

MATHEMATICS
WORKSHEET_290823
CHAPTER-09 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS

MAX. MARKS : 40

CLASS : IX

DURATION : 1½ hrs

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

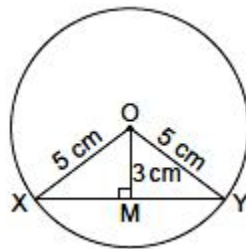
SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Given a circle of radius 5 cm and centre O. OM is drawn perpendicular to the chord XY. If OM = 3 cm, then length of chord XY is

(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

Ans: Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so XM = MY



In right-angled triangle OMX, we have

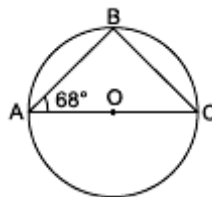
$(OX)^2 = (OM)^2 + (XM)^2$ (By Pythagoras theorem)

$$\Rightarrow (5)^2 = (3)^2 + (XM)^2 \Rightarrow (XM)^2 = 25 - 9 = 16 \Rightarrow XM = 4 \text{ cm}$$

So, the length of chord XY = $2XM = 2 \times 4 = 8 \text{ cm}$

\therefore Correct option is (c).

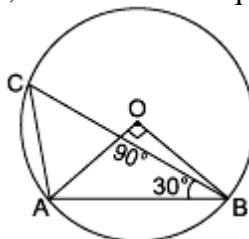
2. In the given figure, O is centre of the circle, $\angle BAO = 68^\circ$, AC is diameter of the circle, then measure of $\angle BCO$ is



(a) 22° (b) 33° (c) 44° (d) 68°

Ans: (a) 22°

3. In figure, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$, then $\angle CAO$ is equal to



- (a) 30° (b) 45° (c) 90° (d) 60°

Ans: We have $\angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 90^\circ = 45^\circ$

Using angle sum property of triangle in $\triangle CAB$, we get
 $\angle CAB = 105^\circ$

Since $OA = OB$ (Radii of the circle)

$\Rightarrow \angle OBA = \angle OAB$

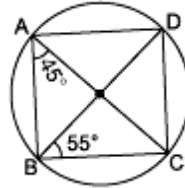
Using angle sum property of triangle in $\triangle AOB$, we get $\angle OAB = 45^\circ$

Now, $\angle CAO = \angle CAB - \angle OAB$

$= 105^\circ - 45^\circ = 60^\circ$

\therefore Correct option is (d).

4. In the given figure, $\angle DBC = 55^\circ$, $\angle BAC = 45^\circ$ then $\angle BCD$ is



- (a) 45° (b) 55° (c) 100° (d) 80°

Ans: We have $\angle BAC = \angle BDC$

(\because Angles in the same segment of a circle are equal)

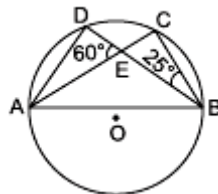
$\Rightarrow \angle BDC = 45^\circ$

Using angle sum property of triangle in $\triangle BDC$, we get

$\angle DBC + \angle BDC + \angle BCD = 180^\circ \Rightarrow 55^\circ + 45^\circ + \angle BCD = 180^\circ \Rightarrow \angle BCD = 80^\circ$

\therefore Correct option is (d).

5. In the given figure, O is the centre of the circle, $\angle CBE = 25^\circ$ and $\angle DEA = 60^\circ$. The measure of $\angle ADB$ is



- (a) 90° (b) 85° (c) 95° (d) 120°

Ans:

We have $\angle DEA = \angle CEB = 60^\circ$ (Vertically opposite angles)

Using angle sum property of triangle in $\triangle CEB$, we have

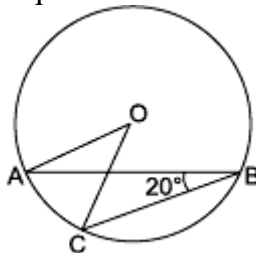
$\angle CEB + \angle CBE + \angle ECB = 180^\circ \Rightarrow 60^\circ + 25^\circ + \angle ECB = 180^\circ \Rightarrow \angle ECB = 95^\circ$

Now, $\angle ADB = \angle ACB$ (\because Angles in the same segment of a circle are equal)

$\Rightarrow \angle ADB = 95^\circ$

\therefore Correct option is (c).

