MATHEMATICS
WORKSHEET_171023
CHAPTER 03 TRIGONOMETRIC FUNCTIONS (ANSWERS)

SUBJECT: MATHEMATICS | **MAX. MARKS : 40**

CLASS : XI | **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

SECTION – A
Questions 1 to 10 carry 1 mark each.

1. The value of $\frac{1-\tan^2 15^0}{1+\tan^2 15^0}$

(a) 1 (b) √3 (c) √3/2 (d) 2

Ans: (c) √3/2

Let θ = 15° ⇒ 2θ = 30°

Now, since we know that,

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \cos 30° = \frac{1-\tan^2 15°}{1+\tan^2 15°} \Rightarrow \frac{\sqrt{3}}{2} = \frac{1-\tan^2 15°}{1+\tan^2 15°}$$

2. If cos x = −1/2 and 0 < x < 2π, then the solutions are:

(a) x = π/3, 4π/3 (b) x = 2π/3, 4π/3 (c) x = 2π/3, 7π/3 (d) x = 2π/3, 5π/3

Ans: (b) x = 2π/3, 4π/3

3. If tan A = $\frac{1}{2}$ and tan B = $\frac{1}{3}$, then tan(2A + B) is equal to

(a) 1 (b) 2 (c) 3 (d) 4

Ans: (c) 3

$$\tan(2A+B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B}$$

$$= \frac{\frac{2\tan A}{1-\tan^2 A} + \tan B}{1 - \left(\frac{2\tan A}{1-\tan^2 A}\right)\tan B} = \frac{\frac{2\times\frac{1}{2}}{1-\frac{1}{4}} + \frac{1}{3}}{1 - \left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right)\times\frac{1}{3}} = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = \frac{5/3}{5/9} = 3$$

4. The value of sin 50° – sin 70° + sin 10° is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

Ans: (b) 0

sin 50° – sin 70° + sin 10°

$= 2\cos\left(\frac{50° + 70°}{2}\right) \cdot \sin\left(\frac{50° - 70°}{2}\right) + \sin 10°$

$= 2 \cos 60° \cdot \sin(-10°) + \sin 10°$

$= -2 \times \frac{1}{2} \times \sin 10° + \sin 10°$

$= -\sin 10° + \sin 10° = 0$

5. If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan\alpha)(1 + \tan\beta)$ is

(a) 1 (b) 2 (c) –2 (d) none of these

Ans: (b) 2

$$\alpha + \beta = \frac{\pi}{4} \quad \text{(given)}$$

$$\Rightarrow \quad \tan(\alpha + \beta) = \tan\frac{\pi}{4} \Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$$

$$\Rightarrow \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 1 \qquad ...(i)$$

Now $(1 + \tan\alpha)(1 + \tan\beta)$

$= 1 + \tan\alpha + \tan\beta + \tan\alpha\tan\beta$

$= 1 + 1 \qquad \{\text{using } (i)\}$

$= 2$

6. If $\cos x + \sqrt{3}\sin x = 2$, then the value of x is:

(a) $\pi/3$ (b) $2\pi/3$ (c) $4\pi/3$ (d) $5\pi/3$

Ans: (a) $\pi/3$

7. The radius of the circle in which a central angle of 30° intercepts an arc of length 22 cm is: (Use π = 22/7)

(a) 43 cm (b) 41 cm (c) 42 cm (d) 40 cm

Ans: (c) 42 cm

8. The angle in the radian through which a pendulum swings its length is 80 cm and tip describes an arc of length 20 cm is:

(a) 1/4 (b) 2/25 (c) 3/25 (d) 4/25

Ans: (a) 1/4

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true & Reason (R) is the correct explanation of Assertion (A)
(b) Both Assertion (A) and Reason (R) are true & Reason (R) is not the correct explanation of Assertion (A)
(c) Assertion (A) is true but Reason (R) is false
(d) Assertion (A) is false but Reason (R) is true

9. **Assertion (A):** The value of $\sin(-690°)\cos(-300°) + \cos(-750°)\sin(-240°) = 1$

Reason (R): The value of sin and cos is negative in the third and fourth quadrant respectively.

