4

RATIONAL NUMBERS

4.1 INTRODUCTION

We know that for two given integers m and n, their sum $m + n$, product $m \times n$ and the difference $m - n$ are always integers. However, it may not always be possible for a given integer to exactly divide another given integer. In other words, the result of division of an integer by a non-zero integer may or may not be an integer. For example, when 9 is divided by 4, the result is not an integer. In fact $\frac{9}{4}$ is a fraction. Thus, there is need to extend the system of integers so that it may also be possible to divide any given integer by any other given integer different from zero (because division by zero is not possible).

In this chapter, we shall introduce the system of rational numbers. We shall also learn comparison of rational numbers and their representation on the number line.

4.2 RATIONAL NUMBERS

In this section, we shall define rational numbers. We shall also show that every integer is a rational number but a rational number need not be an integer.

RATIONAL NUMBER *A number of the form* $\frac{p}{q}$ *or a number which can be expressed in the form* $\frac{p}{q}$, *where p and q are integers and* $q \neq 0$, *is called a rational number.*

In other words, a rational number is any number that can be expressed as the quotient of two integers with the condition that the divisor is not zero.

Each of the numbers $\frac{2}{3}, \frac{-5}{7}, \frac{-11}{-5}, \frac{7}{-9}$ is a rational number.

SOME USEFUL RESULTS

Result I: *Every natural number is a rational number but a rational number need not be a natural number.*

Explanation: We know that

$$1 = \frac{1}{1}, 2 = \frac{2}{1}, 3 = \frac{3}{1} \text{ and so on.}$$

In other words, every natural number n can be written as $n = \frac{n}{1}$, which is the quotient of two integers. Thus, every natural number is a rational number.

Clearly, $\frac{3}{2}, \frac{2}{5}$, etc. are rational numbers but they are not natural numbers.

Hence, every natural number is a rational number but a rational number need not be a natural number.

Result II: *Zero is a rational number.*

Explanation: We know that the integer 0 can be written in any one of the following forms:

$$\frac{0}{1}, \frac{0}{-1}, \frac{0}{2}, \frac{0}{-2}, \frac{0}{3}, \frac{0}{-3}, \frac{0}{4}, \frac{0}{-4} \text{ and so on.}$$

In other words, $0 = \frac{0}{q}$, where q is any non-zero integer.

Thus, 0 can be written as $\frac{p}{q}$, where $p = 0$ and q is any non-zero integer.

Hence, 0 is a rational number.

Result III: *Every integer is a rational number but a rational number need not be an integer.*

Explanation: We know that

$$1 = \frac{1}{1}, 2 = \frac{2}{1}, 3 = \frac{3}{1}, 4 = \frac{4}{1} \text{ and so on. Also, } -1 = \frac{-1}{1}, -2 = \frac{-2}{1}, -3 = \frac{-3}{1} \text{ and so on.}$$

In other words, any integer p can be written as $p = \frac{p}{1}$, which is a rational number. Thus, every integer is a rational number.

Clearly, $\frac{3}{2}, \frac{-5}{3}$, etc. are rational numbers but they are not integers.

Hence, every integer is a rational number but a rational number need not be an integer.

Result IV: *Every fraction is a rational number but a rational number need not be a fraction.*

Explanation: Let $\frac{p}{q}$ be any fraction. Then, p and q are natural numbers.

Since every natural number is an integer. Therefore, p and q are integers. Thus, the fraction $\frac{p}{q}$ is the quotient of two integers such that $q \neq 0$.

Hence, $\frac{p}{q}$ is a rational number.

We know that $\frac{3}{-5}$ is a rational number but it is not a fraction because its denominator is not a natural number.

Since every mixed fraction consisting of an integer part and a fractional part can be expressed as an improper fraction, which is quotient of two integers. Thus, every mixed fraction is also a rational number.

Hence, every fraction is also a rational number.

NUMERATOR AND DENOMINATOR *If $\frac{p}{q}$ is a rational number, then the integer p is known as its numerator and the integer q is called the denominator.*

POSITIVE RATIONAL NUMBER *A rational number is said to be positive if its numerator and denominator are either both positive integers or both negative integers.*

In other words, a rational number is positive, if its numerator and denominator are of the same sign.

Each of the rational numbers $\frac{2}{3}, \frac{5}{9}, \frac{-7}{-12}, \frac{-3}{-11}$ is a positive rational number, but $\frac{3}{-7}, \frac{-4}{5}$ are not positive rational numbers.

ILLUSTRATION 1 Every natural number is a positive rational number.

Solution We know that

$1 = \frac{1}{1}, 2 = \frac{2}{1}, 3 = \frac{3}{1}$ and so on.

In other words, any natural number n can be written as

$n = \frac{n}{1}$, where n and 1 are positive integers.

Hence, every natural number is a positive rational number.

NEGATIVE RATIONAL NUMBER *A rational number is said to be negative if its numerator and denominator are such that one of them is positive integer and another one is a negative integer.*

In other words, a rational number is negative, if its numerator and denominator are of the opposite signs.

Each of the rational numbers $\frac{-3}{7}, \frac{5}{-9}, \frac{-15}{26}$ is a negative rational number.

ILLUSTRATION 2 Every negative integer is a negative rational number.

Solution We know that

$-1 = \frac{-1}{1}, -2 = \frac{-2}{1}, -3 = \frac{-3}{1}$ and so on.

In other words, any negative integer n can be written as $n = \frac{n}{1}$, here n is negative and 1 is positive.

Hence, every negative integer is a negative rational number.

Remark: *The rational number 0 is neither positive nor negative.*

EXERCISE 4.1

1. Write down the numerator of each of the following rational numbers:

   (i) $\frac{-7}{5}$ (ii) $\frac{15}{-4}$ (iii) $\frac{-17}{-21}$ (iv) $\frac{8}{9}$ (v) 5

2. Write down the denominator of each of the following rational numbers:

   (i) $\frac{-4}{5}$ (ii) $\frac{11}{-34}$ (iii) $\frac{-15}{-82}$ (iv) 15 (v) 0

3. Write down the rational number whose numerator is $(-3) \times 4$, and whose denominator is $(34-23) \times (7-4)$.

4. Write the following rational numbers as integers:

   $$\frac{7}{1}, \frac{-12}{1}, \frac{34}{1}, \frac{-73}{1}, \frac{95}{1}$$

5. Write the following integers as rational numbers with denominator 1:

   $$-15, 17, 85, -100$$

6. Write down the rational number whose numerator is the smallest three digit number and denominator is the largest four digit number.

7. Separate positive and negative rational numbers from the following rational numbers:

   $$\frac{-5}{-7}, \frac{12}{-5}, \frac{7}{4}, \frac{13}{-9}, 0, \frac{-18}{-7}, \frac{-95}{116}, \frac{-1}{-9}$$

8. Which of the following rational numbers are positive:

(i) $\frac{-8}{7}$ (ii) $\frac{9}{8}$ (iii) $\frac{-19}{-13}$ (iv) $\frac{-21}{13}$

9. Which of the following rational numbers are negative?

(i) $\frac{-3}{7}$ (ii) $\frac{-5}{-8}$ (iii) $\frac{9}{-83}$ (iv) $\frac{-115}{-197}$

ANSWERS

1. (i) –7 (ii) 15 (iii) –17 (iv) 8 (v) 5
2. (i) 5 (ii) –34 (iii) –82 (iv) 1 (vi) any non-zero integer
3. $\frac{-12}{33}$
4. 7, –12, 34, –73, 95
5. $\frac{-15}{1}, \frac{17}{1}, \frac{85}{1}, \frac{-100}{1}$
6. $\frac{100}{9999}$
7. Positive: $\frac{-5}{-7}, \frac{7}{4}, \frac{-18}{-7}, \frac{-1}{-9}$; Negative: $\frac{12}{-5}, \frac{13}{-9}, \frac{-95}{116}$
8. (ii) $\frac{9}{8}$ (iii) $\frac{-19}{-13}$
9. (i) $\frac{-3}{7}$ (iii) $\frac{9}{-83}$

4.3 SOME USEFUL PROPERTIES OF RATIONAL NUMBERS

In chapter 2, we have learnt that if we multiply the numerator and denominator of a fraction by the same positive integer, the value of the fraction does not change. For example, the fractions $\frac{2}{3}$ and $\frac{6}{9}$ are equal because the numerator and denominator of $\frac{6}{9}$ can be obtained by muliplying each of the numerator and denominator of $\frac{2}{3}$ by 3.

