MENSURATION - I (Perimeter and area of rectilinear figures)

_{20.1} INTRODUCTION

Class VI, we have learnt about perimeters of plane figures and areas of square and rectangle. We have learnt that the perimeter is the length of the boundary of a closed figure and the magnitude of the measurement of the region enclosed by it is its area. In this chapter, we will learn about perimeter and area of few more plane figures.

20.2 UNITS OF MEASUREMENT OF AREA

 $f_{ollowing}$ are various units of measurement of area:

SQUARE MILLIMETRE The area of a region formed by a square of side 1 millimetre is called a square millimetre and is written as 1 mm^2 .

SQUARE CENTIMETRE The area of a region formed by a square of side 1 cm is called a square centimetre and is written as 1 cm².

 $1 \, \mathrm{cm} = 10 \, \mathrm{mm}$

 $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm} = 10 \times 10 \text{ mm}^2 = 100 \text{ mm}^2$

Thus, $1 \text{ cm}^2 = 100 \text{ mm}^2$

SQUARE DECIMETRE The area of a region formed by a square of side 1 decimetre is called a square decimetre, written as $1 dm^2$.

 $1 \, \mathrm{dm} = 10 \, \mathrm{cm}$

 $1 \text{ dm}^2 = 10 \times 10 \text{ cm}^2 = 100 \text{ cm}^2$

Also, $1 \text{ dm}^2 = 100 \times 100 \text{ mm}^2$ $= 10000 \, \text{mm}^2$

 $[:: 1 \text{ cm}^2 = 100 \text{ mm}^2]$

SQUARE METRE The area of a region formed by a square of side 1 metre is called a square metre, written as $1 m^2$.

 $1 \, \text{m} = 100 \, \text{cm}$

 $1 \text{ m}^2 = 100 \times 100 \text{ cm}^2 = 10000 \text{ cm}^2$

Also, 1 m = 10 dm

 $1 \text{ m}^2 = 10 \times 10 \text{ dm}^2 = 100 \text{ dm}^2$

and, $1 \text{ m}^2 = 100 \times 10000 \text{ mm}^2$

 $[:: 1 \text{ dm}^2 = 10000 \text{ mm}^2]$

 $= 1000000 \,\mathrm{mm}^2$

ARE The area of a region formed by a square of side one decametre (1 dam) is called an are, written as 1 dam² or 1 are.

 $1 \, \text{dam} = 10 \, \text{m}$

 $1 \, dam^2 = 10 \times 10 \, m^2 = 100 \, m^2$

or,
$$1 \text{ are } = 100 \text{ m}^2$$

Also, $1 \text{ m}^2 = 100 \text{ dm}^2$
 $\therefore 1 \text{ are } = 100 \times 100 \text{ dm}^2$
 $= 10^4 \text{ dm}^2$
 $= 10^6 \text{ cm}^2$ $\left[\because 1 \text{ dm}^2 = 100 \text{ cm}^2\right]$

HECTARE The area of a region formed by a square of side 1 hectometre (1 hm) is called a hectare, written as 1 hm^2 .

∴
$$1 \text{ hm} = 100 \text{ m}$$

∴ $1 \text{ hm}^2 = 100 \times 100 \text{ m}^2$
 $= 10^4 \text{ m}^2$

or, $1 \text{ hectare} = 10^4 \text{ m}^2$

Also,
$$100 \text{ m}^2 = 1 \text{ are}$$

$$\therefore$$
 1 hectare = 100 are or, 1 ha = 100 a.

SQUARE KILOMETRE The area of a region formed by a square of side 1 km is called a square kilometer, written as 1 km^2 .

$$∴ 1 \text{ km} = 1000 \text{ m}$$

∴ $1 \text{ km}^2 = 1000 \times 1000 \text{ m}^2 = 10^6 \text{ m}^2$.

$$100 \,\mathrm{m}^2 = 1 \,\mathrm{are}$$

$$\therefore$$
 1 km² = $\frac{1000000}{100}$ are = 10000 are

Also, 1 ha =
$$10000 \,\mathrm{m}^2$$

$$\therefore 1 \text{ km}^2 = \frac{1000000}{10000} \text{ ha} = 100 \text{ ha}$$

In the above discussion, we have introduced various standard units each of which can be converted into others. The following table provides ready reference for the conversion of these units.

Conversion of units

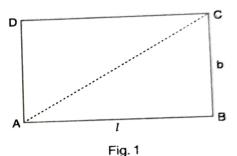
Units of Length	Units of Area
1 cm = 10 mm	$1 \text{ cm}^2 = (10 \times 10) \text{ mm}^2 = 100 \text{ mm}^2$
1 m = 10 dm	$1 \text{ m}^2 = (10 \times 10) \text{ dm}^2 = 100 \text{ dm}^2$
$1 \mathrm{dm} = 10 \mathrm{cm}$	$1 dm^2 = (10 \times 10) cm^2 = 100 cm^2$
1 m = 100 cm	$1 \text{ m}^2 = (100 \times 100) \text{ cm}^2 = 10000 \text{ cm}^2$
1 dam = 10 m	$1 dam^2 = (10 \times 10) m^2 = 100 m^2 = 1 are$
1 hm = 100 m	$1 \text{ hm}^2 = (100 \times 100) \text{ m}^2 = 10000 \text{ m}^2 = 1 \text{ hectare}$
1 km = 10 hm	$1 \text{ km}^2 = (10 \times 10) \text{ hm}^2 = 100 \text{ hm}^2$
1 km = 1000 m	$1 \text{ km}^2 = (1000 \times 1000) \text{ m}^2 = 1000000 \text{ m}^2 = 100 \text{ hectare}$

PERIMETER AND AREA OF SQUARES AND RECTANGLES M.3 VI, we have learnt about the perimeters and areas of rectangles and squares. In class we will review the problems related to class vi, we will review the problems related to them.

perimeter and area of a rectangle

perimeter = 2(l + b)

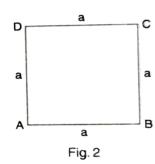
- (i) Perimeter = 2(l + b)
- (ii) Area = $l \times b$
- (iii) Length = $\frac{Area}{Breadth}$
- (iv) Breadth = $\frac{\text{Area}}{\text{Length}}$
- (v) Diagonal = $\sqrt{l^2 + b^2}$



ii) Perimeter and area of a square

 $_{\text{let }a}^{\text{re}}$ be the length of each side of a square. Then,

- (i) Perimeter = 4a
- (ii) Area = α^2 , Area = $\left(\frac{\text{Perimeter}}{4}\right)^2$
- (iii) Side of the square = $\sqrt{\text{Area}}$
- (iv) Diagonal = $\sqrt{2} a$
- (v) Area = $\frac{1}{2}$ (Diagonal)²



(iii) Area of four walls of a room

Let l, b and h denote respectively the length, breadth and height of a room. Then.

- Area of the 4 walls = $2(l+b) \times h$ (i)
- Diagonal of the room $=\sqrt{l^2+b^2+h^2}$ (ii)

ILLUSTRATIVE EXAMPLES

Find the area, in square metres, of a rectangle whose Example 1

- (i) length = 4.5 m, breadth = 1.6 m(ii) length = 720 cm, breadth = 25 cm
- (iii) length = 3 dm 6 cm, breadth = 2 dm 9 cm.

Solution (i) We have,

Length = 4.5 m and, Breadth = 1.6 m

- Area of the rectangle = Length \times Breadth = 4.5 \times 1.6 m² = 7.20 m²
- (ii) We have,

Length =
$$720 \text{ cm} = \frac{720}{100} \text{ m} = 7.2 \text{ m}$$
, Breadth = $25 \text{ cm} = \frac{25}{100} \text{ m} = 0.25 \text{ m}$

Area of the rectangle = Length × Breadth = $7.2 \times 0.25 \text{ m}^2 = 1.8 \text{ m}^2$

(iii) We have,

Length =
$$3 \text{ dm } 6 \text{ cm} = (3 \times 10 + 6) \text{ cm} = 36 \text{ cm} = 0.36 \text{ m}$$

$$[\because 1 \, dm = 10 \, cm]$$

Breadth =
$$2 \text{ dm } 9 \text{ cm} = (2 \times 10 + 9) \text{ cm} = 29 \text{ cm} = 0.29 \text{ m}$$

$$\therefore$$
 Area of the rectangle = $0.36 \times 0.29 \,\mathrm{m}^2 = 0.1044 \,\mathrm{m}^2$

Find the area, in square centimetres, of a square whose side is Example 2

Solution

(i) We have.

Side of the square
$$= 2.4 \text{ dm} = (2.4 \times 10) \text{ cm} = 24 \text{ cm}$$

$$\therefore$$
 Area of the square = $(\text{Side})^2 = (24)^2 \text{ cm}^2 = 576 \text{ cm}^2$

(ii) We have,

Side of the square
$$= 20 \text{ m m} = 2 \text{ cm}$$

$$[:: 10 \text{ m m} = 1 \text{ cm}]$$

$$\therefore$$
 Area of the square = $(\text{Side})^2 = (2)^2 \text{ cm}^2 = 4 \text{ cm}^2$

Find the area, in hectare, of a field whose length is 240 m and breadth 110 m. Example 3 Solution We have,

Length of the field = 240 m, Breadth of the field = 110 m

$$\therefore$$
 Area of the field = $(240 \times 110) \text{ m}^2$

$$= 26400 \,\mathrm{m}^2$$

$$= \frac{26400}{10000} \text{ hectare} = 2.64 \text{ hectare} \quad [\because 10000 \text{ m}^2 = 1 \text{ hectare}]$$

Find the area of a rectangular plot one side of which is 48 m and its diagonal Example 4 $50 \, \mathrm{m}$

Let the other side be x metres. Since $\triangle ABC$ is a right triangle. Therefore, Solution

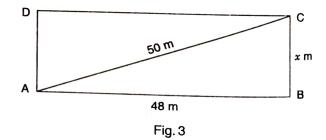
$$AC^{2} = AB^{2} + BC^{2}$$

$$\Rightarrow 50^{2} = 48^{2} + x^{2}$$

$$\Rightarrow x^{2} = (50)^{2} - (48)^{2}$$

$$\Rightarrow x^{2} = (50 + 48)(50 - 48)$$

$$\Rightarrow x^{2} = 98 \times 2$$



$$\Rightarrow x^2 = 98 \times 1$$

$$\Rightarrow$$
 $x^2 = 14^2$

$$\Rightarrow$$
 $x = 14.$

Thus, the other side of the rectangle is 14 m.