Ans: (c) Assertion (A) is true but Reason (R) is false

10. **Assertion (A):** $\sin(x + y)\cos(x - y) - \cos(x + y)\sin(x - y) = \sin 2x$

Reason (R): $\sin A \cos B - \cos A \sin B = \sin(A - B)$

Ans: (d) Assertion (A) is false but Reason (R) is true

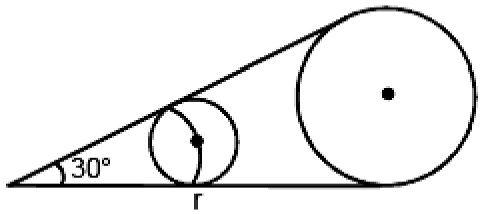
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If the angular diameter of the moon be 30′, how far from the eye a coin of diameter 2.2 cm be kept to hide the moon?

Ans:

$\theta = 30'; l = 2.2$ cm; $r = ?$



$$\theta = 30' = \frac{30}{60} \times \frac{\pi}{180}$$

$$\therefore \quad \frac{\pi}{360} = \frac{2.2}{r} \Rightarrow r = \frac{22 \times 360 \times 7}{22 \times 10} = 252 \text{ cm}.$$

12. Find the value of $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{3\pi}{4} - 2\tan^2 \frac{3\pi}{4}$.

Ans:

Consider, $2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{3\pi}{4} - 2 \tan^2 \frac{3\pi}{4}$ ...(*i*)

$$\sin \frac{3\pi}{4} = \sin 135° = \sin (180° - 45°)$$

$$= \sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos \frac{3\pi}{4} = \cos 135° = \cos (180° - 45°)$$

$$= -\cos 45° = -\frac{1}{\sqrt{2}}$$

$$\tan \frac{3\pi}{4} = \tan 135° = \tan (180° - 45°)$$

$$= -\tan 45° = -1$$

Substituting in (*i*), we get

$$= 2 \left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(-\frac{1}{\sqrt{2}}\right)^2 - 2 (-1)^2$$

$$= 2 \times \frac{1}{2} + 2 \times \frac{1}{2} - 2 \times 1 = 1 + 1 - 2 = 0$$

13. Prove the following : sin 10° + sin 20° + sin 40° + sin 50° = sin 70° + sin 80°.

Ans: (sin 50° + sin 10°) + (sin 40° + sin 20°)

= 2 sin 30° cos 20° + 2 sin 30° cos 10°.

= cos 20° + cos 10°

= cos (90° – 70°) + cos (90° – 80°)

= sin 70° + sin 80°.

14. Find the value of tan 22° 30′.

Ans:

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Let $\theta = 45°$

$$\therefore \quad \tan\frac{45°}{2} = \tan 22° \, 30'$$

$$= \sqrt{\frac{1-\cos 45°}{1+\cos 45°}} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{2}-1$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and x, y both lie in the second quadrant, find the values of $\cos(x - y)$

Ans: Given, $\sin x = \frac{3}{5}$, $\cos y = \frac{-12}{13}$ and x, y both lie in the second quadrant.

We know that $\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} \Rightarrow \cos x = \pm\frac{4}{5}$

Since, x lies in 2nd quadrant, $\cos x$ is (–ve).

$\therefore \cos x = \frac{-4}{5}$

Also, $\sin^2 y = 1 - \cos^2 y = 1 - \left(\frac{-12}{13}\right)^2 = \frac{25}{169} \Rightarrow \sin y = \pm\frac{5}{13}$

Since, y lies in 2nd quadrant, $\sin y$ is (+ve)

$\therefore \sin y = \frac{5}{13}$

$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

$$= \left(\frac{-4}{5}\right)\left(\frac{-12}{13}\right) + \frac{3}{5}\times\frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$$

16. If $\sin\alpha = k\sin\beta$, prove that, $\tan\left(\frac{\alpha-\beta}{2}\right) = \frac{k-1}{k+1}\tan\frac{\alpha+\beta}{2}$.