Also, $\frac{2}{3}=\frac{2\times 5}{3\times 5}=\frac{10}{15}, \frac{2}{3}=\frac{2\times 11}{3\times 11}=\frac{22}{33}$ and so on.

Similarly,

$$\frac{-3}{4}=\frac{-3\times(-1)}{4\times(-1)}=\frac{3}{-4}, \frac{-3}{4}=\frac{-3\times(2)}{4\times 2}=\frac{-6}{8}, \frac{-3}{4}=\frac{-3\times(-2)}{4\times(-2)}=\frac{6}{-8} \text{ and so on.}$$

$$\therefore \quad \frac{-3}{4}=\frac{-3\times(-1)}{4\times(-1)}=\frac{-3\times 2}{4\times 2}=\frac{-3\times(-2)}{4\times(-2)} \text{ and so on.}$$

These illustrations suggest us the following:

Property 1 *If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then*

$$\frac{p}{q}=\frac{p\times m}{q\times m}$$

In other words, a rational number remains unchanged, if we multiply its numerator and denominator by the same non-zero integer.

EQUIVALENT RATIONAL NUMBERS If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p \times m}{q \times m}$ is a rational number equivalent to $\frac{p}{q}$.

For example, rational numbers $\frac{6}{8}, \frac{9}{12}, \frac{15}{20}, \frac{-21}{-28}, \frac{-36}{-48}$ are equivalent to the rational number $\frac{3}{4}$

NOTE: *If the denominator of a rational number is a negative integer, then by using the above property, we can make it positive by multiplying its numerator and denomiator by –1.*

For example, $\frac{3}{-4} = \frac{3 \times (-1)}{(-4) \times (-1)} = \frac{-3}{4}$

We have learnt in chapter 2, that if we divide the numerator and denominator of a fraction by a common divisor, then the value of the fraction does not change. For example,

$$\frac{12}{32} = \frac{12 \div 4}{32 \div 4} = \frac{3}{8}$$

Also,

$$\frac{210}{462} = \frac{210 \div 2}{462 \div 2} = \frac{105}{231} = \frac{105 \div 3}{231 \div 3} = \frac{35}{77} = \frac{35 \div 7}{77 \div 7} = \frac{5}{11}$$

Similarly, we have

$$\frac{-27}{45} = \frac{(-27) \div 3}{45 \div 3} = \frac{-9}{15} = \frac{(-9) \div 3}{15 \div 3} = \frac{-3}{5}, \text{ and } \frac{70}{-84} = \frac{70 \div 2}{(-84) \div 2} = \frac{35}{-42} = \frac{35 \div (-7)}{(-42) \div (-7)} = \frac{-5}{6}$$

These illustrations suggest us the following:

Property 2 *If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q, then*

$$\frac{p}{q} = \frac{p \div m}{q \div m}$$

In other words, if we divide the numerator and denominator of a rational number by a common divisor of both, the rational number remains unchanged.

We shall now use the above properties in expressing a given rational number in different forms. The following examples will illustrate the same.

ILLUSTRATIVE EXAMPLES

Example 1 Write each of the following rational numbers with positive denominator:

$$\frac{5}{-7}, \frac{15}{-28}, \frac{-17}{-13}$$

Solution In order to express a rational number with positive denominator, we multiply its numerator and denominator by –1. Therefore,

$\frac{5}{-7} = \frac{5 \times (-1)}{(-7) \times (-1)} = \frac{-5}{7}$; $\frac{15}{-28} = \frac{15 \times (-1)}{(-28) \times (-1)} = \frac{-15}{28}$, and $\frac{-17}{-13} = \frac{(-17) \times (-1)}{(-13) \times (-1)} = \frac{17}{13}$

Example 2 Express $\frac{-5}{6}$ as a rational number with numerator:

(i) -15 (ii) 10

Solution (i) In order to express $\frac{-5}{6}$ as a rational number with numerator -15, we first find a number which when multiplied by -5 gives -15.

Clearly, such number is $(-15) \div (-5) = 3$

Multiplying the numerator and denominator of $\frac{-5}{6}$ by 3, we have

$$\frac{-5}{6} = \frac{(-5) \times 3}{6 \times 3} = \frac{-15}{18}$$

Thus, the required rational number is $\frac{-15}{18}$.

(ii) In order to express $\frac{-5}{6}$ as a rational number with numerator 10, we first find a number which when multiplied by -5 gives 10.

Clearly, such a number is $10 \div (-5) = -2$.

Multiplying the numerator and denominator of $\frac{-5}{6}$ by (-2), we have

$$\frac{-5}{6} = \frac{(-5) \times (-2)}{6 \times (-2)} = \frac{10}{-12}$$

Thus, the required rational number is $\frac{10}{-12}$

Example 3 Express $\frac{-4}{5}$ as a rational number with denominator:

(i) 20 (ii) -30

Solution (i) In order to express $\frac{-4}{5}$ as a rational number with denominator 20, we first find a number which when multiplied with 5 gives 20.

Clearly, such a number $= 20 \div 5 = 4$

Multiplying the numerator and denominator of $\frac{-4}{5}$ by 4, we have

$$\frac{-4}{5} = \frac{-4 \times 4}{5 \times 4} = \frac{-16}{20}$$

(ii) In order to express $\frac{-4}{5}$ as a rational number with denominator -30, we first find the number which when multiplied by 5 gives -30.

Clearly, such a number $= (-30) \div 5 = -6$

Multiplying the numerator and denominator of $\frac{-4}{5}$ by –6, we have

$$\frac{-4}{5}=\frac{(-4)\times(-6)}{5\times(-6)}=\frac{24}{-30}$$

Example 4 Express $\frac{-48}{60}$ as a rational number with denominator 5.

Solution In order to express $\frac{-48}{60}$ as a rational number with denominator 5, we first find a number which gives 5 when 60 is divided by it.

Clearly, such a number = $(60 \div 5) = 12$

Dividing the numerator and denominator of $\frac{-48}{60}$ by 12, we have

$$\frac{-48}{60}=\frac{-48\div 12}{60\div 12}=\frac{-4}{5}$$

Example 5 Express $\frac{42}{-63}$ as a rational number with denominator 3.

Solution In order to express $\frac{42}{-63}$ as a rational number with denominator 3, we first find a number which gives 3 when –63 is divided by it.

Clearly, such a number = $(-63) \div 3 = -21$

Dividing the numerator and denominator of $\frac{42}{-63}$ by –21, we get

$$\frac{42}{-63}=\frac{42\div(-21)}{(-63)\div(-21)}=\frac{-2}{3}$$

Example 6 Fill in the blanks:

(i) $\frac{5}{-7}=\frac{\dots}{35}=\frac{\dots}{-77}$ (ii) $\frac{7}{13}=\frac{35}{\dots}=\frac{-63}{\dots}$

Solution (i) We have, $35 \div (-7) = -5$

$$\therefore \frac{5}{-7}=\frac{5\times(-5)}{-7\times(-5)}=\frac{-25}{-35}$$

Similarly, we have $(-77) \div (-7) = 11$

$$\therefore \frac{5}{-7}=\frac{5\times 11}{-7\times 11}=\frac{55}{-77}$$

Hence, $\frac{5}{-7}=\frac{-25}{35}=\frac{55}{-77}$

(ii) We have, $35 \div 7 = 5$

$$\therefore \frac{7}{13}=\frac{7\times 5}{13\times 5}=\frac{35}{65}$$

Similarly, we have $(-63) \div 7 = -9$

$$\therefore \quad \frac{7}{13} = \frac{7 \times (-9)}{13 \times (-9)} = \frac{-63}{-117}$$

Hence, $\frac{7}{13} = \frac{35}{65} = \frac{-63}{-117}$

Example 7 In each of the following, find an equivalent form of the rational numbers having a common denominator

(i) $\frac{5}{6}$ and $\frac{7}{9}$ (ii) $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$

Solution (i) We have to convert $\frac{5}{6}$ and $\frac{7}{9}$ into equivalent rational numbers having common denominator.