Area of the rectangle = (48×14) m² = 672 m²

The length and breadth of a rectangular piece of land are 500 m and 300 m $\,$ Example 5 respectively. Find:

- (i) its area
- (ii) its cost, if 1 m² of the land costs Rs 10000.

solution

- (i) We have, l = Length = 500 m, b = Breadth = 300 m
- Area = $l \times b = 500 \times 300 \,\mathrm{m}^2 = 150000 \,\mathrm{m}^2$
- (ii) Since 1 m² of the land costs Rs 10000.
- Cost of 150000 m² of land = $Rs(150000 \times 10000)$

= Rs 15000000000 = Rs 1500 millions

Example 6 solution

Find the area of a square park whose perimeter is 320 m.

Let the length of each side of the square park be a metre. Then,

Perimeter = 320 m

$$4a = 320$$

$$\Rightarrow a = \frac{320}{4} = 80 \text{ m}$$

[: Perimeter of a square = $4 \times \text{Side}$]

$$\therefore$$
 Area = a^2 = (80 × 80) m² = 6400 m²

Example 7

Find the breadth of a rectangular plot of land, if its area is 440 sq. m and length is 22 m. Also, find its perimeter.

Solution

We have, l = Length of the plot = 22 m, Area of the plot = 440 sq. metre

Let the breadth of the plot be b metres. Then,

Breadth =
$$\frac{\text{Area}}{\text{Length}}$$
 $b = \frac{440}{22} = 20 \text{ m}$

Perimeter = $2(l+b) = 2 \times (22+20) \text{ m} = 2 \times 42 \text{ m} 84 \text{ m}$.

Hence, the breadth of the plot is 20 m and the perimeter is 84 m.

Example 8

The perimeter of a rectangular sheet is 100 cm. If the length is 35 cm, find its breadth. Also, find the area.

Solution

 $l={
m Length}$ of the sheet = 35 cm, $P={
m Perimeter}$ of the sheet = 100 cm

Let the breadth of the sheet be b cm. Then,

Perimeter = $2 \times (l + b)$

$$100 = 2 \times (35 + b)$$

$$100 = 70 + 2b$$

$$2b = 100 - 70 \Rightarrow 2b = 30 \Rightarrow b = 15 \text{ cm}$$

Area of sheet = $l \times b = 35 \times 15 \text{ cm}^2 = 525 \text{ cm}^2$

Example 9

The areas of a square and rectangle are equal. If the side of the square is 40 cm and the breadth of the rectangle is 25 cm, find the length of the rectangle. Also, find the permeter of the rectangle.

Solution

We have,

Side of the square = 40 cm.

 \therefore Area of the square = $(\text{Side})^2 = 40 \text{ cm} \times 40 \text{ cm} = 1600 \text{ cm}^2$

 \Rightarrow Area of the rectangle = 1600 cm²

: Area of the rectangle: Area of the square

It is given that the breadth of the rectangle is 25 cm.

$$\therefore \quad \text{Length} = \frac{\text{Area}}{\text{Breadth}} = \frac{1600}{25} \text{ cm} = 64 \text{ cm}$$

Perimeter of the rectangle = 2 (Length + Breadth)

$$= 2 \times (64 + 25) \text{ cm} = 2 \times 89 \text{ cm} = 178 \text{ cm}$$

Example 10 Anu wants to fence the garden infront of her house as shown in Fig. 4 on three sides. Find the cost of fencing at the rate of Rs 150 per metre.

Solution Anu wants to fence the garden on three sides, as one of the side of the house will serve the purpose of fence. Therefore, length of the fence required is the perimeter of the garden excluding the one side facing the house.

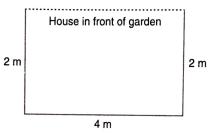


Fig. 4

$$\therefore$$
 Length of fence = $4m+2m+2m=8m$

Hence, cost of fencing = $Rs(8 \times 150) = Rs 1200$

Example 11 A wire is in the shape of a square of side 10 cm. If the wire is rebent into a rectangle of length 12 cm, find its breadth. Which encloses more area, the square or the rectangle?

Solution We have,

Side of the square = 10 cm.

Length of the wire = Perimeter of the square = $4 \times \text{Side} = 4 \times 10 \text{ cm} = 40 \text{ cm}$ l = Length of the rectangle = 12 cm.

Let b be the breadth of the rectangle.

Now,

Perimeter of the rectangle = Perimeter of the square

$$\Rightarrow$$
 2 $(l+b) = 40$

$$\Rightarrow$$
 2 (12 + b) = 40

$$\Rightarrow 24 + 2b = 40 \Rightarrow 2b = 40 - 24 \Rightarrow 2b = 16 \Rightarrow b = \frac{16}{2}$$
 cm = 8 cm

So, Breadth of the rectangle = 8 cm.

Area of the square = $(\text{Side})^2 = 10 \text{ cm} \times 10 \text{ cm} = 100 \text{ cm}^2$

Area of the rectangle = $l \times b = 12 \text{ cm} \times 8 \text{ cm} = 96 \text{ cm}^2$

the square encloses more area even though its perimeter is the same as that of the $\frac{120 \text{ m} \times 60 \text{ cm}^2}{20 \text{ le}}$.

A door of length 1 m and breadth 0.5 m is on a wall. The length of the wall is 4.5 m and the breadth is 2.6 m. 4.5 m and the breadth is 3.6 m as shown in Fig. 5. Find the cost of white washing the wall, if the rate of white washing the wall is Rs 20 per m². We have,

l = Length of the wall = 4.5 m, b = Breadth of the wall = 3.6 m

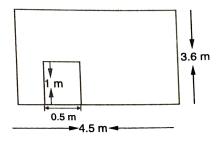


Fig. 5

Area of the wall = $l \times b = 4.5 \text{ m} \times 3.6 \text{ m} = 16.2 \text{ m}^2$

Area of the door = $1 \text{ m} \times 0.5 \text{ m} = 0.5 \text{ m}^2$

Area to be white washed = $16.2 \text{ m}^2 - 0.5 \text{ m}^2 = 15.7 \text{ m}^2$

Cost of white washing the wall = $Rs(15.7 \times 20) = Rs(314)$.

Example 13 A wall 4.84 m long and 3.1 m high is covered with rectangular tiles of size 22 cm by 10 cm. Find the total cost of the tiles at the rate of Rs 1.50 per tile.

We have, Solution

√o₩,

solution

Area of the wall = $4.84 \times 3.1 \,\mathrm{m}^2$ $= 15.004 \,\mathrm{m}^2$ $1 \cdot 1 \text{ m}^2 = 10000 \text{ cm}^2$ $=15.004\times10000$ cm² $= 150040 \, \text{cm}^2$

Area of one tile $=22\times10\,\mathrm{cm}^2=220\,\mathrm{cm}^2$

Number of tiles = $\frac{\text{Area of the wall}}{\text{Area of one tile}} = \frac{150040}{220} = 682$ Cost of one tile = Rs 1.50

Total cost = Number of tiles \times Cost of one tile = Rs (682 \times 1.50) = Rs 1023

Example 14 A black board of sides 5 m 20 cm and 3 m 40 cm is to be painted, find the cost at the rate of Rs 12 per square metre.

Solution We have,

Length of the black board = 5 m 20 cm = 5.20 m

Breadth of the black board = 3m 40 cm = 3.40 m

 \therefore Area of the black board = $(5.20 \times 3.40) \text{ m}^2 = 17.68 \text{ m}^2$

Rate of painting the black board = ₹12 per m²

∴ Cost of painting the black board = ₹ $(17.68 \times 12) = ₹ 212.16$

Example 15 Find the height of the wall whose length is 4 m and which can be covered by 2400 tiles of size 25 cm by 20 cm.

Solution Area of a tile = $25 \times 20 \text{ cm}^2 = 500 \text{ cm}^2$

 \therefore Area of 2400 tiles = 2400 × 500 cm²

=
$$1200000 \text{ cm}^2 = \frac{1200000}{10000} \text{m}^2$$
 [:: $10000 \text{ cm}^2 = 1 \text{ m}^2$]

$$= 120 \,\mathrm{m}^2$$

Let the height of the wall be h metres. Then,

Area of the wall = $4h \text{ m}^2$

Since 2400 tiles completely cover the wall.

 \therefore Area of the wall = Area of 2400 tiles

$$\Rightarrow$$
 4 $h = 120$

$$\Rightarrow \frac{4h}{4} = \frac{120}{4} = 30$$

[Dividing both sides by 4]

Hence, the height of the wall is 30 metres.

EXERCISE 20.1

- 1. Find the area, in square metres, of a rectangle whose
 - (i) Length = 5.5 m, breadth = 2.4 m
- (ii) Length = 180 cm, breadth = 150 cm
- 2. Find the area, in square centimetres, of a square whose side is
 - (i) 2.6 cm
- (ii) 1.2 dm
- 3. Find in square metres, the area of a square of side 16.5 dam.
- 4. Find the area of a rectangular field in ares whose sides are:
 - (i) 200 m and 125 m
- (ii) 75 m 5 dm and 120 m
- 5. Find the area of a rectangular field in hectares whose sides are:
 - (i) 125 m and 400 m
- (ii) 75 m 5 dm and 120 m
- 6. A door of dimensions 3 m \times 2m is on the wall of dimension 10 m \times 10 m. Find the cost of painting the wall if rate of painting is $\stackrel{?}{=}$ 2.50 per sq. m.
- 7. A wire is in the shape of a rectangle. Its length is 40 cm and breadth is 22 cm. If the same wire is bent in the shape of a square, what will be the measure of each side. Also, find which shape encloses more area?
- 8. How many square metres of glass will be required for a window, which has 12 panes, each pane measuring 25 cm by 16 cm?
- 9. A marble tile measures 10 cm × 12 cm. How many tiles will be required to cover a wall of size 3 m × 4 m? Also, find the total cost of the tiles at the rate of ₹ 2 per tile.
- 10. A table top is 9 dm 5 cm long 6 dm 5 cm broad. What will be the cost to polish it at the rate of 20 paise per square centimetre?