Ans: Given, $\sin\alpha = k\sin\beta \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{k}{1}$

Applying componendo and dividendo, we get, $\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta} = \frac{k+1}{k-1}$

$$\frac{2\sin\frac{\alpha+\beta}{2}\,.\,\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\,.\,\sin\frac{\alpha-\beta}{2}} = \frac{k+1}{k-1} \Rightarrow \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}} = \frac{k+1}{k-1} \Rightarrow \tan\frac{\alpha-\beta}{2} = \frac{k-1}{k+1}\,.\,\tan\frac{\alpha+\beta}{2}$$

17. Prove that $\dfrac{\sin A.\sin 2A+\sin 3A.\sin 6A}{\sin A.\cos 2A+\sin 3A.\cos 6A}=\tan 5A$

Ans: LHS = $\dfrac{\sin A.\sin 2A+\sin 3A.\sin 6A}{\sin A.\cos 2A+\sin 3A.\cos 6A}$

$=\dfrac{2\sin 2A\sin A+2\sin 6A\sin 3A}{2\cos 2A\sin A+2\cos 6A\sin 3A}$

$=\dfrac{\cos A-\cos 3A+\cos 3A-\cos 9A}{\sin 3A-\sin A+\sin 9A-\sin 3A}$

$=\dfrac{\cos A-\cos 9A}{\sin 9A-\sin A}=\dfrac{2\sin 5A\sin 4A}{2\cos 5A\sin 4A}$

= tan 5A = RHS

OR

Prove that $\sqrt{2+\sqrt{2+2\cos 4x}}=2\cos x, 0<x<\dfrac{\pi}{4}$.

Ans: LHS

$=\sqrt{2+\sqrt{2+2\cos 4x}}=\sqrt{2+\sqrt{2+2\left(2\cos^2 2x-1\right)}}$

$=\sqrt{2+\sqrt{2+4\cos^2 2x-2}}=\sqrt{2+\sqrt{4\cos^2 2x}}=\sqrt{2+2\cos 2x}$

$=\sqrt{2\left(1+\cos 2x\right)}=\sqrt{2\times 2\cos^2 x}=\sqrt{4\cos^2 x}$

= 2cos x = RHS

SECTION – D

Questions 18 carry 5 marks.

18. If α, β are two distinct roots of the equation a tanθ + b secθ = c, prove that tan (α + β) = $\dfrac{2ac}{a^2-c^2}$.

Ans: Changing into sin θ, cos θ, we get

a sin θ + b = c cos θ, α, β are the roots.

⇒ a sin α + b = c cos α and a sin β + b = c cos β.

⇒ a (sin α – sin β) = c (cos α – cos β)

$\Rightarrow a\ .\ 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$=-c.2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\Rightarrow \tan\dfrac{\alpha+\beta}{2}=-\dfrac{a}{c}$

$$\Rightarrow \tan(\alpha+\beta)=\frac{2\tan\frac{\alpha+\beta}{2}}{1-\tan^2\frac{\alpha+\beta}{2}}=\frac{-2\frac{a}{c}}{1-\frac{a^2}{c^2}}=\frac{2ac}{a^2-c^2}$$

OR

Prove that: $\sin^3 x+\sin^3\left(\dfrac{2\pi}{3}+x\right)+\sin^3\left(\dfrac{4\pi}{3}+x\right)=-\dfrac{3}{4}\sin 3x$.

Ans: LHS = $\sin^3 x + \sin^3\left(\frac{2\pi}{3}+x\right) + \sin^3\left(\frac{4\pi}{3}+x\right)$

$$= \frac{3\sin x - \sin 3x}{4} + \frac{3\sin\left(\frac{2\pi}{3}+x\right) - \sin(2\pi+3x)}{4}$$

$$+ \frac{3\sin\left(\frac{4\pi}{3}+x\right) - \sin(4\pi+3x)}{4}$$

$$= \frac{1}{4}\left[3\left\{\sin x + \sin\left(\frac{2\pi}{3}+x\right) + \sin\left(\frac{4\pi}{3}+x\right)\right\}\right.$$

$$\left. - \{\sin 3x + \sin 3x + \sin 3x\}\right]$$

$$\left[\begin{array}{l} \because\ \sin(2\pi+x) = \sin x \\ \quad\ \sin(4\pi+x) = \sin x \end{array}\right]$$

$$= \frac{1}{4}\left[3\left\{\sin x + 2\,.\,\sin(\pi+x)\cos\left(\frac{\pi}{3}\right)\right\} - 3\sin 3x\right]$$

$$= \frac{1}{4}\left[3\left\{\sin x - 2\,.\,\sin x\,.\left(\frac{1}{2}\right)\right\} - 3\sin 3x\right]$$

$$[\because\ \sin(\pi+x) = -\sin x]$$

$$= \frac{1}{4}[3\{\sin x - \sin x\} - 3\sin 3x]$$

$= -\frac{3}{4}\sin 3x$ = RHS

<u>SECTION – E (Case Study Based Questions)</u>

Questions 19 to 20 carry 4 marks each.

