

MATHEMATICS
WORKSHEET_071125

CHAPTER 07 BINOMIAL THEOREM (ANSWERS)

SUBJECT: MATHEMATICS

MAX. MARKS : 40

CLASS : XI

DURATION : 1½ hrs

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. For what value of k, coefficients of x^2 and x^3 will be equal in the expansion of $(3 + kx)^9$?
- (a) 4/7 (b) 5/7 (c) 9/7 (d) 11/7

Ans. (c) 9/7

Given $(3 + kx)^9$

$$\begin{aligned}\text{General term} &= {}^9C_r (3)^{9-r} (kx)^r \\ &= {}^9C_r \cdot 3^{9-r} k^r x^r\end{aligned}$$

$$\text{Coefficient of } x^2 = {}^9C_2 \cdot 3^7 \cdot k^2$$

$$\text{and coefficient of } x^3 = {}^9C_3 \cdot 3^6 \cdot k^3$$

$$\text{If } {}^9C_2 \cdot 3^7 \cdot k^2 = {}^9C_3 \cdot 3^6 \cdot k^3$$

$$\Rightarrow \frac{9 \times 8}{2} \times 3^7 \times k^2 = \frac{9 \times 8 \times 7}{6} \times 3^6 \times k^3$$

$$\Rightarrow 3 = \frac{7}{3}k \Rightarrow k = \frac{9}{7}$$

2. Find the number of terms in the expansion of the following : $(1 + 2x + x^2)^{20}$
- (a) 41 (b) 42 (c) 43 (d) 44

Ans. (a) 41

$$(1 + 2x + x^2)^{20} = ((1 + x)^2)^{20} = (1 + x)^{40}$$

$\Rightarrow$ 41 terms in the expansion.

3. Find the number of terms in the expansion of the following : $(1 + 5\sqrt{2}x)^9 + (1 - 5\sqrt{2}x)^9$
- (a) 4 (b) 5 (c) 10 (d) 11

Ans. (b) 5

$$\text{Number of terms} = \frac{9+1}{2} = 5$$

4. Given the integers $r > 1$, $n > 2$ and coefficients of $(3r)$ th and $(r + 2)$ th terms in the binomial expansion of $(1 + x)^{2n}$ are equal then
- (a) $n = 2r$ (b) $n = 3r$ (c) $n = 2r + 1$ (d) none of these

Ans. (a) $n = 2r$

We know $(r + 1)$ th term in the expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$\text{Coefficient of } (r + 1)\text{th term} = {}^nC_r$$

According to the given condition, $T_{3r} = T_{r+2}$

$$\text{i.e. } {}^{2n}C_{(3r-1)} = {}^{2n}C_{(r+1)}$$

$$\Rightarrow 3r - 1 = r + 1 \text{ or } 3r - 1 + r + 1 = 2n$$

$$\Rightarrow r = 1 \text{ or } 4r = 2n$$

$$\Rightarrow r = 1 \text{ (rejected) or } n = 2n$$

5. The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is
 (a) 50 (b) 202 (c) 51 (d) none of these

Ans. (c) 51

$$\text{we know } (n + a)^n + (n - a)^n = 2[{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \dots]$$

Here $n = 100$, even numbers from 1 to 100 are 50

$$\therefore \text{Total terms} = 50 + 1 = 51$$

6. The coefficient of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ are in the ratio
 (a) 1 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

Ans: (d) 2 : 1

$$\text{coefficient of } x^n \text{ in the expansion of } (1 + x)^{2n} = {}^{2n}C_n$$

According to given condition

$${}^{2n}C_n : {}^{2n-1}C_n = ?$$

$$\text{i.e. } \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{(2n)!}{n!n!}}{\frac{(2n-1)!}{n!(2n-1-n)!}} = \frac{2n}{n} = \frac{2}{1}$$

$$\text{i.e. } {}^{2n}C_n : {}^{2n-1}C_n = 2 : 1$$

7. If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal then n is equal to

- (a) 56 (b) 55 (c) 45 (d) 15

Ans. (b) 55

8. The total number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is
 (a) 102 (b) 25 (c) 26 (d) none of these

Ans: (c) 26

Total number of terms in the expansion of $(x + a)^{51}$ is 52.

So, $(x + a)^{51} - (x - a)^{51}$ contains 26 terms (out of 52 terms 26 terms will be cancelled)

[or 1 to 51 there are 26 odd numbers]

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
 (b) Both A and R are true but R is not the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false but R is true.

9. **Assertion (A):** The coefficient of a^4b^5 in the expansion of $(a + b)^9$ is 9C_4 .

Reason (R): The formula of $(a + b)^n$ is ${}^nC_0a^nb^0 + {}^nC_1a^{n-1}b^1 + \dots + {}^nC_na^n$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

10. **Assertion (A):** The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$

Reason (R): The coefficient of x^6 in the expansion of $(1 + x)^n$ is nC_6 .

