rightarrow$ 4, 12, 36 are in continued proportion.

Example 4 If $3 : x :: 12 : 20$, find the value of x.

Solution We have,

$3 : x :: 12 : 20$

$\Rightarrow$ 3, x, 12, 20 are in proportion

$\Rightarrow$ Product of extremes = Product of means

$\Rightarrow$ $3 \times 20 = x \times 12$

$\Rightarrow$ $60 = 12x$

$\Rightarrow$ $\frac{12x}{12} = \frac{60}{12}$ [Dividing both sides by 12]

$\Rightarrow$ $x = 5$

Example 5 The first three terms of a proportion are 3, 5 and 21 respectively. Find its fourth term.

Solution Let the fourth term be x. Then,

3, 5, 21, x are in proportion.

$\Rightarrow$ Product of extremes = Product of means

$\Rightarrow$ $3 \times x = 5 \times 21$

$\Rightarrow$ $3x = 5 \times 21$

$\Rightarrow$ $\frac{3x}{3} = \frac{5 \times 21}{3}$ [Dividing both sides by 3]

$\Rightarrow$ $x = 5 \times 7 = 35$

Example 6 What must be added to the numbers 6, 10, 14 and 22 so that they are in proportion?

Solution Let the required number be x. Then,

$6 + x, 10 + x, 14 + x, 22 + x$ are in proportion.

$\Rightarrow$ Product of extremes = Product of means

$\Rightarrow$ $(6 + x)(22 + x) = (10 + x)(14 + x)$

$\Rightarrow$ $132 + 6x + 22x + x^2 = 140 + 10x + 14x + x^2$

$\Rightarrow$ $132 + 28x = 140 + 24x$

$\Rightarrow$ $28x - 24x = 140 - 132$

$\Rightarrow$ $4x = 8$

$\Rightarrow$ $x = \frac{8}{4} = 2.$

Hence, the required number is 2.

Example 7 For every 12 mangoes that I buy, 3 turn out to be rotten. At this rate, how many rotten mangoes will I have if I buy 100 mangoes?

Solution Suppose I have x rotten mangoes if I buy 100 mangoes. Then, the ratio of rotten mangoes to mangoes bought is $x : 100$.

Since for every 12 mangoes that I buy 3 turn out to be rotten. Therefore, the ratio of rotten mangoes to the mangoes bought is $3 : 12$.

$\therefore \quad x : 100 = 3 : 12$

$\Rightarrow \quad x \times 12 = 100 \times 3$ [$\because$ Product of extremes = Product of means]

$\Rightarrow \quad 12x = 300$

$\Rightarrow \quad x = \frac{300}{12} = 25$

Thus, there will be 25 rotten mangoes, if I buy 100 mangoes.

Example 8 The scale of a map is 1 : 3000000. What is the actual distance between the two towns, if they are 3 cm apart on the map?

Solution The scale of a map is 1 : 3000000 means that if the distance on the map is 1 cm, then the actual distance is 3000000 cm.

We have, $3000000\,\text{cm} = \frac{3000000}{1000 \times 100}\,\text{km} = 30\,\text{km}$

Thus, if the distance on the map is 1 cm, the actual distance is 30 km.

Let the actual distance between the two towns be x km. Thus, we have the following information.

Distance on the map	Actual distance
1 cm	30 km
3 cm	x km

$\therefore \quad 1 : 3 = 30 : x$

$\Rightarrow \quad 1 \times x = 3 \times 30$

$\Rightarrow \quad x = 90$

Hence, the actual distance between the two towns is 90 km.

Example 9 If three loaves of bread are consumed by 9 people, how many people will consume 9 loaves of bread?

Solution Suppose x people will consume 9 loaves of bread.

We have,

Number of People	Number of Loaves of bread
9	3
x	9

$\therefore \quad 9 : x = 3 : 9$

$\Rightarrow \quad 9 \times 9 = x \times 3$

$\Rightarrow \quad 81 = 3x$

$\Rightarrow \quad x = \frac{81}{3} = 27$

Hence, 27 people will consume 9 loaves of bread.

EXERCISE 9.3

1. Find which of the following are in proportion?
   (i) 33, 44, 66, 88 (ii) 46, 69, 69, 46 (iii) 72, 84, 186, 217
2. Find x in the following proportions:
   (i) $16 : 18 = x : 96$ (ii) $x : 92 = 87 : 116$

3. The ratio of the income to the expenditure of a family is 7 : 6. Find the savings if the income is ₹ 1400.

4. The scale of a map is 1 : 4000000. What is the actual distance between the two towns if they are 5 cm apart on the map?

5. The ratio of income of a person to his savings is 10 : 1. If his savings of one year are ₹ 6000, what is his income per month?

6. An electric pole casts a shadow of length 20 metres at a time when a tree 6 metres high casts a shadow of length 8 metres. Find the height of the pole.

