21

MENSURATION – II
(Area of Circle)

21.1 INTRODUCTION

In this chapter, we shall discuss problems on circumference and area of a circle. We shall also discuss problems on areas of segment and a sector of a circle.

21.2 CIRCLE

A circle is a geometrical figure consisting of all those points in a plane which are at a given distance from a fixed point in the same plane.

The fixed point is called the centre of the circle and the constant distance is known as its radius.

In Fig. 1, O is the centre and r is the radius of the circle.

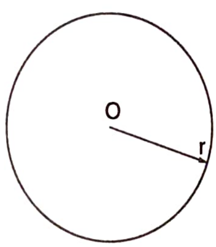

Fig. 1

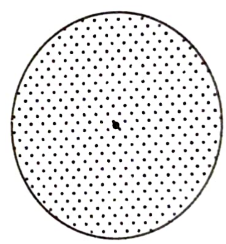

Fig. 2

A circle with centre O and radius r is generally denoted by $C(O, r)$.

CIRCULAR REGION *The part of the circle that consists of the circle and its interior is called the circular region.*

A circular region is also called a circular discuss shown in Fig. 2.

CHORD OF A CIRCLE *A line segment joining any two points on a circle is called a chord of the circle.*

It should be noted that a chord is not a part of the circle.

In Fig. 3, PQ is a chord of the circle.

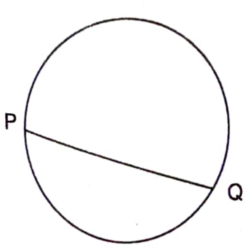

Fig. 3

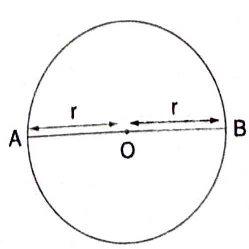

Fig. 4

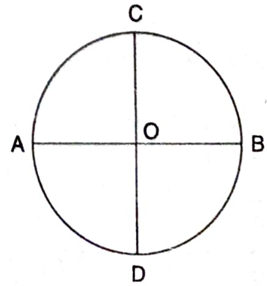

Fig. 5

DIAMETER *A chord passing through the centre of a circle is known as its diameter.*

Note that a circle has many diameters and a diameter of a given circle is one of the largest chords of the circle. Also, all diameters are of the same length.

In this book, the word 'diameter' will be used for a chord passing through the centre, and also for its length.

Clearly, if d is diameter of a circle of radius $= r$, then $d = 2r$.

A diameter of a circle divides the circumference of a circle into two equal parts each of which is called a *semi-circle*.

Two perpendicular diameters of a circle divide its circumference into four equal parts each of which is known as a *quadrant*.

CONCENTRIC CIRCLES *Circles having the same centre but with different radii are said to be concentric circles.*

Fig. 6, shows two concentric circles.

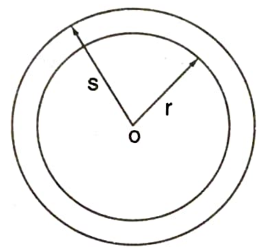

Fig. 6

CONGRUENT CIRCLES *Two circles are said to be congruent if and only if either of them can be super imposed on the other so as to cover it exactly.*

It follows from the above definition that two circles are congruent if and only if their radii are equal.

21.3 CIRCUMFERENCE OF A CIRCLE

The perimeter of a circle is called its circumference.

The length of the thread that winds around the circle exactly once, gives the circumference of the circle.

RELATION BETWEEN DIAMETER AND CIRCUMFERENCE The ratio of the circumference of a circle and its diameter is always constant.

The above result can be verified with the help of the following experiment :

Experiment Draw three circles of different radii. Use a tape or a piece of thread to measure the circumference of each circle. Also, measure the diameter of each one of them.

Arrange your observations as under :

Circle	Circumference C	Diameter $d = 2r$	$\frac{\text{Circumference}}{\text{Diameter}}\left(\frac{C}{2r}\right)$
(i)			
(ii)			
(iii)			

For each circle, compute the ratio of the circumference and diameter and enter the same against each circle in the last column of the table. You will find that in each case

$$\frac{\text{Circumference}}{\text{Diameter}} = 3.14 \text{ (approximately)}$$

This ratio is denoted by π(Pi).

Thus, we have

$$\pi = 3.14 \text{ (approximately)} = \frac{22}{7} \text{ (approximately)}$$

Now, $\frac{\text{Circumference}}{\text{Diameter}} = \pi$

$\Rightarrow$ $\frac{C}{2r} = \pi$

$\Rightarrow$ $C = 2\pi r$

Thus, circumference C of a circle of radius r is given by

$$C = 2\pi r$$

If d denotes the diameter of the circle. Then, $d = 2r$

$\therefore$ $C = \pi d$

Remark *The number π is not a rational number, but its value upto two decimal places coincides with $\frac{22}{7}$. So, we take the value of π as $\frac{22}{7}$. In the remaining part of this chapter, unless stated otherwise, the value of π will be taken as $\frac{22}{7}$.*

21.4 THE NUMBER π

In the previous section, we have observed that the circumference of a circle is in constant ratio to its diameter. This constant is denoted by the Greek letter π which is read as "pie" in the word apple-pie. It has been proved that π is not a rational number. Therefore, it cannot be written as a terminating (finite) or a non-terminating repeating decimal. The values of π correct to 2, 3, 4, 5 and 6, places of decimal are respectively 3.14, 3.142, 3.1416, 3.14159 and 3.141593. Throughout this chapter the value of π will be taken as $\frac{22}{7}$ unless specified otherwise.