- A room is 9.68 m long and 6.2 m wide. Its floor is to be covered with rectangular tiles of size 11. 22 cm by 10 cm. Find the total cost of the tiles at the rate of D. 6.7 A room by 10 cm. Find the total cost of the tiles at the rate of Rs 2.50 per tile.
- One side of a square field is 179 m. Find the cost of raising a lawn on the field at the rate of a square metre. Rs 1.50 per square metre.
- A rectangular field is measured 290 m by 210 m. How long will it take for a girl to go two times and the field, if she walks at the rate of 1.5 mind the A round the field, if she walks at the rate of 1.5 m/sec?
- A corridor of a school is 8 m long and 6 m wide. It is to be covered with convas sheets. If the A compared to available canvas sheets have the size 2 m x 1 m, find the cost of canvas sheets required to cover the corridor at the rate of Rs 8 per sheet.
- The length and breadth of a playground are 62 m 60 cm and 25 m 40 cm respectively. Find the cost of turfing it at Rs 2.50 per square metre. How long will a man take to go three times round the field, if he walks at the rate of 2 metres per second.
- 16. A lane 180 m long and 5 m wide is to be paved with bricks of length 20 cm and breadth 15 cm. Find the cost of bricks that are required, at the rate of Rs 750 per thousand.
- 17. How many envelopes can be made out of a sheet of paper 125 cm by 85 cm; supposing one envelope requires a piece of paper of size 17 cm by 5 cm?
- 18. The width of a cloth is 170 cm. Calculate the length of the cloth required to make 25 diapers, if each diaper requires a piece of cloth of size 50 cm by 17 cm.
- 19. The carpet for a room 6.6 m by 5.6 m costs Rs 3960 and it was made from a roll 70 cm wide. Find the cost of the carpet per metre.
- 20. A room is 9 m long, 8 m broad and 6.5 m high. It has one door of dimensions 2 m \times 1.5 m and three windows each of dimensions 1.5 m \times 1 m. Find the cost of white washing the walls at Rs 3.80 per square metre.
- 21. A hall 36 m long and 24 m broad allowing 80 m² for doors and windows, the cost of papering the walls at Rs 8.40 per m² is Rs 9408. Find the height of the hall.

ANSWER

- 3. 27225 m² (ii) 144 cm² 2. (i) 6.76 cm² (ii) $2.7 \, \text{m}^2$ (i) 13.2 m² (ii) 90.6 ares 5. (i) 5 hectares (ii) 0.906 hectares 6. Rs. 235 4. (i) 250 ares 10. Rs 1235 9. 1000, Rs 2000 7. 31 cm, Square/8. 0.48 m²
- 13. $22\frac{2}{9}$ minutes 14. Rs 192 12. Rs 48061.50 11. Rs 6820
- 17, 125 16. Rs 22500 Rs 3975.10,4 minutes 24 seconds 15.
- 21. 10 m 20. Rs 811.30 19. Rs 75 per metre 18. 125 cm

20.4 AREA BETWEEN RECTANGLES

In this section, we shall apply the formulae for the areas of a rectangle and a square to determine the areas of the paths around (in or outside) a rectangle and also of the central paths.

Following examples will illustrate the same.

ILLUSTRATIVE EXAMPLES

A rectangular grassy lawn measuring 30 m by 28 m is to be surrounded A rectangular grass, such is 2 m wide. Find the cost of levelling the path at externally by a path which is 2 m Example 1 the rate of Rs 5 per square metre.

Solution

Let ABCD be the grassy lawn, and let PQRS be the external boundaries of the path.

We have,

Length AB = 30 m, Breadth BC = 28 m

 \therefore Area of lawn $ABCD = 30 \times 28 \text{ m}^2 = 840 \text{ m}^2$

Length
$$PQ = (30 \text{ m} + 2 \text{ m} + 2 \text{ m}) = 34 \text{ m}$$

Breadth
$$QR = 28 \text{ m} + 2 \text{ m} + 2 \text{ m} = 32 \text{ m}$$

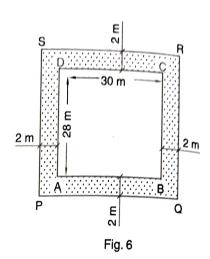
$$\therefore$$
 Area $PQRS = 34 \times 32 \,\text{m}^2 = 1088 \,\text{m}^2$

Now,

Area of the path

= Area *PQRS* – Area of the lawn

$$= (1088 - 840) \,\mathrm{m}^2 = 248 \,\mathrm{m}^2$$



 \therefore Cost of levelling the path = Rs (248×5) = Rs 1240.

Example 2 A rectangular park is 45 m long and 30 m wide. A path 2.5 m wide is constructed outside the park. Find the area of the path.

Solution

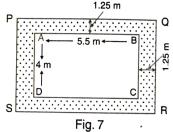
Let *ABCD* be the rectangular park and the shaded region represents the 2.5 wide path to be constructed outside the park.

We have,

$$PQ = (45 + 2.5 + 2.5) \,\mathrm{m} = 50 \,\mathrm{m}$$

$$PS = (30 + 2.5 + 2.5) \,\mathrm{m} = 35 \,\mathrm{m}$$

Clearly,



Area of the path = Area of rectangle PQRS - Area of rectangle ABCD = $(50\times35)\,\mathrm{m}^2-(45\times30)\,\mathrm{m}^2$, = $1750\,\mathrm{m}^2-1350\,\mathrm{m}^2=400\,\mathrm{m}^2$

Example 3 A verandah 1.25 m wide is constructed all along the outside of a room 5.5 m long and 4 m wide. Find (i) the area of the verandah (ii) the cost of cementing the floor of the verandah at the rate of Rs 200 per m².

Solution

Let ABCD denote the floor of the room and the shaded region represents the floor of the 1.25 m wide verandah.

We have,

$$PQ = 5.5 + 1.25 + 1.25$$
) m = 8 m and $PS = (4 + 1.25 + 1.25)$ m = 6.5 m We have,

Area of verandah = Area of rectangle
$$PQRS$$
 - Area of rectangle $ABCD$
= $PQ \times PS - AB \times AD$
= $8 \times 6.5 \,\text{m}^2 - 5.5 \times 4 \,\text{m}^2 = (52 - 22) \,\text{m}^2 = 30 \,\text{m}^2$

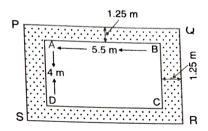


Fig. 8

(ii) We have,

Rate of cementing the floor of verandah = $Rs 200 per m^2$

 \therefore Cost of cementing the floor of verandah = Rs (30×200) = Rs 6000

A path 1 m wide is built along the border inside a square garden of side 30 m. Find (i) the area of the path (ii) the cost of planting the grass in the remaining portion of the garden at the rate of Rs 40 per m².

Let *ABCD* be the square of side 30 m and the shaded region represents 1 m wide path which is built along the border inside the garden.

We have,

_{xample} 4

solution

$$PQ = (30-1-1) \text{ m} = 28 \text{ m}$$
, $PS = (30-1-1) \text{ m} = 28 \text{ m}$

(i) Area of the path

= Area of square ABCD – Area of square PQRS

$$= (30 \times 30) \, \text{m}^2 - (28 \times 28) \, \text{m}^2 = (900 - 784) \, \text{m}^2 = 116 \, \text{m}^2$$

(ii) Area of the remaining portion in which the grass is planted

= Area of square $PQRS = 28 \times 28 \,\mathrm{m}^2 = 784 \,\mathrm{m}^2$

Since the grass is planted in the garden at the rate of Rs 40 per m².

 \therefore Cost of planting the grass in the region $PQRS = Rs(40 \times 784) = Rs 31360$

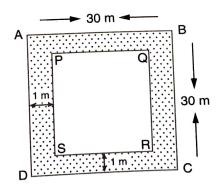


Fig. 9

Example 5

A grassy plot is $80~\text{m} \times 60~\text{m}$. Two cross paths each 4~m wide are constructed at right angles through the centre of the field, such that each path is parallel to one of the sides of the rectangle. Find the total area used as path. Also, find the cost of gravelling them at Rs 5 per square metre (Fig.11)

Solution

Let ABCD and EFGH be the cross paths.

We have, AB = 80 m and BC = 4 m.

a

F G 80 m

Fig. 10

В

s

P

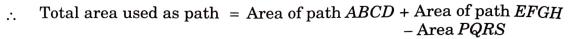
Area of path $ABCD = (80 \times 4) \,\text{m}^2 = 320 \,\text{m}^2$ Again,

 $EF = 60 \,\mathrm{m}$ and $FG = 4 \,\mathrm{m}$

Area of path $EFGH = (60 \times 4) \text{ m}^2 = 240 \text{ m}^2$. Clearly, area PQRS is common to both the paths.

We have,

Area
$$PQRS = (4 \times 4) \,\text{m}^2 = 16 \,\text{m}^2$$



$$= (320 + 240 - 16) \,\mathrm{m}^2 = 544 \,\mathrm{m}^2$$

m 09

Rate of gravelling the path = Rs 5 per square metre

Total cost of gravelling the path = $Rs(5 \times 544) = Rs 2720$

A rectangular lawn is 30 m by 20 m. It has two roads each 2 m wide running in Example 6 the middle of it, one parallel to the length and the other parallel to the breadth. Find the area of the roads.

Solution

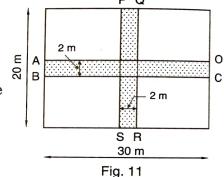
Area of road
$$ABCD = 30 \times 2 = 60 \,\mathrm{m}^2$$

Area of road
$$PQRS = 20 \times 2 = 40 \text{ m}^2$$

Clearly, the shaded portion is common to the two roads.

