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DATA HANDLING – IV (Probability)

25.1 INTRODUCTION

In our everyday life, we come across statements such as:

(i) Probably it may rain today.

(ii) He is probably right.

(iii) Most probably he may join politics.

(iv) Indian cricket team has good chances of winning world-cup.

In such statements, we generally use the words: 'probably', 'most probably', chances, etc. All these words convey the same sense that there is some uncertainty in the occurrence or happening of the event in the question. For example, in first statement 'probably it may rain' will mean it may rain or it may not rain today. We are predicting rain today on the basis of our past experiences. Similar predictions are made in other statements. Thus, the word probably connotes that there is uncertainty about what has happened or what is going to happen. The uncertainty of 'probably' etc. can be measured by means of 'probability' in many cases. The concept of probability originated in the beginning of eighteenth century in problems pertaining to games of chance such as tossing a coin, throwing a die etc. Let us try to understand and introduce the concept of probability by performing the following activities.

25.2 PROBABILITY

Let us perform the following activities which will help us in understanding the concept of probability.

ACTIVITY 1 Take a coin, toss it 10 times and note down the number of times head and tail come up. Record your observations in the form of the following table.

Number of times the coin is tossed	10	20	30	50
Number of times head comes up	4	13	16	24
Number of times tail comes up	6	7	14	26
$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$	$\frac{4}{10} = 0.4$	$\frac{13}{20} = 0.65$	$\frac{16}{30} = 0.533$	$\frac{24}{50} = 0.48$
$\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$	$\frac{6}{10} = 0.6$	$\frac{7}{20} = 0.35$	$\frac{14}{30} = 0.466$	$\frac{26}{50} = 0.52$

Repeat the same experiment by increasing the number of tosses to 20, 30 and 50 and record your observations in the above table.

You will find that as the number of tosses gets larger, the values of fractions in fourth and fifth row of the table come closer to 0.5.

A die is a well balanced cube with its six faces marked with numbers 1 to 6, one number on one face.

Take a die and perform the following activity.

ACTIVITY 2 Throw a die 20 times and note down the number of times the numbers 1, 2, 3, 4, 5, 6 come up. Record your observations in the form of table given below.

Number of times a die is thrown	Number of times the scores turn up					
20	1	2	3	4	5	6
30						
40						

Repeat the same experiment 30 and 40 times and record your observations in the above table. Also, find the values of the following fractions in each case.

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

⋮

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

You will find that as the number of throws of the die increases, the value of each of the above fractions comes closer to $\frac{1}{6} = 0.1666$

In activity 1, each toss of a coin is called a trial. Similarly, in activity 2, each throw of a die is called a trial. Thus, we may define the term trial as follows:

TRIAL *A trial is an action which results in one or several outcomes.*

The possible outcomes in activity 1 were Head (H) and Tail (T); whereas in activity 2, the possible outcomes were 1, 2, 3, 4, 5 and 6.

In activity 1, we have considered an experiment in which the outcome of a trial is not certain. In other words, we were not certain whether outcome of a trial will be a head (H) or a tail (T). Similarly, in activity 2 we were not sure about the number on the upper face of the die. Such experiments are called random experiments. So, we define the term random experiment as follows.

RANDOM EXPERIMENT *An experiment in which the result of a trial cannot be predicted in advance is called a random experiment.*

Tossing a coin, Throwing a die, Tossing a pair of coins, Throwing a pair of dice, Drawing a ball from a bag containing 5 red and 4 white balls etc. are random experiments.

In tossing a coin, possible outcomes of a trial are: Head (H), Tail (T)

In tossing a pair of coins, possible outcomes of a trial are: HH, HT, TH, TT

In throwing a die, possible outcomes of a trial are: 1, 2, 3, 4, 5, 6.

In activity 1, getting of a head in a particular toss is an event with outcome 'head'. Similarly, getting a tail is an event with outcome 'tail'.

Similarly, in Activity 2, the getting of a particular number, say 1, is an event with outcome 1. Similarly, getting 2 is an event with outcome 2 and so on.

In the experiment of throwing a die 'getting an even number' is an event which consists of three outcomes, namely, 2, 4 and 6. Similarly, 'getting an odd number' is an event which consists of 1, 3, 5, as outcomes.

Thus, we may define the term event as follows:

EVENT *An event associated to a random experiment is the collection of some outcomes of the experiment.*

In the experiment of throwing a die 'getting a multiple of 3' is an event which consists of outcomes 3 and 6. Similarly, 'getting a prime number' is an event consisting of outcomes 2, 3 and 5.