ILLUSTRATIVE EXAMPLES

Example 1 Find the circumference of a circle of radius 14 cm.

Solution The circumference C of a circle of radius r is given by

$C = 2\pi r$.

Here, $r = 14$ cm

$$\therefore \quad C = 2 \times \frac{22}{7} \times 14 \text{ cm} = 88 \text{ cm}$$

Example 2 Find the circumference of a circle whose diameter is 42 cm.

Solution We have,

Diameter = 42 cm

$$\therefore \quad \text{Radius} = \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

Let C be the circumfernce of the circle. Then,

$$C = 2\pi r$$

$$\Rightarrow \quad C = 2 \times \frac{22}{7} \times 21 \text{ cm} = 132 \text{ cm}$$

Example 3 Find the diameter of a circle whose circumference is 15.7 cm.

Solution We know that the circumference C of a circle of a diameter d is given by

$$C = \pi d$$

$$\Rightarrow \quad d = \frac{C}{\pi}$$

Here, $C = 15.7$ cm

$$\therefore \quad d = \frac{15.7}{\pi} \text{ cm} = \frac{15.7}{22} \times 7 \text{ cm} = \frac{109.9}{22} \text{ cm} = 5 \text{ cm (approx.)}$$

Example 4 The ratio of the radii of two circles is 2 : 5. What is the ratio of their circumferences ?

Solution We have, ratio of radii = 2 : 5.

So, let the radii of two circles be $2r$ and $5r$ respectively.

Let C_1 and C_2 be the circumferences of two circles of radii $2r$ and $5r$ respectively. Then,

$$C_1 = 2\pi \times 2r = 4\pi r \quad \text{and,} \quad C_2 = 2\pi \times 5r = 10\pi r$$

$$\therefore \quad \frac{C_1}{C_2} = \frac{4\pi r}{10\pi r} = \frac{2}{5} \Rightarrow C_1 : C_2 = 2 : 5$$

Example 5 A piece of wire in the form of a rectangle 8.9 cm long and 5.4 cm broad is reshaped and bent into the form of a circle. Find the radius of the circle.

Solution We have,

Length of the wire = Perimeter of the rectangle

$$= 2(l + b) = 2 \times (8.9 + 5.4) \text{ cm} = 28.6 \text{ cm}$$

Let the wire be bent into the form of a circle of radius r cm. Then,

Circumference = 28.6 cm

$$\Rightarrow \quad 2\pi r = 28.6$$

$$\Rightarrow \quad 2\times\frac{22}{7}\times r = 28.6$$

$$\Rightarrow \quad r = \frac{28.6\times 7}{2\times 22}\text{ cm} = \frac{286\times 7}{2\times 22\times 10}\text{ cm} = 4.55\text{ cm}.$$

Example 6 The diameter of a wheel of a cycle is 70 cm. It moves slowly along a road. How far will it go in 24 complete revolutions?

Solution It may be noted that in one revolution, the cycle covers a distance equal to the circumference of the wheel?

Now, diameter of the wheel = 70 cm

$\therefore$ Circumference of the wheel $= \pi d = \left(\frac{22}{7}\times 70\right)$ cm $= 220$ cm

Thus, the cycle travels 220 cm in one revolution.

$\therefore$ Distance covered by the cycle in 24 revolutions $= (220\times 24)$ cm

$= 5280$ cm $= 52.80$ m

Example 7 The diameter of the wheel of a car is 77 cm. How many revolutions will it make to travel 121 km?

Solution We have,

Diameter of the wheel of the car = 77 cm

$\therefore$ Circumference of the wheel of the car $= \pi d = \left(\frac{22}{7}\times 77\right)$ cm $= 242$ cm

Note that in one revolution of the wheel, the car travels a distance equal to the circumference of the wheel.

$\therefore$ Distance travelled by the car in one revolution of the wheel = 242 cm

Total distance travelled by the car $= 121$ km $= 121000$ m $= 12100000$ cm

$\therefore$ Number of revolutions $= \frac{12100000}{242} = 50000$

Example 8 There is a circular pond and foot-path runs along its boundary. A man walks around it, exactly once, keeping close to the edge. If his step is 66 cm long and he takes exactly 400 steps to go around the pond, what is the diameter of the pond?

Solution We have, length of one step = 66 cm

$\therefore$ Distance covered in 400 steps $= (400\times 66)$ cm $= \frac{400\times 66}{100}$ m $= 264$ m

Let d metre be the diameter of the pond. Then,

Circumference = Distance covered in 400 steps

$$\Rightarrow \quad \pi d = 264$$

$$\Rightarrow \quad \frac{22}{7}\times d = 264$$

$$\Rightarrow \quad d = \frac{264 \times 7}{22} \text{ m} = 84 \text{ m}$$

Hence, the diameter of the pond is 84 metres.

Example 9 The circumference of a circle exceeds the diameter by 30 cm. Find the radius of the circle.

