CHAPTER 6

GRAPHS OF TRIGONOMETRIC FUNCTIONS

6.1 INTRODUCTION

In the previous chapters, we have learnt that all trigonometric functions are periodic functions. We have also learnt that sine, cosine, cosecant and secant functions are periodic with period 2π whereas tangent and cotangent functions are periodic with period π. We also know that if a function $f(x)$ is periodic with period T, then $f(ax+b)$ is periodic with period $T/|a|$. Therefore, $\sin(ax+b)$, $\cos(ax+b)$, $\operatorname{cosec}(ax+b)$ and $\sec(ax+b)$ are periodic with period $2\pi/|a|$ and $\tan(ax+b)$ and $\cot(ax+b)$ are periodic with period $\pi/|a|$. For example, $\sin 2x$ and $\cos 3\pi x$ are periodic functions with periods π and $2/3$ respectively.

If the graph of a periodic function with period T is to be drawn in a given interval, then it is sufficient to draw its graph only in an interval of length T. Because, once it is drawn in one such interval, it can-easily be drawn completely by repeating it over the intervals of lengths T. As mentioned in the above paragraph all trigonometric functions are periodic functions. So, we will draw their graphs on the intervals of lengths equal to their periods.

6.2 GRAPH OF SINE FUNCTION

We know that $f(x) = \sin x$ is a periodic function with period 2π. Therefore, it is sufficient to know the graph of $f(x) = \sin x$ in the interval $[0, 2\pi]$. Using the periodicity of the function, we can draw the graph of $f(x) = \sin x$ in other intervals such as $[-2\pi, 0]$, $[2\pi, 4\pi]$ etc.

In order to draw the graph of $f(x) = \sin x$ in the interval $[0, 2\pi]$, we require the values of $\sin x$ at some points in $[0, 2\pi]$. These values are listed in the following table.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$y = \sin x$	0	0.5	0.71	0.86	1	0.86	0.71	0.5	0	−0.5	−0.71	−0.86	−1	−0.86	−0.71	−0.5	0

On suitable scale we plot the points $(0, 0)$ $(\pi/6, 0.5)$, $(\pi/4, 0.71)$, $(\pi/3, 0.86)$ $(\pi/2, 1)$, $(2\pi/3, 0.86)$, $(3\pi/4, 0.71)$, $(5\pi/6, 0.5$, $(\pi, 0)$, $(7\pi/6, -0.5)$, $(5\pi/4, -0.71)$, $(4\pi/3, -0.86)$ $(3\pi/2, -1)$, $(5\pi/3, -0.86)$, $(7\pi/4, -0.71)$, $(11\pi/6, -0.5)$ and $(2\pi, 0)$ in the xy-plane and join them by a free hand curve to obtain the curve $y = \sin x$ i.e. the graph of $f(x) = \sin x$ in the interval $[0, 2\pi]$ as shown in Fig. 6.1.

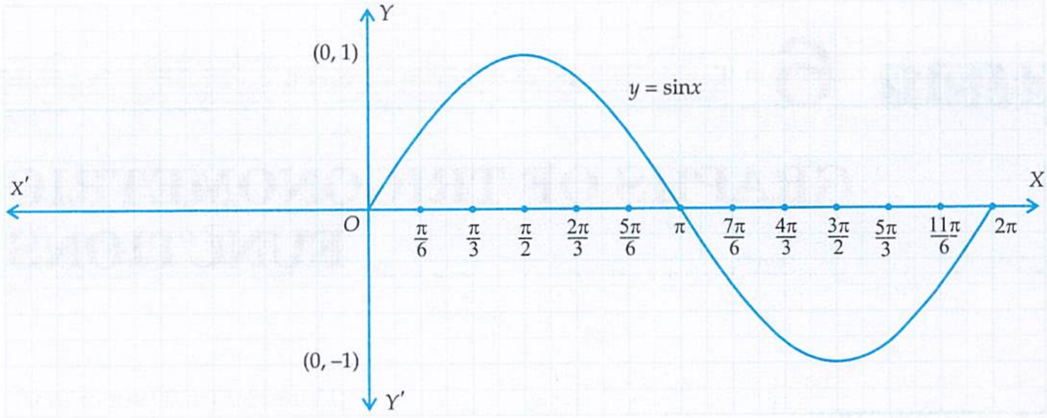

Fig. 6.1 Graph of $f(x) = \sin x$ in $[0, 2\pi]$

As $f(x) = \sin x$ is a periodic function with period 2π. The graph of $f(x) = \sin x$ in the interval $[-2\pi, 0]$ is identical to its graph in $[0, 2\pi]$ as shown in Fig. 6.2.