Area of the shaded portion = $2 \times 2 = 4 \text{ m}^2$

Hence, Area of the roads = $60 + 40 - 4 = 96 \,\mathrm{m}^2$.



A table cover, 4 m × 2 m, is spread on a meeting table. If 25 cm of the table Example 7 cover is hanging all around the table, find the cost of polishing the table top at Rs 2.25 per square metre.

Solution

To find the cost of polishing the table top we have to find its area for which we require its length and breadth.

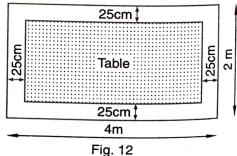
We have,

Length of the cloth = 4 m

Breadth of the cloth = 2 m

Since 25 cm width of cloth is outside the table on each side.

- Length of the table = $4 2 \times 0.25 = 3.5 \text{ m}$ Breadth of the table = $2-2\times0.25 = 1.5 \text{ m}$
- Area of the top of the table = (3.5×1.5) m²



It is given that the cost of polishing the table top is at the rate of Rs 2.25 per square metre. Therefore,

Cost of polishing the top = Area × Rate per sq. metre

$$= \text{Rs} (3.5 \times 1.5 \times 2.25)$$

$$= \operatorname{Rs}\left(\frac{7}{2} \times \frac{3}{2} \times \frac{9}{4}\right) = \operatorname{Rs} 11.81$$

The length and breadth of a park are in the ratio 2:1 and its perimeter is 240 m. A path of 2 m wide runs inside it, along boundary. Find the cost of paving the path at Rs 3 per m².

Let ABCD be the park whose length AB and breadth AD are in the ratio 2:1. Let the shaded region represent the 2 m inside path that runs inside the park ABCD. Since length and breadth of the park are in the ratio 2:1. So, let the length and breadth of the park be 2x and x metre respectively. Then,

Perimeter =
$$2(2x + x)$$

But, Perimeter = $240 \, \text{m}$

$$2(2x + x) = 240$$

$$\Rightarrow$$
 $6x = 240$

_{Exam}ple 8

solution

$$\Rightarrow \quad x = \frac{240}{6} = 40$$

Length $AB = (2 \times 40) \,\mathrm{m} = 80 \,\mathrm{m}$

and Breadth = AD = 40 m

We have,

$$PQ = (80 - 2 - 2) \,\mathrm{m} = 76 \,\mathrm{m}$$
 and, $PR = (40 - 2 - 2) \,\mathrm{m} = 36 \,\mathrm{m}$

Area of the path = Area of rectangle ABCD – Area of rectangle PQRS

=
$$(80 \times 40 - 76 \times 36) \,\mathrm{m}^2 = (3200 - 2736) \,\mathrm{m}^2 = 464 \,\mathrm{m}^2$$

Hence, cost of paving the path = $Rs(464 \times 3) = Rs1392$

EXERCISE 20.2

- 1. A rectangular grassy lawn measuring 40 m by 25 m is to be surrounded externally by a path which is 2 m wide. Calcualte the cost of levelling the path at the rate of Rs 8.25 per square metre.
- One metre wide path is built inside a square park of side 30 m along its sides. The remaining part of the park is covered by grass. If the total cost of covering by grass is Rs 1176, find the rate per square metre at which the park is covered by the grass.
- 3. Through a rectangular field of sides 90 m × 60 m, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of the field. If the width of the roads is 3 m, find the total area covered by the two roads.
- 4. From a rectangular sheet of tin, of size 100 cm by 80 cm, are cut four squares of side 10 cm from each corner. Find the area of the remaining sheet.

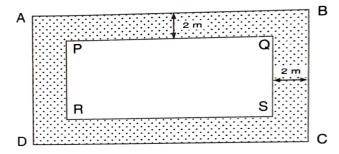


Fig. 13

16. 196 m²

- 5. A painting 8 cm long and 5 cm wide is painted on a cardboard such that there is a margin of 1.5 cm along each of its sides. Find the total area of the margin.
- 6. Rakesh has a rectangular field of length 80 m and breadth 60 m. In it, he wants to make a garden 10 m long and 4 m broad at one of the corners and at another corner, he wants to grow flowers in two floor-beds each of size 4 m by 1.5 m. In the remaining part of the field, he wants to apply manures. Find the cost of applying the manures at the rate of Rs 300 per are.
- 7. Each side of a square flower bed is 2 m 80 cm long. It is extended by digging a strip 30 cm wide all around it. Find the area of the enlarged flower bed and also the increase in the area of the flower bed.
- 8. A room 5 m long and 4 m wide is surrounded by a verandah. If the verandah occupies an area of $22 \, \text{m}^2$, find the width of the varandah.
- 9. A square lawn has a 2 m wide path surrounding it. If the area of the path is 136 m², find the area of the lawn.
- 10. A poster of size 10 cm by 8 cm is pasted on a sheet of cardboard such that there is a margin of width 1.75 cm along each side of the poster. Find (i) the total area of the margin (ii) the cost of the cardboard used at the rate of Re 0.60 per cm².
- 11. A rectangular field is 50 m by 40 m. It has two roads through its centre, running parallel to its sides. The width of the longer and shorter roads are 1.8 m and 2.5 m respectively. Find the area of the roads and the area of the remaining portion of the field.
- 12. There is a rectangular field of size $94 \text{ m} \times 32 \text{ m}$. Three roads each of 2 m width pass through the field such that two roads are parallel to the breadth of the field and the third is parallel to the length. Calculate: (i) area of the field covered by the three roads (ii) area of the field not covered by the roads.
- 13. A school has a hall which is 22 m long and 15.5 m broad. A carpet is laid inside the hall leaving all around a margin of 75 cm from the walls. Find the area of the carpet and the area of the strip left uncoverd. If the width of the carpet is 82 cm, find the cost at the rate of Rs 18 per metre.
- 14. Two cross roads, each of width 5 m, run at right angles through the centre of a rectangular park of length 70 m and breadth 45 m parallel to its sides. Find the area of the roads. Also, find the cost of constructing the roads at the rate of Rs 105 per m².
- 15. The length and breadth of a rectangular park are in the ratio 5:2. A 2.5 m wide path running all around the outside the park has an area $305 \, \text{m}^2$. Find the dimensions of the park.
- 16. A square lawn is surrounded by a path 2.5 m wide. If the area of the path is 165 m², find the area of the lawn.

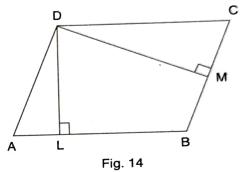
ANSWERS

2. Rs 1.50 per square metre 1. Rs 2277 3. 441 m² 4. 7600 cm² 5. 30 cm² 6. Rs 14244 7. 11.56 m², 3.72 m² 8. 1 m 9. 225 m² 10. (i) 75.25 cm² (ii) Rs 93.15 11. 185.5 m², 1814.5 m² 12. (i) 308 m² (ii) 2700 m² 13. 287 m², 54 m², Rs 6300 14. Area = $550 \text{ m}^2 \text{ Cost} = \text{Rs} 57750$ 15. Length = 40 cm, Breadth = 16 m MENSURATION - I (Perimeter and area of rectilinear figures)

AREAS OF A PARALLELOGRAM AND A RHOMBUS A HHOMBUS a quadrilateral, whose each pair of opposite sides are parallel. In A = ABCD is a parallelogram such that AB and BC are a pair of its opposite sides so AB = BC. Similarly, BC and AD are a pair of opposite sides so ABCD is a parameter and such that AB and DC are a pair of its opposite sides such that $BC \parallel AD$.

 $\mathcal{P}^{L\perp}AB$, then we find that any line segment with its $\mathcal{P}^{L\perp}AB$ on the two sides AB and DCIf pL is any line segment with its and pC perpendicular to has the same length pL. So we saw that polynomial the same length DL. So, we call AB as the base the northeterm of the corresponding altitude them, has corresponding altitude. So, and DL the corresponding altitude.

Similarly, if $DM \perp BC$, then any line segment with its similar on the two sides AD and BC and perpendicular end-point has the same length DM. So, we can call BC as to them, and DM the correspondition to the base and DM the corresponding altitude.



We have,

Area of a parallelogram = $Base \times Altitude$

$$Base = \frac{Area of the parallelogram}{Altitude of the parallelogram}$$

$$Altitude = \frac{Area of the parallelogram}{Corresponding base}$$

Since a rhombus is a parallelogram with all its sides equal, therefore, the above formulae are applicable in case of a rhombus also.

Also, Area of rhombus = $\frac{1}{2}$ (Product of the diagonals)

Following examples will illustrate the use of the above formulae.

ILLUSTRATIVE EXAMPLES

Example 1 Find the area of a parallelogram with base 5 cm and altitude 4.2 cm.

Solution We have, Base = 5 cm and altitude = 4.2 cm

Area of the parallelogram = Base \times height = (5×4.2) cm² = 21 cm²

Example 2 Find the area in square metres of the parallelogram whose base and altitudes are as under:

Solution

- Base = 10 dm, altitude = 4.6 dm (ii) Base = 2 m 20 cm, altitude = 60 cm
- (i) We have,

Base = 10 dm = 1 m,

- Area of the parallelogram = Base \times Altitude = (1 \times 0.46) m² = 0.46 m²
- (ii) We have, Base = 2 m 20 cm = 2.2 mAltitude = $60 \text{ cm} = \frac{60}{100} \text{ m} = 0.6 \text{ m}$
 - Area of the parallelgram = $(2.2 \times 0.6) \,\mathrm{m}^2 = 1.32 \,\mathrm{m}^2$ ٠.

