23

DATA HANDLING – II
(Central Values)

23.1 CENTRAL VALUE

In our day-to-day life we frequently use words average height, average price, average weight, etc. What do we mean by the word average? An average is a number indicating the central value of a group of observation i.e. data. In other words, an average is the number representing the entire data. Let us consider the following statements to understand what the term average means.

(i) The average score of class VII students in a class test is 65 marks.

(ii) The average temperature of January month in Delhi is 18°C.

(iii) The average age of pupils in class VII is 13 years.

The first statement means that the maximum number of students have scored around 65 marks. Only a few of them might have scored very high marks and some of them might have obtained very low marks.

In the second statement, the average temperature of 18 degree celsius means that, in general, the temperature in the month of January remains around 18 degree celsius. Sometimes, in the month it may be less than 18 degree celsius and at other times in the month it may be more than 18°C.

When we say that the average age of pupils in class VII is 13 years, it does not mean that every student of class VII is 13 years old. There may be some students of 14 years age and some may be of 12 years age. But, the maximum number of students are around 13 years old.

It follows from the above discussion that an average is a number indicating the representative or central value of a group of observations or data. Maximum number of observations of a data are very close to the central value and only few observations are either less than or greater than the central value. Since an average represents the entire data, therefore, its value lies somewhere in between the largest and smallest values of the variable. For this reason an average is called a measure of central value.

23.2 MEASURES OF CENTRAL VALUE

As discussed above that the central value or representative value of a group of observations or data is the value representing the entire data. It gives us a number around maximum number of observations are clustered. The question now is, how to find the central value of a group of observations or data. Different forms of data need different methods to determine the central value representing it. Various methods of determining the central values are known as the measures of central value. The most commonly used measures of central value are:

(i) Arithmetic mean (ii) Median (iii) Mode

Let us now learn about these measures.

23.3 ARITHMETIC MEAN

The most common representative value or central value of a group of observations is the arithmetic mean or the mean. The mean of a group of observations is the value which is equally shared out among all the observations. Thus, the arithmetic mean of a group of observations is defined as

$$\text{Arithmetic mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Thus, if $x_1, x_2, x_3, \ldots, x_n$ are the values of n observations, then the arithmetic mean of these observations is given by

$$\text{Arithmetic mean} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

In order to have an idea of the spread of the observations in a group of observations, we compute its range which is defined as follows:

RANGE *The difference between the highest and the lowest observations of a group of observations is defined as its range.*

i.e., Range = Value of the largest observation – Value of the smallest observation.

Following examples will illustrate the computation of arithmetic mean of given observations.

ILLUSTRATIVE EXAMPLES

Example 1 A batsman scored the following number of runs in six innings:

36, 35, 50, 46, 60, 55

Calculate the mean runs scored in an inning.

Solution We have,

$$\text{Mean runs scored} = \frac{36+35+50+46+60+55}{6} = \frac{282}{6} = 47$$

Thus, the mean runs scored in an inning is 47.

Example 2 If the heights of 5 persons are 144 cm, 152 cm, 151 cm, 158 cm and 155 cm respectively. Find the mean height.

Solution

$$\text{Mean height} = \frac{144+152+151+158+155}{5} = \frac{760}{5} \text{ cm} = 152 \text{ cm}$$

Example 3 Find the arithmetic mean of first 6 natural numbers.

Solution First six natural numbers are 1, 2, 3, 4, 5, 6. Let $\overline{X}$ denote their arithmetic mean. Then,

$$\overline{X} = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2} = 3.5$$

Example 4 Find the mean of first five whole numbers.

Solution First five whole numbers are 0, 1, 2, 3, 4. Let $\overline{X}$ denote their arithmetic mean. Then,

$$\overline{X} = \frac{0+1+2+3+4}{5} = \frac{10}{5} = 2$$

Example 5 The ages in years of 10 teachers in a school are: 32, 41, 28, 54, 35, 26, 23, 33, 38, 40.

(i) What is the age of the oldest teacher and that of the youngest teacher?

(ii) What is the range of the ages of the teachers?

(iii) What is the mean age of these teachers?

Solution (i) In order to find the ages of oldest and youngest teachers, Let us arrange their ages in ascending order as follows:

23, 26, 28, 32, 33, 35, 38, 40, 41, 54

We observe that the age of the oldest teacher is 54 years and the age of the youngest teacher is 23 years.

(ii) We have,

Age of the oldest teacher = 54 years

Age of the youngest teacher = 23 years

$\therefore$ Range of the ages of teachers = (54 – 23) = 31 years

(iii) We have,

$$\text{Mean age of teachers} = \frac{\text{Sum of the ages of teachers}}{\text{Total number of teachers}}$$

$$\Rightarrow \text{Mean age of teachers} = \frac{23+26+28+32+33+35+38+40+41+54}{10}$$

$$\Rightarrow \text{Mean age of teachers} = \frac{350}{10}\text{ years} = 35 \text{ years}$$

Example 6 The heights of 10 girls were measured in cm and the results are as follows:

135, 150, 139, 128, 151, 132, 146, 149, 143, 141

(i) What is the height of the tallest girl?

(ii) What is the height of the shortest girl?

(iii) What is the range of the data?

(iv) What is the mean height of the girls?

(v) How many girls have heights more than the mean height?

