18

SYMMETRY

18.1 INTRODUCTION

In class VI, we have studied about symmetry and lines of symmetry of various mathematical and non-mathematical figures and objects. Symmetry is such a nice thing that it is used almost in every field of activity. Architects, designers of jewellery or clothing, artists and many others use the idea of symmetry in their profession. Even nature has also gifted many living and non-living symmetrical objects to us. Some of them are shown below.

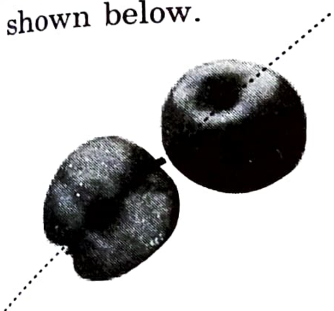


Fig. 1

Fig. 2

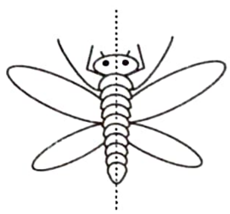


Fig. 3

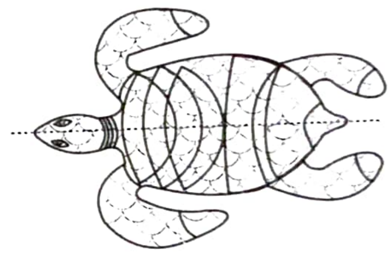


Fig. 4

Following are the pictures of some man made architectural marvels which are beautiful because of their symmetry.

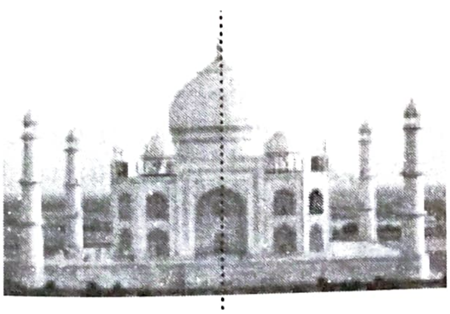


Fig. 5 Tajmahal

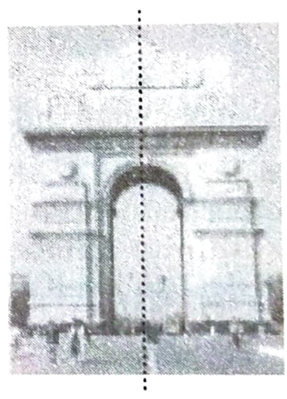
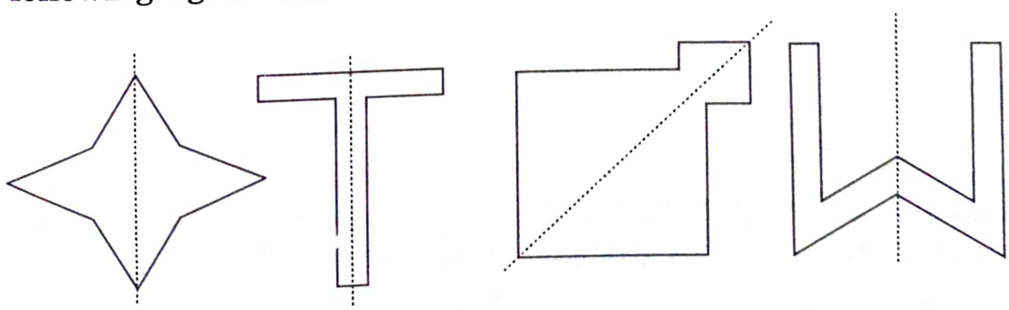


Fig. 6

In each of the following figures dotted lines are lines of symmetry.



(i) (ii) (iii) (iv)

Fig. 7

18.2 LINES OF SYMMETRY

We observe that each one of the figures 1 to 7 is divided into two identical parts by the dotted lines. When we fold these figures along the dotted line, one half of the figures would fit exactly cover the other half. The dotted lines are known as the lines of symmetry of the respective figures. Thus, we may define a line of symmetry of a figure as follows.

LINE OF SYMMETRY *If a line divides a figure into two parts such that when the figure is folded about the line the two parts of the figure coincide, then the line is known as the line of symmetry.*

The line of symmetry is also known as the axis of symmetry.

In each one of the figure 1 to 7 dotted lines are lines of symmetry of the respective figures.

A figure may or may not have a line of symmetry. There may be figures having more than one line of symmetry.

The following figures have more than one line of symmetry. Such figures are said to have multiple lines of symmetry.

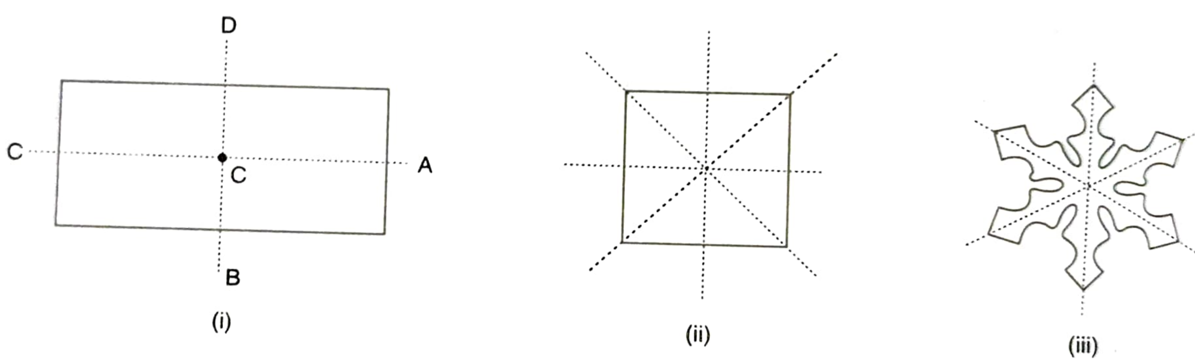


Fig. 8

Let us now discuss about the lines of symmetry of some geometrical figures.

LINES OF SYMMETRY OF A LINE A line has infinite length and hence it can be considered that each line perpendicular to the given line divide the line into two equal halves (parts). So, a line has infinite number of symmetrical lines which are perpendicular to it. Also, a line is symmetrical to itself.

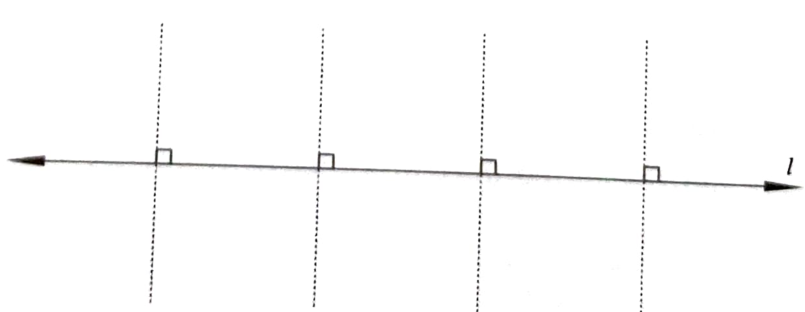


Fig. 9

LINES OF SYMMETRY OF A LINE SEGMENT A line segment has two lines of symmetry, namely, the segment itself and the perpendicular bisector of the segment.

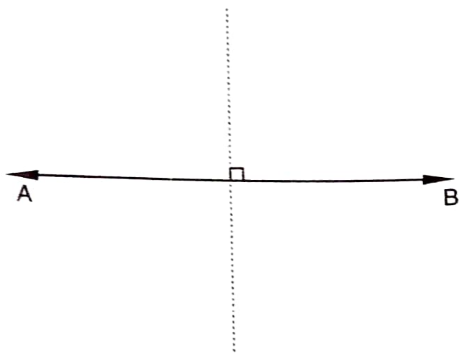


Fig. 10

LINES OF SYMMETRY OF AN ANGLE An angle with equal arms has one line of symmetry which is along the internal bisector of the angle.