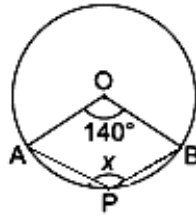
6. In figure, if $\angle ABC = 20^\circ$, then $\angle AOC$ is equal to:



- (a) 20° (b) 40° (c) 60° (d) 10°

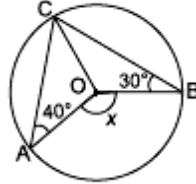
Ans: (b) 40°

7. In the given figure, value of x is



- (a) 140° (b) 70° (c) 110° (d) 280°
 Ans: (c) 110°

8. In the given figure, O is the centre of the circle. The value of x is

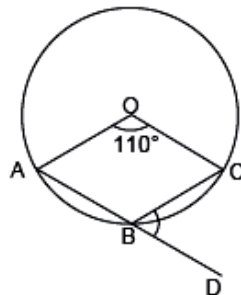


- (a) 140° (b) 70° (c) 290° (d) 210°
 Ans: (a) 140°

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

9. **Assertion (A):** If O is the centre of the circle as shown in figure, then $\angle CBD = 55^\circ$.

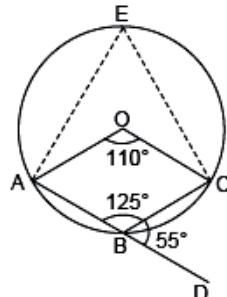


Reason (R): Exterior angle of cyclic quadrilateral is equal to interior opposite angle

Ans: (a) Both A and R are true and R is the correct explanation of A.

O is the centre of the circle and $\angle AOC = 110^\circ$.

Take point E on major arc, join AE and EC.

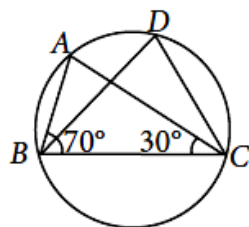


$\angle AEC = \frac{1}{2} \angle AOC = 55^\circ$ [\because Arc ABC subtends $\angle AEC$ in the alternate segment and $\angle AOC$ at the centre]

But $\angle AEC = \angle CBD$ [Exterior angle of cyclic quadrilateral is equal to interior opposite angle]

$\therefore \angle CBD = 55^\circ$

10. **Assertion (A):** In the given figure, $\angle ABC = 70^\circ$ and $\angle ACB = 30^\circ$. Then, $\angle BDC = 80^\circ$.



Reason (R): Angles in the same segment of a circle are equal.

Ans: (a) Both A and R are true and R is the correct explanation of A.

Consider the $\triangle ABC$, the sum of all angles will be 180° .

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\Rightarrow 70^\circ + \angle BAC + 30^\circ = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 100^\circ = 80^\circ$$

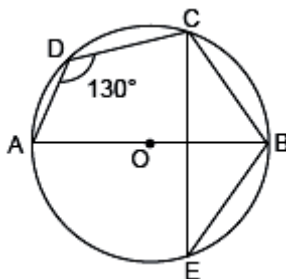
We know that angles in the same segment of a circle are equal.

So, $\angle BDC = \angle BAC = 80^\circ$

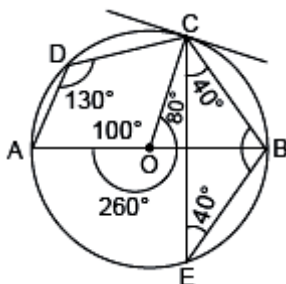
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, $\angle ADC = 130^\circ$ and chord $BC =$ chord BE . Find $\angle CBE$.



Ans: Join OC



$$\text{Reflex } \angle AOC = 2 \times \angle ADC = 2 \times 130^\circ = 260^\circ$$

$$\therefore \angle AOC = 100^\circ$$

$$\text{Now, } \angle COB = 180^\circ - 100^\circ = 80^\circ$$

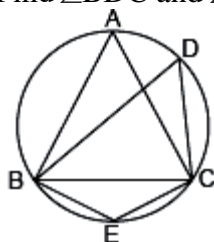
$$\angle CEB = \frac{1}{2} \angle COB$$

$$\Rightarrow \angle CEB = \frac{1}{2} \times 80^\circ = 40^\circ$$

$$\text{Also, } \angle CEB = \angle ECB = 40^\circ \quad (\because BC = BE \text{ given})$$

