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PRACTICE PAPER 08 (2024-25)
CHAPTER 07 BINOMIAL THEOREM (ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**
CLASS : XI **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. For what value of k, coefficients of x^2 and x^3 will be equal in the expansion of $(3 + kx)^9$?
(a) 4/7 (b) 5/7 (c) 9/7 (d) 11/7
Ans. (c) 9/7
Given $(3 + kx)^9$

$$\text{General term} = {}^9C_r\,(3)^{9-r}\,(kx)^r = {}^9C_r\,.\,3^{9-r}\,k^r\,x^r$$

$$\text{Coefficient of } x^2 = {}^9C_2.\ 3^7.\ k^2$$

and coefficient of $x^3 = {}^9C_3\ .\ 3^6\ .\ k^3$

If ${}^9C_2\ .\ 3^7.\ k^2 = {}^9C_3\ .\ 3^6.\ k^3$

$$\Rightarrow \frac{9\times 8}{2}\times 3^7\times k^2 = \frac{9\times 8\times 7}{6}\times 3^6\times k^3$$

$$\Rightarrow 3 = \frac{7}{3}k \Rightarrow k = \frac{9}{7}.$$

2. Find the number of terms in the expansion of the following : $(1 + 2x + x^2)^{20}$
(a) 41 (b) 42 (c) 43 (d) 44
Ans. (a) 41
$(1 + 2x + x^2)^{20} = ((1 + x)^2)^{20} = (1 + x)^{40}$
$\Rightarrow$ 41 terms in the expansion.

3. Find the number of terms in the expansion of the following : $(1 + 5\sqrt{2}\,x)^9 + (1 - 5\sqrt{2}\,x)^9$
(a) 4 (b) 5 (c) 10 (d) 11
Ans. (b) 5
Number of terms = $\frac{9+1}{2} = 5$

4. Given the integers r > 1, n > 2 and coefficients of (3r)th and (r + 2)th terms in the binomial expansion of $(1 + x)^{2n}$ are equal then
(a) n = 2r (b) n = 3r (c) n = 2r + 1 (d) none of these
Ans. (a) n = 2r
We know (r + 1)th term in the expansion of $(a + b)^n$ is given by
$T_{r+1} = {}^nC_r a^{n-r} b^r$
Coefficient of (r + 1)th term = ${}^nC_r$
According to the given condition, $T_{3r} = T_{r+2}$
i.e. ${}^{2n}C_{(3r-1)} = {}^{2n}C_{(r+1)}$

$\Rightarrow 3r - 1 = r + 1$ or $3r - 1 + r + 1 = 2n$
$\Rightarrow r = 1$ or $4r = 2n$
$\Rightarrow r = 1$ (rejected) or $n = 2n$

5. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is
(a) 50 (b) 202 (c) 51 (d) none of these
Ans. (c) 51
we know $(n + a)^n + (n - a)^n = 2[{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + ...]$
Here $n = 100$, even numbers from 1 to 100 are 50
$\therefore$ Total terms $= 50 + 1 = 51$

6. The coefficient of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ are in the ratio
(a) 1 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1
Ans: (d) 2 : 1
coefficient of x^n in the expansion of $(1 + x)^{2n} = {}^{2n}C_n$
According to given condition
${}^{2n}C_n : {}^{2n-1}C_n = ?$

i.e. $$\frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{(2n)!}{n!n!}}{\frac{(2n-1)!}{n!(2n-1-n)!}} = \frac{2n}{n} = \frac{2}{1}$$

i.e. ${}^{2n}C_n : {}^{2n-1}C_n = 2 : 1$

7. If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal then n is equal to
(a) 56 (b) 55 (c) 45 (d) 15
Ans. (b) 55

8. The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is
(a) 102 (b) 25 (c) 26 (d) none of these
Ans: (c) 26
Total number of terms in the expansion of $(x + a)^{51}$ is 52.
So, $(x + a)^{51} - (x - a)^{51}$ contains 26 terms (out of 52 terms 26 terms will be cancelled)
[or 1 to 51 there are 26 odd numbers]

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** The coefficient of a^4b^5 in the expansion of $(a + b)^9$ is ${}^9C_4$.
Reason (R): The formula of $(a + b)^n$ is ${}^nC_0a^nb^0 + {}^nC_1a^{n-1}b^1 + \ldots\ldots. + {}^nC_n a^n$.
Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

10. **Assertion (A):** The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$
Reason (R): The coefficient of x^6 in the expansion of $(1 + x)^n$ is ${}^nC_6$.
Ans: (d) A is false but R is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