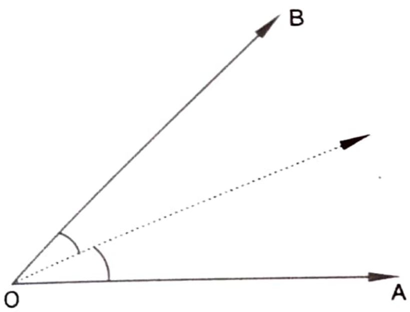


Fig. 11

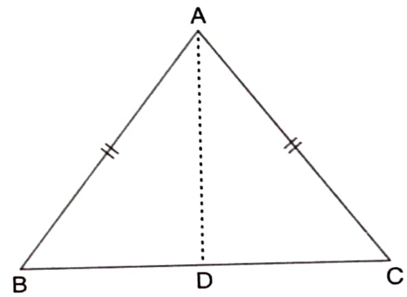


Fig. 12

LINES OF SYMMETRY OF AN ISOSCELES TRIANGLE An isosceles triangle has one line of symmetry which is along the median through the vertex. Median AD is a line of symmetry of isosceles triangle ABC as the $AB = AC$ in Fig.12.

LINES OF SYMMETRY OF A PARALLELOGRAM A parallelogram has no line of symmetry. In Fig. 13, parallelogram $ABCD$ has no line of symmetry.

Fig. 13

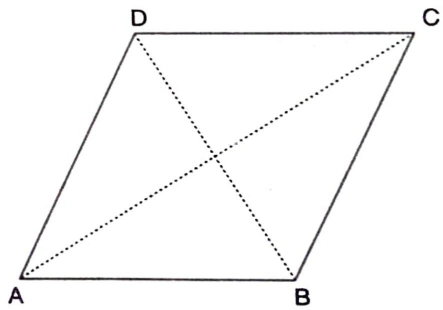


Fig. 14

LINES OF SYMMETRY OF A RHOMBUS A rhombus has two lines of symmetry along the diagonals of the rhombus.

In rhombus $ABCD$, diagonals AC and BD are two lines of symmetry. As shown in Fig 14.

LINES OF SYMMETRY OF A RECTANGLE *A rectangle has two lines of symmetry along the line segments joining the mid-points of the opposite sides.*

In the fig. 15, PQ and RS are two lines of symmetry of rectangle $ABCD$.

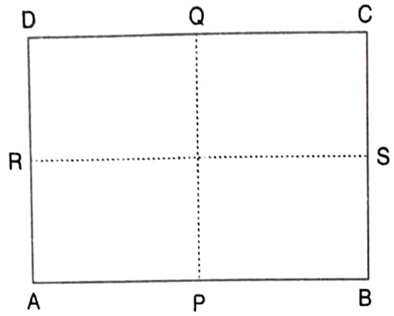


Fig. 15

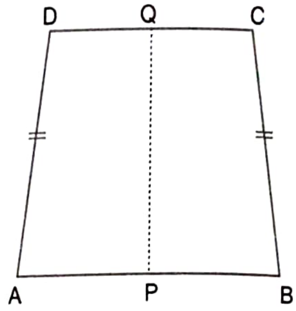


Fig. 16

LINES OF SYMMETRY OF AN ISOSCELES TRAPEZIUM An isosceles trapezium has two parallel sides and two non-parallel sides of equal length. In Fig. 16, $ABCD$ is an isosceles trapezium such that $BC = AD$ and AB is parallel to CD. It has only one line of symmetry along the line segment joining the mid-points of two parallel sides AB and CD.

LINES OF SYMMETRY OF A KITE A kite, as shown in Fig. 17 has one line of symmetry along the diagonal BD.

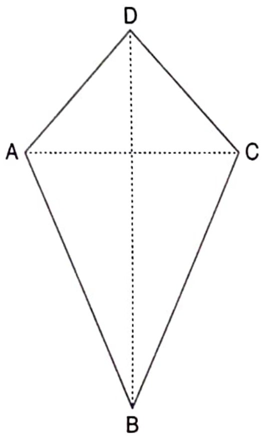


Fig. 17

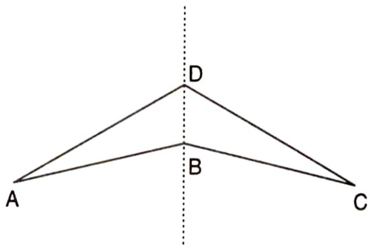


Fig. 18

LINES OF SYMMETRY OF AN ARROW HEAD An arrow head, as shown in Fig. 18, has the diagonal BD as the only line of symmetry.

LINES OF SYMMETRY OF A SEMI-CIRCLE A semi-circle, as shown in Fig. 19, has only one line of symmetry which is perpendicular to the perpendicular bisector of the diameter AB.

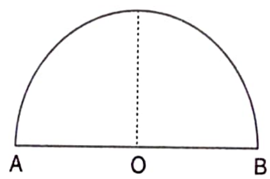


Fig. 19

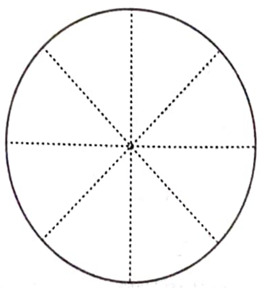


Fig. 20

LINES OF SYMMETRY OF A CIRCLE A circle has an infinite number of lines of symmetry all along the diameters.

18.3 LINES OF SYMMETRY OF SOME REGULAR POLYGONS

Regular polygons are symmetrical beauties and hence their lines of symmetry are quite interesting. In this section, we will discuss about the lines of symmetry of some regular polygons. We know that a closed figure made up of several line segments is called a polygon. Polygons made up of three, four, five and six line segments are known as triangles, quadrilaterals, pentagons and hexagons respectively. A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure.

LINES OF SYMMETRY OF AN EQUILATERAL TRIANGLE All sides of an equilateral triangle are equal in length and each angle is of measure 60°. An equilateral triangle has three lines of symmetry along the three medians of the triangle as shown in Fig. 21.

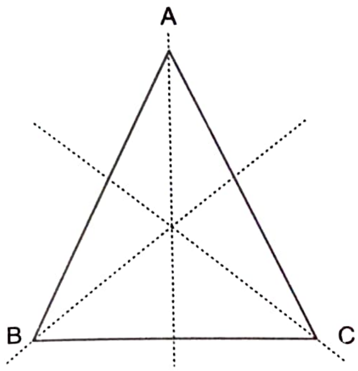


Fig. 21

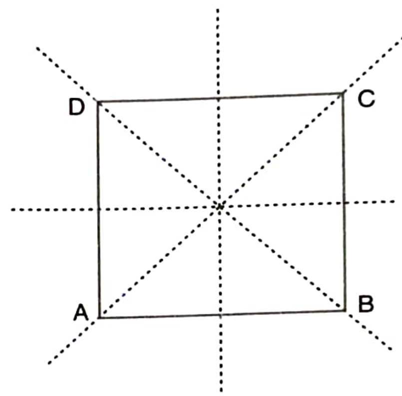


Fig. 22

LINES OF SYMMETRY OF A SQUARE A square is a quadrilateral whose all sides are of equal length and each of its angles is a right-angle i.e. 90°. Diagonals of a square are perpendicular bisector of each other.

A square has four lines of symmetry two along the diagonals and two along the line segments joining the mid-points of opposite sides as shown Fig. 22.