In the experiment of tossing two coins simultaneously possible outcomes are:

HH, HT, TH, TT

Associated to this experiment 'getting exactly one head' is an event which consists of outcomes HT and TH. Similarly, 'getting at least one head' is an event consisting of outcomes HH, HT and TH.

HAPPENING OF AN EVENT *An event associated with a random experiment is said to happen if any one of the outcomes satisfying the definition of the event is an outcome of the experiment when it is performed.*

In random experiment of throwing a die, if the outcome of the throw is 4, then we say that each one of the following events happens:

(i) Getting 4.
(ii) Getting an even number.
(iii) Getting a number greater than 3.

However, on the basis of the same outcome, one can say that none of the following events have happened:

(i) Getting an odd number.

(ii) Getting a multiple of 3.

(iii) Getting a number less than 3.

(iv) Getting a prime number.

On the basis of the above discussion, we define the experimental or empirical probability of happening of an event as follows:

EMPIRICAL PROBABILITY *Let n be the total number of trials of a random experiment. Then the empirical probability P(E) of happening of an event E is defined as*

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

Let us now discuss some examples on finding the empirical probabilities.

ILLUSTRATIVE EXAMPLES

Example 1 A coin is tossed 100 times in which head is obtained 55 times. On tossing a coin at random, find the probability of getting (i) a head (ii) a tail.

Solution We have

Total number of trials = 100

Number of times a head come up = 55

Number of times a tail come up = (100 – 55) = 45

$\therefore$ Probability of getting a head $= \dfrac{\text{Number of heads}}{\text{Total number of trials}} = \dfrac{55}{100} = 0.55$

Probability of getting a tail $= \dfrac{\text{Number of tails}}{\text{Total number of trials}} = \dfrac{45}{100} = 0.45$

Example 2 A die is thrown 200 times and the outcomes are noted as shown below:

Outcome:	1	2	3	4	5	6
Frequency:	35	30	31	28	37	39

If a die is thrown at random, find the probability of getting *a/an*

(i) 1 (ii) 4 (iii) 6 (iv) even number (v) odd number

(vi) multiple of 3

Solution We have,

Total number of trials = 200

(i) Probability of getting a 1 $= \dfrac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}}$

$$= \frac{35}{200} = \frac{7}{40}$$

(ii) Probability of getting a 4 $= \dfrac{\text{Frequency of 4}}{\text{Total number of times the die is thrown}}$

$$= \frac{28}{200} = \frac{7}{50}$$

(iii) Probability of getting a 6 $= \dfrac{\text{Frequency of 6}}{\text{Total number of times the die is thrown}}$

$$= \frac{39}{200}$$

(iv) We have,

Frequency of getting an even number

= Frequency of Getting 2 + Frequency of 4 + Frequency of 6

$= 30 + 28 + 39 = 97$

$\therefore$ Probability of getting an even number

$$= \frac{\text{Frequency of getting an even number}}{\text{Total number of times the die is thrown}} = \frac{97}{200}$$

(v) We have,

Frequency of getting an odd number

= Frequency of getting 1 + Frequency of getting 3 + Frequency of getting 5

$= 35 + 31 + 37 = 103$

$\therefore$ Probability of getting an odd number

$$= \frac{\text{Frequency of getting an odd number}}{\text{Total number of times the die is thrown}} = \frac{103}{200}$$

Example 3 In a cricket match, a batsman hits a boundary 6 times out of 90 balls he plays. Find the probability that he (i) hit a boundary (ii) did not hit a boundary.

Solution We have,

Total number of trials = 90

Number of trials in which the batsman hit a boundary = 6

Number of trials in which the batsman did not hit a boundary = 90 – 6 = 84

$\therefore$ Probability that the batsman hit a boundary

$$= \frac{\text{Number of times he hit the boundary}}{\text{Total number of trials}} = \frac{6}{90} = \frac{1}{15}$$

Probability that the batsman did not hit a boundary

$$= \frac{\text{Number of times he did not hit the boundary}}{\text{Total number of trials}} = \frac{84}{90} = \frac{14}{15}$$

Example 4 There are 6 marbles in a bag with numbers from 1 to 6 marked on each of them. What is the probability of drawing a marble with number (i) 2 ? (ii) 5 ?