Ans: (d) A is false but R is true.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

Ans. Given coefficient $(1 - 3x + 7x^2)(1 - x)^{16}$

We know that, $T_{r+1} = {}^nC_r a^{n-r} b^r$

$$(1 - 3x + 7x^2)(1 - x)^{16}$$

$$= (1 - 3x + 7x^2)({}^{16}C_0 - {}^{16}C_1 x^1 + {}^{16}C_2 x^2 + \dots + {}^{16}C_{16} x^{16})$$

$$= (1 - 3x + 7x^2)(1 - 16x + 120x^2 + \dots)$$

$$\text{Coefficient of } x = -16 + 3 = -13$$

12. Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans: $(1.1)^{10000} = (1 + 0.1)^{10000}$

$$= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + \dots + {}^{10000}C_{10000}(0.1)^{10000}$$

$$= 1 + 1000 + (\text{+ve terms}) > 1000$$

13. Using Binomial theorem, find the value of $(0.98)^{14}$ upto 4 places of decimal.

Ans. $(0.98)^{14} = (1 - 0.02)^{14}$

$$= 1 + {}^{14}C_1(-0.02)^1 + {}^{14}C_2(-0.02)^2 + {}^{14}C_3(-0.02)^3$$

[Neglecting higher powers of (0.01)]

$$= 1 - 14(0.02) + 91(0.0004) - 364(0.000008)$$

$$= 1 - 0.28 + 0.0364 - 0.002912 = 0.753488.$$

14. If the coefficients of $(r - 5)^{\text{th}}$ and $(2r - 1)^{\text{th}}$ terms in the expansion of $(1 + x)^{34}$ are equal, find r .

Ans: The coefficients of $(r - 5)^{\text{th}}$ and $(2r - 1)^{\text{th}}$ terms of the expansion $(1 + x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$, respectively. Since they are equal so ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

Therefore, either $r - 6 = 2r - 2$ or $r - 6 = 34 - (2r - 2)$

[Using the fact that if ${}^nC_r = {}^nC_p$, then either $r = p$ or $r = n - p$]

So, we get $r = -4$ or $r = 14$. r being a natural number, $r = -4$ is not possible. So, $r = 14$.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Evaluate: $(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6$

Ans:

Consider $(\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6$

We know $(x + y)^n + (x - y)^n$

$$= 2[{}^nC_0 x^n + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + \dots]$$

Here $x = \sqrt{3}$, $y = \sqrt{2}$, $n = 6$

$$\therefore (\sqrt{3} + \sqrt{2})^6 + (\sqrt{3} - \sqrt{2})^6$$

$$= 2[{}^6C_0 (\sqrt{3})^6 + {}^6C_2 (\sqrt{3})^4 (\sqrt{2})^2 + {}^6C_4 (\sqrt{3})^2$$

$$(\sqrt{2})^4 + {}^6C_6 (\sqrt{2})^6]$$

$$= 2[1 \times 27 + 15 \times 9 \times 2 + 15 \times 3 \times 4 + 1 \times 8]$$

$$= 2[27 + 270 + 180 + 8] = 970$$

16. Simplify: $(x + \sqrt{x-1})^6 + (\sqrt{x} - \sqrt{x-1})^6$

Ans: Use $(x + y)^n + (x - y)^n$
 $= 2[{}^nC_0 x^n + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + \dots]$
 $= 2[{}^6C_0 x^6 + {}^6C_2 x^4 (x-1) + {}^6C_4 x^2 (x-1)^2 + {}^6C_6 (x-1)^3]$
 $= 2(x^6 + 15x^5 - 29x^3 + 12x^2 + 3x - 1)$

17. By using binomial theorem show that : $6^n - 5n - 1$ is divisible by 25, $n \in \mathbb{N}$.

Ans: $6^n - 5n - 1 = (1 + 5)^n - 5n - 1$
 $= [1 + 5n + {}^nC_2 \cdot 5^2 + {}^nC_3 \cdot 5^3 + \dots 5^n] - 5n - 1$
 $= 25[{}^nC_2 + 5 \cdot {}^nC_3 + \dots 5^{n-2}]$
 which is divisible by 25.

SECTION – D

Questions 18 carry 5 marks.

18. Using the binomial theorem, show that $6^n - 5n$ always leaves remainder 1 when divided by 25.

Ans. For any two numbers, say a and b, we can find numbers x and y such that $a = bx + y$, then we say that c divides a with x as quotient and y as remainder. Thus, in order to show that $6^n - 5n$ leaves remainder 1 when divided by 25, we should prove that $6^n - 5n = 25k + 1$, where K is some natural number.