[: Perimeter = 4 (Side)]

Find the altitude of parallelogram whose area is $2.25~\mathrm{m}^2$ and base is $25~\mathrm{dm}$ Example 3 Solution We have,

Area of the given parallelogram = 2.25 m^2

Base of the given parallelogram = $25 \,\mathrm{dm} = \frac{25}{10} \,\mathrm{m} = 2.5 \,\mathrm{m}$

.. Altitude of the given parallelogram =
$$\frac{\text{Area}}{\text{Base}} = \frac{2.25}{2.5} \text{m} = 0.9 \text{ m}$$

Example 4 The side of a rhombus is 6.5 cm and its altitude is 4 cm. Find its area.

Solution We have,

Side = 6.5 cm, Altitude = 4 cm.

$$\therefore$$
 Area of the rhombus = Base × Altitude = (6.5×4) cm² = 26 cm²

The area of a rhombus is 72 cm². If its perimeter is 32 cm, find its altitude. Example 5

Solution We have,

Perimeter of the rhombus = 32 cm

$$\Rightarrow$$
 Side = $\frac{32}{4}$ cm = 8 cm

4 (Side) = 32 cm

Now,

Area of the rhombus = 72 cm^2

$$\Rightarrow$$
 (Side × Altitude) = 72

$$\Rightarrow$$
 8 × Altitude = 72

$$\Rightarrow$$
 Altitude = $\frac{72}{8}$ cm = 9 cm

In Fig. 15 ABCD is a parallelogram $CM \perp AB$ and $BL \perp AD$. Example 6

(i) If
$$AB = 16$$
 cm, $AD = 12$ cm and $CM = 10$ cm, find BL .

(ii) If
$$AD = 10$$
 cm, $CM = 8$ cm and $BL = 12$ cm, find AB .

Solution

(i) We have,

Base AB = 16 cm and Altitude CM = 10 cm

$$= (16 \times 10) \, \mathrm{cm}^2$$

$$= 160 \text{ cm}^2$$

D C B

Fig. 15

Now, taking AD as the base, we have

Area of parallelogram $ABCD = Base \times Altitude = (12 \times BL) cm^2$

...(i)

...(ii)

From (i) and (ii), we have

$$12 \times BL = 160 \Rightarrow BL = \frac{160}{12} = 13.33 \,\mathrm{cm}$$

(ii) We have,

_{Exam}ple 7

Example 8

^{Exam}ple 9

Solution

Solution

Solution

$$AD = 10 \text{ cm}, BL = 12 \text{ cm}$$

Area of parallelogram $ABCD = \text{Base} \times \text{height} = 10 \times 12 \text{ cm}^2 = 120 \text{ cm}^2 \dots (iii)$ Now, taking AB as the base, we have

Area of parallelogram $ABCD = AB \times CM = (AB \times 8) \text{ cm}^2$...(iv) From (iii) and (iv), we get

$$AB \times 8 = 120 \Rightarrow AB = \frac{120}{8} \text{ cm} \Rightarrow AB = 15 \text{ cm}$$

A field in the form of a parallelogram has base 15 dam and altitude 8 dam. Find the cost of watering the field at the rate of 50 paise per square metre. We have,

Base =
$$15 \text{ dam} = (15 \times 10) \text{ m} = 150 \text{ m}$$

$$[:: 1 dam = 10 m]$$

Altitude = $8 \text{ dam} = (8 \times 10) \text{ m} = 80 \text{ m}$

Area of the field = $(150 \times 80) \,\text{m}^2 = 12000 \,\text{m}^2$

Rate of watering the field = 50 paise per $m^2 = \text{Re} \frac{1}{2} \text{per } m^2$

Cost of watering the field = Rs $\left(12000 \times \frac{1}{2}\right)$ = Rs 6000

The area of a parallelogram is 338 m². If its altitude is twice the corresponding base, determine the base and the altitude.

Let the base be x metre long. Then, Altitude = 2x metres

Area = Base × Altitude = $(x \times 2x)$ m² = $2x^2$ m²

But, the area is given to be 338 m².

$$\therefore 2x^2 = 338$$

$$\Rightarrow x^2 = \frac{338}{2} \Rightarrow x^2 = 169 \Rightarrow x^2 = 13^2 \Rightarrow x = 13$$

Hence, base $\stackrel{\backslash}{=}$ 13 m and altitude = (2×13) m = 26 m.

The base of a parallelogram is thrice its height. If the area is 876 cm², find the base and height of the parallelogram.

Let the height of the parallelogram be x cm. Then, base = 3x cm.

Area of the parallelogram = $(x \times 3x)$ cm² = $3x^2$ cm²

But, area of the parallelogram is given as 867 cm²

$$\therefore 3x^2 = 867 \implies x^2 = 289 \implies x^2 = 17^2 \implies x = 17$$

Thus, height = 17 cm and base = (3×17) cm = 51 cm.

Example 10 Find the area of a rhombus having each side equal to 13 cm and one of wh_{0se} diagonals is 24 cm.

Let ABCD be the given rhombus whose diagonals intersect at O. Then, Solution

$$AB = 13 \,\mathrm{cm}$$
 and $AC = 24 \,\mathrm{cm}$

Since the diagonals of a rhombus bisect each other at right angles. Therefore

 $\triangle AOB$ is a right triangle, right angled at O such that $OA = \frac{1}{2}AC = 12$ cm and

$$AB = 13 \text{ cm}.$$

By Pythagoras theorem, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow 13^2 = 12^2 + OB^2$$

$$\Rightarrow OB^2 = 13^2 - 12^2$$

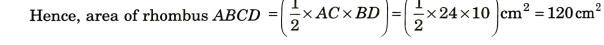
$$\Rightarrow OB^2 = 169 - 144 = 25$$

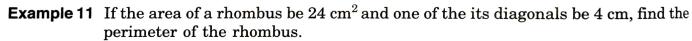
$$\Rightarrow OB^2 = 5^2$$

$$\Rightarrow OB = 5 \text{ cm}$$

$$BD = 2 \times OB = 2 \times 5 \text{ cm} = 10 \text{ cm}$$

Hence, area of rhombus $ABCD = \left(\frac{1}{2} \times AC \times BD\right) = \left(\frac{1}{2} \times 24 \times 10\right) \text{cm}^2 = 120 \text{ cm}^2$





Solution Let ABCD be a rhombus such that its one diagonal AC = 4 cm. Suppose the diagonals AC and BD intersect at O.

Now, Area of rhombus $ABCD = 24 \text{ cm}^2$

$$\Rightarrow \frac{1}{2} \times AC \times BD = 24$$

$$\Rightarrow \frac{1}{2} \times 4 \times BD = 24$$

$$\Rightarrow 2 \times BD = 24$$

$$\Rightarrow BD = 12 \text{ cm}$$

Thus, we have AC = 4 cm and BD = 12 cm

$$\therefore OA = \frac{1}{2}AC = 2 \text{ cm and } OB = \frac{1}{2}BD = 6 \text{ cm}$$

Since the diagonals of a rhombus bisect each other at right angles. Therefore, $\triangle AOB$ is a right triangle, right angled at O.

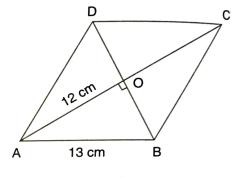


Fig. 16

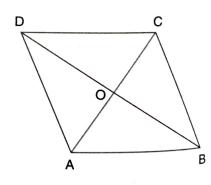


Fig. 17

By Pythagoras theorem, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = 2^2 + 6^2 = 40 = 4 \times 10 = 2^2 \times 10$$

$$\Rightarrow AB = 2\sqrt{10} \text{ cm}$$

Hence, perimeter of rhombus $ABCD = (4 \times 2\sqrt{10}) \text{ cm} = 8\sqrt{10} \text{ cm}$

If the area of a rhombus be 48 cm² and one of its diagonals is 6 cm, find its altitude.

Let ABCD be the given rhombus such that diagonal AC = 6 cm.

Now, Area of rhombus $ABCD = 48 \text{ cm}^2$

$$\Rightarrow \frac{1}{2} \times AC \times BD = 48$$

$$\Rightarrow \frac{1}{2} \times 6 \times BD = 48$$

$$\Rightarrow 3BD = 48$$

ution

$$\Rightarrow BD = \frac{48}{3} = 16 \text{ cm}$$

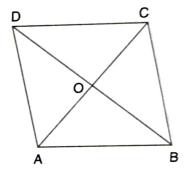


Fig. 18

[Using Pythagoras Theorem]

Since diagonals of a rhombus bisect each other at right angles.

$$\therefore OA = \frac{1}{2}AC = 3 \text{ cm and } OB = \frac{1}{2}BD = 8 \text{ cm.}$$

Also, $\triangle OAB$ is a right triangle, right angled at O.

$$\therefore AB^2 = OA^2 + OB^2$$

$$\Rightarrow AB^2 = 3^2 + 8^2$$
$$\Rightarrow AB^2 = 9 + 64$$

$$\Rightarrow AB = 9 + 6$$
$$\Rightarrow AB^2 = 73$$

$$\Rightarrow AB = \sqrt{73}$$
 cm

Now, Area of rhombus $ABCD = 48 \text{ cm}^2$

$$\Rightarrow$$
 Base \times Altitude = 48

$$\Rightarrow$$
 AB × Altitude = 48 \Rightarrow $\sqrt{73}$ × Altitude = 48 \Rightarrow Altitude = $\frac{48}{\sqrt{73}}$ cm

If the side of a square is 4 m and it is converted into a rhombus whose major diagonal is 6 m, find the other diagonal and the area of the rhombus.

Let AB=4 m be the side of a square ABPQ which is converted into a rhombus ABCD such that diagonal AC=6 m.