Solution In order to find the heights of the tallest and the shortest girls, let us arrange the heights in ascending order as follows:

128, 132, 135, 139, 141, 143, 146, 149, 150, 151

(i) Clearly, the height of the tallest girl is 151 cm

(ii) The height of the shortest girl is 128 cm.

(iii) The range of the heights = (151 – 128) cm = 23 cm.

(iv) We have,

$$\text{Mean height} = \frac{\text{Sum of the heights}}{\text{Total number of girls}}$$

$$\Rightarrow \text{Mean height} = \frac{128+132+135+139+141+143+146+149+150+151}{10}\text{ cm}$$

$$\Rightarrow \text{Mean height} = \frac{1414}{10}\text{ cm} = 141.4 \text{ cm}$$

Hence, mean height of the girls is 141.4 cm.

(v) Clearly, 5 girls have heights more than the mean height.

Example 7 If the mean of 6, 8, 5, x and 4 is 7, find the value of x.

Solution The number of observations = 5

The mean of the given observations is $\dfrac{6+8+5+x+4}{5}$

But, the mean is given as 7.

$$\therefore \quad \frac{6+8+5+x+4}{5}=7$$

$$\Rightarrow \quad \frac{23+x}{5}=7$$

$$\Rightarrow \quad 23+x=7\times 5 \Rightarrow x=35-23=12.$$

Example 8 If the mean of 6, 4, 7, p and 10 is 8, find the value of p.

Solution Since 8 is the mean of 6, 4, 7, p, 10.

$$\therefore \quad 8=\frac{6+4+7+p+10}{5} \Rightarrow 40=27+p \Rightarrow p=13$$

Example 9 The mean of 10 numbers is 20. If 5 is subtracted from every number, what will be the new mean?

Solution Let $x_1, x_2, x_3, \ldots x_{10}$ be 10 numbers with their mean equal to 20. Then,

$$20=\frac{x_1+x_2+x_3+\ldots+x_{10}}{10}$$

$$\Rightarrow \quad x_1+x_2+x_3+\ldots+x_{10}=20\times 10$$

$$\Rightarrow \quad x_1+x_2+x_3+\ldots+x_{10}=200 \qquad \ldots \text{(i)}$$

New numbers are $x_1-5, x_2-5, x_3-5, \ldots, x_{10}-5$. Let M be their arithmetic mean. Then,

$$M=\frac{(x_1-5)+(x_2-5)+(x_3-5)+\ldots+(x_{10}-5)}{10}$$

$$\Rightarrow \quad M=\frac{(x_1+x_2+x_3+\ldots+x_{10})-5\times 10}{10} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad M=\frac{200-50}{10}=15$$

Example 10 The mean of 16 numbers is 8. If 2 is added to every number, what will be the new mean?

Solution Let $x_1, x_2, x_3, \ldots, x_{16}$ be 16 numbers with their mean equal to 8. Then,

$$8=\frac{x_1+x_2+x_3+\ldots+x_{16}}{16}$$

$$\Rightarrow \quad x_1+x_2+x_3+\ldots+x_{16}=8\times 16$$

$$\Rightarrow \quad x_1+x_2+x_3+\ldots+x_{16}=128 \qquad \ldots \text{(i)}$$

New numbers are $x_1+2, x_2+2, x_3+2, \ldots, x_{16}+2$. Let M be arithmetic mean of new numbers. Then,

$$M = \frac{(x_1+2)+(x_2+2)+(x_3+2)+\ldots+(x_{16}+2)}{16}$$

$$\Rightarrow \quad M = \frac{(x_1+x_2+\ldots x_{16})+(2\times 16)}{16}$$

$$\Rightarrow \quad M = \frac{128+32}{16} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad M = \frac{160}{16} = 10$$

Example 11 If the mean of five observations $x, x+2, x+4, x+6, x+8,$ is 11. find the mean of first three observations.

Solution We have,

$$11 = \frac{x+(x+2)+(x+4)+(x+6)+(x+8)}{5}$$

$$\Rightarrow \quad 11 = \frac{5x+20}{5}$$

$$\Rightarrow \quad 55 = 5x+20$$

$$\Rightarrow \quad 5x = 35$$

$$\Rightarrow \quad x = 7$$

$\therefore$ Mean of first three observations

$$= \frac{x+(x+2)+(x+4)}{3} = \frac{3x+6}{3} = x+2 = 7+2 = 9 \qquad [\because x = 7]$$

Example 12 The mean of 40 observations was 160. It was detected on re-checking that the value of 165 was wrongly copied as 125 for computation of mean. Find the correct mean.

Solution We have,

n = Number of observations = 40, Mean = 160

$$\therefore \quad \text{Mean} = \frac{\text{Sum of the observations}}{\text{Number of observations}}$$

$$\Rightarrow \quad 160 \times 40 = \frac{\text{Sum of the observations}}{40}$$

$\Rightarrow$ 160×40 = Sum of the observations.

Thus, incorrect sum of observations = 160×40

Now,

Correct sum of the observations = Incorrect sum of the observations
− Incorrect observation
+ correct observation

$\Rightarrow$ Correct sum of the observations = $160 \times 40 - 125 + 165$

$\Rightarrow$ Correct sum of the observations = $6400 + 40 = 6440$.

$$\therefore \quad \text{Correct mean} = \frac{\text{Correct sum of the observations}}{\text{Number of observations}} = \frac{6440}{40} = 161$$

Example 13 Following table shows the points of scores each player plays:

Player	Game 1	Game 2	Game 3	Game 4
A	14	16	10	10
B	0	8	6	4
C	8	11	Did not play	13

(i) Find the average scores of three players.