Ans. Given coefficient $(1 - 3x + 7x^2)(1 - x)^{16}$

We know that, $T_{r+1} = {}^nC_r a^{n-r} b^r$

$(1 - 3x + 7x^2)(1 - x)^{16}$

$= (1 - 3x + 7x^2)({}^{16}C_0 - {}^{16}C_1 x^1 + {}^{16}C_2 x^2 + ... + {}^{16}C_{16} x^{16})$

$= (1 - 3x + 7x^2)(1 - 16x + 120x^2 + . . .)$

Coefficient of x = $-16 + 3 = -19$

12. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans: $(1.1)^{10000} = (1 + 0.1)^{10000}$

$= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + ... + {}^{10000}C_{10000}(0.1)^{10000}$

$= 1 + 1000 + (+\text{ve terms}) > 1000$

13. Using Binomial theorem, find the value of $(0.98)^{14}$ upto 4 places of decimal.

Ans. $(0.98)^{14} = (1 - 0.02)^{14}$

$= 1 + {}^{14}C_1 (-0.02)^1 + {}^{14}C_2 (-0.02)^2 + {}^{14}C_3 (-0.02)^3$

[Neglecting higher powers of (0. 01)]

$= 1 - 14(0.02) + 91(0.0004) - 364(0.000008)$

$= 1 - 0.28 + 0.0364 - 0.002912 = 0.753488.$

14. If the coefficients or $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms in the expansion of $(1 + x)^{34}$ are equal, find r.

Ans: The coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms of the expansion $(1 + x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$, respectively. Since they are equal so ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

Therefore, either $r - 6 = 2r - 2$ or $r - 6 = 34 - (2r - 2)$

[Using the fact that if ${}^nC_r = {}^nC_p$, then either $r = p$ or $r = n - p$]

So, we get $r = -4$ or $r = 14$. r being a natural number, $r = -4$ is not possible. So, $r = 14$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Evaluate: $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6$

Ans:

Consider $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6)$

We know $(x+y)^n + (x-y)^n$

$= 2\,[{}^nC_0 x^n + {}^nC_2 x^{n-2}y^2 + {}^nC_4 x^{n-4}y^4 + ...]$

Here $x = \sqrt{3}$, $y = \sqrt{2}$, $n = 6$

$\therefore \quad (\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^6$

$= 2[{}^6C_0 (\sqrt{3})^6 + {}^6C_2(\sqrt{3})^4 (\sqrt{2})^2 + {}^6C_4(\sqrt{3})^2$

$(\sqrt{2})^4 + {}^6C_6 (\sqrt{2})^6]$

$= 2[1 \times 27 + 15 \times 9 \times 2 + 15 \times 3 \times 4 + 1 \times 8]$

$= 2[27 + 270 + 180 + 8] = 970$

16. Simplify: $(x+\sqrt{x-1})^6 + (\sqrt{x}-\sqrt{x-1})^6$

Ans: Use $(x + y)^n + (x - y)^n$
$= 2[{}^nC_0\, x^n + {}^nC_2\, x^{n-2}\, y^2 + {}^nC_4\, x^{n-4}\, y^4 + ...]$
$= 2[{}^6C_0\, x^6 + {}^6C_2\, x^4(x - 1) + {}^6C_4\, x^2\, (x - 1)^2\, {}^6C_6(x - 1)^3]$
$= 2(x^6 + 15x^5 - 29x^3 + 12x^2 + 3x - 1)$

17. By using binomial theorem show that : $6^n - 5n - 1$ is divisible by 25, $n \in N$.
Ans: $6^n - 5n - 1 = (1 + 5)^n - 5n - 1$
$= [1 + 5n + {}^nC_2 . 5^2 + {}^nC_3 . 5^3 + ...5^n]-5n - 1$
$= 25[{}^nC_2 + 5.{}^nC_3 + ...5^{n-2}]$
which is divisible by 25.

SECTION – D

Questions 18 carry 5 marks.

18. Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25.
Ans. For any two numbers, say a and b, we can find numbers x and y such that a = bx + y, then we say that c divides a with x as quotient and y as remainder. Thus, in over to show that $6^n - 5n$ leaves remainder 1 when divided by 25, we should prove that $6^n - 50 = 25k + 1$, where K is some natural number.
We know that, $(1 + a)^n = {}^nC_0 + {}^nC_1\, a + {}^nC_2\, a^2 + ... + {}^nC_n\, a^n$
Now, for a = 5, we get:
$(1 + 5)^n = {}^nC_0 + {}^nC_1\, 5 + {}^nC_2\, (5)^2 +... + {}^nC_n\, (5)^n$
Now the above from can be written as :
$6^n = 1 + 5n + 5^2\, {}^nC_2 + 5^3\, {}^nC_3 + ... + 5^n$
Now, bring 5n to the L.H.S., we get
$6^n - 5n = 1 + 5^2\, {}^nC_2 + 5^3\, {}^nC_3 +... + 5^n$
$6^n - 5n = 1 + 5^2\, ({}^nC_2 + 5\, {}^nC_3 + ... + 5^{n-2})$
$6^n - 5n = 1 + 25\, ({}^nC_2 + 5\, {}^nC_3 + ... + 5^{n-2})$
$6^n - 5n = 1 + 25$ K (Where K $= {}^nC_2 + 5\, {}^nC_3 + ... + 5^{n-2}$)
Hence, proved.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Four friends applied the knowledge of Binomial Theorem while playing a game to make the equations by observing some conditions they make some equations.

