17

CONSTRUCTIONS

17.1 INTRODUCTION

In the previous class, we have learnt how to draw a line segment, an angle, perpendicular at a point given on a line, perpendicular from a point to a line when the point is outside the line, perpendicular bisector of a line segment, bisector of an angle, a line parallel to a given line etc. In this chapter, we shall learn the construction of triangles when sufficient data is given. In the previous class, we have studied that a triangle has six elements or parts, namely, three sides and three angles. A triangle can always be drawn if we have three elements one of which must be a side.

In the following sections, we shall learn about the simple cases of construction of triangles.

17.2 DRAWING A LINE PARALLEL TO A GIVEN LINE THROUGH A GIVEN POINT OUTSIDE IT

In the previous chapters, we have learnt that if a transversal cuts two parallel lines, then the alternate angles are equal. Therefore, to draw a line parallel to a given line XY through a point P outside it, we proceed as follows:

Steps of Construction

STEP I *Take any point Q on XY.*

STEP II *Join PQ.*

STEP III *With Q as centre, draw an arc cutting XY and PQ at A and B respectively.*

STEP IV *With centre P and the same radius as in step III, draw an arc on the opposite side of QP to cut QP at C.*

STEP V *With centre C and radius equal to AB draw an arc cutting the arc drawn in step IV at D.*

STEP VI *Join PD and produce it in both directions to obtain the required line.*

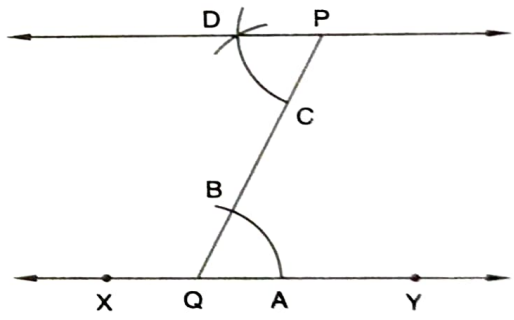

Fig. 1

Validity: Since $\angle DPQ = \angle AQP$ and these are alternate angles. Therefore, $PD \parallel XY$ and PD contains P.

EXERCISE 17.1

1. Draw an $\angle BAC$ of measure 50° such that $AB = 5$ cm and $AC = 7$ cm. Through C draw a line parallel to AB and through B draw a line parallel to AC, intersecting each other at D. Measure BD and CD.
2. Draw a line PQ. Draw another line parallel to PQ at a distance of 3 cm from it.
3. Take any three non-collinear points A, B, C and draw ΔABC. Through each vertex of the triangle, draw a line parallel to the opposite side.

4. Draw two parallel lines at a distance 5 cm apart.

17.3 SSS TRIANGLE CONSTRUCTION

In order to construct a triangle when the lengths of its sides are given, we follow the following steps:

Steps of Construction

STEP I *Draw a line segment of length equal to one of the sides, say BC of the triangle.*

STEP II *With centre B and radius equal to the length of side AB, draw an arc.*

STEP III *With centre C and radius equal to the length of side AC, draw an arc cutting the arc drawn in Step II at A.*

STEP IV *Join AB and AC to obtain the desired triangle ABC.*

The following examples will illustrate the above procedure:

ILLUSTRATIVE EXAMPLES

Example 1 Construct a triangle ABC if the lengths of its sides are given by $AB = 6$ cm, $BC = 7$ cm and $AC = 5$ cm.