ANSWERS

1. (i), (iii) 2. (i) $\frac{256}{3}$ (ii) 69 3. ₹ 200

4. 200 km 5. ₹5000 6. 15 metres

HINTS TO SELECTED PROBLEMS

3. Let the expenditure be of ₹ x. Then, $7 : 6 = 1400 : x$.
Now, use, saving = Income – Expenditure

5. Savings per month $= ₹ \frac{6000}{12} = ₹\ 500$. Let the income per month be ₹ x.
Then, $x : 500 = 10 : 1$.

6. Use : Height of the tree : Length of the shadow of tree = Height of the pole: Length of shadow of pole.

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. If $a : b = 3 : 4$, then $4a : 3b =$
(a) 4:3 (b) 3:4 (c) 1:1 (d) None of these

2. $\frac{1}{12} : \frac{1}{60} =$
(a) 4:1 (b) 1:4 (c) 5:1 (d) 1:5

3. The simplest form of 24 : 36 is
(a) 9:4 (b) 4:9 (c) 3:2 (d) 2:3

4. If $a : b = 4 : 5$ and $b : c = 2 : 3$, then $a : c$
(a) 4:3 (b) 8:15 (c) 8:9 (d) 5:3

5. If $p : q = 2 : 5$, then $\frac{25p + 14q}{5p + 7q} =$
(a) 8:5 (b) 5:8 (c) 8:3 (d) 3:8

6. A ratio equivalent to 2 : 5 is
(a) 6:15 (b) 4:5 (c) 5:2 (d) 5:4

7. If $2a = 3b = 4c$, then $a : b : c =$
(a) 2:3:4 (b) 3:4:6 (c) 4:3:2 (d) 6:4:3

8. If $2x = 3y$ and $4y = 5z$, then $x : z =$
(a) 4:3 (b) 8:15 (c) 3:4 (d) 15:8

9. If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$, then $a:b:c=$
(a) 2:3:4 (b) 4:3:2 (c) 3:2:4 (d) None of these

10. If $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}=3:4:5$, then $a:b:c$
(a) 5:4:3 (b) 20:15:12 (c) 9:12:15 (d) 12:15:20

11. If $a:b=5:7$ and $b:c=6:11$, then $a:b:c=$
(a) 35:49:66 (b) 30:42:77 (c) 30:42:55 (d) None of these

12. If $x:y=1:1$, then $\frac{3x+4y}{5x+6y}=$
(a) $\frac{7}{11}$ (b) $\frac{17}{11}$ (c) $\frac{17}{23}$ (d) $\frac{4}{5}$

13. If a : b = 2 : 5, then $\frac{3a+2b}{4a+b}=$
(a) $\frac{16}{13}$ (b) $\frac{13}{16}$ (c) $\frac{25}{22}$ (d) $\frac{20}{21}$

14. The mean proportional of *a* and *b* is 10 and the value of *a* is four times the value of *b*. The value of $a+b$ $(a>0, b>0)$ is
(a) 20 (b) 25 (c) 101 (d) 29

15. If $8:x::16:35$
(a) 35 (b) 70 (c) $\frac{35}{2}$ (d) 24

16. The mean proportional of 6 and 24 is
(a) 15 (b) 12 (c) 8 (d) 144

17. The boys and girls in a school are in the ratio 9 : 5. If the number of girls is 320, then the total strength of the school is
(a) 840 (b) 896 (c) 920 (d) 576

18. If the first three terms of a proportion are 3, 5 and 21 respectively, then its fourth term is
(a) 21 (b) 35 (c) 15 (d) None of these

19. What must be added to each term of the ratio 9 : 16 to make the ratio 2 : 3 ?
(a) 5 (b) 3 (c) 4 (d) 6

20. What least number is to be subtracted from each term of the ratio 15 : 19 to make the ratio 3:4?
(a) 3 (b) 5 (c) 6 (d) 9

21. If ₹ 840 is divided between *P* and *Q* in the ratio 3 : 4, then P's share is
(a) ₹ 340 (b) ₹ 480 (c) ₹ 360 (d) ₹ 400

22. The ages of Ravish and Shikha are in the ratio 3 : 8. Six years hence, their ages will be in the ratio 4 : 9. The present age of Ravish is
(a) 18 years (b) 15 years (c) 12 years (d) 21 years

23. The present ages of Renu and Ravi are in the ratio 5 : 6. The sum of their present ages is 44 in years. The difference of their ages (in years) is
(a) 4 (b) 5 (c) 8 (d) 2

24. The third proportional of 3 and 27 is
    (a) 243 (b) 256 (c) 289 (d) 225

ANSWSERS

1. (c)	2. (c)	3. (d)	4. (b)	5. (c)	6. (a)	7. (d)
8. (d)	9. (a)	10. (b)	11. (b)	12. (a)	13. (a)	14. (b)
15. (c)	16. (b)	17. (b)	18. (b)	19. (a)	20. (a)	21. (c)
22. (a)	23. (a)	24. (a)				

THINGS TO REMEMBER

1. *The ratio of a number 'a' to another number 'b' (b ≠ 0) is a fraction* $\frac{a}{b}$ *and is written as a : b.*
2. *In the ratio a : b, the first term is a and the second term is b.*
3. *A ratio is said to be in the simplest form if its two terms have no common factor other than 1.*
4. *The ratio of two numbers is usually expressed in its simplest form.*
5. *The ratio of two quantities is an abstract quantity, i.e., it has no units in itself.*
6. *An equality of two ratios is called a proportion. If a : b = c : d, then we write a : b : : c : d.*
7. *The numbers a, b, c, d are in proportion if the ratio of the first two is equal to the ratio of the last two, i.e., a : b = c : d.*
8. *If four numbers a, b, c, d are in proportion, then a and d are known as extreme terms and b and c are called middle terms.*
9. *Four numbers are in proportion if the product of extreme terms is equal to the product of middle terms, i.e., a : b : : c : d if and only if ad = bc.*
10. *From the terms of a given proportion, we can make three more proportions.*
11. *If a : b = b : c, then a, b, c are said to be in continued proportion.*
12. *If a, b, c are in continued proportion, i.e., a : b : : b : c, then b is called the mean proportional between a and c.*