(ii) Who is the best performer?

Solution (i) We have,

$$\text{Average score of player } A = \frac{14+16+10+10}{4} = \frac{50}{4} = 12.5$$

$$\text{Average score of player } B = \frac{0+8+6+4}{4} = \frac{18}{4} = 4.5$$

$$\text{Average score of player } C = \frac{8+11+13}{3} = 10.66$$

(ii) Clearly, average score of player A is more. So, player A is the best performer.

EXERCISE 23.1

1. Ashish studies for 4 hours, 5 hours and 3 hours on three consecutive days. How many hours does he study daily on an average?
2. A cricketer scores the following runs in 8 innings: 58, 76, 40, 35, 48, 45, 0, 100. Find the mean score.
3. The marks (out of 100) obtained by a group of students in science test are 85, 76, 90, 84, 39, 48, 56, 95, 81 and 75. Find the
   (i) highest and the lowest marks obtained by the students.
   (ii) range of marks obtained.
   (iii) mean marks obtained by the group.
4. The enrolment of a school during six consecutive years was as follows:
   1555, 1670, 1750, 2019, 2540, 2820
   Find the mean enrollment of the school for this period.
5. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.3	1.0

   (i) Find the range of the rainfall from the above data.
   (ii) Find the mean rainfall for the week.
   (iii) On how many days was the rainfall less than the mean rainfall.
6. If the heights of 5 persons are 140 cm, 150 cm, 152 cm, 158 cm and 161 cm respectively, find the mean height.

7. Find the mean of 994, 996, 998, 1002 and 1000.
8. Find the mean of first five natural numbers.
9. Find the mean of all factors of 10.
10. Find the mean of first 10 even natural numbers.
11. Find the mean of $x, x+2, x+4, x+6, x+8$.
12. Find the mean of first five multiples of 3.
13. Following are the weights (in kg) of 10 new born babies in a hospital on a particular day:

    3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6. Find the mean $\overline{X}$.
14. The percentage of marks obtained by students of a class in mathematics are:

    64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1. Find their mean.
15. The numbers of children in 10 families of a locality are:

    2, 4, 3, 4, 2, 3, 5, 1, 1, 5. Find the mean number of children per family.
16. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.
17. The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.
18. The mean weight per student in a group of 7 students is 55 kg. The individual weights of 6 of them (in kg) are 52, 54, 55, 53, 56 and 54. Find the weight of the seventh student.
19. The mean weight of 8 numbers is 15 kg. If each number is multiplied by 2, what will be the new mean?
20. The mean of 5 numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.
21. The mean of 200 items was 50. Later on, it was discovered that the two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.
22. The mean of 5 numbers is 27. If one more number is included, then the mean is 25. Find the included number.
23. The mean of 75 numbers is 35. If each number is multiplied by 4, find the new mean.

ANSWERS

1. 4 hrs 2. 50.25 runs 3. (i) 95, 39 marks (ii) 56 marks (iii) 72.9 marks
4. 2059 5. (i) 20.5 mm (ii) 5.87 mm (iii) 5 days
6. 152.2 cm 7. 998 8. 3 9. 4.5 10. 11 11. $x+4$
12. 9 13. 4 kg 14. 39.5 15. 3 16. 39.7 17. 35
18. 61 kg 19. 30 kg 20. 26 21. 50.9 22. 15 23. 140

23.4 ARITHMETIC MEAN OF GROUPED DATA

In the previous subsection, we have learnt about the method of finding the arithmetic mean of individual observations or ungrouped data. We have learnt in the previous chapter that in case of large number of observations, data can be presented in the form of a frequency table which exhibits the values taken by the variable and the corresponding frequencies. In such a form data is known as grouped data. Sometimes, we also call it discrete frequency distribution. If $x_1, x_2, x_3, \ldots, x_n$ are n values of a variate with corresponding frequencies $f_1, f_2, \ldots, f_n$ respectively. Then, the arithmetic mean of these values is defined as

$$\text{Mean} = \frac{f_1 \times x_1 + f_2 \times x_2 + \ldots + f_n \times x_n}{f_1 + f_2 + \ldots + f_n}$$

or, $$\text{Mean} = \frac{\Sigma f_i \, x_i}{\Sigma f_i}$$, where Σ(called sigma) is the Greek letter showing summation.

In order to compute the arithmetic mean of grouped data, we may use the following steps:

STEP I — *Prepare frequency table in such a way that its first column consists of the values of the variate and the second column the corresponding frequencies.*

STEP II — *Multiply the frequency of each row with the corresponding values of variate to obtain third column containing* $f_i \, x_i$.

STEP III — *Find the sum of all entries in III column to obtain* $\Sigma f_i \, x_i$.

STEP IV — *Find the sum of all the frequencies in II column to obtain* Σf_i or, N

STEP V — *Use the formula:* $\overline{X} = \dfrac{\Sigma f_i \, x_i}{N}$.

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

Example 1 Organise the following grades in a class assessment in a tabular form:

4, 6, 7, 5, 3, 5, 4, 5, 8, 6, 2, 5, 1, 9, 6, 7, 8, 4, 6, 7

Find the arithmetic mean of grades. Also, find the highest grade, the lowest grade and the range of the data.