Solution To construct the ΔABC, we follow the following steps:

Steps of Construction

STEP I *Draw a line segment $BC = 7$ cm.*

STEP II *With centre B and radius $AB = 6$ cm, draw an arc of the circle.*

STEP III *With centre C and radius $AC = 5$ cm, draw another arc intersecting the arc drawn in step III at A.*

STEP IV *Join AB and AC to obtain the desired triangle.*

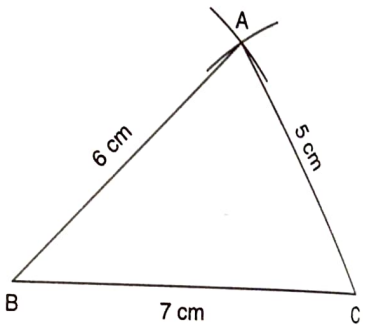

Fig. 2

Example 2 Draw ΔABC in which $AB = 4.5$ cm, $BC = 5$ cm and $CA = 6$ cm. Also, draw the perpendicular bisector of BC.

Solution In order to draw the ΔABC and the perpendicular bisector of BC, we follow the following steps:

Steps of Construction:

STEP I *Draw a line segment $BC = 5$ cm.*

STEP II *With centre B and radius $AB = 4.5$ cm, draw an arc of the circle.*

STEP III *With centre C and radius $AC = 6$ cm, draw an arc intersecting the previously drawn arc at A.*

STEP IV *Join AB and AC to obtain the desired triangle.*

STEP V *With centre B and radius more than $\frac{1}{2}(BC)$, draw two arcs on both sides of BC.*

STEP VI *With centre C and the same radius as in step V, draw two arcs intersecting the arcs drawn in step V at D and E.*

STEP VII *Join DE to obtain the required perpendicular bisector of BC.*

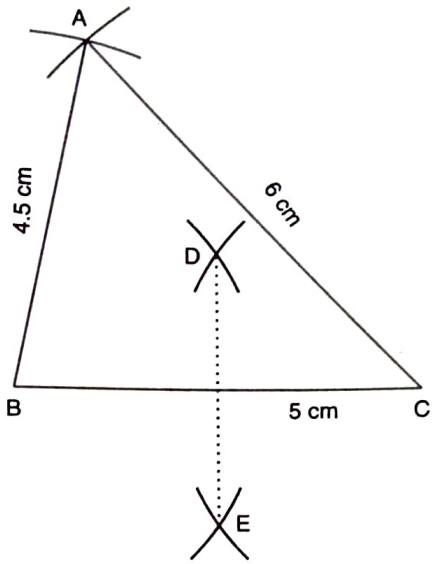

Fig. 3

EXERCISE 17.2

1. Draw Δ *ABC* in which *AB* = 5.5 cm, *BC* = 6 cm and *CA* = 7 cm. Also, draw perpendicular bisector of side *BC*.
2. Draw Δ *PQR* in which *PQ* = 3 cm, *QR* = 4 cm and *RP* = 5 cm. Also, draw the bisector of $\angle Q$.
3. Draw an equilateral triangle one of whose sides is of length 7 cm.
4. Draw a triangle whose sides are of lengths 4 cm, 5 cm and 7 cm. Draw the perpendicular bisector of the largest side.
5. Draw a triangle *ABC* with *AB* = 6 cm, *BC* = 7 cm and *CA* = 8 cm. Using ruler and compass alone, draw (i) the bisector *AD* of $\angle A$ and (ii) perpendicular *AL* from *A* on *BC*.

   Measure *LAD*.
6. Draw Δ *DEF* such that *DE* = *DF* = 4 cm and EF = 6 cm. Measure $\angle E$ and $\angle F$.
7. Draw any trianlge *ABC*. Bisect side *AB* at *D*. Through *D*, draw a line parallel to *BC*, meeting *AC* in *E*. Measure *AE* and *EC*.

17.4 SAS TRIANGLE CONSTRUCTION

In order to construct a triangle when two of its sides, say *AB* and *BC* and the included angle $\angle B$ are given, we follow the following steps:

Steps of Construction

STEP I *Draw $\angle XBY$ of measure equal to that of $\angle B$.*

STEP II *From ray BX, cut off line segment equal to BC.*

STEP III *From ray BY, cut off line segment equal to BA.*

STEP IV *Join AC to obtain the triangle ABC.*

Following examples will illustrate the above procedure.