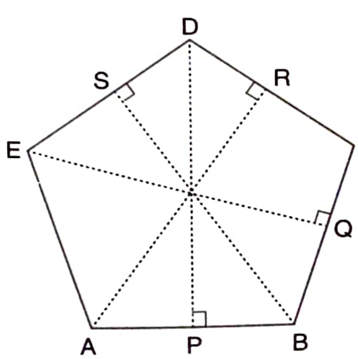


Fig. 23

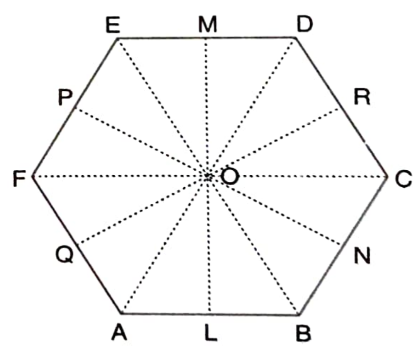


Fig. 24

LINES OF SYMMETRY OF A REGULAR PENTAGON A regular pentagon has all its sides equal in length and the measure of each of its angles is 108°.

A regular pentagon has five lines of symmetry as shown in fig. 23.

LINES OF SYMMETRY OF A REGULAR HEXAGON A regular hexagon has all its sides equal and each of its anlgles measures 120°. A regular hexagon has six lines of symmetry. Three along the lines joining the mid-points of opposite sides and three along the diagonals as shown in fig. 24.

We observe from the above discussion that each regular polygon has as many lines of symmetry as the number of its sides.

ILLUSTRATIVE EXAMPLES

Example 1 Following letters of English alphabet are symmetrical about a line. Identify, a line of symmetry in each case:

(i) A (ii) B (iii) C (iv) D (v) E (vi) M
(vii) T (viii) U (ix) V (x) W (xi) X (xii) Y.

Solution The dotted line is the line of symmetry.

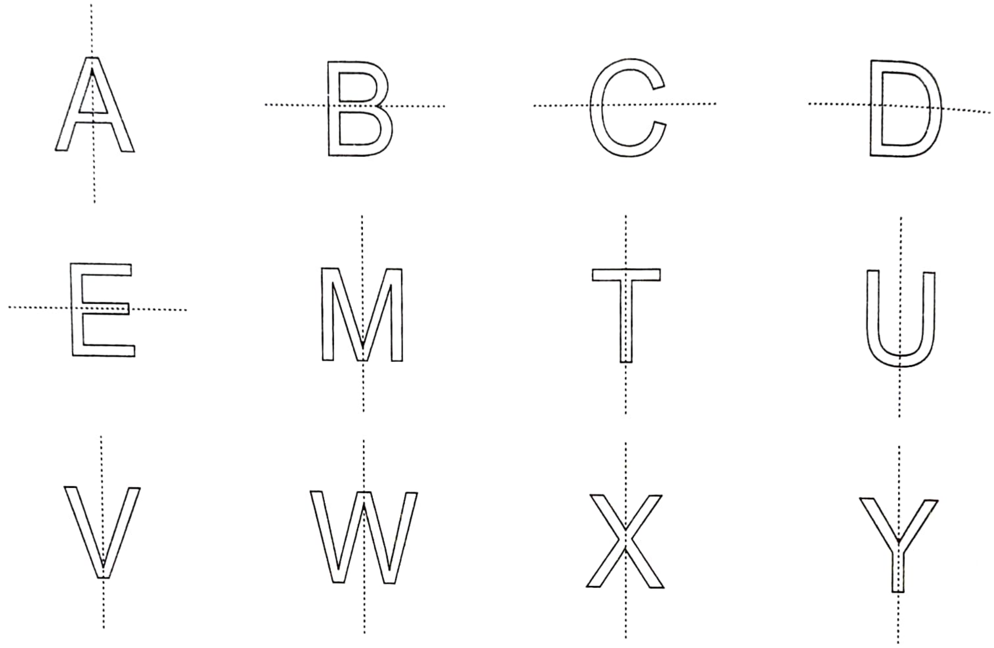


Fig. 25

Example 2 Each of the following letters from English alphabet has two lines of symmetry. Identify lines of symmetry in each case:

(i) H (ii) I (iii) O

Solution The dotted lines are lines of symmetry in each case:

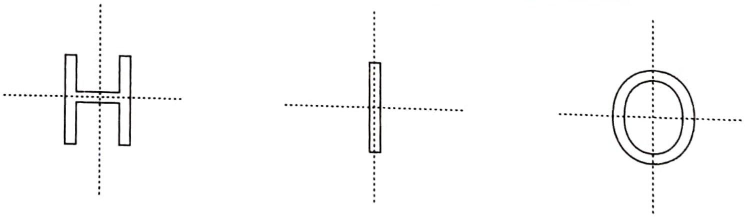


Fig. 26

EXERCISE 18.1

1. State the number of lines of symmetry for the following figures:

(i) An equilateral triangle (ii) An isosceles triangle
(iii) A scalene tringle (iv) A rectangle
(v) A rhombus (vi) A square
(vii) A parallelogram (viii) A quadrilateral
(ix) A regular pentagon (x) A regular hexagon
(xi) A circle (xii) A semi-circle

2. What other name can you give to the line of symmetry of
   (i) An isosceles triangle? (ii) A circle?
3. Identify three examples of shapes with no line of symmetry.
4. Identify multiple lines of symmetry, if any, in each of the following figures:

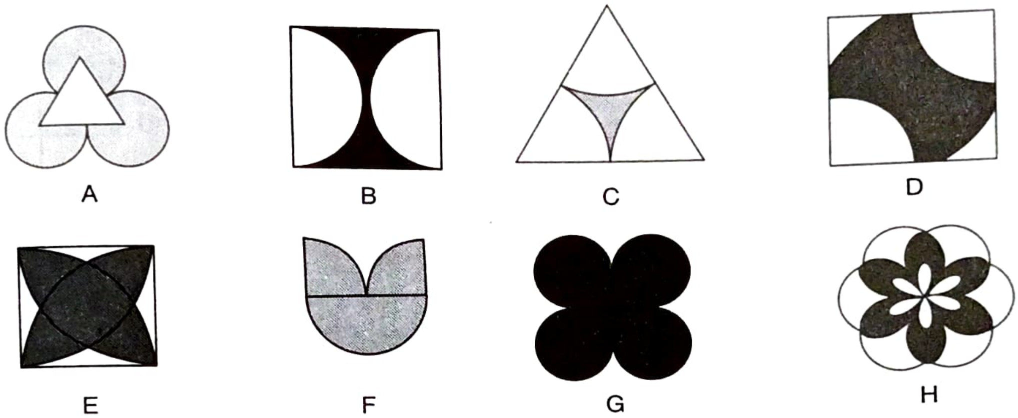


Fig. 27

ANSWERS

1. (i) 3 (ii) 1 (iii) 0 (iv) 2 (v) 2 (vi) 4
   (vii) 0 (viii) 0 (ix) 5 (x) 6 (xi) Infinitely many
   (xii) 1
2. (i) Altitude (ii) Diameter
3. Parallelogram, A scalene triangle, A quadrilateral.

18.4 LINE OF SYMMETRY AND REFLECTION

The concept of line of symmetry is closely related to the mirror reflection. If a figure has a line of symmetry and we place a mirror along the line of symmetry, then the image or reflection of one side of the figure is exactly same as the figure on the other side of the line of symmetry. That is why line symmetry of a figure is also known as the reflection symmetry. Using this, we can also draw the complete figure, if we are given a line of symmetry and one half on one side of it. For example, if the dotted line in the following figure is a line of symmetry and we wish to draw the complete figure. Then, we take the image of the figure in the mirror placed along the line of symmetry. The complete figure will look like the figure shown below.