Clearly, such a denominator is the LCM of 6 and 9.

We have, $6 = 2 \times 3$ and $9 = 3 \times 3$

$\therefore$ LCM of 6 and 9 is $2 \times 3 \times 3 = 18$

Now, $18 \div 6 = 3$ and $18 \div 9 = 2$

$$\therefore \quad \frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18} \text{ and } \frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$$

Hence, given rational numbers with common denominator are $\frac{15}{18}$ and $\frac{14}{18}$.

(ii) LCM of 3, 6 and 12 is 12

Also, $12 \div 3 = 4$, $12 \div 6 = 2$

$$\therefore \quad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \text{ and } \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

Hence, the given rational numbers with common denominator are $\frac{8}{12}, \frac{10}{12}$ and $\frac{7}{12}$.

EXERCISE 4.2

1. Express each of the following as a rational number with positive denominator:

   (i) $\frac{-15}{-28}$ (ii) $\frac{6}{-9}$ (iii) $\frac{-28}{-11}$ (iv) $\frac{19}{-7}$

2. Express $\frac{3}{5}$ as a rational number with numerator:

   (i) 6 (ii) −15 (iii) 21 (iv) −27

3. Express $\frac{5}{7}$ as a rational number with denominator:

   (i) −14 (ii) 70 (iii) −28 (iv) −84

4. Express $\frac{3}{4}$ as a rational number with denominator:
   (i) 20 (ii) 36 (iii) 44 (iv) –80
5. Express $\frac{2}{5}$ as a rational number with numerator
   (i) –56 (ii) 154 (iii) –750 (iv) 500
6. Express $\frac{-192}{108}$ as a rational number with numerator:
   (i) 64 (ii) –16 (iii) 32 (iv) –48
7. Express $\frac{168}{-294}$ as a rational number with denominator:
   (i) 14 (ii) –7 (iii) –49 (iv) 1470
8. Write $\frac{-14}{42}$ in a form so that the numerator is equal to:
   (i) –2 (ii) 7 (iii) 42 (iv) –70
9. Select those rational numbers which can be written as a rational number with numerator 6:
$$\frac{1}{22}, \frac{2}{3}, \frac{3}{4}, \frac{4}{-5}, \frac{5}{6}, \frac{-6}{7}, \frac{-7}{8}$$
10. Select those rational numbers which can be written as a rational number with denominator 4:
$$\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{-16}{17}, \frac{5}{-4}, \frac{140}{28}$$
11. In each of the following, find an equivalent form of the rational number having a common denominator:
   (i) $\frac{3}{4}$ and $\frac{5}{12}$ (ii) $\frac{2}{3}, \frac{7}{6}$ and $\frac{11}{12}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$

ANSWERS

1. (i) $\frac{15}{28}$ (ii) $\frac{-6}{9}$ (iii) $\frac{28}{11}$ (iv) $\frac{-19}{7}$
2. (i) $\frac{6}{10}$ (ii) $\frac{-15}{-25}$ (iii) $\frac{21}{35}$ (iv) $\frac{-27}{-45}$
3. (i) $\frac{-10}{-14}$ (ii) $\frac{50}{70}$ (iii) $\frac{-20}{-28}$ (iv) $\frac{-60}{-84}$
4. (i) $\frac{15}{20}$ (ii) $\frac{27}{36}$ (iii) $\frac{33}{44}$ (iv) $\frac{-60}{-80}$
5. (i) $\frac{-56}{-140}$ (ii) $\frac{154}{385}$ (iii) $\frac{-750}{-1875}$ (iv) $\frac{500}{1250}$
6. (i) $\frac{64}{-36}$ (ii) $\frac{-16}{9}$ (iii) $\frac{32}{-18}$ (iv) $\frac{-48}{27}$

7. (i) $\frac{-8}{14}$ (ii) $\frac{4}{-7}$ (iii) $\frac{28}{-49}$ (iv) $\frac{-840}{1470}$

8. (i) $\frac{-2}{6}$ (ii) $\frac{7}{-21}$ (iii) $\frac{42}{-126}$ (iv) $\frac{-70}{210}$ 9. $\frac{1}{22}, \frac{2}{3}, \frac{3}{4}, \frac{-6}{7}$

10. $\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$

11. (i) $\frac{9}{12}$ and $\frac{5}{12}$ (ii) $\frac{8}{12}, \frac{14}{12}$ and $\frac{11}{12}$ (iii) $\frac{120}{168}, \frac{63}{168}, \frac{108}{168}$ and $\frac{160}{168}$

4.4 LOWEST FORM OF A RATIONAL NUMBER

DEFINITION *A rational number $\frac{p}{q}$ is said to be in the lowest form or simplest form if p and q have no common factor other than 1.*

In other words, a rational number $\frac{p}{q}$ is said to be in the lowest form, if the HCF of p and q is 1, i.e., p and q are relatively prime.

The rational number $\frac{2}{7}$ is in the lowest form, because 2 and 7 have no common factor other than 1. However, the rational number $\frac{12}{40}$ is not in the lowest form, because 4 is a common factor to both numerator and denominator.

Every rational number can be put in the lowest form using the following steps:

STEP I *Obtain the rational number $\frac{p}{q}$ (say).*

STEP II *Find the HCF of p and q.*

STEP III *If $m = 1$, then $\frac{p}{q}$ is in lowest form.*

STEP IV *If $m \neq 1$, then $\frac{p \div m}{q \div m}$ is the lowest form of $\frac{p}{q}$.*

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Find whether the following rational numbers are in the lowest form or not.

(i) $\frac{17}{79}$ (ii) $\frac{24}{320}$

Solution (i) We observe that 17 and 79 have no common factor, i.e., their HCF is 1.

Therefore, $\frac{17}{79}$ is in the lowest form.

(ii) We have,

$24 = 2 \times 2 \times 2 \times 3$ and $320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

Thus, HCF of 24 and 320 is $2 \times 2 \times 2 = 8$.

Therefore, $\frac{24}{320}$ is not in the lowest form.

Example 2 Express each of the following rational numbers to the lowest form.

(i) $\frac{12}{16}$ (ii) $\frac{-60}{72}$ (iii) $\frac{-24}{-36}$ (iv) $\frac{91}{-364}$

Solution (i) We have,

$12 = 2 \times 2 \times 3$ and $16 = 2 \times 2 \times 2 \times 2$

$\therefore$ HCF of 12 and 16 is $2 \times 2 = 4$.

So, $\frac{12}{16}$ is not in lowest form.

Dividing numerator and denominator by 4, we have

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

$\therefore$ $\frac{3}{4}$ is the lowest form of $\frac{12}{16}$.

(ii) We have,

$60 = 2 \times 2 \times 3 \times 5$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

$\therefore$ HCF of 60 and 72 is $2 \times 2 \times 3 = 12$

Dividing numerator and denominator of $\frac{-60}{72}$ by 12, we have

$$\frac{-60}{72} = \frac{-60 \div 12}{72 \div 12} = \frac{-5}{6}$$

Hence, $\frac{-5}{6}$ is the lowest form of $\frac{-60}{72}$.

(iii) We have,

$24 = 2 \times 2 \times 2 \times 3$ and $36 = 2 \times 2 \times 3 \times 3$

$\therefore$ HCF of 24 and 36 is $2 \times 2 \times 3 = 12$

Dividing numerator and denominator of $\frac{-24}{-36}$ by 12, we have

$$\frac{-24}{-36} = \frac{-24 \div 12}{-36 \div 12} = \frac{-2}{-3}$$

$\therefore$ $\frac{-2}{-3}$ is the lowest form of $\frac{-24}{-36}$.

(iv) We have,

$91 = 7 \times 13$ and $364 = 2 \times 2 \times 7 \times 13$

$\therefore$ HCF of 91 and 364 is $13 \times 7 = 91$.