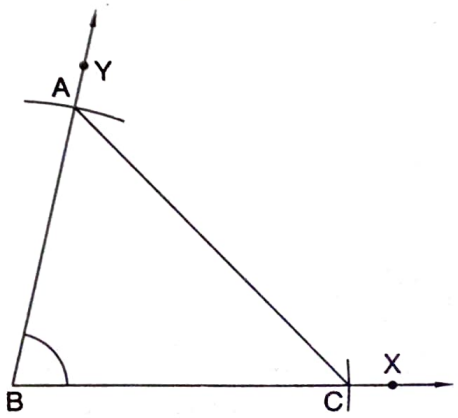

Fig. 4

ILLUSTRATIVE EXAMPLES

Example 1 Construct ΔABC in which $\angle B = 60°$; $AB = 5$ cm and $BC = 6$ cm.

Solution In order to construct the ΔABC, we follow the following steps:

Steps of Construction:

STEP I *Draw $\angle XBY$ of measure 60°.*

STEP II *From ray BX, cut off line segment BC of length 6 cm.*

STEP IV *From ray BY, cut off line segment BA of length 5 cm.*

STEP IV *Join AC to obtain the required triangle ABC, where $\angle B = 60°$, $AB = 5$ cm and $BC = 6$ cm.*

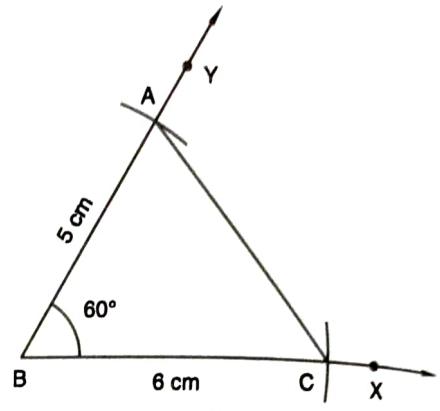

Fig. 5

Example 2 Draw a triangle ABC with $BC = 3.2$ cm, $AB = 3.6$ cm and $\angle B = 120°$. Also draw a perpendiculr from A on BC.

Solution We follow the following steps to construct the required triangle:

Steps of Construction:

STEP I *Draw $\angle XBY$ of measure 120°.*

STEP II *From ray BX, cut off line segment BC of length 3.2 cm.*

STEP III *From ray BY, cut off line segment BA of length 3.6 cm.*

STEP IV *Join CA to obtain the required triangle.*

STEP V *Draw ray BZ.*

STEP VI *With centre A, draw an arc intersecting rays BX and BZ at P and Q respectively.*

STEP VII *With centre P and radius more than $\frac{1}{2}$ (PQ), cut an arc on the opposite of A.*

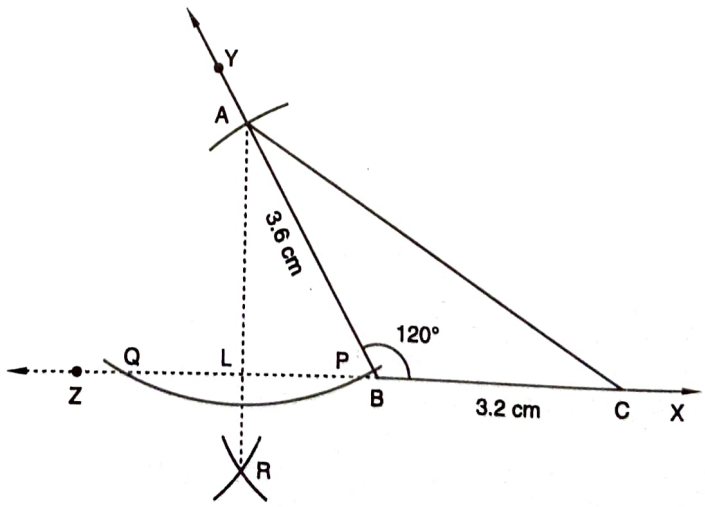

Fig. 6

STEP VIII *With centre Q and the same radius as in step VII, cut an arc which intersects the arc drawn in step VII at R.*