(a) Expand, $(1 - x + x^2)^4$.
(b) Expand the expression, $(1 - 3x)^7$.
(c) Show that $11^9 + 9^{11}$ is divisible by 10.
Ans. (a) We have, $(1 - x + x^2)^4 = [(1 - x) + x^2]^4$
$= {}^4C_0(1 - x)^4 + {}^4C_1(1 - x)^3\, (x^2) + {}^4C_2(1 - x)^2(x^2)^2 + {}^4C_3(1 - x)\, (x^2)^3 + {}^4C_4(x^2)^4$
$= (1 - x)^4 + 4x^2\, (1 - x)^3 + 6x^4(1 - x)^2 + 4x^6\, (1 - x) + 1.x^8$
$= (1 - 4x + 6x^2 - 4x^3 + x^4) + 4x^2\, (1 - 3x + 3x^2 - x^3) + 6x^4\, (1 - 2x + x^2) + 4\, (1 - x)x^6 + x^8$

$= 1 – 4x + 6x^2 – 4x^3 + x^4 + 4x^2 – 12x^3 + 12x^4 – 4x^5 + 6x^4 –12x^5 + 6x^6 + 4x^6 – 4x^7 + x^8$
$= 1 – 4x + 10x^2 – 16x^3 + 19x^4 – 16x^5 + 10x^6 – 4x^7 + x^8$
(b) Here, a = 1, b = 3x, and n = 7
Given, $(1 – 3x)^7 = {}^7C_0(1)^7 – {}^7C_1(1)^6 (3x)^1 + {}^7C_2(1)^5 (3x)^2 – {}^7C_3(1)^4 (3x)^3 + {}^7C_4(1)^3 (3x)^4 – {}^7C_5(1)^2 (3x)^5 + {}^7C_6(1)^1 (3x)^6 – {}^7C_7(1)^0 (3x)^7$
$= 1 – 21x + 189x^2 – 945x^3 + 2835x^4 – 5103x^5 + 5103x^6 – 2187x^7$.
(c) $11^9 + 9^{11} = (10 + 1)^9 + (10 – 1)^{11}$
$= ({}^9C_0. 10^9 + {}^9C_1.10^8 + ... {}^9C_9) + ({}^{11}C_0.10^{11} – {}^{11}C_1.10^{10} + ... –{}^{11}C_{11})$
$= {}^9C_0.10^9 + {}^9C_1.10^8 + ... + {}^9C_8.10 + 1 + 10^{11} – {}^{11}C_1.10^{10} + ... + {}^{11}C_{10}.10 – 1$
$= 10[{}^9C_0.10^8 + {}^9C_1.10^7 + ... + {}^9C_8 + {}^{11}C_0.10^{10} – {}^{11}C_1.10^9 + ... + {}^{11}C_{10}]$
= 10 K, which is divisible by 10.

20. Pascal's triangle defines the coefficients which appear in binomial expansions. That means the nth row of Pascal's triangle comprises the coefficients of the expanded expression of the polynomial $(x + y)^n$.

Exponent	Pascal's Triangle												
0							1						
1						1		1					
2					1		2		1				
3				1		3		3		1			
4			1		4		6		4		1		
5		1		5		10		10		5		1	
6	1		6		15		20		15		6		1

The expansion of $(x + y)^n$ is:
$(x + y)^n = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \dots + a_{n-1}xy^{n-1} + a_ny^n$
Based on the above data answer any four of the following questions.
(a) Find the number of terms in the expansion of $(x + y)^6$
(b) Find the coefficient of the fifth term in the expansion of $(x + y)^6$
(c) Find the number of terms in the expansion of $(x + y)^{10} (x – y)^{10}$
(d) Find the number of terms in $(x + y)^{10} + (x – y)^{10}$
Ans. (a) Here, n = 6
Number of terms = n + 1 = 6 + 1 = 7
(b) $T_5 = {}^6C_4x^2y^4 = 15x^2y^4$
Co-efficient of fifth term = 15
(c) $(x + y)^{10}(x – y)^{10} = (x^2 – y^2)^{10}$
Number of terms = 10 + 1 = 11
(d) n = 10
Number of terms = n/2 + 1 = 5 + 1 = 6