Dividing numerator and denominator by 91, we have

$$\frac{91}{-364} = \frac{91 \div 91}{-364 \div 91} = \frac{1}{-4}$$

Hence, $\frac{1}{-4}$ is the lowest form of $\frac{91}{-364}$

Example 3 Fill in the blanks:

$$\frac{90}{165} = \frac{-6}{\ldots\ldots} = \frac{\ldots\ldots}{-55}$$

Solution Here, $90 = 2 \times 3 \times 3 \times 5$ and $165 = 3 \times 5 \times 11$

$\therefore$ HCF of 90 and 165 is 15.

So, $\frac{90}{165}$ is not in lowest form.

Dividing numerator and denominator by 15, we get

$$\frac{90}{165} = \frac{90 \div 15}{165 \div 15} = \frac{6}{11}$$

Thus, the rational number $\frac{90}{165}$ in the lowest form equals $\frac{6}{11}$

Now, $(-6) \div 6 = -1$

$$\therefore \quad \frac{6}{11} = \frac{6 \times -1}{11 \times -1} = \frac{-6}{-11}$$

Similarly, we have $(-55) \div 11 = -5$

$$\therefore \quad \frac{6}{11} = \frac{6 \times -5}{11 \times -5} = \frac{-30}{-55}$$

Hence, $\frac{90}{165} = \frac{-6}{-11} = \frac{-30}{-55}$

EXERCISE 4.3

1. Determine whether the following rational numbers are in the lowest form or not:

   (i) $\frac{65}{84}$ (ii) $\frac{-15}{32}$ (iii) $\frac{24}{128}$ (iv) $\frac{-56}{-32}$

2. Express each of the following rational numbers to the lowest form:

   (i) $\frac{4}{22}$ (ii) $\frac{-36}{180}$ (iii) $\frac{132}{-428}$ (iv) $\frac{-32}{-56}$

3. Fill in the blanks:

   (i) $\frac{-5}{7} = \frac{\ldots}{35} = \frac{\ldots}{49}$ (ii) $\frac{-4}{-9} = \frac{\ldots}{18} = \frac{12}{\ldots}$

   (iii) $\frac{6}{-13} = \frac{-12}{\ldots} = \frac{24}{\ldots}$ (iv) $\frac{-6}{\ldots} = \frac{3}{11} = \frac{\ldots}{-55}$

ANSWERS

1. (i), (ii) 2. (i) $\frac{2}{11}$ (ii) $\frac{-1}{5}$ (iii) $\frac{33}{-107}$ (iv) $\frac{4}{7}$

3. (i) $-25 - 35$ (ii) 8, 27 (iii) 26, -52 (iv) $-22, -15$

4.5 STANDARD FORM OF A RATIONAL NUMBER

DEFINITION *A rational number $\frac{p}{q}$ is said to be in the standard form if q is positive, and the integers p and q have no common divisor other than 1.*

In order to express a given rational number in the standard form, we follow the following steps:

STEP I *Obtain the rational number.*

STEP II *See whether the denominator of the rational number is positive or not. If it is negative, multiply or divide numerator and denominator both by –1 so that the denominator becomes positive.*

STEP III *Find the greatest common divisor (GCD) of the absolute values of the numerator and the denominator.*

STEP IV *Divide the numerator and denominator of the given rational number by the GCD (HCF) obtained in step III. The rational number so obtained is the standard form of the given rational number.*

The following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Express each of the following rational numbers in the standard form:

(i) $\frac{-8}{28}$ (ii) $\frac{-12}{-30}$ (iii) $\frac{14}{-49}$ (iv) $\frac{-16}{-56}$

Solution (i) The denominator of the rational number $\frac{-8}{28}$ is positive. In order to express it in the standard form, we divide its numerator and denominator by the greatest common divisor of 8 and 28.

The greatest common divisor of 8 and 28 is 4.

Dividing the numerator and denominator of $\frac{-8}{28}$ by 4, we have

$$\frac{-8}{28} = \frac{(-8) \div 4}{28 \div 4} = \frac{-2}{7}$$

Thus, the standard form of $\frac{-8}{28}$ is $\frac{-2}{7}$.

(ii) The denominator of the rational number $\frac{-12}{-30}$ is negative. So, we first make it positive.

Multiplying the numerator and denominator of $\frac{-12}{-30}$ by –1, we have

$$\frac{-12}{-30} = \frac{(-12) \times (-1)}{(-30) \times (-1)} = \frac{12}{30}$$

The greatest common divisor of 12 and 30 is 6.

Dividing the numerator and denominator of $\frac{12}{30}$ by 6, we get

$$\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = \frac{2}{5}$$

Hence, the standard form of $\frac{-12}{-30}$ is $\frac{2}{5}$.

(iii) The denominator of $\frac{14}{-49}$ is negative. So, we first make it positive.

Multiplying the numerator and denominator of $\frac{14}{-49}$ by –1, we have

$$\frac{14}{-49} = \frac{14 \times (-1)}{(-49) \times (-1)} = \frac{-14}{49}$$

The greatest common divisor of 14 and 49 is 7.

Dividing the numerator and denominator of $\frac{14}{-49}$ by 7, we have

$$\frac{14}{-49} = \frac{(-14) \div 7}{49 \div 7} = \frac{-2}{7}$$

Hence, the standard form of $\frac{14}{-49}$ is $\frac{-2}{7}$.

(iv) The denominator of $\frac{-16}{-56}$ is negative. So, we first make it positive.

Multiplying the numerator and denominator of $\frac{-16}{-56}$ by –1, we have

$$\frac{-16}{-56} = \frac{(-16) \times (-1)}{(-56) \times (-1)} = \frac{16}{56}$$

The greatest common divisor of 16 and 56 is 8.

Dividing the numerator and denominator of $\frac{16}{56}$ by 8, we get

$$\frac{16}{56} = \frac{16 \div 8}{56 \div 8} = \frac{2}{7}$$

Hence, the standard form of $\frac{-16}{-56}$ is $\frac{2}{7}$.

Example 2 Express each one of the following rational numbers in the standard form:

(i) $\frac{-247}{-228}$ (ii) $\frac{299}{-161}$

Solution (i) The denominator of $\frac{-247}{-228}$ is negative. So, we first make it positive.

Multiplying the numerator and denominator of $\frac{-247}{-228}$ by –1, we get

$$\frac{-247}{-228} = \frac{(-247) \times (-1)}{(-228) \times (-1)} = \frac{247}{228}$$

Now, we find the greatest common divisor of 247 and 228 by division method as follows:

12	228 −228	247 −228	1
	0	19	

Clearly, the greatest common divisor of 228 and 247 is the last divisor 19.

Dividing the numerator and denominator of $\frac{247}{228}$ by 19, we get

$$\frac{247}{228} = \frac{247 \div 19}{228 \div 19} = \frac{13}{12}$$

Hence, the standard form of $\frac{-247}{-228}$ is $\frac{13}{12}$.

(ii) The denominator of $\frac{299}{-161}$ is negative. So, we first make it positive.

Multiplying the numerator and denominator of $\frac{299}{-161}$ by −1, we have

$$\frac{299}{-161} = \frac{299 \times (-1)}{(-161) \times (-1)} = \frac{-299}{161}$$

Now, we find the greatest common divisor of 299 and 161 by division method:

1	161 −138	299 −161	1
	23	138 −138	6
		0	

Clearly, the greatest common divisor of 299 and 161 is the last divisor equal to 23.

Dividing the numerator and denominator of $\frac{-299}{161}$ by 23, we get

$$\frac{-299}{161} = \frac{(-299) \div 23}{161 \div 23} = \frac{-13}{7}$$

Hence, the standard form of $\frac{-299}{161}$ is $\frac{-13}{7}$.