Solution Let the radius of the circle be r cm. Then,

Circumference of the circle $= 2\pi r$ cm

Diameter of the circle $= 2r$ cm

It is given that the circumference of the circle exceeds its diameter by 30 cm.

$$\therefore \quad 2\pi r = 2r + 30$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 2r + 30$$

$$\Rightarrow \quad \frac{44r}{7} = 2r + 30$$

$$\Rightarrow \quad \frac{44r}{7} - 2r = 30$$

$$\Rightarrow \quad \frac{44r - 14r}{7} = 30 \Rightarrow \frac{30r}{7} = 30 \Rightarrow r = \frac{30 \times 7}{30} = 7 \text{ cm}$$

Example 10 A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.

Solution Let the outer and inner radii of the ring be R metres and r metres respec-tively. Then,

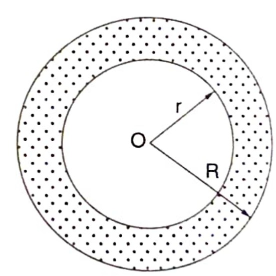

Fig. 7

$$2\pi R = 396 \quad \text{and} \quad 2\pi r = 352$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times R = 396 \quad \text{and} \quad 2 \times \frac{22}{7} \times r = 352$$

$$\Rightarrow \quad R = 396 \times \frac{7}{22} \times \frac{1}{2} \quad \text{and} \quad r = 352 \times \frac{7}{22} \times \frac{1}{2}$$

$$\Rightarrow \quad R = 63 \text{ m} \quad \text{and} \quad r = 56 \text{ m}.$$

Hence, width of the track $= (R - r)$ metres $= (63 - 56)$ metres $= 7$ metres.

Example 11 The inner circumference of a circular track is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of Rs 2 per metre. (Use $\pi = 22/7$)

Solution Let the inner and outer radii of the circular track be r metres and R metres respectively. Then,

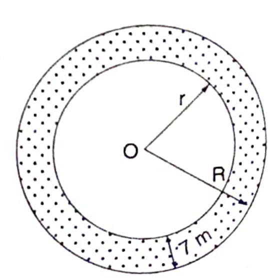

Fig. 8

Inner circumference = 220 metres

$$\Rightarrow \quad 2\pi r = 220 \Rightarrow 2 \times \frac{22}{7} \times r = 220 \Rightarrow r = 35 \text{ m}$$

Since the track is 7 metre wide everywhere.

$$\therefore \quad R = \text{Outer radius} = r + 7 = (35 + 7) \text{ m} = 42 \text{ m}$$

$\therefore$ Outer circumference $= 2\pi R = 2\times\frac{22}{7}\times 42\text{m} = 264\text{m}$

Rate of fencing = Rs 2 per metre

$\therefore$ Total cost of fencing = (Circumference $\times$ Rate) = Rs (264×2) Rs528

EXERCISE 21.1

1. Find the circumference of a circle whose radius is
   (i) 14 cm (ii) 10 m (iii) 4 km
2. Find the circumference of a circle whose diameter is
   (i) 7 cm (ii) 4.2 cm (iii) 11.2 km
3. Find the radius of a circle whose circumference is
   (i) 52.8 cm (ii) 42 cm (iii) 6.6 km
4. Find the diameter of a circle whose circumference is
   (i) 12.56 cm (ii) 88 m (iii) 11.0 km
5. The ratio of the radii of two circles is 3 : 2. What is the ratio of their circumferences?
6. A wire in the form of a rectangle 18.7 cm long and 14.3 cm wide is reshaped and bent into the form of a circle. Find the radius of the circle so formed.
7. A piece of wire is bent in the shape of an equilateral triangle of each side 6.6 cm. It is re-bent to form a circular ring. What is the diameter of the ring?
8. The diameter of a wheel of a car is 63 cm. Find the distance travelled by the car during the period, the wheel makes 1000 revolutions.
9. The diameter of a wheel of a car is 98 cm. How many revolutions will it make to travel 6160 metres.
10. The moon is about 384400 km from the earth and its path around the earth is nearly circular. Find the circumference of the path described by the moon in lunar month.
11. How long will John take to make a round of a circular field of radius 21 m cycling at the speed of 8 km/hr ?
12. The hour and minute hands of a clock are 4 cm and 6 cm long respectively. Find the sum of the distances travelled by their tips in 2 days.
13. A rhombus has the same perimeter as the circumference of a circle. If the side of the rhombus is 2.2 m, find the radius of the circle.
14. A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.
15. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.
16. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour with which the boy is cycling.
17. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour ?

18. A water sprinkler in a lawn sprays water as far as 7 m in all directions. Find the length of the outer edge of wet grass.

19. A well of diameter 150 cm has a stone parapet around it. If the length of the outer edge of the parapet is 660 cm, then find the width of the parapet.

20. An ox in a kolhu (an oil processing apparatus) is tethered to a rope 3 m long. How much distance does it cover in 14 rounds?