Solution In the tabular form, the above data can be represented as follows:

Grade	*Tally Bars*	*Frequency* (f_i)
1	|	1
2	|	1
3	|	1
4	|||	3
5	||||	4
6	||||	4
7	|||	3
8	||	2
9	|	1

In order to compute the arithmetic mean of grades i.e. average grade, we prepare the following table.

Computation of Arithmetic mean

Grade (x_i)	Frequency (f_i)	$f_i \times x_i$
1	1	$1 \times 1 = 1$
2	1	$1 \times 2 = 2$
3	1	$1 \times 3 = 3$
4	3	$3 \times 4 = 12$
5	4	$4 \times 5 = 20$
6	4	$4 \times 6 = 24$
7	3	$3 \times 7 = 21$
8	2	$2 \times 8 = 16$
9	1	$1 \times 9 = 9$
	$\Sigma f_i = 20$	$\Sigma f_i\, x_i = 108$

$\therefore$ Mean grade $= \dfrac{\Sigma f_i\, x_i}{\Sigma f_i} = \dfrac{108}{20} = 5.4$

Hence, the mean grade is 5.4.

It is evident from the frequency table that the highest and lowest grades are 9 and 1 respectively.

$\therefore$ Range of the data $= 9 - 1 = 8$

Example 2 Given below are the ages of 25 students of a class in a school. Prepare a frequency table and hence find the mean age.

15, 16, 16, 14, 17, 17, 16, 15, 15, 15, 16, 16, 17, 17, 16, 16, 17, 17, 16, 16, 16, 15, 16, 16, 17

Solution In the tabular form, the given data can be presented as follows:

Age (x_i)	Tally Bars	Frequency (f_i)
14	|	1
15	~~||||~~	5
16	~~||||~~ ~~||||~~ ||	12
17	~~||||~~ ||	7

In order to compute the average age of students in the class, we prepare following table:

Computation of Arithmetic mean

Age (x_i)	Frequency (f_i)	$f_i \times x_i$
14	1	$1 \times 14 = 14$
15	5	$5 \times 15 = 75$
16	12	$12 \times 16 = 192$
17	7	$7 \times 17 = 119$
Total	$\Sigma f_i = 25$	$\Sigma f_i\, x_i = 400$

We have, $\Sigma f_i = 25$ and $\Sigma f_i\, x_i = 400$

$$\therefore \quad \text{Mean age} = \frac{\Sigma f_i\, x_i}{\Sigma f_i} = \frac{400}{25} = 16 \text{ years.}$$

Example 3 Find the mean of the following distribution:

x:	4	6	9	10	15
f:	5	10	10	7	8

Solution

Calculation of Arithmetic Mean

x_i	f_i	$f_i\, x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \Sigma f_i = 40$	$\Sigma f_i\, x_i = 360$

$$\therefore \quad \text{Mean} = \overline{X} = \frac{\Sigma f_i\, x_i}{\Sigma f_i} = \frac{360}{40} = 9.$$

Example 4 Following table shows the weights of 12 students:

Weight (in kgs)	67	70	72	73	75
Number of Students:	4	3	2	2	1

Find the mean weight.

Solution

Calculation of Arithmetic Mean

Weight (in kgs) x_i	*Frequency* f_i	$f_i\, x_i$
67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
	$N = \Sigma f_i = 12$	$\Sigma f_i\, x_i = 843$

$$\therefore \quad \text{Mean} = \overline{X} = \frac{\Sigma f_i\, x_i}{N} = \frac{843}{12} = 70.25 \text{ kg}.$$

Example 5 Find the mean of the following distribution:

x:	10	30	50	70	89
f:	7	8	10	15	10

Solution

Calculation of Mean

x_i	f_i	$f_i\ x_i$
10	7	70
30	8	240
50	10	500
70	15	1050
89	10	890
	$\Sigma f_i = N = 50$	$\Sigma f_i x_i = 2750$

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{N} = \dfrac{2750}{50} = 55.$

Example 6 If the mean of the following distribution is 6, find the value of p.

x:	2	4	6	10	$p+5$
f:	3	2	3	1	2

Solution

Calculation of Mean

x_i	f_i	$f_i\ x_i$
2	3	6
4	2	8
6	3	18
10	1	10
$p+5$	2	$2p+10$
	$N = \Sigma f_i = 11$	$\Sigma f_i\ x_i = 2p+52$

We have,

$N = \Sigma f_i = 11, \Sigma f_i\ x_i = 2p + 52$

$\therefore$ Mean $= \dfrac{\Sigma f_i\ x_i}{N}$

$\Rightarrow$ $6 = \dfrac{2p+52}{11} \Rightarrow 66 = 2p + 52 \Rightarrow 2p = 14 \Rightarrow p = 7$

Example 7 Find the value of p, if the mean of the following distribution is 7.5.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Calculation of Mean

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	p	$9p$
11	8	88
13	4	52
	$N = \Sigma f_i = 41 + p$	$\Sigma f_i x_i = 303 + 9p$

We have,

$\Sigma f_i = 41 + p$, $\Sigma f_i x_i = 303 + 9p$.