Solution We have,

Total number of marbles in the bag = 6

Number of marbles, in the bag, marked with number 2 = 1

Number of marbles, in the bag, marked with number 5 = 1

$\therefore$ Probability of drawing a marble with number $2 = \frac{1}{6}$

Probability of drawing a marble with number $5 = \frac{1}{6}$

Example 5 In a survey of 200 girls it was found that 85 like tea while 115 dislike it. Out of these girls, one girl is chosen at random. What is the probability that the chosen girl (i) likes tea? (ii) dislikes tea?

Solution We have,

Total number of girls = 200

Number of girls who like tea = 85

Number of girls who dislike tea = 200 – 85 = 115

$\therefore$ Probability that a chosen girl likes tea $= \frac{85}{200} = \frac{17}{40}$

Probability that a chosen girl dislikes tea $= \frac{115}{200} = \frac{23}{40}$

EXERCISE 1

1. A coin is tossed 1000 times with the following frequencies:

   Head: 445, Tail: 555

   When a coin is tossed at random, what is the probability of getting (i) a head? (ii) a tail?

2. A die is thrown 100 times and outcomes are noted as given below:

Outcome:	1	2	3	4	5	6
Frequency:	21	9	14	23	18	15

   If a die is thrown at random, find the probability of getting *a*/*an*.

   (i) 3 (ii) 5 (iii) 4 (iv) Even number (v) Odd number (vi) Number less than 3.

3. A box contains two pair of socks of two colours (black and white). I have picked out a white sock. I pick out one more with my eyes closed. What is the probability that I will make a pair?

4. Two coins are tossed simultaneously 500 times and the outcomes are noted as given below:

Outcome:	Two heads (HH)	One head (HT or TH)	No head (TT)
Frequency:	105	275	120

   If same pair of coins is tossed at random, find the probability of getting (i) Two heads (ii) One head (iii) No head.

ANSWERS

1. (i) 0.445 (ii) 0.555

2. (i) $\frac{7}{50}$ (ii) $\frac{9}{50}$ (iii) $\frac{23}{100}$ (iv) $\frac{47}{100}$ (v) $\frac{53}{100}$ (vi) $\frac{3}{10}$

3. $\frac{1}{3}$

4. (i) $\frac{21}{100}$ (ii) $\frac{11}{20}$ (iii) $\frac{6}{25}$

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. An unbiased coin is tossed once, the probability of getting head is

   (a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. There are 10 cards numbered from 1 to 10. A card is drawn randomly. The probability of getting an even numbered card is

   (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$

3. A dice is rolled. The probability of getting an even prime is

   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$

4. There are 100 cards numbered from 1 to 100 in a box. If a card is drawn from the box and the probability of an event is $\frac{1}{2}$, then the number of favourable cases to the event is

   (a) 20 (b) 25 (c) 40 (d) 50

5. When a dice is thrown, the total number of possible outcomes is

   (a) 6 (b) 1 (c) 3 (d) 4

6. There are 10 marbles in a box which are marked with the distinct numbers from 1 to 10. A marble is drawn randomly. The probability of getting prime numbered marble is

   (a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{9}{3}$ (d) $\frac{3}{10}$

7. The probability of getting a red card from a well shuffled pack of cards is

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

8. A coin is tossed 100 times and head is obtained 59 times. The probability of getting a tail is

   (a) $\frac{59}{100}$ (b) $\frac{41}{100}$ (c) $\frac{29}{100}$ (d) $\frac{43}{100}$

9. A dice is tossed 80 times and number 5 is obtained 14 times. The probability of not getting the number 5 is

   (a) $\frac{7}{40}$ (b) $\frac{7}{80}$ (c) $\frac{33}{40}$ (d) None of these

10. A bag contains 4 green balls, 4 red balls and 2 blue balls. If a ball is drawn from the bag, the probability of getting neither green nor red ball is

    (a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

ANSWERS

1. (a) 2. (c) 3. (a) 4. (d) 5. (a) 6. (b) 7. (b)
8. (b) 9. (c) 10. (d)

THINGS TO REMEMBER

1. *A trial is an action which results in one or several outcomes.*
2. *An experiment in which the result of a trial cannot be predicted in advance is called a random experiment.*
3. *An event associated to a random experiment is the collection of some outcomes of the experiment.*
4. *An event associated with a random experiment is said to happen if any one of the outcomes satisfying the definition of the event is an outcome of the experiment when it is performed.*
5. *The Empirical probability of happening of an event E is defined as*

$$P(E) = \frac{\text{Number of trials in which the event happend}}{\text{Total number of trials}}$$