ANSWER

1.	(i) 88 cm		(ii) 62.86 m		(iii) 25.142 km		
2.	(i) 22 cm		(ii) 13.2 cm		(iii) 35.2 km		
3.	(i) 8.4 cm		(ii) 6.68 cm		(iii) 1.05 km		
4.	(i) 3.99 cm		(ii) 28 m		(iii) 3.5 km		
5.	3:2	6.	10.5 cm	7.	6.3 cm	8.	1980 m
9.	2000	10.	2416228.57 km	11.	59.4 seconds	12.	1910.8 cm
13.	1.4 m	14.	44 cm	15.	70 cm	16.	15.84 km/hr
17.	250	18.	44 m	19.	30 cm	20.	264 m

HINTS TO SELECTED PROBLEMS

10. Take radius = 384000 km.

11. Required time $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{2 \times \frac{22}{7} \times 21}{8}$ hr

12. The hour hand travels $\left(2 \times \dfrac{22}{7} \times 4\right)$ cm distance in 12 hours and the minute hand travels $\left(2 \times \dfrac{22}{7} \times 6\right)$ cm in one hour.

18. Wet grass forms a circular region of radius 7 m.

$\therefore$ Length of the outer edge of wet grass $= 2 \times \dfrac{22}{7} \times 7 \text{ m} = 44 \text{ m}$

19. Let the width of the parapet be x cm. Clearly, outer edge of the parapet forms of a circle of radius $(x + 75)$ cm.

$\therefore$ Length of the outer edge of parapet = 660 cm.

$$\Rightarrow \quad 2 \times \frac{22}{7} \times (x + 75) = 660$$

$$\Rightarrow \quad x + 75 = \frac{660 \times 7}{2 \times 22} \Rightarrow x + 75 = 105 \Rightarrow x = 30 \text{ cm}$$

21.5 AREA OF A CIRCLE

In this section, we shall first obtain the formula for the area of a circle and then the same will be used to solve some simple problems.

To obtain the formula for the area of a circle, let us consider the following experiment.

Experiment On a thick sheet of a paper, draw a circle with any radius, say r cm. With a pair of scissors, cut the sheet of paper along the circle. Thus, we get a circular disc of radius r cm.

ld the circular disc so that the two parts cover each other exactly, and to get a crease. old again this folded disc, so that the two parts cover each other exactly and press to get nother crease. Repeat the process of folding and pressing to obtain creases dividing the rcular disc into 16 equal parts. Number the parts from 1 to 16, in serial order as shown in ig. 9. Carefully cut out the sixteen parts and arrange them on a sheet of paper as shown Fig. 10.

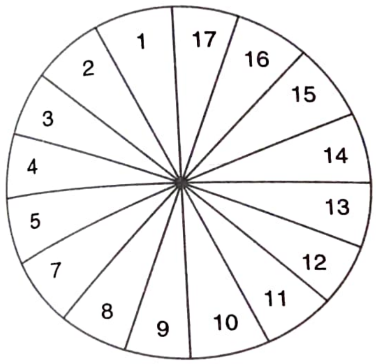

Fig. 9

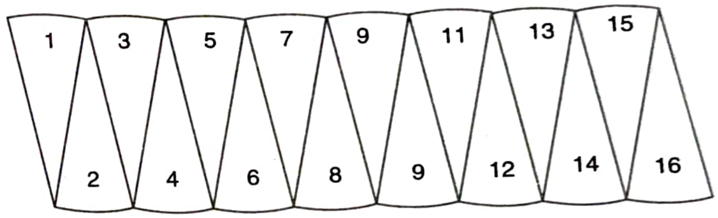

Fig. 10

This looks like a rectangular region. But it is not exactly so. However, if the disc is divided into a very large number of equal sectors and then these sectors are arranged as shown above, then the above figure will become very near to a rectangle.

Thus, the given circular region is equal in area to a near to a rectangle whose length and breadth are respectively half the circumference and the radius of the circle.

$\therefore$ Area of the circle = Area of the rectangle with length equal to half the circumference and breadth equal to its radius

$$= \frac{1}{2}(\text{Circumference}) \times \text{Radius}$$

$$= \left(\frac{1}{2} \times 2\pi r \times r\right) \text{ sq. cm}$$

$$= \pi r^2 \text{ sq. cm}$$

Hence, area A of a circle of radius r cm is given by

$$A = \pi r^2 \quad \text{Also,} \quad r = \sqrt{\frac{A}{\pi}}$$

Remark *Area of a semi-circle* $= \frac{1}{2}(\textit{Area of the circle}) = \frac{1}{2}\pi r^2$

Area of a quadrant of a circle $= \frac{1}{4}(\textit{Area of the circle}) = \frac{1}{4}\pi r^2$.

ILLUSTRATIVE EXAMPLES

Example 1 Find the area of a circle of radius 4.2 cm.

Solution We know that the area A of a circle of radius r is given by $A = \pi r^2$

Here, $r = 4.2$ cm

$\therefore \quad A = \frac{22}{7} \times (4.2)^2 \text{ cm}^2 \qquad \left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow \quad A = \left(\frac{22}{7} \times 4.2 \times 4.2\right) \text{cm}^2 = (22 \times 0.6 \times 4.2) \text{ cm}^2 = 55.44 \text{ cm}^2$

Example 2 Find the area of a circle of diameter 7 cm.

Solution Let r be the radius of the circle. Then, $r = \frac{7}{2}$ cm $= 3.5$ cm

$\therefore \quad$ Area of the circle $= \pi r^2$

$\Rightarrow \quad$ Area of the circle $= \frac{22}{7} \times (3.5)^2 \text{ cm}^2$

$\Rightarrow \quad$ Area of the circle $= \left(\frac{22}{7} \times 3.5 \times 3.5\right) \text{cm}^2 = (22 \times 0.5 \times 3.5) \text{ cm}^2 = 38.5 \text{ cm}^2$

Example 3 The circumference of a circle is 44 cm. Find its area.