STEP IX *Join AR. If it meets BZ at L, then AL is the required perpendicular from A on BC.*

EXERCISE 17.3

1. Draw ΔABC in which $AB = 3$ cm, $BC = 5$ cm and $\angle B = 70°$.
2. Draw ΔABC in which $\angle A = 70°$, $AB = 4$ cm and $AC = 6$ cm. Measure BC.
3. Draw an isosceles triangle in which each of the equal sides is of length 3 cm and the angle between them is 45°.
4. Draw ΔABC in which $\angle A = 120°$, $AB = AC = 3$ cm. Measure $\angle B$ and $\angle C$.
5. Draw ΔABC in which $\angle C = 90°$ and $AC = BC = 4$ cm.
6. Draw a triangle ABC in which $BC = 4$ cm, $AB = 3$ cm and $\angle B = 45°$. Also, draw a perpendicular from A on BC.
7. Draw a triangle ABC with $AB = 3$ cm, $BC = 4$ cm and $\angle B = 60°$. Also, draw the bisector of angles C and A of the triangle, meeting in a point O. Measure $\angle COA$.

17.5 ASA TRIANGLE CONSTRUCTION

To construct a triangle when two of its angles, say B and C, and the included side BC are given, we proceed as follows (Fig. 6):

Steps of Construction

STEP I *Draw line segment BC.*

STEP II *Draw $\angle CBX$ of measure equal to that of $\angle B$.*

STEP III *Draw $\angle BCY$ with Y on the same side of BC as X, such that its measure is equal to that of $\angle C$. Let BX and CY intersect at A. Then, ΔABC is the required triangle.*

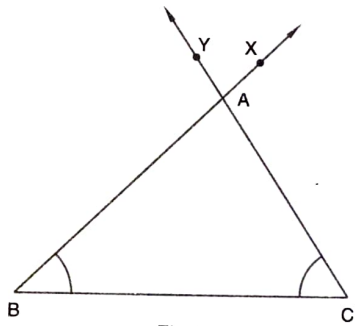

Fig. 7

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Draw ΔABC in which $BC = 6$ cm, $\angle B = 35°$ and $\angle C = 100°$. Measure $\angle A$.

Solution To draw the ΔABC, we follow the following steps:

Steps of Construction:

STEP I *Draw a line segment BC = 6 cm.*

STEP II *Draw $\angle CBX$, such that $\angle CBX = 35°$.*

STEP III *Draw $\angle BCY$ with Y on the same side of BC as X, such that $\angle BCY = 100°$.*

STEP IV *Let BX and CY intersect at A. Then ΔABC is the required triangle.*

By measurement we find that $\angle A = 45°$.

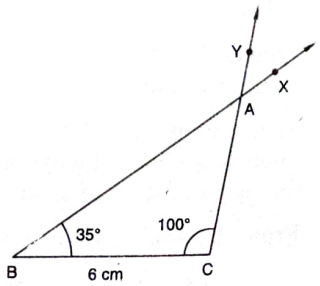

Fig. 8

Example 2 Draw a triangle ABC in which $BC = 5.2$ cm, $\angle B = 60°$ and $\angle A = 100°$.

Solution Here, we are given the side BC, $\angle B$ and $\angle A$. But to draw the triangle, we require $\angle C$.

We know that

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 100° + 60° + \angle C = 180°$

$\Rightarrow \quad 160° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 160° = 20°$

Thus, we have, $BC = 5.2$ cm, $\angle B = 60°$ and $\angle C = 20°$.

Now, to draw the ΔABC, we follow the following steps:

Steps of Construction:

STEP I *Draw a line segment $BC = 5.2$ cm.*

STEP II *Draw $\angle CBX$, such that $\angle CBX = 60°$.*

STEP III *Draw $\angle BCY$, with Y on the same side of BC as X such that $\angle BCY = 20°$*

Let BX and CY intersect at A.