EXERCISE 4.4

1. Write each of the following rational numbers in the standard form:

(i) $\frac{2}{10}$ (ii) $\frac{-8}{36}$ (iii) $\frac{4}{-16}$ (iv) $\frac{-15}{-35}$

(v) $\frac{299}{-161}$ (vi) $\frac{-63}{-210}$ (vii) $\frac{68}{-119}$ (viii) $\frac{-195}{275}$

ANSWERS

1. (i) $\frac{1}{5}$ (ii) $\frac{-2}{9}$ (iii) $\frac{-1}{4}$ (iv) $\frac{3}{7}$ (v) $\frac{-13}{7}$ (vi) $\frac{3}{10}$
(vii) $\frac{-4}{7}$ (viii) $\frac{-39}{55}$

4.6 EQUALITY OF RATIONAL NUMBERS

In section 4.3, we have learnt that a rational number remains unchanged if we multiply or divide its numerator and denominator by the same non–zero integer. It follows from this that a rational number can be written in several equivalent forms. Two rational numbers are said to be equivalent if one can be obtained from the other either by multiplying or by dividing its numerator and denominator by the same non–zero integer. Now, a very natural question arises: "Given two rational numbers, how to test whether they are equal?". There are many methods to test the equality of two rational numbers. In this section, we shall discuss various methods to test the equality of two rational numbers.

METHOD I

In order to test the equality of two rational numbers, we express both the rational numbers in the standard form. If they have the same standard form they are equal, otherwise they are not equal.

ILLUSTRATION 1 Are the rational numbers $\frac{8}{-12}$ and $\frac{-50}{75}$ equal?

Solution First we express the given rational numbers in the standard form.

We have,

$$\frac{8}{-12} = \frac{8 \times (-1)}{(-12) \times (-1)}$$ [Multiplying the numerator and denominator by –1]

$$\Rightarrow \frac{8}{-12} = \frac{-8}{12}$$

$$\Rightarrow \frac{8}{-12} = \frac{(-8) \div 4}{12 \div 4}$$ [Dividing the numerator and denominator by the greatest common divisor of 8 and 12 i.e. 4]

$$\Rightarrow \frac{8}{-12} = \frac{-2}{3}$$

and, $$\frac{-50}{75} = \frac{(-50) \div 25}{75 \div 25}$$ [Dividing the numerator and denominator by the greatest common divisor of 50 and 75 i.e. 25]

$$\Rightarrow \frac{-50}{75} = \frac{-2}{3}$$

Clearly, the given rational numbers have the same standard form.

$$\therefore \frac{8}{-12} = \frac{-50}{75}$$

ILLUSTRATION 2 Are the rational numbers $\frac{-8}{28}$ and $\frac{28}{-49}$ equal?

Solution In order to test the equality of the given rational numbers, we first express them in the standard form.

We have,

$$\frac{-8}{28} = \frac{(-8) \div 4}{28 \div 4}$$ [Dividing the numerator and denominator by the greatest common divisor of 8 and 28 i.e. 4]

$$\Rightarrow \frac{-8}{28} = \frac{-2}{7}$$

and, $$\frac{28}{-49} = \frac{28 \times (-1)}{-49 \times (-1)}$$ [Multiplying the numerator and denominator by −1]

$$\Rightarrow \frac{28}{-49} = \frac{-28}{49}$$

$$\Rightarrow \frac{28}{-49} = \frac{(-28) \div 7}{(49) \div 7}$$ [Dividing the numerator and denominator by the greatest common divisor of 28 and 49 i.e. 7]

$$\Rightarrow \frac{28}{-49} = \frac{-4}{7}$$

Clearly, the standard forms of two rational numbers are not same.

Hence, $\frac{-8}{28}$ is not equal to $\frac{28}{-49}$

METHOD II

In this method, denominators of the given rational numbers are made equal by using the following steps:

STEP I *Obtain the two numbers.*

STEP II *Multiply the numerator and denominator of the first number by the denominator of the second number.*

STEP III *Multiply the numerator and denominator of the second number by the denominator of the first number.*

STEP IV *Check the numerators of the two numbers obtained in steps II and III. If their numerators are equal, then the given rational numbers are equal, otherwise they are not equal.*

ILLUSTRATION 3 Are the rational numbers $\frac{-4}{6}$ and $\frac{16}{-24}$ equal?

Solution Multiplying the numerator and denominator of $\frac{-4}{6}$ by the denominator of $\frac{16}{-24}$ i.e. by −24, we have

$$\frac{-4}{6} = \frac{-4\times(-24)}{6\times(-24)} = \frac{96}{-144}$$

Multiplying the numerator and denominator of $\frac{16}{-24}$ by the denominator of $\frac{-4}{6}$ i.e. by 6, we have

$$\frac{16}{-24} = \frac{16\times 6}{(-24)\times 6} = \frac{96}{-144}$$

Clearly, the numerators of the above obtained rational numbers are equal. Hence, the given rational numbers are equal.

ILLUSTRATION 4 Show that the rational numbers $\frac{-15}{35}$ and $\frac{4}{-6}$ are not equal.

Solution Multiplying the numerator and denominator of $\frac{-15}{35}$ by the denominator of $\frac{4}{-6}$ i.e. –6, we get

$$\frac{-15}{35} = \frac{(-15)\times(-6)}{35\times(-6)} = \frac{90}{-210}$$

Multiplying the numerator and denominator of $\frac{4}{-6}$ by the denominator of $\frac{-15}{35}$ i.e. 35, we get

$$\frac{4}{-6} = \frac{4\times 35}{-6\times 35} = \frac{140}{-210}$$

We find that the numerators of rational numbers $\frac{90}{-210}$ and $\frac{140}{-210}$ are unequal. Hence, the given rational numbers are unequal.

METHOD III

In this method, to test the equality of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we use the following result:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow a\times d = b\times c$$

$\Leftrightarrow$ Numerator of first × Denominator of second

= Numerator of second × Denominator of first

ILLUSTRATION 5 Which of the following pairs of rational numbers are equal?

(i) $\frac{-7}{21}$ and $\frac{3}{-9}$ (ii) $\frac{-8}{-14}$ and $\frac{13}{21}$

Solution (i) The given rational numbers are $\frac{-7}{21}$ and $\frac{3}{-9}$

We have,

Numerator of first × Denominator of second = (–7) × (–9) = 63

and, Numerator of second × Denominator of first = 3 × 21 = 63.

Clearly,

Numerator of first × Denominator of second

= Numerator of second × Denominator of first

Hence, $\frac{-7}{21} = \frac{3}{-9}$.

(ii) The given rational numbers are $\frac{-8}{-14}$ and $\frac{13}{21}$

We have,

Numerator of first × Denominator of second = $-8 \times 21 = -168$

and, Numerator of second × Denominator of first $= 13 \times (-14) = -182$

Clearly,

Numerator of first × Denominator of second

≠ Numerator of second × Denominator of first.

Hence, $\frac{-8}{-14} \neq \frac{13}{21}$.

ILLUSTRATION 6 If $\frac{-5}{7} = \frac{x}{28}$, find the value of x.

Solution We know that $\frac{a}{b} = \frac{c}{d}$ if $ad = bc$.

$\therefore \quad \frac{-5}{7} = \frac{x}{28}$

$\Rightarrow \quad -5 \times 28 = 7 \times x$

$\Rightarrow \quad 7x = -140$

$\Rightarrow \quad \frac{7x}{7} = \frac{-140}{7}$ [Dividing both sides by 7]

$\Rightarrow \quad x = -20$

Aliter In order to write $\frac{-5}{7}$ as a rational number with denominator 28, we first find a number which when multiplied with 7 gives 28.

Clearly, such a number is $28 \div 7 = 4$.

Multiplying the numerator and denominator of $\frac{-5}{7}$ by 4, we have

$$\frac{-5}{7} = \frac{-5 \times 4}{7 \times 4} = \frac{-20}{28}$$

$$\therefore \quad \frac{-5}{7} = \frac{x}{28} \Rightarrow \frac{-20}{28} = \frac{x}{28} \Rightarrow x = -20$$

ILLUSTRATION 7 Fill in the blank: $\frac{-3}{8} = \frac{\cdots}{48}$

Solution In order to fill the required blank, we have to express -3 as a rational number with denominator 48. For this, we first find an integer which when multiplied with 8 gives us 48.