$\text{Mean} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 7.5 = \dfrac{303 + 9p}{41 + p}$

$\Rightarrow \quad 7.5 \times (41 + p) = 303 + 9p$

$\Rightarrow \quad 307.5 + 7.5p = 303 + 9p$

$\Rightarrow \quad 9p - 7.5p = 307.5 - 303$

$\Rightarrow \quad 1.5p = 4.5 \Rightarrow p = \dfrac{4.5}{1.5} = 3$

EXERCISE 23.2

1. A die was thrown 20 times and the following scores were recorded:
   5, 2, 1, 3, 4, 4, 5, 6, 2, 2, 4, 5, 5, 6, 2, 2, 4, 5, 5, 1
   Prepare the frequency table of the scores on the upper face of the die and find the mean score.
2. The daily wages (in Rs) of 15 workers in a factory are given below:
   200, 180, 150, 150, 130, 180, 180, 200, 150, 130, 180, 180, 200, 150, 180
   Prepare the frequency table and find the mean wage.
3. The following table shows the weights (in kg) of 15 workers in a factory:

Weight (in kg) :	60	63	66	72	75
Numbers of workers :	4	5	3	1	2

   Calculate the mean weight.

4. The ages (in years) of 50 students of a class in a school are given below:

Age (in years) :	14	15	16	17	18
Numbers of students:	15	14	10	8	3

   Find the mean age.

5. Calculate the mean for the following distribution:

x :	5	6	7	8	9
f :	4	8	14	11	3

6. Find the mean of the following data:

x :	19	21	23	25	27	29	31
f :	13	15	16	18	16	15	13

7. The mean of the following data is 20.6. Find the value of p.

x :	10	15	p	25	35
f :	3	10	25	7	5

8. If the mean of the following data is 15, find p.

x :	5	10	15	20	25
f :	6	p	6	10	5

9. Find the value of p for the following distribution whose mean is 16.6

x :	8	12	15	p	20	25	30
f :	12	16	20	24	16	8	4

10. Find the missing value of p for the following distribution whose mean is 12.58

x :	5	8	10	12	p	20	25
f :	2	5	8	22	7	4	2

11. Find the missing frequency (p) for the following distribution whose mean is 7.68

x :	3	5	7	9	11	13
f :	6	8	15	p	8	4

12. Find the value of p, if the mean of the following distribution is 20

x :	15	17	19	$20+p$	23
f :	2	3	4	$5p$	6

ANSWERS

1.

Score (x_i) :	1	2	3	4	5	6
Frequency (f_i) :	2	5	1	4	6	2

Mean score = 3.65

2.

Wages (x_i) :	130	150	180	200
No. of workers (f_i) :	2	4	6	3

Mean wage = Rs 169.33

3. 65 kg 4. 15.4 years 5. 7.025 6. 25 7. $p = 20$ 8. 8

9. 18 10. 15 11. 9 12. 1

23.5 MEDIAN

Mean is not the only tool or method to determine the representative value or central value of a group of observations. In different situations, we use different methods or tools to find the central value of a group of observations. Also, for different requirements from a group of observations (data) we use different measures of central value. Let us consider a group of 17 students of a school with the following heights (in cms):

101, 106, 116, 110, 109, 123, 118, 127, 120, 117, 102, 122, 115, 112, 121, 125, 111

The mean height of this group of students is

$$\frac{101+106+116+110+109+123+118+127+120+117+102+122+115+112+121+125+111}{17}$$

$$= \frac{1955}{17} = 115.$$

Clearly, there are 7 students of heights less than the mean height and 9 students are of heights greater than the mean height.

If the games teacher of the school wants to divide this group of 17 students into two groups on the basis of mean height so that one group has students of height less than the mean height and the other group has students of height more than the mean height, then the groups would be of unequal size. This means that the mean is unable to divide the group of students into two equal sub-groups. So, there must be some other representative value (or central value) to divide a group of observations into two equal groups. Such a representative value or central value is known as the median of the data. Median is the value which lies in the middle of the data with half of the observations above it and the other half below it.

Let us again look at the given heights of students and arrange them in ascending order as follows:

101, 102, 106, 109, 110, 111, 112, 115, 116, 117, 118, 120, 121, 122, 123, 125, 127

We find that the middle value in this data is 116. This value is the median. Thus, we define the median as follows:

MEDIAN *Median of a group of observations is the value of the variable which divides the group into two equal parts.*

In other words, median is the value which exceeds and is exceeded by the same number of observations i.e., it is the value such that the number of observations above it is equal to the number of observations below it.

In case of individual observations i.e., ungrouped data, we may use the following steps to find the median.

STEP I *Arrange the observations in ascending or descending order of magnitude.*

STEP II *Determine the total number of observations, say, n.*

STEP III *If n is odd, then use the following formula:*

$$\textit{Median} = \textit{Value of} \left(\frac{n+1}{2}\right)^{th} \textit{observation.}$$

If n is even, then use the following formula:

$$\text{Median} = \frac{\text{Value of} \left(\frac{n}{2}\right)^{th} \text{observation} + \text{Value of} \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$$

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Find the median of the data: 24, 36, 46, 17, 18, 25, 35

Solution Arranging the data in ascending order, we get

17, 18, 24, 25, 35, 36, 46

Here, $n = 7$, which is odd.

$$\therefore \quad \text{Median} = \text{Value of} \left(\frac{7+1}{2}\right)^{th} \text{observation} = \text{Value of } 4^{th} \text{ observation} = 25$$

Example 2 The runs scored in a cricket match by 11 players are as follows:

6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 10

Find the median score.