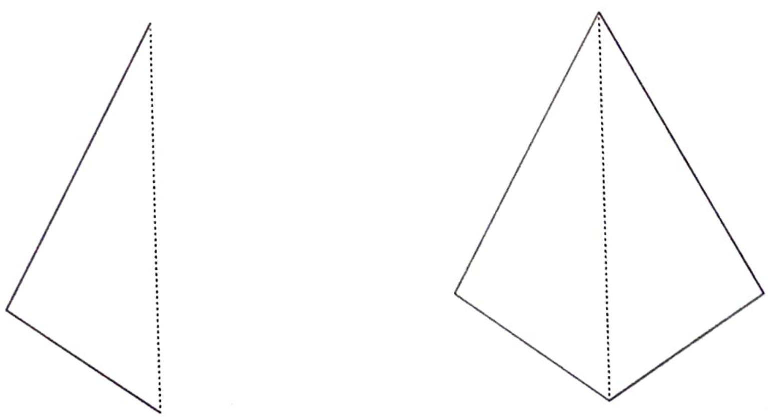


Fig. 28

ACTIVITY

STEP I *Take sheet of paper and fold it into 2 halves (Fig. 29).*

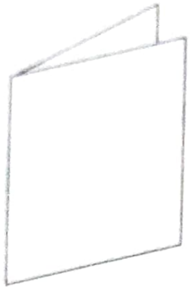


Fig. 29

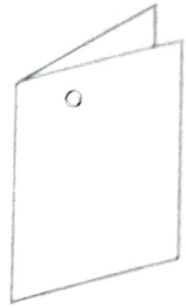


Fig. 30

STEP II *Punch a hole as shown in Fig. 30.*

STEP III *Unfold the sheet to separate the two halves as shown in Fig. 31.*

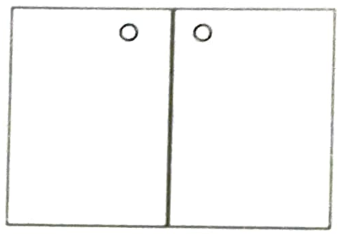


Fig. 31

You will find the fold is a line (or axis) of symmetry.

EXERCISE 18.2

1. In the following figures, the mirror line (i.e. the line of symmetry) is given as dotted line. Complete each figure performing reflection in the dotted (mirror) line. Also, try to recall the name of the complete figure.

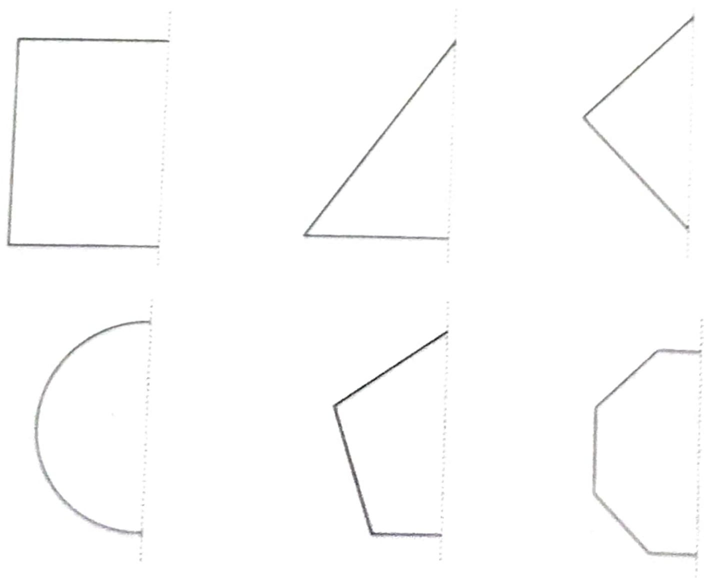


Fig. 32

2. Each of the following figures shows paper cuttings with punched holes. Copy these figures on a plane sheet and mark the axis of symmetry so that if the paper is folded along it, then the wholes on one side of it coincide with the holes on the other side.