$$\therefore \angle CBE = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$$

12. In the given figure, $\triangle ABC$ is equilateral. Find $\angle BDC$ and $\angle BEC$.



Ans: $\angle BAC = 60^\circ$

$\therefore \angle BAC = \angle BDC$

$\Rightarrow \angle BDC = 60^\circ$

Now, $\square DBEC$ is a cyclic quadrilateral

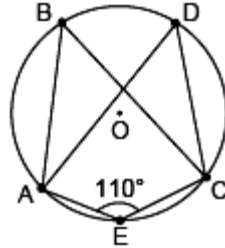
$\therefore \angle BDC + \angle BEC = 180^\circ$ [\because Opposite angles of a cyclic quadrilateral are supplementary]

$60^\circ + \angle BEC = 180^\circ \Rightarrow \angle BEC = 180^\circ - 60^\circ = 120^\circ$

[$\because \triangle ABC$ is an equilateral triangle]

[\because Angles in the same segment of a circle are equal]

13. In the given figure, ABCE is a cyclic quadrilateral and O is the centre of circle. If $\angle AEC = 110^\circ$, then find (a) $\angle ABC$ (b) $\angle ADC$



Ans: In cyclic quadrilateral ABCE,

$\angle ABC + \angle AEC = 180^\circ$ (Opp. \angle s of a cyclic quadrilateral)

$\Rightarrow \angle ABC + 110^\circ = 180^\circ$

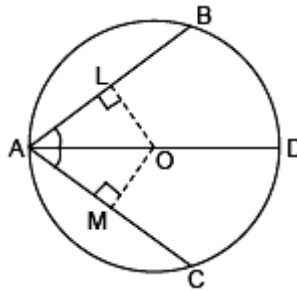
$\therefore \angle ABC = 70^\circ$

$\angle ABC = \angle ADC$ (Angles in the same segment)

$\therefore \angle ADC = 70^\circ$

14. If two chords of a circle are equally inclined to the diameter passing through their point of intersection, prove that the chords are equal.

Ans: Two chords AB and AC of a circle are equally inclined to diameter AOD, i.e. $\angle DAB = \angle DAC$



Draw $OL \perp AB$ and $OM \perp AC$

In $\triangle OLA$ and $\triangle OMA$

$\angle OLA = \angle OMA$ (each 90°)

$AO = AO$ (common)

$\angle OAL = \angle OAM$ (given)

$\triangle OLA \cong \triangle OMA$ (AAS rule)

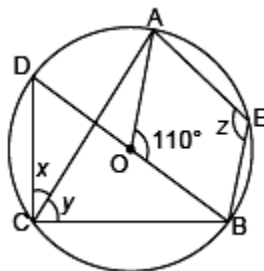
$OL = OM$ (CPCT)

$AB = AC$ (chords equidistant from the centre are equal)

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In the given figure, O is the centre of the circle and $\angle AOB = 110^\circ$, find the value of x, y and z.



Ans: Given: A circle with centre O in which $\angle AOB = 110^\circ$

Now, $\angle AOB = 2\angle ACB = 2y$

(Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$$\Rightarrow y = \frac{1}{2} \angle AOB = \frac{1}{2} \times 110^\circ = 55^\circ$$

From the given figure, ACBE is a cyclic quadrilateral

$\therefore \angle ACB + \angle AEB = 180^\circ$ (Sum of opposite angles of a cyclic quadrilateral is 180°)

$$\Rightarrow y + z = 180^\circ$$

$$\Rightarrow z = 180^\circ - 55^\circ = 125^\circ$$

Now, $\angle DOA + \angle AOB = 180^\circ$ (Linear pair)

$$\Rightarrow \angle DOA = 180^\circ - \angle AOB = 180^\circ - 110^\circ = 70^\circ$$

Also, $\angle DOA = 2\angle DCA$

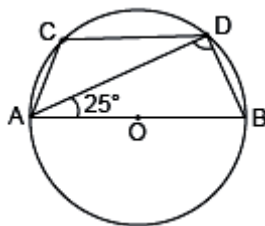
(Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$$\Rightarrow \angle DCA = \frac{1}{2} \angle DOA$$

$$\Rightarrow x = \frac{1}{2} \times 70^\circ = 35^\circ$$

$$\therefore x = 35^\circ, y = 55^\circ \text{ and } z = 125^\circ$$

16. In the given figure, AB is diameter of the circle with centre O and $CD \parallel AB$. If $\angle DAB = 25^\circ$, then find the measure of $\angle CAD$.