Solution Arranging runs scored in ascending order, we get

6, 8, 10, 10, 10, 15, 15, 50, 80, 100, 120

Here, $n = 11$, which is odd.

$$\therefore \quad \text{Median} = \text{Value of } \left(\frac{11+1}{2}\right)^{\text{th}} \text{ i.e., } 6^{\text{th}} \text{ observation} = 15$$

Example 3 Find the median of the data: 25, 34, 31, 23, 22, 26, 35, 28, 20, 32

Solution Arranging the data in ascending order, we get

20, 22, 23, 25, 26, 28, 31, 32, 34, 35

Here, the number of observation $n = 10$ (even).

$$\Rightarrow \quad \text{Median} = \frac{\text{Value of } \left(\frac{10}{2}\right)^{\text{th}} \text{observation} + \text{Value of } \left(\frac{10}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$\Rightarrow \quad \text{Median} = \frac{\text{Value of } 5^{\text{th}} \text{ observation} + \text{Value of } 6^{\text{th}} \text{ observation}}{2}$$

$$\Rightarrow \quad \text{Median} = \frac{26+28}{2} = 27$$

Hence, median of the given data is 27.

Example 4 Find the median of the values: 37, 31, 42, 43, 46, 25, 39, 45, 32

Solution Arranging the data in ascending order, we have

25, 31, 32, 37, 39, 42, 43, 45, 46

Here, the number of observations $n = 9$ (odd)

$$\therefore \quad \text{Median} = \text{Value of } \left(\frac{9+1}{2}\right)^{\text{th}} \text{ observation}$$

$$\Rightarrow \quad \text{Median} = \text{Value of } 5^{\text{th}} \text{ observation} = 39$$

Example 5 The median of the observations 11, 12, 14, 18, $x + 2$, $x + 4$, 30, 32, 35, 41 arranged in ascending order is 24. Find the value of x.

Solution Here, the number of observations n is 10. Since n is even.

$$\therefore \quad \text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$\Rightarrow \quad 24 = \frac{5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}}{2}$$

$$\Rightarrow \quad 24 = \frac{(x+2)+(x+4)}{2}$$

$$\Rightarrow \quad 24 = \frac{2x+6}{2} \Rightarrow 24 = x + 3 \Rightarrow x = 21$$

Hence, $x = 21$.

Example 6 Find the median of the data: 19, 25, 59, 48, 35, 31, 30, 32, 51. If 25 is replaced by 52, what will be the new median.

Solution Arranging the given data in ascending order, we have

19, 25, 30, 31, 32, 35, 48, 51, 59

Here, the number of observations $n = 9$ (odd)

Since the number of observations is odd. Therefore,

$$\text{Median} = \text{Value of} \left(\frac{9+1}{2}\right)^{th} \text{observation} = \text{Value of } 5^{th} \text{ observation} = 32.$$

Hence, median = 32

If 25 is replaced by 52, then the new observations arranged in ascending order are:

19, 30, 31, 32, 35, 48, 51, 52, 59

$\therefore$ New median = Value of 5^{th} observation = 35.

EXERCISE 23.3

Find the median of the following data (1-8)

1. 83, 37, 70, 29, 45, 63, 41, 70, 34, 54
2. 133, 73, 89, 108, 94, 104, 94, 85, 100, 120
3. 31, 38, 27, 28, 36, 25, 35, 40
4. 15, 6, 16, 8, 22, 21, 9, 18, 25
5. 41, 43, 127, 99, 71, 92, 71, 58, 57
6. 25, 34, 31, 23, 22, 26, 35, 29, 20, 32
7. 12, 17, 3, 14, 5, 8, 7, 15
8. 92, 35, 67, 85, 72, 81, 56, 51, 42, 69
9. Numbers 50, 42, 35, $2x + 10$, $2x - 8$, 12, 11, 8, 6 are written in descending order and their median is 25, find x.
10. Find the median of the following observations : 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced by 99 and 41 by 43 in the above data, find the new median?
11. Find the median of the following data : 41, 43, 127, 99, 61, 92, 71, 58, 57, If 58 is replaced by 85, what will be the new median?
12. The weights (in kg) of 15 students are : 31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30. Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.
13. The following observations have been arranged in ascending order. If the median of the data is 63, find the value of x :

    29, 32, 48, 50, x, $x + 2$, 72, 78, 84, 95

ANSWERS

1. 49.5 2. 97 3. 33 4. 16 5. 71 6. 27.5
7. 10 8. 68 9. 16.5 10. 58, 58 11. 61,71 12. 35 kg, 35 kg
13. 62

23.6 MODE

In the previous two sections, we have seen that in some situations arithmetic mean is an appropriate measure of central value where as in some other situations, median is the appropriate measure of central value. Let us now consider the following table showing the sizes of shoes and the number of pairs of shoes sold in a week by a shopkeeper.

Size of shoes:	5	6	7	8	9	10	11
No. of pairs of shoes sold:	10	20	38	42	10	4	2

Suppose the shop-keeper finds the arithmetic mean of the number of pairs of shoes sold in a week. He finds that the mean number of shoes sold in a week is

$$\frac{10+20+38+42+10+4+2}{7}=\frac{126}{7}=18$$

On the basis of the mean if the shopkeeper places the order for 18 pairs of shoes of each size, then he will not be able to cater the needs of the customers demanding 6, 7, 8 sizes of shoes.