Solution Let the radius of the circle be r cm. Then,
Circumference = 44 cm

$\Rightarrow \quad 2\pi r = 44 \qquad [\because \text{Circumference} = 2\pi r]$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm} = 7 \text{ cm}$

$\therefore \quad$ Area of the circle $= \pi r^2 = \left(\frac{22}{7} \times 7^2\right) \text{cm}^2 = \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2 = 154 \text{ cm}^2$

Example 4 The area of a circle is 616 cm^2. Find the radius of the circle.

Solution Let the radius of the circle be r cm.
We have,
Area of the circle = 616 cm^2

$\Rightarrow \quad \pi r^2 = 616 \qquad [\because \textit{Area} = \pi r^2]$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 616$

$\Rightarrow \quad r^2 = \frac{616 \times 7}{22} = 28 \times 7 = 196 = 14^2$

$\Rightarrow \quad r = 14$ cm

Hence, radius of the circle = 14 cm.

Example 5 A copper wire, when bent in the form of a square, encloses an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it. (Use $\pi = 22/7$).

Solution We have,

Area of the square = 484 cm^2

$\Rightarrow \quad (\text{Side})^2 = (22)^2 \text{ cm}^2$

$\Rightarrow \quad \text{Side} = 22$ cm

So, Perimeter of the square $= 4(\text{Side}) = (4 \times 22)$ cm $= 88$ cm.

Let r be the radius of the circle. Then,

Circumference of the circle = Perimeter of the square

$\Rightarrow \quad 2\pi r = 88$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14$ cm

$\therefore$ Area of the circle $\pi r^2 = \frac{22}{7} \times (14)^2 \text{ cm}^2 = 616 \text{ cm}^2$.

Example 6 A circular grassy plot of land, 42 m in diameter, has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at Rs 4 per square metre.

Solution Radius of the plot $= \frac{42}{2}$ m $= 21$ m

Radius of the plot including the path $= (21 + 3.5)$ m $= 24.5$ m

$\therefore$ Area of the path $= \left[\pi(24.5)^2 - \pi(21)^2\right] \text{m}^2$

$= \pi\left[(24.5)^2 - \pi(21)^2\right] \text{m}^2$

$= \pi\left[(24.5 + 21)(24.5 - 21)\right] \text{m}^2$

$= \left[\pi(45.5)(3.5)\right] \text{m}^2$

$= \left(\frac{22}{7} \times 45.5 \times 3.5\right) \text{m}^2 = 500.5 \text{ m}^2$

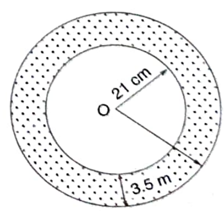

Fig. 11

Hence, cost of gravelling the path $=$ Rs $(500.5 \times 4) =$ Rs 2002

Example 7 A paper is in the form of a rectangle $ABCD$ in which AB = 20 cm and BC = 14 cm. A semicircular portion with BC as diameter is cut off. Find the area of the remaining part.

Solution Length of the rectangle $ABCD = AB = 20$ cm

Breadth of the rectangle $ABCD = BC = 14$ cm

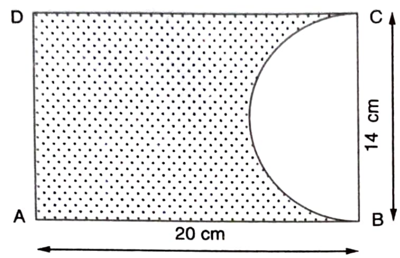

Fig. 12

$\therefore$ Area of rectangle $ABCD = (20 \times 14)\ \text{cm}^2 = 280\ \text{cm}^2$.

Diameter of the semi-circle $= BC = 14$ cm

$\therefore$ Radius of the semi-circle = 7 cm

Area of the semi-circular portion cut off from the rectangle $ABCD$

$$= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2\ \text{cm}^2 = 77\ \text{cm}^2$$

$\therefore$ Area of the remaining part

= Area of rectangle $ABCD$ – Area of semi-circle $= (280 - 77)\ \text{cm}^2 = 203\ \text{cm}^2$

Example 8 In Fig. 13, find the area of the shaded region [Use $\pi = 3.14$]

Solution Clearly, diameter of the circle = Diagonal BD of rectangle $ABCD$

$\therefore$ Diameter $= BD = \sqrt{BC^2 + CD^2} = \sqrt{6^2 + 8^2}\ \text{cm} = 10\ \text{cm}$

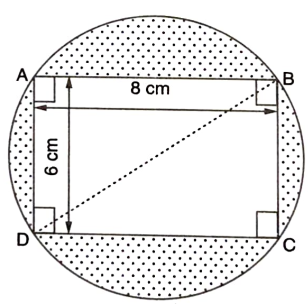

Fig. 13

Let r be the radius of the circle. Then,

r = Radius of the circle = $(10/2)$ cm = 5 cm

Area of rectangle $ABCD = AB \times BC = (8 \times 6)\ \text{cm}^2 = 48\ \text{cm}^2$