Since the diagonal of a rhombus bisect each other at right angles, therefore

$$OA = \frac{1}{2}AC = 3 \text{ m and } \angle AOB = 90^{\circ}$$

In right $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

 $\Rightarrow 4^2 = 3^2 + OB^2$

 $\Rightarrow OB^2 = 16 - 9$

 $\Rightarrow OB = \sqrt{7} \text{ m}$

 $BD = 2OB = 2\sqrt{7} \text{ m}$

Using Pythagoras
Theorem

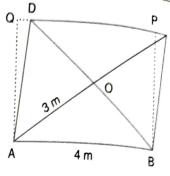


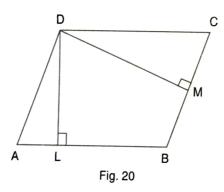
Fig. 19

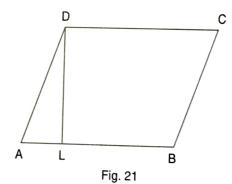
Thus, the other diagonal, $BD = 2 \times \sqrt{7}$ m

Hence, area of rhombus $ABCD = \frac{1}{2} \times AC \times BD = \left(\frac{1}{2} \times 6 \times 2\sqrt{7}\right) m^2 = 6\sqrt{7} m^2$

EXERCISE 20.3

- 1. Find the area of a parallelogram with base 8 cm and altitude 4.5 cm.
- 2. Find the area in square metres of the parallelogram whose base and altitudes are as under:
 - (i) Base = 15 dm, altitude = 6.4 dm (ii) Base = 1 m 40 cm, altitude = 60 cm
- 3. Find the altitude of a parallelogram whose area is 54 dm² and base is 12 dm.
- 4. The area of a rhombus is $28\,\text{m}^2$. If its perimeter be $28\,\text{m}$, find its altitude.
- 5. In Fig. 20, ABCD is a parallelogram, $DL \perp AB$ and $DM \perp BC$. If AB = 18 cm, BC = 12 cm and DM = 9.3 cm, find DL.
- 6. The longer side of a parallelogram is 54 cm and the corresponding altitude is 16 cm. If the altitude corresponding to the shorter side is 24 cm, find the length of the shorter side.





- 7. In Fig. 21, ABCD is a parallelogram, $DL \perp AB$. If AB = 20 cm, AD = 13 cm and area of the parallelogram is 100 cm^2 , find AL.
- 8. In Fig. 21, if AB = 35 cm, AD = 20 cm and area of the parallelogram is 560 cm^2 , find LB.
- 9. The adjacent sides of a parallelogram are 10 m and 8 m. If the distance between the longer sides is 4 m, find the distance between the shorter sides.
- The base of a parallelogram is twice its height. If the area of the parallelogram is 512 cm², find the base and height.
- 11. Find the area of a rhombus having each side equal to 15 cm and one of whose diagonals is 24 cm.
- 12. Find the area of a rhombus, each side of which measures 20 cm and one of whose diagonals is 24 cm.
- 13. The length of a side of a square field is 4 m. What will be the altitude of the rhombus, if the area of the rhombus is equal to the square field and one of its diagonals is 2 m?

Two sides of a parallelogram are 20 cm and 25 cm. If the altitude corresponding to the sides of 14. Length 25 cm is 10 cm, find the altitude corresponding to the sides of Two sides cm is 10 cm, find the altitude corresponding to the other pair of sides.

The base and corresponding altitude of a parallelogram are 10 cm and 12 cm respectively. If the other altitude is 8 cm, find the length of the other pair of parallel sides.

A floral design on the floor of a building consists of 280 tiles. Each tile is in the shape of a parallelogram of altitude 3 cm and base 5 cm. Find the cost of polishing the design at the rate of 50 paise per cm².

ANSWERS

1. 36 m²

2. (i) 0.96 m²

(ii) $0.84 \,\mathrm{m}^2$

4.5 dm

4. 4 m

5. 6.2 cm

6. 36 cm 10. Base = 32 cm, Height = 16 cm

7. 12 cm

8. 23 cm

9.5 m

12. 384 cm^2 13. $\frac{16}{\sqrt{65}} \text{ cm}$

14. 12.5 cm

11. 216 cm² 15. 15 cm

16. ₹2100

20.6 AREA OF A TRIANGLE

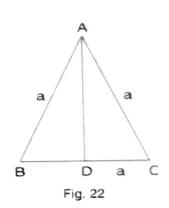
(i) Area of a triangle We have,

Area of
$$\triangle ABC = \frac{1}{2} (BC \times AL)$$

Area of
$$\triangle ABC = \frac{1}{2}$$
 (Base× Height)

From the above formula, we also obtain the following results.

$$Base = \frac{2 \times Area}{Height}, Height = \frac{2 \times Area}{Base}$$



(ii) Area of an equilateral triangle

Let ABC be an equilateral triangle each of whose side is a units in length. Let $AD \pm BC$.

Then,
$$BD = DC = \frac{a}{2}$$
.

Applying Pythagoras theorem in $\triangle ABD$, we have

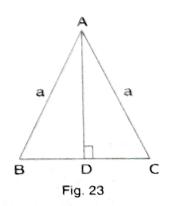
$$AB^2 = AD^2 + BD^2$$

$$AD = \sqrt{AB^2 - BD^2}$$

$$AD = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}a}{2} \text{ units}$$

Area of
$$\triangle ABC = \frac{1}{2} \times (\text{Base} \times \text{Height})$$

Area of
$$\triangle ABC = \frac{1}{2} \times BC \times AD$$



Area of
$$\triangle ABC = \left(\frac{1}{2} \times a \times \frac{\sqrt{3}a}{2}\right) \text{sq. units } = \left(\frac{\sqrt{3}}{4} \times (\text{Side})^2\right) \text{sq. units.}$$

Thus, area of an equilateral triangle = $\left(\frac{\sqrt{3}}{4} \times (\text{Side})^2\right) \text{sq. units}$ (iii) Area of an isosceles triangle

Let ABC be an isosceles triangle such that AB = AC = b units and BC = a units.

Draw $AD \perp BC$. Then, $BD = DC = \frac{a}{2}$.

Applying Pythagoras theorem in $\triangle ABD$, we have

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow b^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow AD^2 = b^2 - \frac{a^2}{4}$$

$$\Rightarrow AD = \sqrt{b^2 - \frac{a^2}{4}}$$

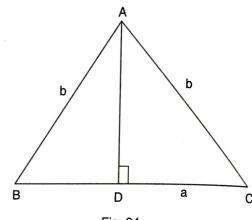


Fig. 24

$$\therefore \qquad \text{Area of } \triangle ABC = \frac{1}{2} \times BC \times AD$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} \times a \times \sqrt{b^2 - \frac{a^2}{4}} = \frac{1}{2} \times \text{Base} \times \sqrt{(\text{Equal Side})^2 - \frac{1}{4} (\text{Base})^2}$$

ILLUSTTRATIVE EXAMPLES

Example 1 Find the area in square centimetres of the triangle whose base and altitude are as under:

- (i) Base = 15 cm, Altitude = 8 cm
- (ii) Base = 1.5 m, Altitude = 0.8 m

Solution We know that the area of a triangle $=\frac{1}{2} \times \text{Base} \times \text{Altitude}$

- (i) Here, Base = 15 cm and Altitude = 8 cm
- \therefore Area of the triangle = $\left(\frac{1}{2} \times 15 \times 8\right)$ cm² = 60 cm^2
- (ii) Here, Base = 1.5 m = 150 cm and Altitude = 0.8 m = 80 cm
- \therefore Area of the triangle = $\left(\frac{1}{2} \times 150 \times 80\right)$ cm² = 6000 cm²

Find the altitude of a triangle whose base is 20 cm and area is 150 cm². Example 2 solution

We have, Altitude of a triangle = $\frac{2 \times \text{Area}}{\text{Rase}}$

Here, Base = 20 cm and Area = 150 cm^2

Altitude = $\frac{2 \times 150}{20}$ cm = 15 cm

Example 3 The area of a right triangle is 50 m². If one of the legs is 20 m, find the length of the other leg.

> In a right angled triangle, if one side is the base, then the other side is its altitude or height.

Let the given leg be the base. Then, the other leg is the altitude.

Here, area of the triangle = 50 m^2

One leg of the triangle = 20 m

The other leg of the triangle = Height of the triangle

$$= \frac{2 \times Area}{Base} = \left(\frac{2 \times 50}{20}\right) m = 5 m$$

Find the area of an isosceles right triangle, if one of the equal sides is 20 cm Example 4 long.

We know that in a right angled triangle, any one of the two sides which are at Solution right angle can be taken as the base and the other perpendicular side is the altitude.

Base = 20 cm and Altitude = 20 cm.

So, area of the given triangle = $\left(\frac{1}{2} \times 20 \times 20\right)$ cm² = 200 cm²

The area of a triangle is equal to that of a square whose each side measures Example 5 60 metres. Find the side of the triangle whose corresponding altitude is 90 metres.

Solution We have,

solution

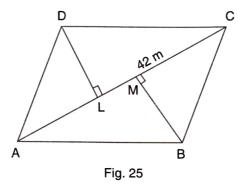
 $= (60 \times 60) \,\mathrm{m}^2 = 3600 \,\mathrm{m}^2$ Area of the square

[: Area of triangle = Area of square] $= 3600 \,\mathrm{m}^2$ Area of the triangle Altitude of the triangle = 90 m

Side of the triangle = $\frac{2 \times Area}{Corresponding altitude} = \left(\frac{2 \times 3600}{90}\right) m = 80 m$

A field in the form of a parallelogram has one of its diagonals 42 m long and Example 6 the perpendicular distance of this diagonal from either of the outlying vertices is 10.8 m (see Fig. 25). Find the area of the field.

Solution We have, AC = 42 m and DL = BM = 10.8 m



Area of the field =
$$2 \times \text{Area of } \triangle ACD$$

= $\left(2 \times \frac{1}{2} \times 42 \times 10.8\right) \text{m}^2$
= 453.6 m^2

Example 7 Find the area of an equilateral triangle having each side 4 cm.

Solution We know that the area of an equilateral triangle is equal to

$$\left(\frac{\sqrt{3}}{4} \times (\text{Side})^2\right)$$
 sq. units

Here, Side = 4 cm

$$\therefore$$
 Area of the given triangle = $\frac{\sqrt{3}}{4} \times 4^2 \text{ cm}^2 = 4\sqrt{3} \text{ cm}^2$

Example 8 Find the area of an isosceles triangle having the base 6 cm and the length of each equal side 5 cm.