If the shopkeeper decides to find the median size of the number of pairs of shoes sold, then median size is 7. On the basis of this if he procures 38 pairs of shoes of each size, then he will not be able to cater the needs of the customers of size 8.

If follows from the above discussion that the mean and median are not appropriate measures of central value (central tendency) in such situations. We, therefore, think of an alternative measure of central tendency.

If the shopkeeper looks at the shoe size 8 that is sold most and decides to procure 42 pairs of shoes of each size, then he is able to cater the needs of customers of all sizes. We find that the highest occuring event in the sale data is the sale of size 8. This representative value of the data is called the mode of the data. Thus, we define the mode of a data as follows:

MODE *Mode of a set of observations is the value of the observation that occurs most frequently.*

The mean, median and mode of a group of observations (data) are connected by the following empirical relation:

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}.$$

In order to find the mode of a group of observations (or ungrouped data), we may follow the following steps:

STEP I *Obtain the set of obserations i.e., ungrouped data.*

STEP II *Count the number of times the various values repeat themselves. In other words, prepare the frequency table.*

STEP III *Find the value which occurs the maximum number of times i.e., obtain the value which has the maximum frequency.*

STEP IV *The value obtained in step III is the mode.*

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Find the mode of the following data:

1, 1, 2, 4, 3, 2, 1, 2, 2, 4

Solution Arranging the number with the same values together, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4.

Clearly, 2 occurs maximum number of times.

Hence, mode of the given data is 2.

Aliter Arranging the data in the form of a frequency table, we have

Value	*Tally bars*	*Frequency*
1					3
2						4
3			1
4				2

Clearly the value 2 occurs maximum number of times i.e. 4.

Hence, the mode is 2.

Example 2 Following are the margins of victory in the football matches of a league.

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2, 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2. Find the mode.

Solution Arranging the margins of victory in the tabular form, we have

Margins of victory	*Tally bars*	*Number of matches (Frequency)*
1	~~||||~~ ||||	9
2	~~||||~~ ~~||||~~ ||||	14
3	~~||||~~ ||	7
4	~~||||~~	5
5					3
6				2
	Total	40

We find that the value 2 has the maximum frequency. So, 2 is the mode which means that most of the matches have been won with a victory margin of 2 goals.

Example 3 Find the mode of the data: 2, 2, 3, 3, 2, 5, 4, 5, 5, 8, 6, 6

Solution Arranging the numbers with the same values together we get

2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

Here, 2 and 5 both occur three times. Therefore, they are both modes of the data. In such a case, we say that the data is bi-modal.

Example 4 The weights (in kg) of 15 students of a class are:

38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47

(i) Find the mode and median of this data.

(ii) Is there more than one mode?

Solution (i) Arranging the weights in the ascending order such that the same weights are put together, we get

32, 35, 36, 37, 38, 38, 38, 40, 42, 43, 43, 43, 45, 47, 50

Here, $n = 15$.

$$\therefore \quad \text{Median} = \text{Value of} \left(\frac{15+1}{2}\right)^{\text{th}} \text{observation} = \text{Value of } 8^{\text{th}} \text{ observation} = 40$$

Here, 38 and 43 both occur three times. Therefore, 38 kg and 43 kg are two modes.

Example 5 Find the mode and median of the following data:

12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15, 17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

Also, find the mean by using the empirical relation.

Solution Arranging the data in tabular form, we have

Value	*Tally bars*	*Frequency*
12	III	3
13	IIII	4
14	~~IIII~~	5
15	~~IIII~~ ~~IIII~~	10
16	~~IIII~~ I	6
17	II	2
18	I	1
19	I	1
	Total	32

Looking at the table, we find that the value 15 occurs most frequently. So, 15 is the mode.

We find that there are 32 observations i.e. $n = 32$.

$$\therefore \quad \text{Median} = \frac{\text{Value of} \left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \text{Value of} \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$\Rightarrow \quad \text{Median} = \frac{\text{Value of } 16^{\text{th}} \text{ observation} + \text{Value of } 17^{\text{th}} \text{ observation}}{2}$$

$$\Rightarrow \quad \text{Median} = \frac{15+15}{2} = 15 \left[\because \text{ Value of } 12^{th} \text{ observation is 14 and each of the observations from } 13^{th} \text{ to } 22^{nd} \text{ is of value 15}\right]$$

Hence, Median = 15

Now, Mode = 3 Median − 2 Mean

$\Rightarrow \quad 15 = 3 \times 15 - 2 \text{ Mean}$

$\Rightarrow \quad 2 \text{ Mean} = 45 - 15$

$\Rightarrow \quad \text{Mean} = \frac{30}{2} = 15$

Hence, Mean = Median = Mode = 15.

EXERCISE 23.4

1. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14
   By using the empirical relation also find the mean.
2. Find the median and mode of the data: 35, 32, 35, 42, 38, 32, 34
3. Find the mode of the data: 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4
4. The runs scored in a cricket match by 11 players are as follows:
   6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 10
   Find the mean, mode and median of this data.
5. Find the mode of the following data:
   12, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14
6. Heights of 25 children (in cm) in a school are as given below:
   168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 163, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162
   What is the mode of heights?
   Also, find the mean and median.
7. The scores in mathematics test (out of 25) of 15 students are as follows:
   19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20
   Find the mode and median of this data. Are they same?
8. Calculate the mean and median for the following data:

Marks :	10	11	12	13	14	16	19	20
Number of students :	3	5	4	5	2	3	2	1

   Using empirical formula, find its mode.
9. The following table shows the weights of 12 persons.