Area of the circle $= \pi r^2 = 3.14 \times (5)^2\ \text{cm}^2 = 78.50\ \text{cm}^2$

Hence, area of the shaded region

$= \text{Area of the circle} - \text{Area of rectangle } ABCD$

$= (78.50 - 48)\ \text{cm}^2 = 30.50\ \text{cm}^2$

Example 9 **The circumferences of two circles are in the ratio 2 : 3. Find the ratio of their areas.**

Solution Let r_1 and r_2 be the radii of two given circles and C_1 and C_2 be their circumferences. Then,

$$C_1 = 2\pi r_1 \text{ and } C_2 = 2\pi r_2$$

Now, $C_1 : C_2 = 2 : 3$

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{2}{3} \Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$$

Let A_1 and A_2 be the areas of two circles. Then,

$$A_1 = \pi r_1^2 \text{ and } A_2 = \pi r_2^2$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{4}{9} \qquad \left[\because \frac{r_1}{r_2} = \frac{2}{3} \therefore \frac{r_1^2}{r_2^2} = \frac{4}{9}\right]$$

$$\Rightarrow \quad A_1 : A_2 = 4 : 9$$

Hence, the areas of two given circles are in the ratio 4 : 9.

Example 10 **The areas of two circles are in the ratio 16 : 25. Find the ratio of their circumferences.**

Solution Let r_1 and r_2 be the radii of two circles and let their areas be A_1 and A_2 respectively. Then,

$$A_1 = \pi r_1^2,\ A_2 = \pi r_2^2$$

Now, $A_1 : A_2 = 16 : 25$ [Given]

$$\Rightarrow \quad \pi r_1^2 : \pi r_2^2 = 16 : 25$$

$$\Rightarrow \quad \frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{25}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{4^2}{5^2} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5} \quad \ldots \text{(i)} \quad \text{[Taking square root of both sides]}$$

Let C_1 and C_2 be the circumferences of two circles. Then,

$$C_1 = 2\pi r_1 \text{ and } C_2 = 2\pi r_2.$$

$$\therefore \quad \frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{4}{5} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad C_1 : C_2 = 4 : 5$$

Hence, the circumferences of the two circles are in the ratio 4 : 5.

Example 11 A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. 14. Find the area of the remaining part of the park. (Take $\pi = 22/7$)

Solution We have,

Area of each quadrant of radius 14 m $= \frac{1}{4}(\pi r^2)$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 \qquad [\because r = 14]$

$= 154 \text{ m}^2$

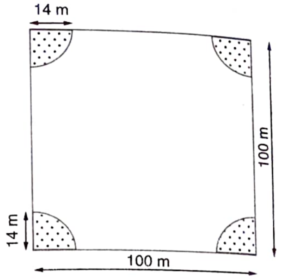

Fig. 14

$\therefore$ Area of 4 quadrants $= (4 \times 154) \text{ m}^2 = 616 \text{ m}^2$

Area of square park having side 100 m long

$= (100 \times 100) \text{ m}^2 = 10000 \text{ m}^2$

Hence, Area of the remaining part of the park $= 10000 - 616 = 9384 \text{ m}^2$.

Example 12 Four equal circles are described about the four corners of a square so that each touches two of the others as shown in Fig. 15. Find the area of the shaded region, each side of the square measuring 14 cm.

Solution Let *ABCD* be the given square each side of which is 14 cm long. Clearly, the radius of each circle is 7 cm.

We have,

Area of the square of side 14 cm long

$= (14 \times 14) \text{ cm}^2 = 196 \text{ cm}^2$

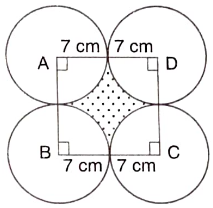

Fig. 15

Area of each quadrant of a circle of radius 7 cm

$= \frac{1}{4}(\pi r^2) = \left(\frac{1}{4} \times \frac{22}{7} \times (7)^2\right) \text{cm}^2 = 38.5 \text{ cm}^2$

$\therefore$ Area of 4 quadrants $= 4 \times 38.5 \text{ cm}^2 = 154 \text{ cm}^2$

Hence,

Area of the shaded region

$=$ Area of the square *ABCD* – Area of 4 quadrants $= (196 - 154) \text{ cm}^2 = 42 \text{ cm}^2$

Example 13 A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze ?

Solution Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius $r = 21$ m

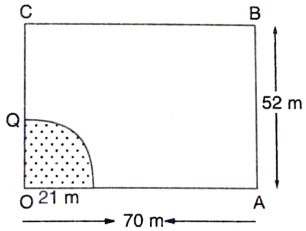

Fig. 16

Hence,

Required area $= \frac{1}{4}\pi r^2 = \left\{\frac{1}{4}\times\frac{22}{7}\times(21)^2\right\} cm^2 = \frac{693}{2} cm^2 = 346.5 cm^2$

Example 14 $PQRS$ is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. 17. Find the area of the shaded region.

Solution We have, $PS = 12$ cm.