19. A railway train is travelling on a circular curve of 1500 metres radius at the rate of 66 km/h.

Now, answer the following:

(i) Find the angle in radians by which it turned in 10 seconds.

(ii) Find the degree measure of the angle turned by railway train in 10 seconds.

(iii) How much degree will train turn in 20 seconds?

OR

(iii) If train changes its speed to 60 km/h then what angle will train turn in 10 seconds.
Ans:

$$\text{Speed of train} = 66 \text{ km/h} = \frac{66 \times 5}{18} \text{ m/sec} = \frac{55}{3} \text{ m/sec}$$

$$\text{Distance} = \text{speed} \times \text{time} = \frac{55 \times 10}{3} \text{ m}$$

$$\text{Now angle} = \frac{\text{arc}}{\text{radius}} = \frac{55 \times 10}{3 \times 1500} = \frac{11}{90} \text{ radian} = \frac{11\pi}{90} \times \frac{7}{22} = \frac{7\pi^c}{180}$$

(*ii*) $$\text{Degree measure} = \frac{7\pi}{180} \times \frac{180°}{\pi} = 7°$$

(*iii*) $$\text{Distance in 20 sec} = \frac{55}{3} \times 20 \text{ m}$$

From (*ii*)

degree measure in 10 sec = 7°

$\therefore$ degree measure in 20 sec 14°

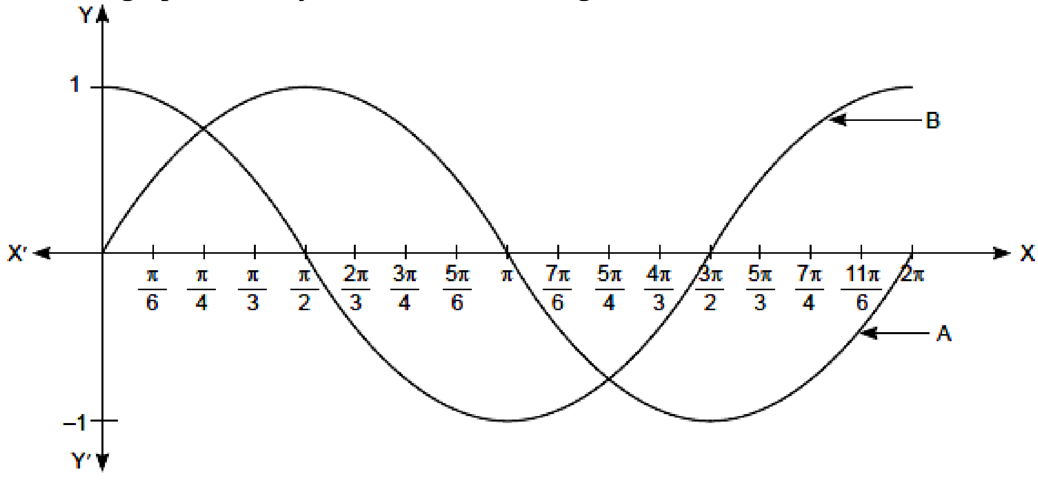
OR

(*iii*) $$\text{If speed} = 60 \text{ km/h} = 60 \times \frac{5}{18} \text{ m/sec} = \frac{50}{3} \text{ m/sec}$$

$$\text{Distance in 10 sec} = \frac{500}{3} \text{ m}$$

$$\text{Angle} = \frac{500}{3 \times 1500} = \frac{1}{9} \text{ radian} = \frac{1}{9}\pi \times \frac{7}{22} = \frac{7\pi^c}{198}$$

20. Observe the below graph carefully answer the following



(i) Graph A represent the graph of which trigonometric function.
(ii) Graph B represent the graph of which trigonometric function.
(iii) From the above graph write the principal value of x if sin x = 1
(iv) From the graph find the angle for which the value of sin x and cos x is same.
Ans: (i) Graph A is of $y = \sin x, x \in [0, 2\pi]$
(ii) Graph B is of $y = \cos x, x \in [0, 2\pi]$
(iii) $\sin x = 1$

$x = \frac{\pi}{2}$, Principal solution, as principal solution lies in [0, 2π] and only for $x = \frac{\pi}{2}$, sin x = 1.

(iv)From graph angles are $\frac{\pi}{4}$ and $\frac{5\pi}{4}$