We know that, $(1 + a)^n = {}^nC_0 + {}^nC_1 a + {}^nC_2 a^2 + \dots + {}^nC_n a^n$

Now, for $a = 5$, we get:

$$(1 + 5)^n = {}^nC_0 + {}^nC_1 5 + {}^nC_2 (5)^2 + \dots + {}^nC_n (5)^n$$

Now the above from can be written as :

$$6^n = 1 + 5n + 5^2 {}^nC_2 + 5^3 {}^nC_3 + \dots + 5^n$$

Now, bring $5n$ to the L.H.S., we get

$$6^n - 5n = 1 + 5^2 {}^nC_2 + 5^3 {}^nC_3 + \dots + 5^n$$

$$6^n - 5n = 1 + 5^2 ({}^nC_2 + 5 {}^nC_3 + \dots + 5^{n-2})$$

$$6^n - 5n = 1 + 25 ({}^nC_2 + 5 {}^nC_3 + \dots + 5^{n-2})$$

$$6^n - 5n = 1 + 25 K \text{ (Where } K = {}^nC_2 + 5 {}^nC_3 + \dots + 5^{n-2})$$

Hence, proved.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Four friends applied the knowledge of Binomial Theorem while playing a game to make the equations by observing some conditions they make some equations.



(a) Expand, $(1 - x + x^2)^4$.

(b) Expand the expression, $(1 - 3x)^7$.

(c) Show that $11^9 + 9^{11}$ is divisible by 10.

Ans. (a) We have, $(1 - x + x^2)^4 = [(1 - x) + x^2]^4$
 $= {}^4C_0 (1 - x)^4 + {}^4C_1 (1 - x)^3 (x^2) + {}^4C_2 (1 - x)^2 (x^2)^2 + {}^4C_3 (1 - x) (x^2)^3 + {}^4C_4 (x^2)^4$
 $= (1 - x)^4 + 4x^2 (1 - x)^3 + 6x^4 (1 - x)^2 + 4x^6 (1 - x) + 1 \cdot x^8$
 $= (1 - 4x + 6x^2 - 4x^3 + x^4) + 4x^2 (1 - 3x + 3x^2 - x^3) + 6x^4 (1 - 2x + x^2) + 4(1 - x)x^6 + x^8$

$$= 1 - 4x + 6x^2 - 4x^3 + x^4 + 4x^2 - 12x^3 + 12x^4 - 4x^5 + 6x^4 - 12x^5 + 6x^6 + 4x^6 - 4x^7 + x^8$$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

(b) Here, $a = 1$, $b = 3x$, and $n = 7$

Given, $(1 - 3x)^7 = {}^7C_0(1)^7 - {}^7C_1(1)^6(3x)^1 + {}^7C_2(1)^5(3x)^2 - {}^7C_3(1)^4(3x)^3 + {}^7C_4(1)^3(3x)^4 - {}^7C_5(1)^2(3x)^5 + {}^7C_6(1)^1(3x)^6 - {}^7C_7(1)^0(3x)^7$

$$= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7.$$

(c) $11^9 + 9^{11} = (10 + 1)^9 + (10 - 1)^{11}$

$$= ({}^9C_0 \cdot 10^9 + {}^9C_1 \cdot 10^8 + \dots + {}^9C_9) + ({}^{11}C_0 \cdot 10^{11} - {}^{11}C_1 \cdot 10^{10} + \dots - {}^{11}C_{11})$$

$$= {}^9C_0 \cdot 10^9 + {}^9C_1 \cdot 10^8 + \dots + {}^9C_8 \cdot 10 + 1 + 10^{11} - {}^{11}C_1 \cdot 10^{10} + \dots + {}^{11}C_{10} \cdot 10 - 1$$

$$= 10[{}^9C_0 \cdot 10^8 + {}^9C_1 \cdot 10^7 + \dots + {}^9C_8 + {}^{11}C_0 \cdot 10^{10} - {}^{11}C_1 \cdot 10^9 + \dots + {}^{11}C_{10}]$$

$$= 10K, \text{ which is divisible by } 10.$$

20. Pascal's triangle defines the coefficients which appear in binomial expansions. That means the n th row of Pascal's triangle comprises the coefficients of the expanded expression of the polynomial $(x + y)^n$.

Exponent	Pascal's Triangle								
0	1								
1	1		1						
2	1		2		1				
3	1		3		3		1		
4	1		4		6		4	1	
5	1		5		10		10	5	1
6	1	6	15	20	15	6	1		

The expansion of $(x + y)^n$ is:

$$(x + y)^n = a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \dots + a_{n-1}xy^{n-1} + a_ny^n$$

Based on the above data answer any four of the following questions.

- (a) Find the number of terms in the expansion of $(x + y)^6$
- (b) Find the coefficient of the fifth term in the expansion of $(x + y)^6$
- (c) Find the number of terms in the expansion of $(x + y)^{10} (x - y)^{10}$
- (d) Find the number of terms in $(x + y)^{10} + (x - y)^{10}$

Ans. (a) Here, $n = 6$

$$\text{Number of terms} = n + 1 = 6 + 1 = 7$$

(b) $T_5 = {}^6C_4x^2y^4 = 15x^2y^4$

$$\text{Co-efficient of fifth term} = 15$$

(c) $(x + y)^{10}(x - y)^{10} = (x^2 - y^2)^{10}$

$$\text{Number of terms} = 10 + 1 = 11$$

(d) $n = 10$

$$\text{Number of terms} = n/2 + 1 = 5 + 1 = 6$$