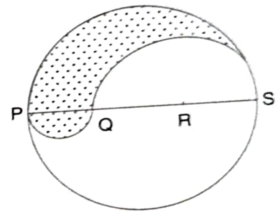

Fig. 17

$\because \quad PQ = QR = RS = \frac{1}{3} PS$

$\therefore \quad PQ = 4$ cm, $QS = 2\,PQ = 8$ cm

$\therefore$ Required Area

= Area of semi-circle with PS as diameter

+ Area of seme-circle with PQ as diameter

– Area of semi-circle with QS as diameter.

$$= \frac{1}{2}\times\frac{22}{7}\times(6)^2 + \frac{1}{2}\times\frac{22}{7}\times2^2 - \frac{1}{2}\times\frac{22}{7}\times(4)^2$$

$$= \frac{1}{2}\times\frac{22}{7}\left(6^2 + 2^2 - 4^2\right)$$

$$= \frac{1}{2}\times\frac{22}{7}\times 24 = \frac{264}{7} cm^2 = 37.71 cm^2$$

EXERCISE 21.2

1. Find the area of a circle whose radius is

   (i) 7 cm (ii) 2.1 m (iii) 7 km

2. Find the area of a circle whose diameter is

   (i) 8.4 cm (ii) 5.6 m (iii) 7 km

3. The area of a circle is 154 cm^2. Find the radius of the circle.

4. Find the radius of a circle, if its area is

   (i) 4π cm^2 (ii) 55.44 m^2 (iii) 1.54 km^2

5. The circumference of a circle is 3.14 m, find its area.

6. If the area of a circle is 50.24 m^2, find its circumference.

7. A horse is tied to a pole with 28 m long string. Find the area where the horse can graze. (Take $\pi = 22/7$).

8. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, find the area of the circle.

9. A road which is 7 m wide surrounds a circular park whose circumference is 352 m. Find the area of of road.

10. Prove that the area of a circular path of uniform width h surrounding a circular region of radius r is $\pi h (2r + h)$.

11. The perimeter of a circle is $4\pi r$ cm. What is the area of the circle?
12. A wire of 5024 m length is in the form of a square. It is cut and made a circle. Find the ratio of the area of the square to that of the circle.
13. The radius of a circle is 14 cm. Find the radius of the circle whose area is double of the area of the circle.
14. The radius of one circluar field is 20 m and that of another is 48 m. Find the radius of the third circular field whose area is equal to the sum of the areas of two fields.
15. The radius of one circular field is 5 m and that of the other is 13 m. Find the radius of the circuiar field whose area is the difference of the areas of first and second field.
16. Two circles are drawn inside a big circle with diameters $\frac{2}{3}$rd and $\frac{1}{3}$rd of the diameter of the big circle as shown in Fig. 18. Find the area of the shaded portion, if the length of the diameter of the circle is 18 cm.

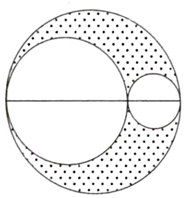

Fig. 18

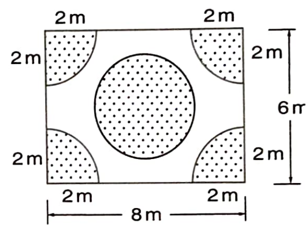

Fig. 19

17. In Fig. 19, the radius of quarter circular plot taken is 2 m and radius of the flower bed is 2 m. Find the area of the remaining field.
18. Four equal circles, each of radius 5 cm, touch each other as shown in Fig. 20. Find the area included between them. (Take $\pi = 3.14$).

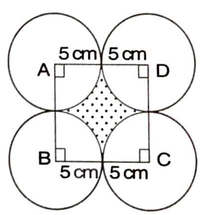

Fig. 20

19. The area of circle is 100 times the area of another circle. What is the ratio of their circumferences?

ANSWER

1. (i) $154\ cm^2$ (ii) $13.86\ m^2$ (iii) $154\ km^2$
2. (i) $55.44\ cm^2$ (ii) $24.64\ m^2$ (iii) $38.5\ km^2$
3. 7 cm
4. (i) 2 cm (ii) 4.2 m (iii) 0.7 km
5. $0.785\ m^2$
6. 25.12 m
7. $2464\ m^2$
8. $154\ cm^2$
9. $2618\ m^2$
11. $4\pi r^2\ cm^2$
12. 11 : 14
13. $14\sqrt{2}$ cm
14. 52 m
15. 12 m
16. $36\pi\ cm^2$
17. $22.86\ m^2$
18. $21.43\ cm^2$
19. 10 : 1

7. Length of the stri… radius 28 m. Henc…
18. Required area = …

Mark the correct altern…
1. The ratio of the p…
(a) π
2. The ratio of the a…
(a) d
3. The cost of fenc…
(a) ₹ 1320
4. If the diameter c…
(a) 7 : 1
5. A circle is inscri… square is
(a) $\pi : 3$
6. How many tim… rectangular fiel…
(a) 3
7. The minute ha… minutes?
(a) 22 cm
8. The cost of fer…
(a) ₹ 1080
9. The area of a … and the radius…
(a) $\sqrt{\pi} : 1$
10. If A is the area…
(a) $\frac{A}{C}$
11. The area of a…
(a) $\frac{C^2}{4\pi}$
12. The circumfe…
(a) $77\ cm^2$
13. Each side of … area of the tri…
(a) $\frac{7\sqrt{3}}{4}$ cr…

HINTS TO SELECTED PROBLEMS

7. Length of the string = 28 m. Area over which the horse can graze is the area of a circle of radius 28 m. Hence, required area $= \pi (28)^2 = 2464 \text{m}^2$