Clearly, such an integer is $48 \div 8 = 6$

Multiplying the numerator and denominator of $\frac{-3}{8}$ by 6, we have

$$\frac{-3}{8} = \frac{-3 \times 6}{8 \times 6} = \frac{-18}{48}$$

Hence, the required number is -18.

EXERCISE 4.5

1. Which of the following rational numbers are equal?

   (i) $\frac{-9}{12}$ and $\frac{8}{-12}$ (ii) $\frac{-16}{20}$ and $\frac{20}{-25}$ (iii) $\frac{-7}{21}$ and $\frac{3}{-9}$ (iv) $\frac{-8}{-14}$ and $\frac{13}{21}$

2. If each of the following pairs represents a pair of equivalent rational numbers, find the values of x:

   (i) $\frac{2}{3}$ and $\frac{5}{x}$ (ii) $\frac{-3}{7}$ and $\frac{x}{4}$ (iii) $\frac{3}{5}$ and $\frac{x}{-25}$ (iv) $\frac{13}{6}$ and $\frac{-65}{x}$

3. In each of the following, fill in the blanks so as to make the statement true:

   (i) A number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and q is not equal to zero, is called a .....

   (ii) If the integers p and q have no common divisor other than 1 and q is positive, then the rational number $\frac{p}{q}$ is said to be in the ....

   (iii) Two rational numbers are said to be equal, if they have the same .... form.

   (iv) If m is a common divisor of a and b, then

   $$\frac{a}{b} = \frac{a \div m}{\ldots}$$

   (v) If p and q are positive integers, then $\frac{p}{q}$ is a ...... rational number and $\frac{p}{-q}$ is a ...... rational number.

   (vi) The standard form of -1 is ...

   (vii) If $\frac{p}{q}$ is a rational number, then q cannot be ....

   (viii) Two rational numbers with different numerators are equal, if their numerators are in the same .... as their denominators.

4. *In each of the following state if the statement is true (T) or false (F):*

   (i) The quotient of two integers is always an integer.

   (ii) Every integer is a rational number.

   (iii) Every rational number is an integer.

   (iv) Every fraction is a rational number.

   (v) Every rational number is a fraction.

   (vi) If $\frac{a}{b}$ is a rational number and m any integer, then $\frac{a}{b} = \frac{a \times m}{b \times m}$

(vii) Two rational numbers with different numerators cannot be equal.

(viii) 8 can be written as a rational number with any integer as denominator.

(ix) 8 can be written as a rational number with any integer as numerator.

(x) $\frac{2}{3}$ is equal to $\frac{4}{6}$.

ANSWERS

1. (ii), (iii)
2. (i) $\frac{15}{2}$ (ii) $\frac{-12}{7}$ (iii) –15 (iv) –30
3. (i) rational number (ii) standard form or lowest form (iii) standard (iv) $b \div m$
   (v) positive, negative (vi) $\frac{-1}{1}$ (vii) zero (viii) ratio
4. (i) F (ii) T (iii) F (iv) T (v) F (vi) F
   (vii) F (viii) F (ix) F (x) T

4.7 REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

In Class VI, we have learnt how to represent integers on the number line. To represent the integers on the number line, we draw a line and mark a point 'O' on it corresponding to the number 'zero' starting from O, we mark on it points at equal distances on right as well as on left of O. Let A, B, C, D, etc. be the points of division on the right of O and A', B', C', D', etc. be the points of division on the left of O as shown in Fig. 1. If we take OA =1 unit, then points A, B, C, D, etc. represent the integers 1, 2, 3, 4, etc respectively and the points A', B', C', D', etc. represent the integers –1 , –2 , –3, –4, etc. respectively.

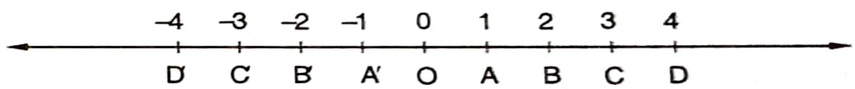

Fig. 1

The point O represents integer 0.

Clearly, every positive integer lies to the right of O and every negative integer lies to the left of 0.

Rational numbers can also be represented on the number line in the same way.

In order to represent rational numbers on the number line, we draw a line and mark a point O on it to represent the rational number zero. The positive rational numbers will be represented by points on the line lying to the right of O and negative rational numbers will be represented by points on the line lying to the left of O. If we mark a point A on the line to the right of O to represent 1, then OA =1 unit. Similarly, if we choose a point A' on the line to the left of O to represent –1, then OA' =1 unit.

Now, suppose we wish to represent the rational number $\frac{1}{2}$ on the number line. For this, we divide the segment OA into two equal parts. Let P be the mid-point of segment OA . Then $OP = PA = \frac{1}{2}$. Since O represents 0 and, A represents 1, therefore P represents the rational number $\frac{1}{2}$ as shown in Fig 2.

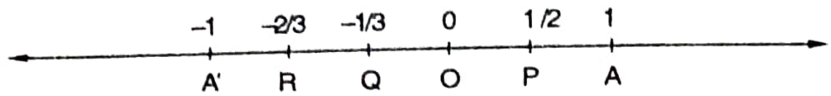

Fig. 2

Similarly, if we want to represent the rational numbers $\frac{-1}{3}$ and $\frac{-2}{3}$ on the number line, we divide the segment OA' into three equal parts. Let Q and R be the points dividing segment OA' into three equal parts. Then, $OQ = QR = RA' = \frac{1}{3}$. Since O represents 0 and A' represents –1. Therefore, Q and R represent $\frac{-1}{3}$ and $\frac{-2}{3}$ respectively.

The following illustrations will illustrate the representation of more rational numbers on the number line.

ILLUSTRATION 1 Represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line.

Solution In order to represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line, we first draw a number line and mark a point O on it to represent zero. Now, we find the points P and Q on the number line representing the positive integers 5 and –5 respectively as shown in Fig. 3.

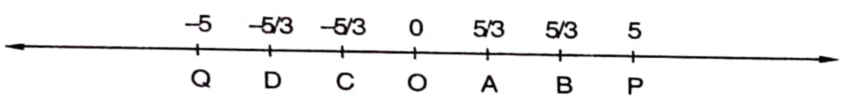

Fig. 3

Now, divide the segment OP into three equal parts. Let A and B be the points of division so that $OA = AB = BP$. By construction, OA is one-third of OP.

Therefore, A represents the rational number $\frac{5}{3}$.

Point Q represents –5 on the number line. Now, divide OQ into three equal parts OC, CD and DQ. The point C is such that OC is one third of OQ. Since Q represents the number –5. Therefore, C represents the rational number $\frac{-5}{3}$.

ILLUSTRATION 2 Represent $\frac{8}{5}$ and $\frac{-8}{5}$ on the number line.

Solution To represent $\frac{8}{5}$ and $\frac{-8}{5}$ on the number line, draw a number line and mark a point O on it to represent zero. Now, mark two points P and Q representing integers 8 and –8 respectively on the number line. Divide the segment OP into five equal parts. Let A, B, C, D be the points of division so that $OA = AB = BC = CD = DP$. By construction, OA is one-fifth of OP. So, A represents the rational number $\frac{8}{5}$.

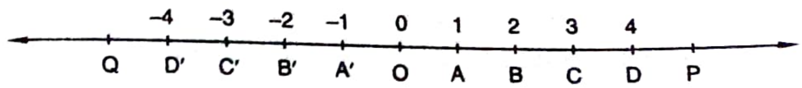

Fig. 4

Now, Q represents –8 on the number line. Divide OQ into five equal parts OA', $A'B'$, $B'C'$, $C'D'$ and $D'Q$. Since Q represents –8. Therefore, A' represents the rational number $\frac{-8}{5}$.

4.8 COMPARISON OF RATIONAL NUMBERS

In the previous classes, we have learnt how to compare two integers and also two fractions. We know that every positive integer is greater than zero and every negative integer is less than zero. Also, every positive integer is greater than every negative integer. Similar to the comparison of integers, we have the following facts about the comparison of rational numbers

(i) Every positive rational number is greater than 0.