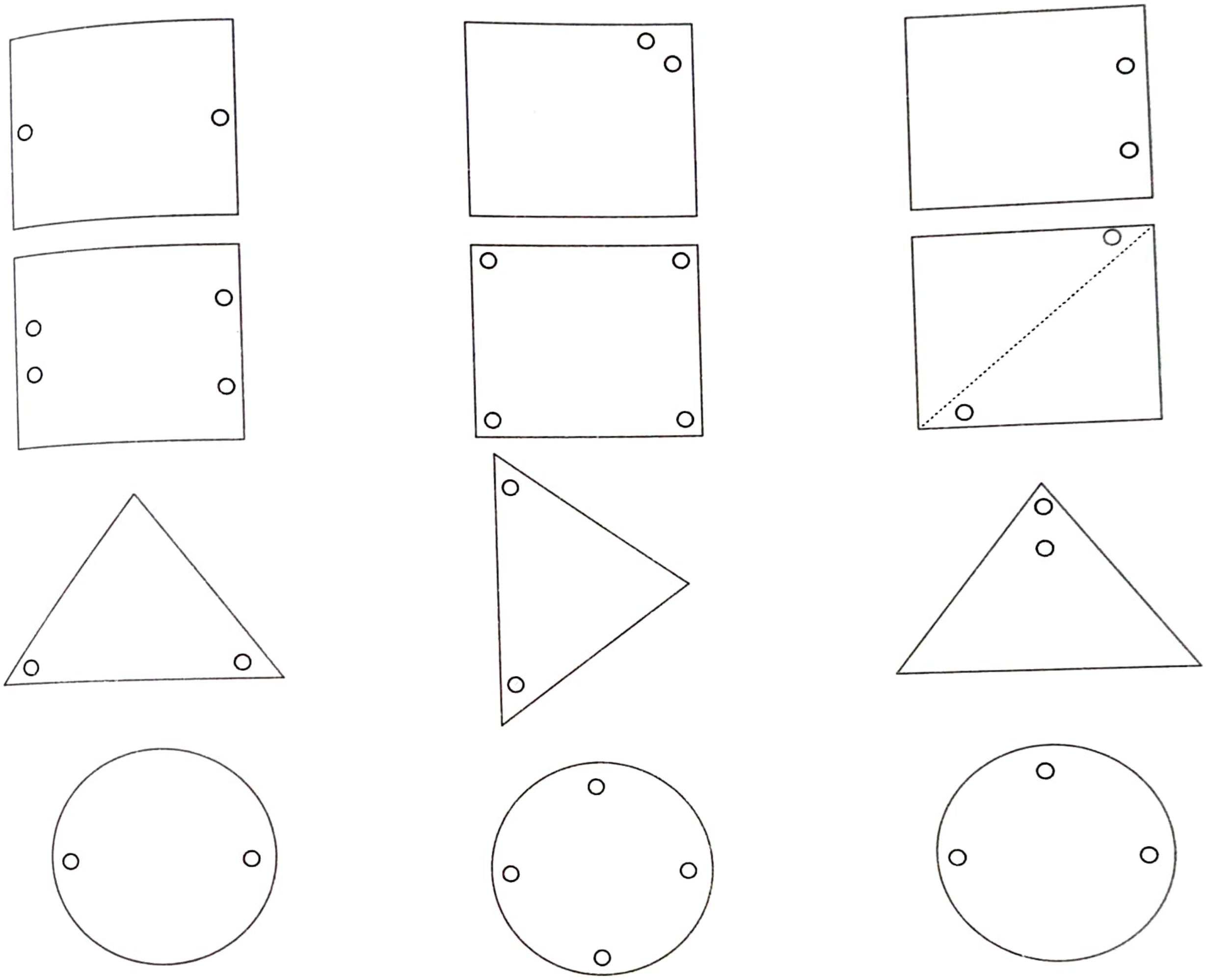


Fig. 33

3. In the following figures if the dotted lines represent the lines of symmetry, find the other hole (s).

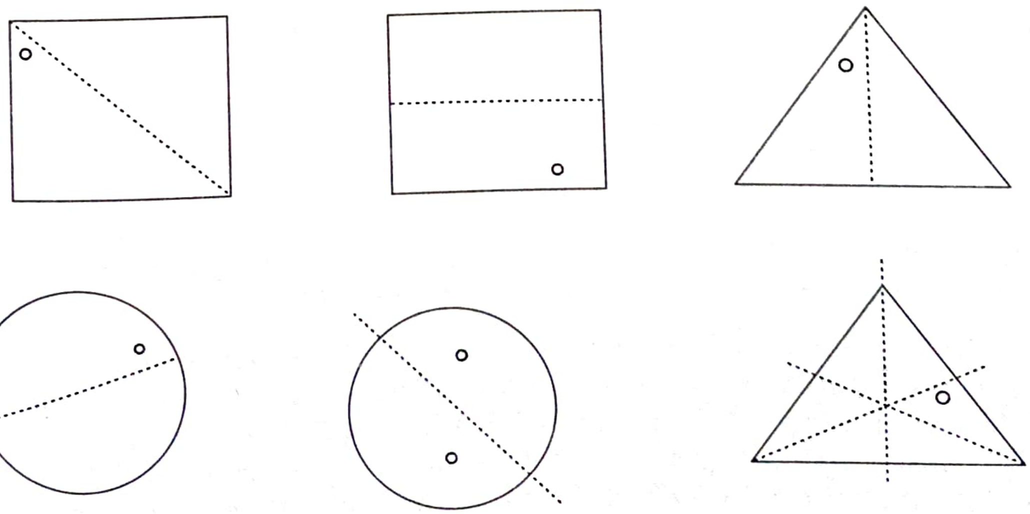


Fig. 34

18.5 ROTATIONAL SYMMETRY

Consider a rotating object, say a wheel of a bicycle or a wind-mill etc. The fixed point about which the object rotates is called the centre of rotation.

Fig. 35

When an object rotates in the direction of motion of hands of a clock, rotation is called clock wise rotation; otherwise it is said to be anti-clockwise rotation.

ANGLE OF ROTATION *The angle through which an object rotates (turns) about a fixed point is known as the angle of rotation.*

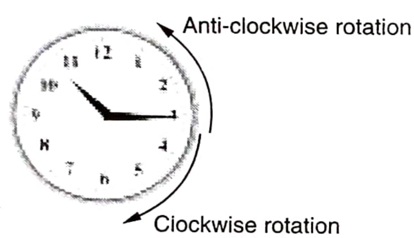


Fig. 36

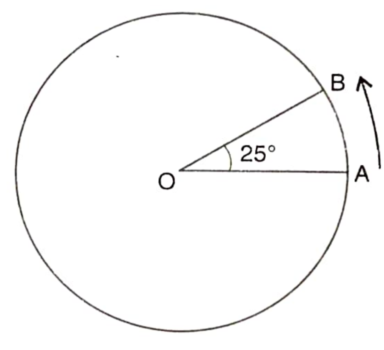


Fig. 37

An object is said to take a full turn, if the angle of rotation is of 360°. A quarter-turn means rotation by 90° and a three quarter-turn means rotation by 270° as shown below.

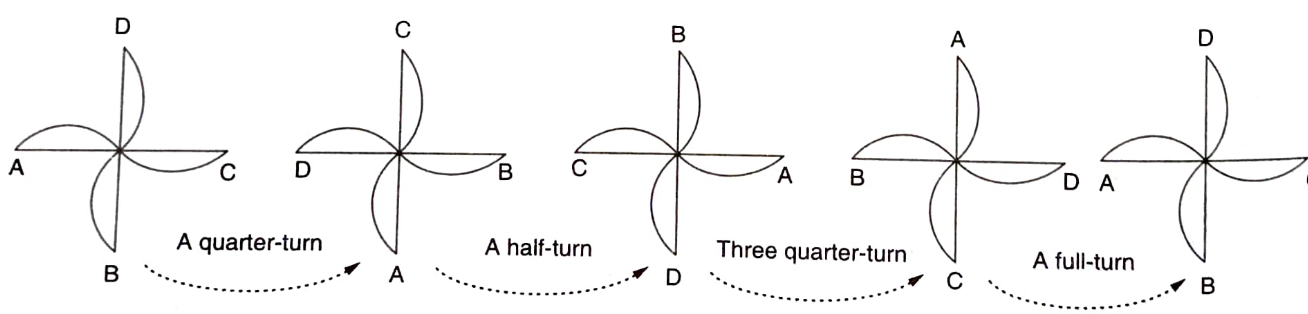


Fig. 38

ROTATIONAL SYMMETRY *A figure is said to have rotational symmetry if it fits onto itself more than once during a full turn i.e. rotation through 360°.*

Consider fig. 38 (i). Let us discuss the rotation of this figure about point O in anti-clockwise direction. When we give it a quarter-turn (i.e. rotation through 90°), we get fig. 38 (ii). Now rotate fig. 38 (ii) through 90° i.e. give a quarter turn to fig. 38 (ii) to get fig. 38 (iii). We observe that fig. 38 (iii) looks exactly as the original fig. 38 (i). Thus, when we rotate fig. 38 (i) about point O by a half-turn i.e. through 180°, we get the same figure. So, we say that fig. 38 (i) has rotational symmetry.

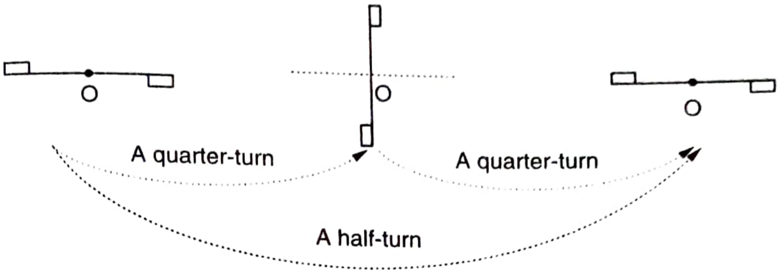


Fig. 39

We also observe that fig. 38 (i) fits onto itself twice in a full-turn i.e. rotation through 360°. So, we say that this figure has rotational symmetry of order 2.

Thus, we define the order of rotational symmetry of a figure as follows:

ORDER OF ROTATIONAL SYMMETRY The number of times a figure fits onto itself in one full-turn is called the order of rotational symmetry.

NOTE 1 *Rotating a figure through 90° clockwise direction is the same as rotating it through 270° in anti-clockwise direction. In other-words, 90° clockwise rotation is equivalent to 270° anti-clockwise rotation.*

NOTE 2 *Rotating a figure through 180° in clockwise direction is the same as rotating it through 180° in anti-clockwise direction i.e., 180° rotations in either direction are equivalent.*

ILLUSTRATIVE EXAMPLES

Example 1 Which of the following figures have rotational symmetry about the marked point (*)? Also, find the order of rotational symmetry.

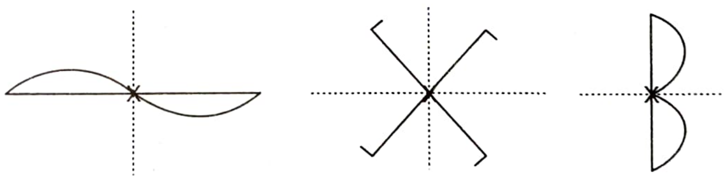


Fig. 40

Solution (i) We observe that fig.40 (i) fits onto itself when we give it a half-turn i.e. when it is rotated through 180° as shown below.