18. Required area = Area of square ABCD – Area of 4 quadrants $= 10 \times 10 - 4\left(\frac{1}{4} \times \frac{22}{7} \times 5^2\right) \text{cm}^2$

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. The ratio of the perimeter (circumference) and diameter of a circle is
   (a) π (b) 2π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$
2. The ratio of the area and circumference of a circle of diameter d is
   (a) d (b) $\frac{d}{2}$ (c) $\frac{d}{4}$ (d) $2d$
3. The cost of fencing a circular garden of radius 21 m at ₹ 10 per metre is
   (a) ₹ 1320 (b) ₹ 132 (c) ₹ 1200 (d) ₹ 660
4. If the diameter of a circle is equal to the diagonal of a square, then the ratio of their areas is
   (a) 7 : 1 (b) 1 : 1 (c) 11 : 7 (d) 22 : 7
5. A circle is inscribed in a square of side 14 m. The ratio of the area of the circle and that of the square is
   (a) $\pi : 3$ (b) $\pi : 4$ (c) $\pi : 2$ (d) $\pi : 1$
6. How many times should a wheel of radius 7 m rotate to go around the perimeter of a rectangular field of length 60 m and breadth 50 m?
   (a) 3 (b) 4 (c) 5 (d) 6
7. The minute hand of a clock is 14 cm long. How far does the tip of the minute hand move in 60 minutes?
   (a) 22 cm (b) 44 cm (c) 33 cm (d) 88 cm
8. The cost of fencing a semi-circular garden of radius 14 m at ₹ 10 per metre is
   (a) ₹ 1080 (b) ₹ 1020 (c) ₹ 700 (d) ₹ 720
9. The area of a square is equal to the area of a circle. The ratio between the side of the square and the radius of the circle is
   (a) $\sqrt{\pi} : 1$ (b) $1 : \sqrt{\pi}$ (c) $1 : \pi$ (d) $\pi : 1$
10. If A is the area and C is the circumference of a circle, then its radius is
    (a) $\frac{A}{C}$ (b) $\frac{2A}{C}$ (c) $\frac{3A}{C}$ (d) $\frac{4A}{C}$
11. The area of a circle of circumference C is
    (a) $\frac{C^2}{4\pi}$ (b) $\frac{C^2}{2\pi}$ (c) $\frac{C^2}{\pi}$ (d) $\frac{4C^2}{\pi}$
12. The circumference of a circle is 44 cm. Its area is
    (a) 77 cm^2 (b) 154 cm^2 (c) 208 cm^2 (d) 144 cm^2
13. Each side of an equilateral triangle is equal to the radius of a circle whose area is 154 cm^2. The area of the triangle is
    (a) $\frac{7\sqrt{3}}{4} \text{cm}^2$ (b) $\frac{49\sqrt{3}}{2} \text{cm}^2$ (c) $\frac{49\sqrt{3}}{4} \text{cm}^2$ (d) $\frac{7\sqrt{3}}{2} \text{cm}^2$

14. The area of a circle is $9\pi\ cm^2$. Its circumference is

(a) 6π cm (b) 36π cm (c) 9π cm (d) $36\pi^2$ cm

15. The area of a circle is increased by 22 cm^2 when its radius is increased by 1 cm. The original radius of the circle is

(a) 6 cm (b) 3 cm (c) 4 cm (d) 3.5 cm

16. The radii of two circles are in the ratio 2 : 3. The ratio of their areas is

(a) 2:3 (b) 4:9 (c) 3:2 (d) 9:4

17. The areas of two circles are in the ratio 49 : 36. The ratio of their circumferences is

(a) 7:6 (b) 6:7 (c) 3:2 (d) 2:3

18. The circumferences of two circles are in the ratio 3 : 4. The ratio of their areas is

(a) 3:4 (b) 4:3 (c) 9:16 (d) 16:9

19. The difference between the circumference and radius of a circle is 37 cm. The area of the circle is

(a) 111 cm^2 (b) 148 cm^2 (c) 154 cm^2 (d) 258 cm^2

ANSWSERS

1. (a)	2. (c)	3. (a)	4. (c)	5. (b)	6. (c)	7. (d)
8. (d)	9. (a)	10. (b)	11. (a)	12. (b)	13. (c)	14. (a)
15. (b)	16. (b)	17. (a)	18. (c)	19. (c)		

THINGS TO REMEMBER

1. *The perimeter of a circle is called its circumference.*
2. *The ratio of the circumference of a circle to its diameter is the same for all circles, regardless of their sizes. The constant ratio is denoted by pi whose approximate value is* $\frac{22}{7}$ *or 3.14.*

i.e. $\frac{\text{Circunference}}{\text{Diameter}} = \pi$

$\Rightarrow$ $\frac{C}{2r} = \pi$, *where C denotes circumference and r radius of the circle.*

$\Rightarrow$ $C = 2\pi r$.

3. *The number π is not a rational number.*
4. *Circumference C of a circle of radius r is given by* $C = 2\pi r$.

or, $C = \pi d$, *where* $d = 2r =$ *diameter*

5. *Area A of a circle of radius r is given by* $A = \pi r^2$
6. *Radius of a circle* $= \sqrt{\frac{A}{\pi}}$.