Then ΔABC is the required triangle.

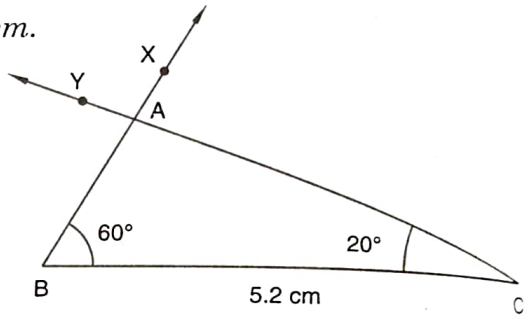

Fig. 9

EXERCISE 17.4

1. Construct ΔABC in which $BC = 4$ cm, $\angle B = 50°$ and $\angle C = 70°$.
2. Draw ΔABC in which $BC = 8$ cm, $\angle B = 50°$ and $\angle A = 50°$.
3. Draw ΔPQR in which $\angle Q = 80°$, $\angle R = 55°$ and $QR = 4.5$ cm. Draw the perpendicular bisector of side QR.
4. Construct ΔABC in which $AB = 6.4$ cm, $\angle A = 45°$ and $\angle B = 60°$.
5. Draw ΔABC in which $AC = 6$ cm, $\angle A = 90°$ and $\angle B = 60°$.

17.6 RHS TRIANGLE CONSTRUCTION

A triangle is said to be a right triangle or a right angled triangle, if one of its three angles is a right angle.

In Fig. 10, ΔABC is a right triangle with $\angle C$ as right angle.

HYPOTENUSE *In a right triangle, the side opposite the right angle is called the hypotenuse of the triangle.*

Each of the other two sides is called a leg or simply, a side of the triangle.

In the above right triangle ABC, $\angle C$ is a right angle. Therefore, AB is the hypotenuse and AC, BC are the sides (or legs) of the right triangle.

From the angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180°$

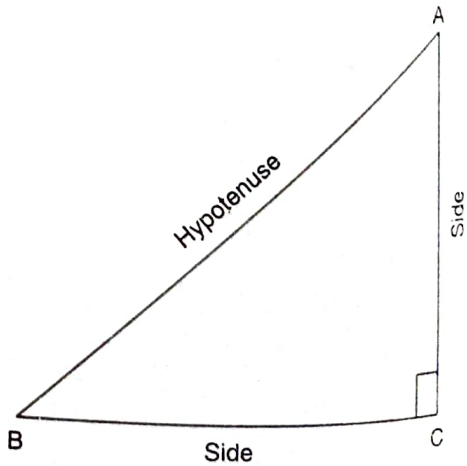

Fig. 10

$\angle A + \angle B + 90° = 180°$

$\Rightarrow \quad \angle A + \angle B = 180° - 90°$

$\Rightarrow \quad \angle A + \angle B = 90°$

$\Rightarrow \quad \angle A$ and $\angle B$ are acute angles.

$\Rightarrow$

Thus, each of the other two angles of a right triangle is acute.

To construct a right triangle ABC right angled at C when its hypotenuse AB and one side BC are given, we follow the following steps:

Steps of Construction

STEP I *Draw a line segment BC of given length.*

STEP II *Draw $\angle BCX$ of measure $90°$.*

STEP III *With centre B and radius equal to the hypotenuse AB, draw an arc of the circle to intersect ray CX at A.*

STEP IV *Join BA to obtain the required triangle ABC.*

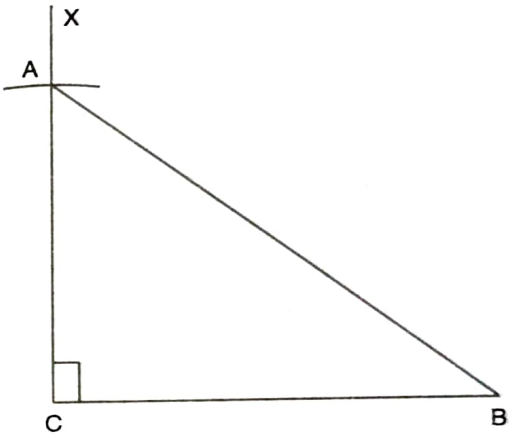

Fig. 11

Following examples will illustrate the above procedure.