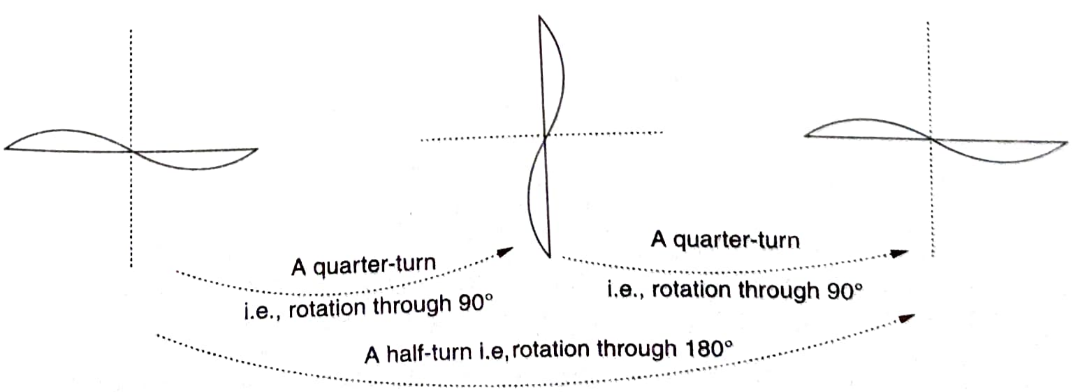


Fig. 41

Thus, the given figure has rotational symmetry of order 2 about the marked point.

(ii) We observe that the given figure when rotated through 90°, 180°, 270° and 360°, it coincides with itself i.e., it fits exactly onto itself in each case. So, it has rotational symmetry of order 4.

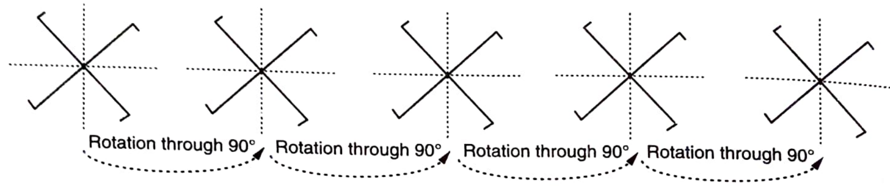


Fig. 42

(iii) Clearly, given figure coincides with itself when it is rotated through 360° i.e., when it takes a full-turn.

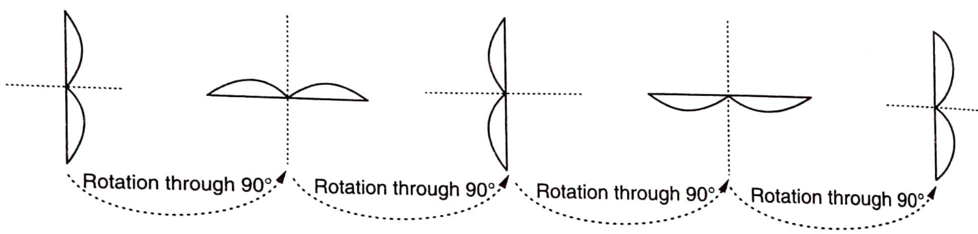


Fig. 43

So, it has rotational symmetry of order 1.

Example 2 Discuss the rotational symmetry and line symmetry of an equilateral triangle.

Solution Let ABC be an equilateral triangle with O as its centroid. We have,

We observe that the triangle ABC fits exactly onto itself when it is rotated through 120°, 240° and 360° about the centroid O. So, it has rotational symmetry of order 3.

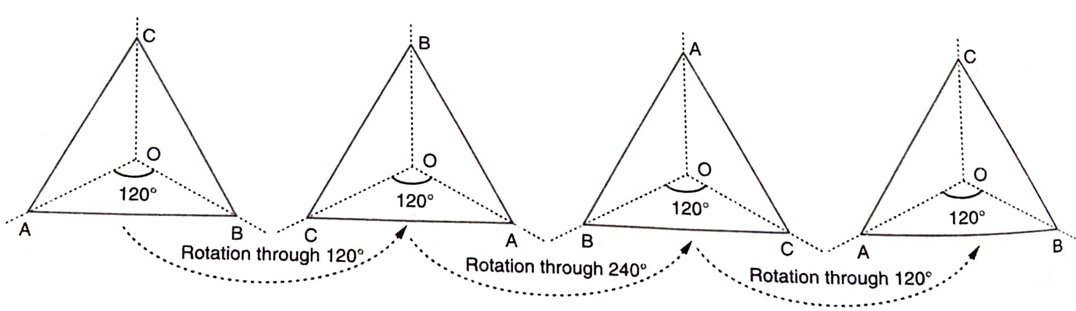


Fig. 44

We also observe that ΔABC has OA, OB and OC as three lines of symmetry. These lines are the bisector of interior angles of the triangle.

Thus, an equilateral triangle has 3 lines of symmetry and rotational symmetry of order 3.

Example 3 Discuss the rotational symmetry of a square. Also, determine its lines of symmetry.

Solution Let *ABCD* be a square with centre *O* as shown in fig. 45. Let us rotate it through 90°, 180°, 270° and 360° about point *O* to attain various positions shown below.

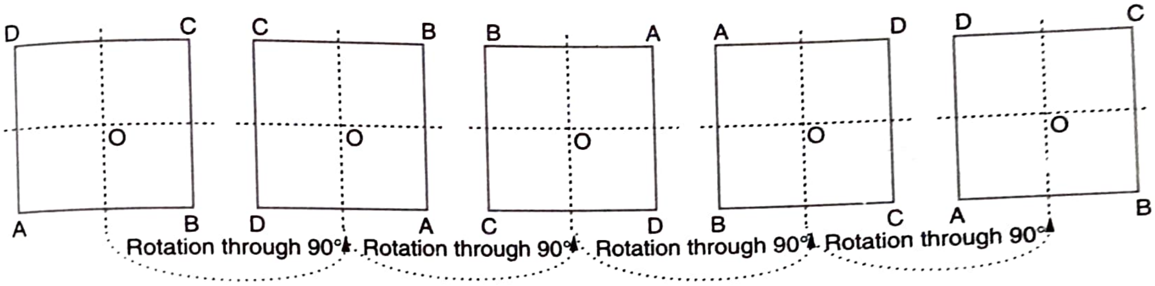


Fig. 45

We observe that the square *ABCD* fits exactly onto itself.

Clearly, after each rotation about point *O* the square attains the position which exactly fits onto itself.

Hence, a square has rotational symmetry of order 4.

In section 18.3, we have seen that a square has 4 lines of symmetry, namely the diagonals and the lines joining the mid-points of opposite sides.

Example 4 Discuss the rotational and line symmetry of a rectangle.

Solution Let *ABCD* be a rectangle such that the line joining the mid-points of opposite sides intersect at *O*.

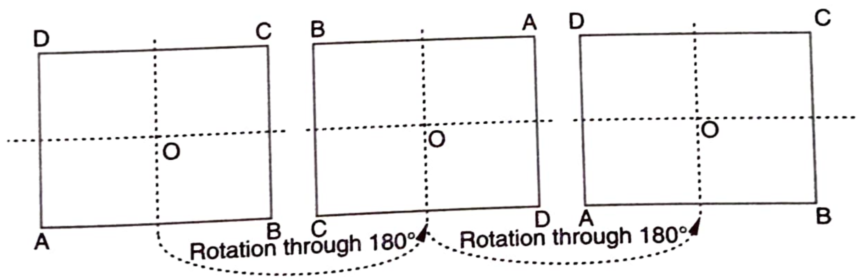


Fig. 46

Clearly, these two lines are lines of symmetry of the rectangle.