Ans: AB is the diameter of the circle with centre O and $CD \parallel AB$. Also, $\angle DAB = 25^\circ$

Now, $\angle ADB = 90^\circ$

[Angle in a semicircle]

$$\angle BAD = \angle ADC = 25^\circ$$

[Alternate interior angles]

$$\therefore \angle BDC = 90^\circ + 25^\circ = 115^\circ$$

Now, $\angle BDC + \angle BAC = 180^\circ$

[opp. \angle s of cyclic quadrilateral]

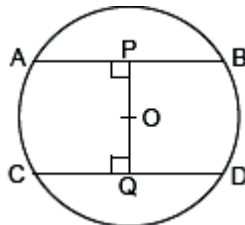
$$\Rightarrow 115^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 115^\circ = 65^\circ$$

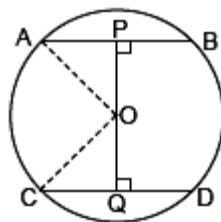
$$\text{Now, } \angle BAC = \angle BAD + \angle CAD \Rightarrow 65^\circ = 25^\circ + \angle CAD$$

$$\therefore \angle CAD = 65^\circ - 25^\circ = 40^\circ$$

17. In the given figure, O is the centre of the circle with radius 5 cm. $OP \perp AB$, $OQ \perp CD$, $AB \parallel CD$, $AB = 6$ cm and $CD = 8$ cm. Determine PQ.



Ans: Join OA and OC.



Since $OP \perp AB$, therefore, P will be the midpoint of AB. Similarly, Q will be the mid-point of CD.
In right-angled $\triangle OAP$, $OP^2 + AP^2 = OA^2$

$$\Rightarrow OP^2 = OA^2 - AP^2 = 25 - 9 = 16 \quad \left[\begin{array}{l} OA = \text{radius} = 5\text{cm} \\ AP = \frac{1}{2}(AB) = \frac{1}{2} \times 6 = 3\text{cm} \end{array} \right]$$

$$\therefore OP = 4 \text{ cm}$$

Similarly, in $\triangle OQC$,

$$OC = \text{radius} = 5 \text{ cm}$$

$$CQ = \frac{1}{2}(CD) = 4 \text{ cm}$$

$$OQ^2 = OC^2 - CQ^2 = 25 - 16 = 9$$

$$\therefore OQ = 3 \text{ cm}$$

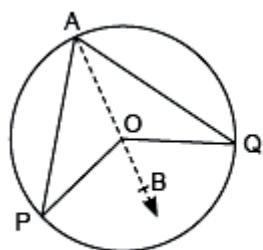
$$\therefore PQ = OP + OQ = 4 + 3 = 7 \text{ cm}$$

SECTION – D

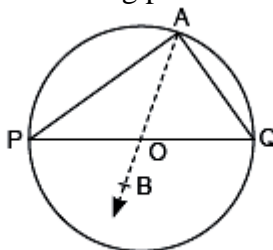
Questions 18 carry 5 marks.

- 18.** Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

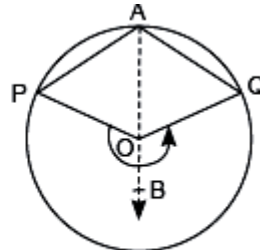
Given : Given an arc PQ of a circle subtending angles POQ at the centre O and $\angle PAQ$ at a point A on the remaining part of the circle.



case (i)



case (ii)



case (iii)

To Prove: $\angle POQ = 2\angle PAQ$

Construction: Join AO and extend it to B.

Proof: Consider three cases

case (i): When arc PQ is a minor arc.

case (ii): When arc PQ is a semicircle.

case (iii): When arc PQ is a major arc.