Weight (in kg) :	48	50	52	54	58
Number of persons :	4	3	2	2	1

   Find the median and mean weights. Using empirical relation, calculate its mode.

ANSWERS

1. Mode = 14, Median = 14, Mean = 14
2. Median = 35, 32 and 35 are two modes.
3. 3 and 4 are two modes.
4. Mode = 10, Median = 15, Mean = 38.54
5. Mode = 14
6. Mode = 163 cm = Mean = Median
7. Mode = 20, Median = 20, Yes
8. Mean = 13.28, Mode = 12.44, Median = 13.
9. Mean = 51 kg Mode = 48 kg Median = 50 kg.

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. If the mean of observations 7, 8, 9, 11 and x is 10, then $x=$
   (a) 10 (b) 15 (c) 12 (d) 13
2. If the mean of observations 20, 42, 35, 45, x is 37, then $x=$
   (a) 43 (b) 42 (c) 44 (d) 45
3. The mean of first five natural numbers is
   (a) 5 (b) 4 (c) 3 (d) 6
4. The mean of first five prime numbers is
   (a) 5.6 (b) 5.5 (c) 5.4 (d) 5.2
5. The mean of first seven even natural numbers is
   (a) 7 (b) 8 (c) 9 (d) 6
6. The mean of first six multiples of 5 is
   (a) 3.5 (b) 18.5 (c) 17.5 (d) 30
7. The mean of 5 numbers is 4. If 1 is added to each number, then the new mean is
   (a) 4 (b) 5 (c) 3 (d) 5.5
8. If the sum of 10 observations is 95, then their mean is
   (a) 9.5 (b) 10 (c) 950 (d) 95
9. If the mean of n observations is 12 and the sum of the observations is 132, then the value of n is
   (a) 9 (b) 10 (c) 11 (d) 12
10. The median of the data 9, 12, 11, 10, 8, 9, 11 is
    (a) 10 (b) 11 (c) 9 (d) None of these
11. The median of the data 5, 7, 9, 10, 11 is
    (a) 7 (b) 9 (c) 11 (d) 10
12. The mean of a data is 15 and the sum of the observations is 195. The number of observations is
    (a) 13 (b) 19 (c) 16 (d) 17
13. The median of 11 observations is 10. The number of possible observations in the data which are less than 10 is
    (a) 5 (b) 6 (c) 3 (d) 10
14. If the mode of 22, 21, 23, 24, 21, 20, 23, 26, x and 26 is 23, then $x=$
    (a) 20 (b) 21 (c) 23 (d) 24
15. If the mean of 5, 7, x, 10, 5 and 7 is 7, then $x=$
    (a) 6 (b) 7 (c) 8 (d) 9
16. The mean of p, q and r is same as the mean of q, $2r$ and s. Then which of the following is correct?
    (a) $p=q=r$ (b) $q=r=s$ (c) $q=r$ (d) $p=r+s$
17. The mean of 10, 15, 19, 30, 43, 69 and x is x. Then the median is
    (a) 19 (b) 43 (c) 30 (d) None of these
18. If the mean of 9, 10, 15, x, 6, 8 and 12 is 11. The median of the observations is
    (a) 4 (b) 10 (c) 13 (d) 5
19. The mode of the unimodular data 7, 8, 9, 8, 9, 10, 9, 10, 11, 10, 11, 12 and x is 10. The value of x is
    (a) 10 (b) 9 (c) 8 (d) 11
20. The mean weight of 21 students is 21 kg. If a student weighing 21 kg is removed from the group, then the mean of the remaining students is
    (a) 20 kg (b) 21 kg (c) 19 kg (d) 18 kg

21. There are 7 observations in the data and their mean is 11. If each observation is multiplied by 2, then the mean of new observations is
   (a) 11 (b) 13 (c) 22 (d) 5.5
22. The mean of 10 observations is 15. If one observation 15 is added, then the new mean is
   (a) 16 (b) 11 (c) 10 (d) 15
23. If the median of 10, 12, x, 6, 18 is 10, then which of the following is correct.
   (a) $6 \leq x \leq 10$ (b) $x < 6$ (c) $x > 18$ (d) Either (a) or (b)
24. The mode of the data 9, x, 6, 3, 4, 9, 8, 6, 4, 6 is 6. Which of the following cannot be the value of x?
   (a) 8 (b) 7 (c) 6 (d) 9
25. Which of the following is correct?
   (a) Mode = 2 Median − 3 Mean (b) Mode = 3 Median − Mean
   (c) Mode − Mean = 3 (Median − Mean) (d) Mode − Median = Median − Mean

ANSWSERS

1. (b)	2. (a)	3. (c)	4. (a)	5. (b)	6. (c)	7. (b)
8. (a)	9. (c)	10. (a)	11. (b)	12. (a)	13. (a)	14. (c)
15. (c)	16. (d)	17. (c)	18. (b)	19. (a)	20. (b)	21. (c)
22. (d)	23. (d)	24. (d)	25. (c)			