If we rotate the rectangle about point *O* through 180° and 360°, we find that it fits onto itself.

So, it has rotational symmetry of order 2.

Example 5 Show that each of the letters *H, I, N, S, X* and *Z* has a rotational symmetry of order 2. Also, mark the point of rotational symmetry in each case.

Solution We observe that each of the letters *H, I, N, S, X* and *Z* fits onto itself when it is rotated through 180° and 360° about point *O* as shown below. Thus each of the letters *H, I, N, S, X* and *Z* has a rotational symmetry of order 2.

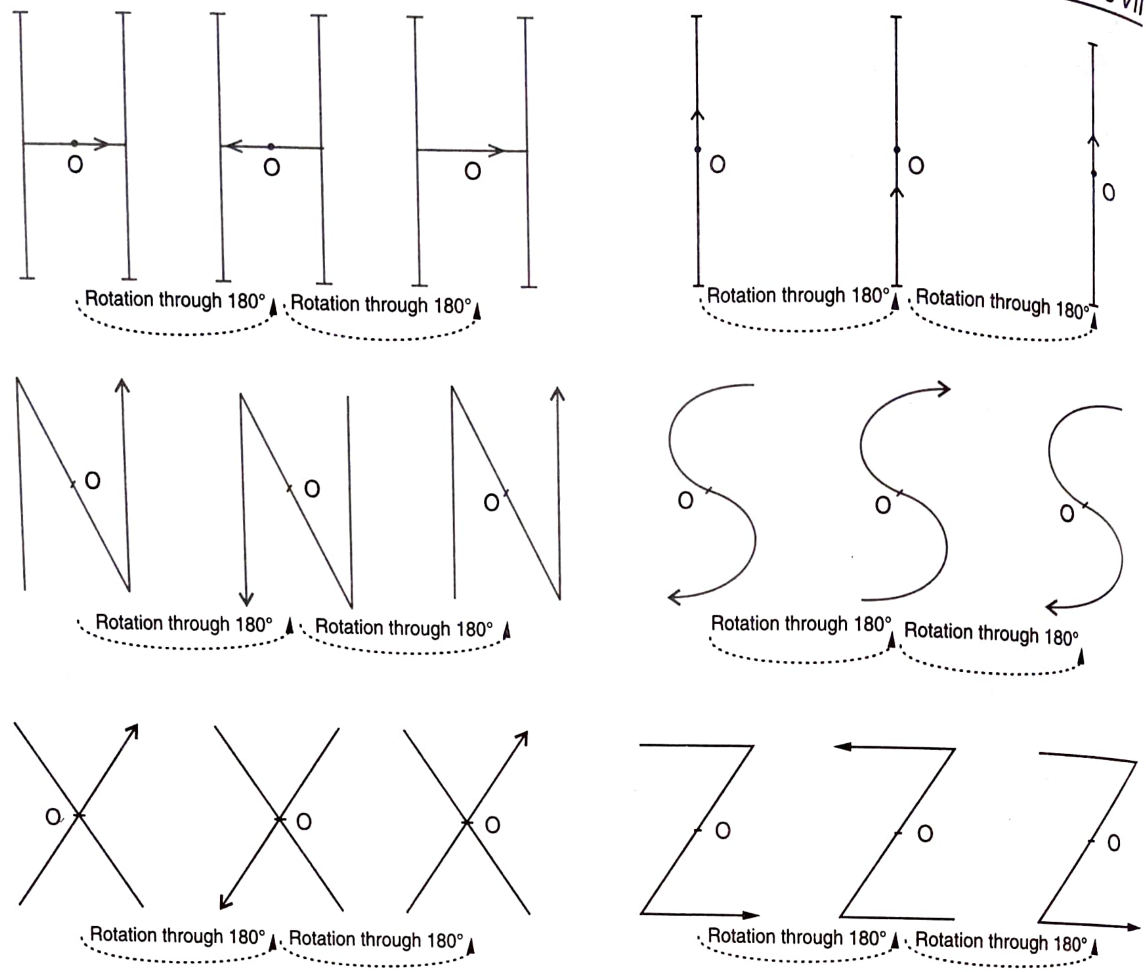


Fig. 47

Example 6 Discuss the rotational and line symmetry of a circle.

Solution Let O be the centre of circle shown in Fig. 48. Clearly, it is symmetrical about every diameter. So, every diameter of it is a line of symmetry.

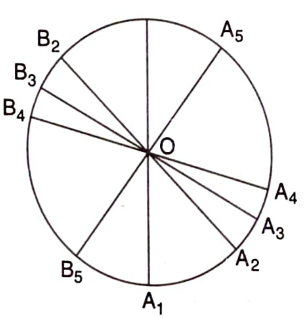


Fig. 48

Thus, a cirlce has unlimited number of lines of symmetry.

If we rotate the circle about its centre O through any angle, then it fits onto itself. Therefore, a circle has rotational symmetry around the centre for every angle. Thus, a circle has unlimited number of angles of symmetry and the order of its rotation is infinite.

Example 7 Discuss the rotational symmetry of a parallelogram. Also, find the centre of rotation, if any.

Solution In order to check the rotational symmetry of a parallelogram, let us perform the following activity.

(i) *Draw two identical parallelograms ABCD on a piece of paper and A′ B′ C′ D′ on a transparent sheet.*

(ii) *Mark the points of intersection of their diagonals as O and O′ respectively.*

(iii) *Place the parallelograms such that A′ lies on A, B′ lies on B, C′ lies on C and D′ lies on D. In such a situation O′ will fall on O.*

(iv) *Stick a pin into the shapes at the point O.*

(v) *Turn the transparent sheet in the clockwise direction so that the two shapes coincide i.e. overlap each other. You will find that the two shapes will coincide in two positions — once when A′ falls on A and secondly when A′ falls on C.*

Thus, a parallelogram has rotational symmetry of order 2. The point where we have stick the pin is the centre of rotation. It is the point of intersection of diagonals of the parallelogram.

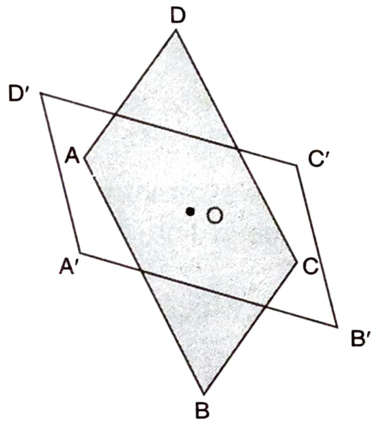


Fig. 49

Example 8 Discuss the rotational and line symmetry of the following figure.

Solution We observe that the figure fits onto itself when it is rotated through 120°, 240° and 360°.

So, it has rotational symmetry of order 3.

We also observe that the given figure has lines l, m and n as three lines of symmetry.