In all the three cases

Taking $\triangle AOQ$

$$\angle BOQ = \angle OAQ + \angle OQA \quad (\text{Exterior angle of } \triangle \text{ is equal to the sum of interior opposite angles})$$

$$\text{Also } OA = OQ \quad (\text{radii of circle})$$

$$\Rightarrow \angle OAQ = \angle OQA \quad (\text{Angles opposite to equal sides})$$

$$\Rightarrow \angle BOQ = \angle OAQ + \angle OAQ$$

$$\Rightarrow \angle BOQ = 2\angle OAQ \quad \dots(i)$$

$$\text{Similarly } \angle BOP = 2\angle OAP \quad \dots(ii)$$

Adding (i) and (ii) we have

$$\angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP = 2(\angle OAQ + \angle OAP)$$

$$\Rightarrow \angle POQ = 2\angle PAQ$$

Specially for case (iii) we can write reflex $\angle POQ = 2\angle PAQ$

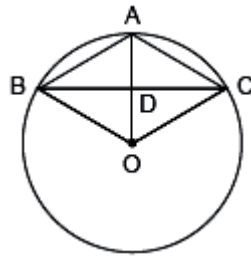
OR

In a circle of radius 18 cm, AB and AC are two chords such that $AB = AC = 12$ cm. Find the length of chord BC.

Ans: Given: A circle with centre O and two chords $AB = AC = 12$ cm

Radii = $OA = OB = OC = 18$ cm

Join OA, OB and OC
 OA and BC intersect at D



In $\triangle OAB$ and $\triangle OAC$

$OB = OC$ (Radii of a circle)

$OA = OA$ (Common)

$AB = AC$ (Given)

$\therefore \triangle OAB \cong \triangle OAC$ (SSS congruence rule)

$\Rightarrow \angle OAB = \angle OAC$... (i) (CPCT)

In $\triangle DAB$ and $\triangle DAC$

$AB = AC$ (given)

$DA = DA$ (common)

$\angle DAB = \angle DAC$ [from (i)]

$\therefore \triangle DAB \cong \triangle DAC$ (SAS congruence rule)

$\Rightarrow \angle ADB = \angle ADC$... (ii) (CPCT)

Now $\angle ADC + \angle ADB = 180^\circ$ (Linear pair)

$\Rightarrow \angle ADB = \angle ADC = 90^\circ$

In right $\triangle ADC$ and $\triangle ODC$

$AC^2 = AD^2 + DC^2$ and $OC^2 = OD^2 + DC^2$

$\Rightarrow DC^2 = AC^2 - AD^2$ and $DC^2 = OC^2 - OD^2$

$\Rightarrow AC^2 - AD^2 = OC^2 - OD^2$

$\Rightarrow 12^2 - (18 - OD)^2 = 18^2 - OD^2$

$\Rightarrow 144 - 324 - OD^2 + 36OD = 324 - OD^2$

$\Rightarrow 36OD = 504 \Rightarrow OD = 14 \text{ cm}$

Now, $DC^2 = OC^2 - OD^2 = 18^2 - (14)^2 = 324 - 196 = 128$

$\Rightarrow DC = 11.31$

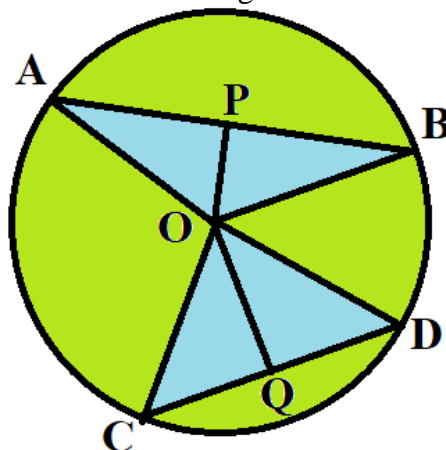
Also $DC = DB$ (Perpendicular from the centre of the circle to the chord bisects the chord)

$\therefore BC = 2DC = 2 \times 11.31 = 22.62 \text{ cm}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Aditya seen one circular park in which two triangular ponds are there whose common vertex is the centre of the park. After coming back to home, he tried to draw the circular park on the paper. He draws a circle of radius 10 cm with the help of a compass and scale. He also draws two chords, AB and CD in such a way that the perpendicular distance from the center to AB and CD are 6 cm and 8 cm respectively. Now, he has some doubts that are given below.