Solution We know that:

Area of an isosceles triangle = $\frac{1}{2} \times Base \times \sqrt{(Equal \ side)^2 - \frac{1}{4}(Base)^2}$

Here, base = 6 cm, equal side = 5 cm.

$$\therefore$$
 Area of the given triangle = $\frac{1}{2} \times 6 \times \sqrt{(5)^2 - \frac{1}{4} \times (6)^2}$ cm²

$$=3 \times \sqrt{25-9} \text{ cm}^2 = 3 \times \sqrt{16} \text{ cm}^2 = 12 \text{ cm}^2$$

Example 9 Find the area of a right angled triangle with base BC = 7 cm and hypotenuse AC = 25 cm.

Solution Let ABC be the right angled triangle with base BC = 7 cm and hypotenuse AC = 25 cm.

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (25)^2 = AB^2 + 7^2$$

$$\Rightarrow AB^2 = 25^2 - 7^2 = 625 - 49 = 576 = 24^2$$

$$\rightarrow$$
 AB = 24 cm

Hence,

solution

Area of
$$\triangle ABC = \frac{1}{2} (\text{Base} \times \text{Height}) = \frac{1}{2} (7 \times 24) = 84 \text{ cm}^2$$

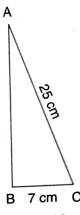


Fig. 26

The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Find its _{xample} 10 area.

We have, Base = 12 cm and Perimeter = 32 cm

Let the length of each of the two equal sides be b cm. Then,

Perimeter = 32 cm

$$\Rightarrow$$
 $2b+12=32$ \Rightarrow $2b=32-12$ \Rightarrow $2b=20$ \Rightarrow $b=10$

Thus, we have

Base = 12 cm and Equal side = 10 cm

Area of the given triangle

$$= \frac{1}{2} \times Base \times \sqrt{(Equal side)^2 - \frac{1}{4} \times (Base)^2}$$
$$= \frac{1}{2} \times 12 \times \sqrt{(10)^2 - \frac{1}{4} \times (12)^2} \text{ cm}^2$$

$$= 6 \times \sqrt{100 - 36} \text{ cm}^2 = 6 \times \sqrt{64} \text{ cm}^2 = 6 \times 8 \text{ cm}^2 = 48 \text{ cm}^2$$

Triangle ABC is right angled at A.AD is perpendicular to BC. If AB = 5 cm, BC = 13 cm and AC = 12 cm. Find the area of $\triangle ABC$. Also, find the length of AD.

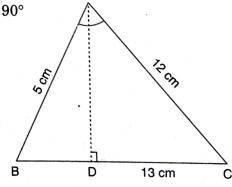
We have, AB = 5 cm, AC = 12 cm and $\angle BAC = 90^{\circ}$ Solution

$$\therefore \quad \text{Area of } \triangle ABC = \frac{1}{2}(AB \times AC)$$

$$\Rightarrow$$
 Area of $\triangle ABC = \frac{1}{2} \times 5 \times 12 \text{ cm}^2$

$$\Rightarrow$$
 Area of $\triangle ABC = 30 \text{ cm}^2$
Also.

Area of
$$\triangle ABC = \frac{1}{2}(BC \times AD)$$



$$\Rightarrow 30 = \frac{1}{2} \times 13 \times AD \Rightarrow AD = \frac{30 \times 2}{13} \text{ cm} = \frac{60}{13} \text{ cm}$$

Example 12 Triangle ABC is isosceles with AB = AC = 7.5 cm and BC = 9 cm. The height from A to BC i.e., AD is 6 cm. Find the area of $\triangle ABC$. What will be the height from C to AB?

Solution We have,

Area of
$$\triangle$$
 $ABC = \frac{1}{2} \times BC \times AD$
$$= \frac{1}{2} \times 9 \times 6 \text{ cm}^2 = 27 \text{ cm}^2$$

Let CF be the height from C to AB. Then,

Area of
$$\triangle$$
 $ABC = \frac{1}{2} \times AB \times CF$

$$\Rightarrow 27 = \frac{1}{2} \times 7.5 \times CF$$

$$\Rightarrow CF = \frac{27 \times 2}{7.5} \Rightarrow CF = 7.2 \text{ cm}$$

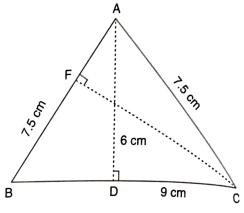


Fig. 28 [:: Area of \triangle $ABC = 27 \, \mathrm{cm}^2$]

EXERCISE 20.4

- 1. Find the area in square centimetres of a triangle whose base and altitude are as under:
 - (i) base = 18 cm, altitude = 3.5 cm
 - (ii) base = 8 dm, altitude = 15 cm
- 2. Find the altitude of a triangle whose area is 42 cm² and base is 12 cm.
- 3. The area of a triangle is 50 cm². If the altitude is 8 cm, what is its base?
- 4. Find the area of a right angled triangle whose sides containing the right angle are of lengths 20.8 m and 14.7 m.
- 5. The area of a triangle, whose base and the corresponding altitude are 15 cm and 7 cm, is equal to area of a right triangle whose one of the sides containing the right angle is 10.5 cm. Find the other side of this triangle.
- 6. A rectangular field is 48 m long and 20 m wide. How many right triangular flower beds, whose sides containing the right angle measure 12 m and 5 m can be laid in this field?
- 7. In Fig. 29, ABCD is a quadrilateral in which diagonal AC = 84 cm; $DL \perp AC$, $BM \perp AC$, DL = 16.5 cm and BM = 12 cm. Find the area of quadrilateral ABCD.

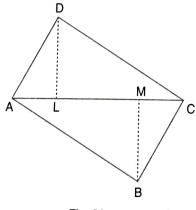
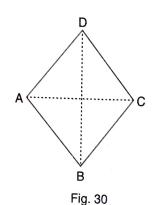
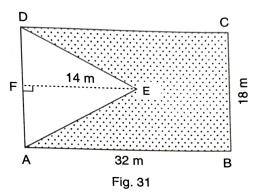


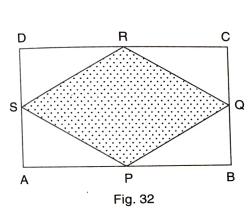
Fig. 29



Find the area of the quadrilateral ABCD given in Fig. 30. The diagonals AC and BD measure 48 m and 32 m respectively and are perpendicular to each other Find the and 32 m respectively and are perpendicular to each other. $_{AB}^{manu}$ $_{ABCD}^{manu}$ is a rectangle with dimensions 32 m by 18 m. $_{ADE}^{manu}$ is a triangle such that $_{AD}^{manu}$ and $_{ADE}^{manu}$ and $_{ADE}^{manu}$ and $_{ADE}^{manu}$ $_{ADE}^{manu}$ and $_{ADE}^{manu}$ $_{ADE}^{$ In Fig. 31, and EF = 14 cm. Calculate the area of the shaded region.

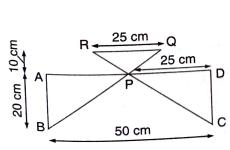


 $|A| = 10^{-10}$ Fig. 32, ABCD is a rectangle of length AB = 40 cm and breadth BC = 25 cm. If P, Q, R, S be the mid-points of the sides AB BC CD and Dthe mid-points of the sides AB, BC, CD and DA respectively, find the area of the shaded region. D



O Fig. 33

- 11. Calculate the area of the quadrilateral ABCD as shown in Fig. 33, given that BD=42 cm, AC = 28 cm, OD = 12 cm and $AC \perp BD$.
- 12. Find the area of a figure formed by a square of side 8 cm and an isosceles triangle with base as one side of the square and perimeter as 18 cm.
- 13. Find the area of Fig. 34 in the following ways:
 - (i) Sum of the areas of three triangles
 - (ii) Area of a rectangle sum of the areas of five triangles



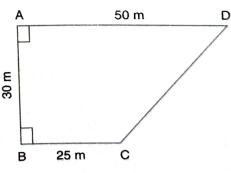
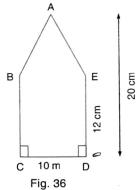


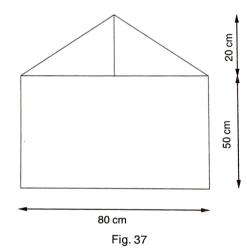
Fig. 35

Fig. 34 Calculate the area of quadrilateral field ABCD as shown in Fig. 35, by dividing it into a rectangle and a triangle.

15. Calculate the area of the pentagon ABCDE, where AB = AE and with dimensions as shown in Fig. 36.



- 16. The base of a triangular field is three times its altitude. If the cost of cultivating the field at ₹24.60 per hectare is ₹332.10, find its base and height.
- 17. A wall is 4.5 m long and 3 m high. It has two equal windows, each having form and dimensions as shown in Fig. 37. Find the cost of painting the wall (leaving windows) at the rate of ₹15 per m^2 .