ILLUSTRATIVE EXAMPLES

Example 1 Draw triangle ABC with $\angle C$ a right angle, $AB = 6.2$ cm and $BC = 4.5$ cm.

Solution To construct the ΔABC, we follow the following steps:

Steps of Construction:

STEP I *Draw a line segment BC of length 4.5 cm.*

STEP II *Draw $\angle BCX$ of measure $90°$.*

STEP III *With centre B and radius $AB = 6.2$ cm, draw an arc of the circle to intersect ray CX at A.*

STEP IV *Join BA to obtain the desired triangle ABC.*

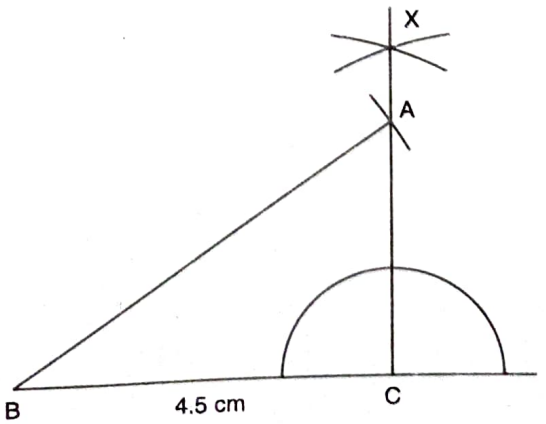

Fig. 12

Example 2 Draw a right triangle having hypotenuse of length 5.4 cm, and one of the acute angles of measure 60° .

Solution Let ΔABC be a right triangle, right angled at C, such that hypotenuse $AB = 5.4$ cm. Further, let $\angle A = 60°$. Then by the angle sum property of ΔABC, we have

$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad 60° + \angle B + 90° = 180°$$

$$\Rightarrow \quad 150° + \angle B = 180°$$

$$\Rightarrow \quad \angle B = 180° - 150° = 30°$$

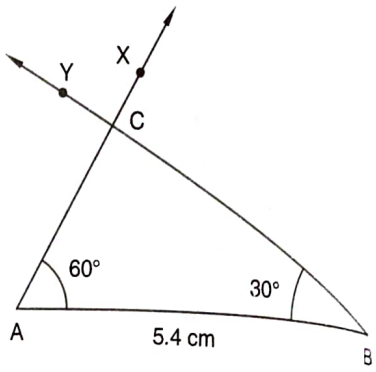

Fig. 13

To draw ΔABC, we follow the following steps:

Steps of Construction:

STEP I *Draw a line segment $AB = 5.4$ cm.*

STEP II *Draw $\angle BAX$ of measure 60°*

STEP III *Draw $\angle ABY$ of measure 30° with Y on the same side of AB as X.*

Let AX and BY intersect at C.

Then, ΔABC is the required triangle.

EXERCISE 17.5

1. Draw a right triangle with hypotenuse of length 5 cm and one side of length 4 cm.
2. Draw a right triangle whose hypotenuse is of length 4 cm and one side is of length 2.5 cm.
3. Draw a right triangle having hypotenuse of length 5.4 cm, and one of the acute angles of measure 30° .
4. Construct a right triangle ABC in which $AB = 5.8$ cm, $BC = 4.5$ cm and $\angle C = 90°$.
5. Construct a right triangle, right angled at C in which $AB = 5.2$ cm and $BC = 4.6$ cm.