(i) Show that the perpendicular drawn from the Centre of a circle to a chord bisects the chord using any one triangle. (2)

(ii) What is the length of CD? (2)

OR

(ii) What is the length of AB? (2)

Ans: (i) In $\triangle AOP$ and $\triangle BOP$

$\angle APO = \angle BPO$ ($OP \perp AB$)

$OP = OP$ (Common)

$AO = OB$ (radius of circle)

$\triangle AOP \cong \triangle BOP$

$\therefore AP = BP$ (CPCT)

(ii) In right $\triangle COQ$

$CO^2 = OQ^2 + CQ^2$

$\Rightarrow 10^2 = 8^2 + CQ^2$

$\Rightarrow CQ^2 = 100 - 64 = 36$

$\Rightarrow CQ = 6$

$\Rightarrow CD = 2CQ = 12 \text{ cm}$

OR

(ii) In right $\triangle AOB$

$AO^2 = OP^2 + AP^2$

$\Rightarrow 10^2 = 6^2 + AP^2$

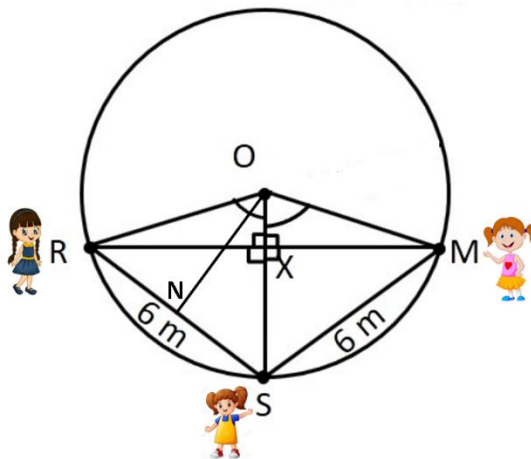
$\Rightarrow AP^2 = 100 - 36 = 64$

$\Rightarrow AP = 8$

$\Rightarrow AB = 2AP$

$\Rightarrow AB = 16 \text{ cm}$

20. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. The distance between Reshma and Salma and between Salma and Mandip is 6m each. In the given below figure Reshma's position is denoted by R, Salma's position is denoted by S and Mandip's position is denoted by M.



(i) Find the area of triangle ORS. [2]

(ii) What is the distance between Reshma and Mandip? [2]

OR

(ii) If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that $CA = 2OD$. [2]

Ans: (i) $NR = NS = \frac{1}{2} = 3 \text{ m}$

$OR = OS = OM = 5 \text{ m}$. (Radii of the circle)

In $\triangle ORN$, by Pythagoras theorem,

$ON^2 + NR^2 = OR^2$

$$\Rightarrow ON^2 + (3^2) = (5)^2$$

$$\Rightarrow ON^2 = (25 - 9) = 16$$

$$\Rightarrow ON = 4\text{m}$$

ORSM will be a kite (OR = OM and RS = SM). We know that diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangle is bisected by another diagonal

$\therefore \angle RXS$ will be of 90° and $RX = XM$

$$\text{Area of } \triangle ORS = \frac{1}{2} \times ON \times RS = \frac{1}{2} \times 4 \times 6 = 12 \text{ m}^2$$

$$(ii) \text{ Area of } \triangle ORS = \frac{1}{2} \times ON \times RS$$

$$\Rightarrow \frac{1}{2} \times RX \times OS = \frac{1}{2} \times 4 \times 6$$

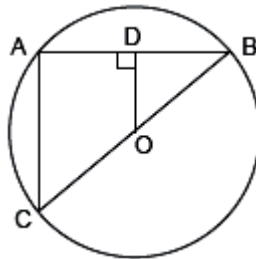
$$\Rightarrow RX \times 5 = 24$$

$$\Rightarrow RX = 4.8$$

$\Rightarrow RM = 2RX = 2(4.8) = 9.6$ Therefore, the distance between Reshma and Mandip is 9.6 m.

OR

(ii)



Since $OD \perp AB$

\therefore D is the mid-point of AB (perpendicular drawn from the centre to a chord bisects the chord)

O is centre \Rightarrow O is the mid-point of BC.

In $\triangle ABC$, O and D are the mid-points of BC and AB, respectively.

$\therefore OD \parallel AC$ and $OD = \frac{1}{2} AC$ (mid-point theorem)

$\therefore CA = 2OD$