ANSWERS

1. (i) 31.5 cm ²	(ii) 600 cm²	2. 7 cm	3. 12.5 cm
4. 152.88 m ²	5. 10 cm		0. 12.0 0111
6. 32	7. 1197 cm ²	8. 768 m ²	9. 450 m ²
10. 500 cm ²	11. 588 cm ²	12. 76 cm ²	13. 625 cm ²
14. 1125 m ²	15. 160 cm ²	16. Base = 900 m, hei	
17. ₹188.1		200 = 000 m, nei	grit = 500 m

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

- 1. If the area of a square is 225 m², then its perimeter is
 - (a) 15 m
- (b) 60 m
- (c) 225 m
- (d) 30 m
- 2. If the perimeter of a square is 16 cm, then its area is
 - (a) 4 cm²
- (b) 8 cm²
- (c) 16 cm²
- (d) 12 cm²
- 3. The length of a rectangle is 8 cm and its area is 48 cm². The perimeter of the rectangle is
 - (a) 14 cm
- (b) 24 cm
- (c) 12 cm
- (d) 28 cm

MENSUHATION 4. The area of a

- - (a) $1:\sqrt{2}$
- 5. The length o
 - (a) d²
- 6. The ratio of (a) 2:1
- 7. If the ratio o
 - (a) 2:1 8. The ratio of
- (a) 2:1
- g. On increas (a) 25 %
- 10. The area
- - (a) $5\sqrt{2}$
- 11. Each diag (a) 196
 - The area area of th
- (a) 1 m A path (levelling
 - (a) ₹ 1
- 14. The len rectang
 - (a) 4
- 15. In Fig. 3 the sha
 - (a) 3

16. The perin

(a) 17. The

> peri (a)

	If the length of a dia	(b)	320 cm ²	(c)	160 cm ²	(d)	its area is 156 cm ²
19.	The area of a recta rectangle is	ingle 1	44 cm long is sa	me a	s that of a square	of side	e 84 cm. The width of the
	(a) 7 cm	(b)	14 cm	(c)	49 cm	(d)	28 cm
20.	The length and bre area of the field is	adth o	f a rectangular fie	eld ar	e in the ratio 5 : 3 a	and its	s perimeter is 480 m. The
		(b)	13500 m ²	(c)	15000 m ²	(d)	54000 m ²
21.	The length of a rec	ctangu	lar field is thrice	its br	eadth and its peri	mete	r is 240 m. The length of
	(a) 30 m	(b)	120 m	(c)	90 m	(d)	80 m
22.							rea of the rectangle is
	(a) 100 cm ²	(b)	110 cm ²	(c)	120 cm ²	(d)	240 cm ²
23.							ively. The field is divided
	into two parts by fe	ncing	diagonally. The c	ost o	of fencing at the rat	e of ₹	10 per metre is
					₹190		
24.							orresponding height is
					10 cm		
25.							en the length of base is
	(a) 16 cm	(b)	32 cm	(c)	48 cm	(d)	64 cm
26.	The lengths of the			٠,			
					202.5 cm ²		
27.							then the length of other
	diagonal is						_
	(a) 6 cm	(b)	8 cm	(c)	12 cm	(d)	18 cm
28.	triangles formed b	y the d	iagonal is				ne area of one of the 4
	(a) 12 cm ²	(b)	8 cm ²	(c)	16 cm ²	(d)	14 cm ²
29.	cm, then the length	า of the	rectangle is				er of the rectangle is 80
	(a) 16 cm	(b)	24 cm	(c)	28 cm	(d)	18 cm
30.	The length of a rec cm, then the area	ctangle of the r	is 8 cm more that ectangle is	an the	breadth. If the pe	rimet	er of the rectangle is 80
	(a) 192 cm ²	(b)	364 cm ²	(c)	384 cm ²	(d)	382 cm ²
31.	The area of a recta	ıngle is	$11.6 \mathrm{m}^2$. If its breather	eadth	is 46.4 cm, then the	ne pei	rimeter is
	(a) 25.464 m	(b)	50.928 m	(c)	101.856 m	(d)	None of those
32.	The area of a rhom	ibus is	119 cm ² and its p	erim	eter is 56 cm. The	heigh	nt of the rhombus is
	(a) 7.5 cm	(b)	6.5 cm	(c)	8.5 cm		9.5 cm
33.	Each side of an eq						
					$24\sqrt{3}$ cm ²		
34.	The area of an equ	ilateral	triangle is $4\sqrt{3}$ (cm ² .	The length of each	of its	side is
	(a) 3 cm	(b)	4 cm	(c)	$2\sqrt{3}$ cm	(d)	$\frac{\sqrt{3}}{2}$ cm
35.	The height of an eq	uilater	al triangle is $\sqrt{6}$ o	m. It	s area is		
	(a) $3\sqrt{3} \text{ cm}^2$	(b)	$2\sqrt{3}$ cm ²	(c)	$2\sqrt{2} \text{ cm}^2$	(4)	$6\sqrt{2} \text{ cm}^2$
36.	If A is the area an e				•	(u)	o√2 cm²
	(a) $A = \sqrt{3} h^2$			_		/ -1\	
	(4) A-VOII	(2)	$\gamma \circ A = H$	(0)	$\sqrt{3}A = n^2$	(a)	$3A = h^2$

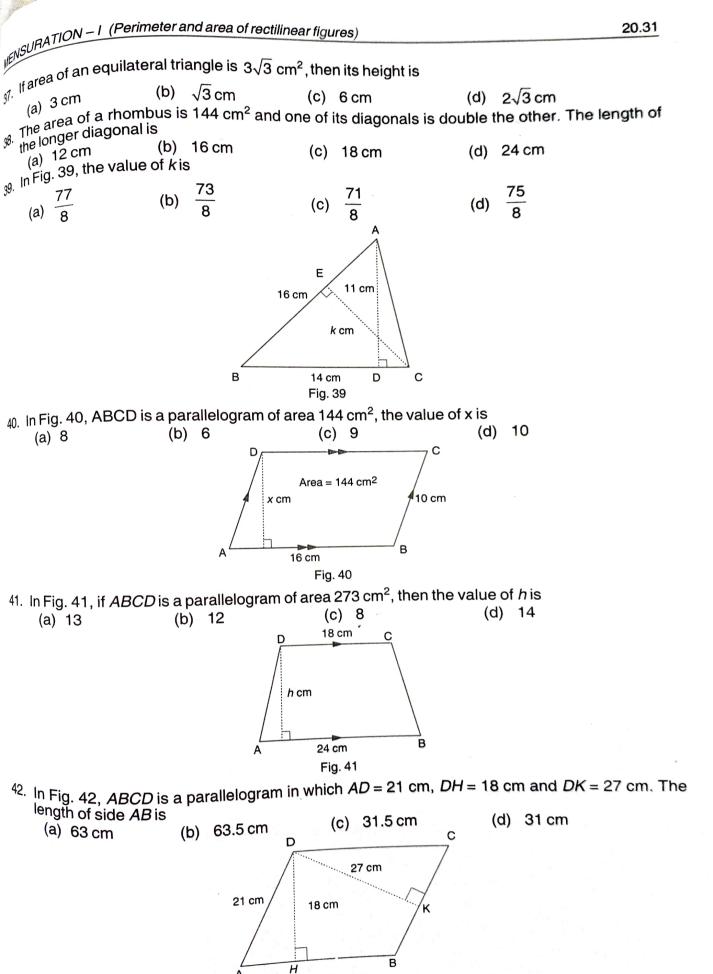


Fig. 42

43.	43. In Fig. 42 ABCD is a parallelogram in which AD = 21 cm, DH = 18 cm and DK = 27 cm. The perimeter of the parallelogram is								
	(a) 105 cm	(b)	84.5 cm	(c)	169 cm	(d) 52.	5 cm		
44.	44. In Fig. 42, the area of the parallelogram is								
	(a) 516 cm ²	(b)	567 cm ²	(c)	416 cm ²	(d) 60 0	6 cm ²		
45.	45. A piece of wire of length 12 cm is bent to form a square. The area of the square is								
	(a) 36 cm ²	(b)	$144 \mathrm{cm}^2$	(c) ⁹	9 cm ²	(d) 12			
46.	46. The area of a right isosceles triangle whose hypotenuse is $16\sqrt{2}$ cm is								
	(a) 125 cm ²	(b)	158cm^2	(c)	128 cm ²	(d) 144	cm ²		
47.	47. A wire is in the form of a square of side 18 m. It is bent in the form of a rectangle, whose length								
	and breadth are in the ratio 3:1. The area of the rectangle is								
	(a) 81 m ²	(b)	243m^2	(c) 1	144 m ²	(d) 324	m ²		
	ANSWSERS								
	1. (b)	2. (c)	3. (d)	4. (b)	5. (b)	6. (d)	7. (b)		
	8. (d)	9. (d)	10. (b)	11. (c)	12. (c)	13. (d)	14. (b)		
		16. (d)	17. (a)	18. (a)	19. (c)	20. (b)	21. (c)		
		23. (a)	24. (a)	25. (b)	26. (b)	27. (c)	28. (d)		
		30. (c)	31. (b)	32. (c)	33. (a)	34. (b)	35. (b)		
		37. (a)	38. (d)	39. (a)	40. (c)	41. (a)	42. (c)		
	43. (a)	44. (b)	45. (c)	46. (c)	47. (b)		• • •		

THINGS TO REMEMBER

A square centimetre is the area of the region formed by a square of side 1 cm. Standard units of area and their relations are:

 $100 \, \text{mm}^2 = 1 \, \text{cm}^2, 100 \, \text{cm}^2 = 1 \, \text{dm}^2$

100 m² = 1 are, 100 ares = 1 hectare $100 \, dm^2 = 1 \, m^2, \, 10000 \, cm^2 = 1 \, m^2$

₁₀₀ hectares = 1 sq. km.

 $_3$ perimeter of a rectangle = 2 (Length + Breadth) or, P = 2(l+b)perimeter of a square = $4 \times (Side)$ or, P = 4I

Area of a rectangle = Length \times Breadth or, $A = I \times b$

or, $l = \frac{A}{b}$ Also, length of a rectangle = $\frac{1}{\text{Breadth}}$ Area

Breadth of a rectangle = $\frac{Area}{Length}$ or, $b = \frac{A}{l}$

Area of a square = $(Side)^2$ or, $A = I \times I$.

4, Area of a parallelogram = Base \times Height or, $A = b \times h$

or, $b = \frac{A}{h}$ Also, Base of parallelogram = $\frac{Area}{height}$

Height of a parallelogram = $\frac{Area}{base}$ or, $h = \frac{A}{b}$

5. Area of a triangle $=\frac{1}{2} \times Base \times Height \ or, \ A = \frac{1}{2} \times b \times h$

Height of a triangle = $\frac{2 \times Area}{Rase}$ or, $h = \frac{2A}{b}$ Base

Base of a triangle = $\frac{2 \times Area}{height}$ or, $b = \frac{2A}{h}$

6. Area of a rhombus = $\frac{1}{2}$ × Product of diagonals

⁷. Area of a trapezium = $\frac{1}{2}$ × (Sum of the parallel sides) × (Distance between the parallel sides)