(ii) Every negative rational number is less than 0.

(iii) Every positive rational number is greater than every negative rational number.

(iv) Every rational number represented by a point on the number line is greater than every rational number represented by points on its left.

(v) Every rational number represented by a point on the number line is less than every rational number represented by points on its right.

In order to compare any two rational numbers, we can use the following steps:

STEP I *Obtain the given rational numbers.*

STEP II *Write the given rational numbers so that their denominators are positive.*

STEP III *Find the LCM of the positive denominators of the rational numbers obtained in step II.*

STEP IV *Express each rational number (obtained in step II) with the LCM (obtained in step III) as common denominator.*

STEP V *Compare the numerators of rational numbers obtained in step IV. The number having greater numerator is the greater rational number.*

Follwoing examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Which of the two rational numbers $\frac{3}{5}$ and $\frac{-2}{3}$ is greater?

Solution Clearly, $\frac{3}{5}$ is a positive rational number and $\frac{-2}{3}$ is a negative rational number. We know that every positive rational number is greater than every negative rational number.

$\therefore \quad \frac{3}{5} > \frac{-2}{3}$

Example 2 Which of the two rational numbers $\frac{5}{7}$ and $\frac{3}{5}$ is greater?

Solution Clearly, denominators of the given rational numbers are positive. The denominators are 7 and 5. The LCM of 7 and 5 is 35. So, we first express each rational number with 35 as common denominator.

$$\therefore \quad \frac{5}{7}=\frac{5\times 5}{7\times 5}=\frac{25}{35} \text{ and } \frac{3}{5}=\frac{3\times 7}{5\times 7}=\frac{21}{35}$$

Now, we compare the numerators of these rational numbers.

$$\therefore \quad 25>21 \Rightarrow \frac{25}{35}>\frac{21}{35} \Rightarrow \frac{5}{7}>\frac{3}{5}$$

Example 3 Which of the two rational numbers $\frac{-4}{9}$ and $\frac{5}{-12}$ is greater?

Solution First we write each one of the given rational numbers with positive denominator.

Clearly, denominator of $\frac{-4}{9}$ is positive. The denominator of $\frac{5}{-12}$ is negative. So, we express it with positive denominator as follows:

$$\frac{5}{-12}=\frac{5\times(-1)}{(-12)\times(-1)}=\frac{-5}{12}$$

Now, LCM of denominators 9 and 12 is 36.

We write the rational numbers so that they have a common denominator 36 as follows:

$$\frac{-4}{9}=\frac{-4\times 4}{9\times 4}=\frac{-16}{36} \text{ and, } \frac{-5}{12}=\frac{-5\times 3}{12\times 3}=\frac{-15}{36}$$

$$\therefore \quad -15>-16 \Rightarrow \frac{-15}{36}>\frac{-16}{36} \Rightarrow \frac{-5}{12}>\frac{-4}{9} \Rightarrow \frac{5}{-12}>\frac{-4}{9}$$

Example 4 Arrange the rational numbers $\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$ in ascending order:

Solution We first write the given rational numbers so that their denominators are positive.

We have,

$$\frac{5}{-8}=\frac{5\times(-1)}{-8\times(-1)}=\frac{-5}{8} \text{ and } \frac{2}{-3}=\frac{2\times(-1)}{(-3)\times(-1)}=\frac{-2}{3}$$

2	10	8	3
2	5	4	3
	5	2	3

Thus, the given rational numbers with positive denomiantors are

$$\frac{-7}{10}, \frac{-5}{8}, \frac{-2}{3}$$

Now, LCM of the denominators 10, 8 and 3 is $2\times 2\times 5\times 2\times 3=120$

We now write the numbers so that they have a common denominators 120 as follows:

$$\frac{-7}{10}=\frac{-7\times 12}{10\times 12}=\frac{-84}{120}, \frac{-5}{8}=\frac{-5\times 15}{8\times 15}=\frac{-75}{120} \text{ and, } \frac{-2}{3}=\frac{-2\times 40}{3\times 40}=\frac{-80}{120}$$

Comparing the numerators of these numbers, we get

$$-84 < -80 < -75$$

$$\therefore \quad \frac{-84}{120} < \frac{-80}{120} < \frac{-75}{120} \Rightarrow \frac{-7}{10} < \frac{-2}{3} < \frac{-5}{8} \Rightarrow \frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$$

Example 5 Arrange the following rational numbers in descending order:

$$\frac{4}{9}, \frac{-5}{6}, \frac{-7}{-12}, \frac{11}{-24}$$

Solution First we express the given rational numbers in the form so that their denominators are positive.

We have,

$$\frac{-7}{-12} = \frac{(-7)\times(-1)}{(-12)\times(-1)}$$ [Multiplying the numerator and denominator by –1]

$$\Rightarrow \frac{-7}{-12} = \frac{7}{12}$$

and, $$\frac{11}{-24} = \frac{11\times(-1)}{(-24)\times(-1)}$$ [Multiplying the numerator and denominator by –1]

$$\Rightarrow \frac{-11}{24} = \frac{-11}{24}$$

Thus, given rational numbers are : $\frac{4}{9}, \frac{-5}{6}, \frac{7}{12}, \frac{-11}{24}$

Now, we find the LCM of 9, 6, 12 and 24.

3	9	6	12	24
2	3	2	4	8
2	3	1	2	4
	3	1	1	2

Required LCM $= 3 \times 2 \times 2 \times 3 \times 2 = 72$.

We now write the rational numbers so that they have a common denominator 72.

We have,

$$\frac{4}{9} = \frac{4\times 8}{9\times 8}$$ [Multiplying the numerator and denominator by 72 ÷9 =8]

$$\Rightarrow \frac{4}{9} = \frac{32}{72}$$

$$\frac{-5}{6} = \frac{-5\times 12}{6\times 12}$$ [Multiplying the numerator and denominator by 72 ÷ 6=12]

$$\Rightarrow \frac{-5}{6} = \frac{-60}{72}$$

$\frac{7}{12} = \frac{7 \times 6}{12 \times 6}$ [Multiplying the numerator and denominator by $72 \div 12 = 6$]

$\Rightarrow \frac{7}{12} = \frac{42}{72}$

$\frac{-11}{24} = \frac{-11 \times 3}{24 \times 3}$ [Multiplying the numerator and denominator by $72 \div 24 = 3$]

$\Rightarrow \frac{-11}{24} = \frac{-33}{72}$

Arranging the numerators of these rational numbers in descending order, we have

$42 > 32 > -33 > -60$

$\Rightarrow \frac{42}{72} > \frac{32}{72} > \frac{-33}{72} > \frac{-60}{72} \Rightarrow \frac{-7}{-12} > \frac{4}{9} > \frac{11}{-24} > \frac{-5}{6}$

Hence, the given numbers when arranged in descending order are:

$\frac{-7}{-12}, \frac{4}{9}, \frac{11}{-24}, \frac{-5}{6}$

4.8.1 SOME USEFUL PROPERTIES OF RATIONAL NUMBERS

We have learnt that every rational number different from zero is either positive or negative. So, we have the following properties of rational numbers:

Property 1 *For each rational number x, exactly one of the following is true:*

(i) $x > 0$ *(ii)* $x = 0$ *(iii)* $x < 0$

As we have seen in the previous section that every pair of rational numbers can be compared. So, we have the following properties:

Property 2 *For any two rational numbers x and y, exactly one of the following is true:*

(i) $x > y$ *(ii)* $x = y$ *(iii)* $x < y$

Property 3 *If x, y, z be rational numbers such that* $x > y$ *and* $y > z$*, then* $x > z$.