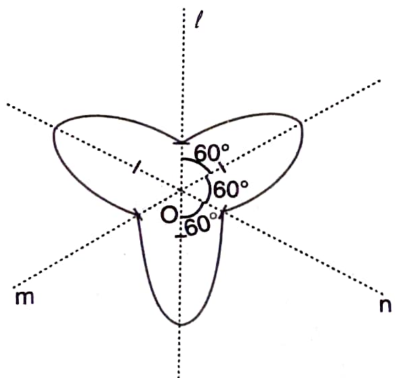


Fig. 50

EXERCISE 18.3

1. Give the order of rotational symmetry for each of the following figures when rotated about the marked point (x):

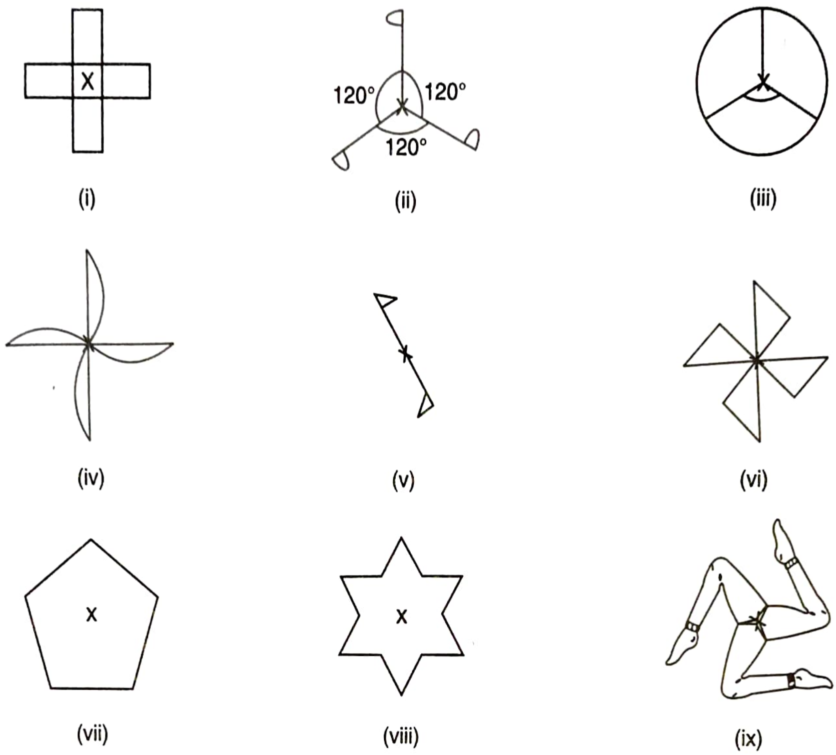


Fig. 51

2. Name any two figures that have both line symmetry and rotational symmetry.
3. Give an example of a figure that has a line of symmetry but does not have rotational symmetry.
4. Give an example of a geometrical figure which has neither a line of symmetry nor a rotational symmetry.
5. Give an example of a letter of the English alphabet which has (i) no line of symmetry and (ii) rotational symmetry of order 2.
6. What is the line of symmetry ot a semi-circle? Does it have rotational symmetry?
7. Draw, whenever possible, a rough sketch of
   (i) a triangle with both line and rotational symmetries.
   (ii) a triangle with only line symmetry and no rotational symmetry.
   (iii) a quadrilateral with a rotational symmetry but not a line of symmetry.
   (iv) a quadrilateral with line symmetry but not a rotational symmetry.
8. Fill in the blanks:

Figures	*Centre of rotation*	*Order of rotation*	*Angle of rotation*
Square			
Rectangle			
Rhombus			
Equilateral triangle			
Regular hexagon			
Circle			
Semi-circle			

9. Fill in the blanks:

English alphabet Letter	Line Symmetry	Number of Lines of symmetry	Rotational Symmetry	Order of rotational Symmetry
Z	Nil	0	Yes	2
S	–	–	–	–
H	Yes	–	Yes	–
O	Yes	–	Yes	–
E	Yes	–	–	–
N	–	–	Yes	–
C	–	–	–	–

ANSWERS

1. (i) 4 (ii) 3 (iii) 3 (iv) 4 (v) 2 (vi) 4 (vii) 5 (viii) 6 (ix) 3
2. An equilateral triangle, A square
3. A semi-circle, An isosceles triangle
4. A scalene triangle
5. (i) Z (ii) N
6. Perpendicular bisector of the diameter, No
7. (i) Draw an equilateral triangle (ii) Draw an isosceles triangle
8.

Figure	Centre of rotation	Order of rotation	Angle of rotation
Square	Point of intersection of the line segments joining the mid-points of opposite sides.	4	90°
Rectangle	Point of intersection of the line segments joining the mid-points of opposite sides.	2	180°
Rhombus	Point of intersection of diagonals	2	180°
Equilateral triangle	Point of intersection of angle bisectors i.e. centroid	3	120°
Regular hexagon	Centre of the hexagon	6	60°
Circle	Centre of the circle	Unlimited	Any angle
Semi-circle	Nil	Nil	Nil

9.

English alphabet Letter	Line Symmetry	Number of Lines of symmetry	Rotational Symmetry	Order of rotational Symmetry
Z	No	0	Yes	2
S	No	0	Yes	2
H	Yes	2	Yes	2
O	Yes	4	Yes	2
E	Yes	1	No	0
N	No	0	Yes	2
C	Yes	1	No	0

OBJECTIVE TYPE QUESTIONS

Mark the correct alternative in each of the following:

1. Which of the following has only 2 lines of symmetry?
   (a) Equilateral triangle (b) Rhombus (c) Circle (d) None of these
2. Which of the following is/are point symmetric?
   (a) Rectangle (b) Square (c) Parallelogram (d) All of these

3. Which of the following has an infinite number of lines of symmetry?
   (a) Equilateral triangle (b) Isosceles triangle (c) Regular hexagon (d) Circle
4. Which of the following is point symmetric?
   (a) Equilateral triangle (b) Trapezium (c) Rectangle (d) None of these
5. The number of lines of symmetry of a square is
   (a) 2 (b) 3 (c) 4 (d) Infinite
6. The number of lines of symmetry of a rectangle is
   (a) 2 (b) 3 (c) 4 (d) 1
7. The number of lines of symmetry of an equilateral triangle is
   (a) 1 (b) 2 (c) 3 (d) 0
8. The order of rotational symmetry of an equilateral triangle is
   (a) 0 (b) 1 (c) 2 (d) 3
9. A rectangle has rotational symmetry of order
   (a) 1 (b) 2 (c) 3 (d) 4
10. The number of lines of symmetry of an isosceles triangle is
   (a) 0 (b) 1 (c) 2 (d) 3

ANSWERS

1. (b) 2. (d) 3. (d) 4. (c) 5. (c) 6. (a) 7. (c)
8. (d) 9. (b) 10. (b)

THINGS TO REMEMBER

1. *If a line divides a figure into two parts such that when the figure is folded about the line the two parts of the figure coin cide, then the line is known as the line of symmetry. The line of symmetry is also known as the axis of symmetry.*
2. *A figure is said to have rotational symmetry if it fits on to itself more than once during a full turn i.e. rotation through 360°.*
3. *The number of times a figure fits onto ifself in one full turn is called the order of rotational symmetry.*
4. *Following table provides the details of linear and rotational symmetries of various figures:*

Figure	*Line Symmetry*	*No. of Line Symmetry*	*Rotational Symmetry*	*Centre of Rotation*	*Order of Rotational Symmetry*
Square	*Yes*	*4*	*Yes*	*Intersection of diagonals*	*4*
Rectangle	*Yes*	*2*	*Yes*	*Intersection of diagonals*	*2*
Equilateral Triangle	*Yes*	*3*	*Yes*	*Centroid*	*3*
Regular Hexagon	*Yes*	*6*	*Yes*	*Centre of the hexagon*	*6*
Circle	*Yes*	*Unlimited*	*Yes*	*Centre*	*Unlimited*
Parallelogram	*Yes*	*2*	*Yes*	*Intersection of diagonals*	*2*
Rhombus	*Yes*	*2*	*Yes*	*Intersection of diagonals*	*2*