EXERCISE 4.6

1. Draw the number line and represent the following rational numbers on it:

   (i) $\frac{2}{3}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{8}$ (iv) $\frac{-5}{8}$

   (v) $\frac{-3}{16}$ (vi) $\frac{-7}{3}$ (vii) $\frac{22}{-7}$ (viii) $\frac{-31}{3}$

2. Which of the two rational numbers in each of the following pairs of rational numbers is greater?

   (i) $\frac{-3}{8}, 0$ (ii) $\frac{5}{2}, 0$ (iii) $\frac{-4}{11}, \frac{3}{11}$ (iv) $\frac{-7}{12}, \frac{5}{-8}$

   (v) $\frac{4}{-9}, \frac{-3}{-7}$ (vi) $\frac{-5}{8}, \frac{3}{-4}$ (vii) $\frac{5}{9}, \frac{-3}{-8}$ (viii) $\frac{5}{-8}, \frac{-7}{12}$

3. Which of the two rational numbers in each of the following pairs of rational numbers is smaller?

   (i) $\frac{-6}{-13}, \frac{7}{13}$ (ii) $\frac{16}{-5}, 3$ (iii) $\frac{-4}{3}, \frac{8}{-7}$ (iv) $\frac{-12}{5}, -3$

4. Fill in the blanks by the correct symbol out of >, =, or <:

(i) $\frac{-6}{7} \ldots\ldots \frac{7}{13}$ (ii) $\frac{-3}{5} \ldots\ldots \frac{-5}{6}$ (iii) $\frac{-2}{3} \ldots\ldots \frac{5}{-8}$ (iv) $0 \ldots\ldots \frac{-2}{5}$

5. Arrange the following rational numbers in ascending order:

(i) $\frac{3}{5}, \frac{-17}{-30}, \frac{8}{-15}, \frac{-7}{10}$ (ii) $\frac{-4}{9}, \frac{5}{-12}, \frac{7}{-18}, \frac{2}{-3}$

6. Arrange the following rational numbers in descending order:

(i) $\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$ (ii) $\frac{-3}{10}, \frac{17}{-30}, \frac{7}{-15}, \frac{-11}{20}$

7. Which of the following statements are true:

(i) The rational number $\frac{29}{23}$ lies to the left of zero on the number line.

(ii) The rational number $\frac{-12}{-17}$ lies to the left of zero on the number line.

(iii) The rational number $\frac{3}{4}$ lies to the right of zero on the number line

(iv) The rational numbers $\frac{-12}{-5}$ and $\frac{-7}{17}$ are on the opposite side of zero on the number line.

(v) The rational numbers $\frac{-21}{5}$ and $\frac{7}{-31}$ are on the opposite side of zero on the number line.

(vi) The rational number $\frac{-3}{-5}$ is on the right of $\frac{-4}{7}$ on the number line.

ANSWERS

2. (i) 0 (ii) $\frac{5}{2}$ (iii) $\frac{3}{11}$ (iv) $\frac{-7}{12}$ (v) $\frac{-3}{-7}$ (vi) $\frac{-5}{8}$

(vii) $\frac{5}{9}$ (viii) $\frac{-7}{12}$

3. (i) $\frac{-6}{-13}$ (ii) $\frac{16}{-5}$ (iii) $\frac{-4}{3}$ (iv) -3

4. (i) $<$ (ii) $>$ (iii) $<$ (iv) $>$

5. (i) $\frac{-7}{10} < \frac{8}{-15} < \frac{-17}{30} < \frac{3}{5}$ (ii) $\frac{2}{-3} < \frac{-4}{9} < \frac{5}{-12} < \frac{7}{-18}$

6. (i) $\frac{140}{28} > \frac{64}{16} > \frac{7}{8} > \frac{5}{-4} > \frac{36}{-12}$ (ii) $\frac{-3}{10} > \frac{7}{-15} > \frac{-11}{20} > \frac{17}{-30}$

7. (i) F (ii) F (iii) T (iv) T (v) F (vi) T

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. $\frac{44}{-77}$ in standard form is

(a) $\frac{4}{-7}$ (b) $-\frac{4}{7}$ (c) $-\frac{44}{77}$ (d) None of these

2. $-\frac{102}{119}$ in standard form is

(a) $-\frac{6}{7}$ (b) $\frac{6}{7}$ (c) $-\frac{6}{17}$ (d) None of these

3. A rational number equal to $\frac{-2}{3}$ is

(a) $\frac{-10}{25}$ (b) $\frac{10}{-15}$ (c) $\frac{-9}{6}$ (d) None of these

4. If $\frac{-3}{7} = \frac{x}{35}$, then $x =$

(a) 15 (b) 21 (c) -15 (d) -21

5. Which of the following is correct?

(a) $\frac{5}{9} > \frac{-3}{-8}$ (b) $\frac{5}{9} < \frac{-3}{-8}$ (c) $\frac{2}{-3} < \frac{-8}{7}$ (d) $\frac{4}{-3} > \frac{-8}{7}$

6. If the rational numbers $\frac{-2}{3}$ and $\frac{4}{x}$ represent a pair of equivalent rational numbers, then

(a) 6 (b) -6 (c) 3 (d) -3

7. What is the additive identity element in the set of whole numbers?

(a) 0 (b) 1 (c) -1 (d) None of these

8. What is the multiplicative identity element in the set of whole numbers?

(a) 0 (b) 1 (c) -1 (d) None of these

9. Which of the following is not zero?

(a) 0×0 (b) $\frac{0}{3}$ (c) $\frac{7-7}{3}$ (d) $9 + 0$

10. The whole number nearest to 457 and divisible by 11 is

(a) 450 (b) 451 (c) 460 (d) 462

11. If $-\frac{3}{8}$ and $\frac{x}{-24}$ are equivalent rational numbers, then $x =$

(a) 3 (b) 6 (c) 9 (d) 12

12. If $\frac{27}{-45}$ is expressed as a rational number with denominator 5, then the numerator is

(a) 3 (b) -3 (c) 6 (d) -6

13. Which of the following pairs of rational numbers are on the opposite sides of the zero on the number line?

(a) $\frac{3}{7}$ and $\frac{5}{12}$ (b) $-\frac{3}{7}$ and $\frac{-5}{12}$ (c) $\frac{3}{7}$ and $\frac{-5}{12}$ (d) None of these

14. The rational number equal to $\frac{2}{-3}$ is

(a) $\frac{14}{-18}$ (b) $\frac{-6}{9}$ (c) $\frac{-8}{-12}$ (d) $\frac{3}{-2}$

15. If $-\frac{3}{4} = \frac{6}{x}$, then $x =$

(a) -8 (b) 4 (c) -4 (d) 8

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1. (b)	2. (a)	3. (b)	4. (c)	5. (a)	6. (b)	7. (a)
8. (b)	9. (d)	10. (b)	11. (c)	12. (b)	13. (c)	14. (b)
15. (a)						

THINGS TO REMEMBER

1. Numbers that can be expressed in the form $\frac{p}{q}$, where q is a non–zero integer and p is any integer are called rational numbers.
2. Every integer is a rational number but a rational number need not be an integer.
3. Every fraction is a rational number but a fraction need not be a rational number.
4. A rational number $\frac{p}{q}$ is said to be in the standard form if q is a positive integer and the integers $\frac{p}{q}$ have no common divisor other than 1.
5. A rational number $\frac{p}{q}$ is positive, if p and q are either both positive or both negative.
6. A rational number $\frac{p}{q}$ is negative, if p and q are of opposite signs.
7. Two rational numbers are equal if they have the same standard form.
8. To convert a rational number to an equivalent rational number, either multiply or divide both its numerator and denominator by a non-zero integer.
9. If $\frac{x}{y}$ is a rational number and m is any non–zero integer, then $\frac{x}{y} = \frac{x \times m}{y \times m}$
10. If $\frac{x}{y}$ is a rational number and m is a common divisor of x and y, then

$$\frac{x}{y} = \frac{x \div m}{y \div m}$$

11. If x and y are positive integers, then the rational numbers $\frac{x}{y}$ and $\frac{-x}{-y}$ are both positive and the rational numbers $\frac{-x}{y}$ and $\frac{x}{-y}$ are both negative.
12. $\frac{a}{b} = \frac{c}{d}$ only when $a \times d = b \times c$
13. If there are two rational numbers with common denominator, then one with the larger numerator is larger than the other.
14. Every positive rational number is greater than zero.
15. Every negative rational number is less than zero.
16. The rational numbers can be represented on the number line.