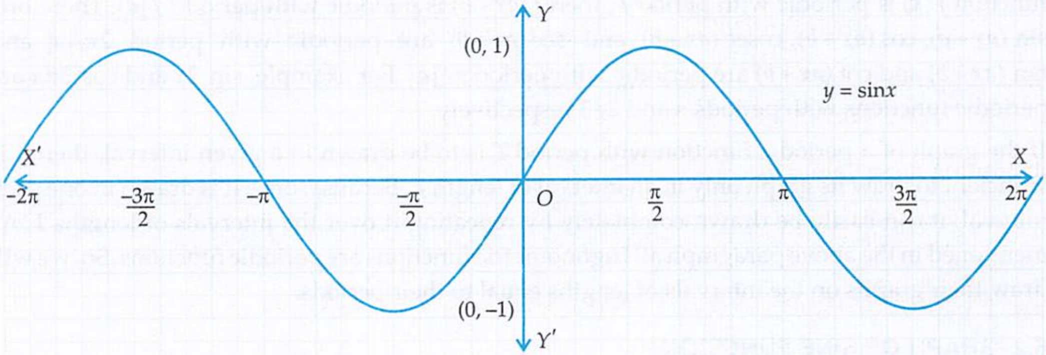

Fig. 6.2 Graph of $f(x) = \sin x$ in $[-2\pi, 2\pi]$

ILLUSTRATIVE EXAMPLES

BASED ON BASIC CONCEPTS (BASIC)

EXAMPLE 1 *Sketch the graph of the function $f(x) = 3 \sin 2x$.*

SOLUTION We know that $g(x) = \sin x$ is periodic function with period 2π. Therefore, $f(x) = 3 \sin 2x$ is periodic function with period π. So, we will draw the graph of $f(x) = 3 \sin 2x$ in the interval $[0, \pi]$ and to draw (or know) its graph in other intervals such as $[-\pi\ 0]$, $[\pi, 2\pi]$ etc, we may use the periodicity of the function. The values of $f(x) = 3 \sin 2x$ at various points in $[0, \pi]$ are listed in the following table.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$	π
$2x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = 3\sin 2x$	0	$\frac{3}{2}$ $= 1.5$	$\frac{3}{\sqrt{2}}$ $= 2.1$	$\frac{3\sqrt{3}}{2}$ $= 2.58$	3	$\frac{3\sqrt{3}}{2}$ $= 2.58$	$\frac{3}{\sqrt{2}}$ $= 2.1$	$\frac{3}{2}$ $= 1.5$	0	$-\frac{3}{2}$ $= -1.5$	$\frac{-3}{\sqrt{2}}$ $= -2.1$	$\frac{-3\sqrt{3}}{2}$ $= -2.58$	-3	$\frac{-3\sqrt{3}}{2}$ $= -2.58$	$\frac{-3}{\sqrt{2}}$ $= -2.1$	$\frac{-3}{2}$ $= -1.5$	0

The points $(0, 0)$ $(\pi/12, 1.5)$, $(\pi/8, 2.1)$, $(\pi/6, 2.58)$, $(\pi/4, 3)$, $(\pi/3, 2.58)$, $(3\pi/8, 2.13)$, $(5\pi/12, 1.5)$, $(\pi/2, 0)$, $(7\pi/12, -1.5)$, $(5\pi/8, -2.13)$, $(2\pi/3, -2.58)$ $(3\pi/4, -3)$, $(5\pi/6, -2.58)$, $(7\pi/8, -2.13)$, $(11\pi/12, -1.5)$ and $(\pi, 0)$ are plotted on a suitable scale in the xy-plane and joined by a free hand curve to obtain the graph of the function $f(x) = 3 \sin 2x$ i.e., the curve $y = 3 \sin 2x$ as shown in Fig. 6.3

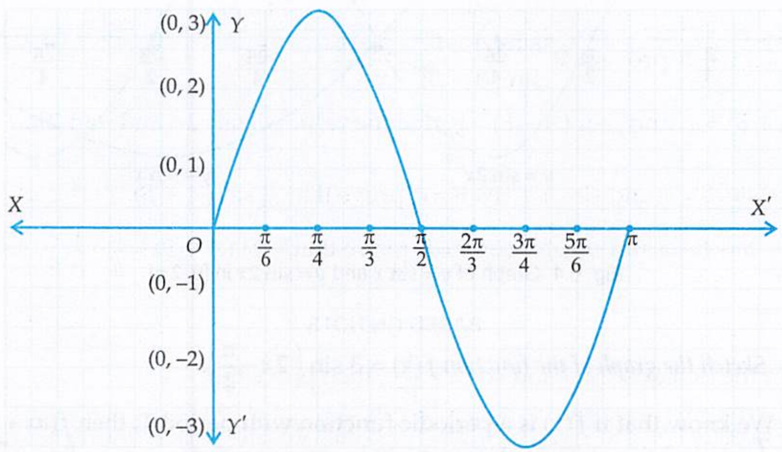

Fig. 6.3 Graph of $f(x) = 3 \sin 2x$ in $[0, \pi]$

EXAMPLE 2 *Sketch the curves* $y = \sin x$ *and* $y = \sin 2x$ *on the same scale and same axes.*

SOLUTION We observe that the functions $f(x) = \sin x$ and $g(x) = \sin 2x$ are periodic functions with periods 2π and π respectively. The values of these functions are tabulated below.

Values of $f(x) = \sin x$ in $[0, 2\pi]$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

Values of $g(x) = \sin 2x$ in $[0, \pi]$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{8}$	2π
$\sin 2x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

In order to draw the curves $y = \sin x$ and $y = \sin 2x$, we plot the points given in the above tables and join them by free hand curves as shown in Fig. 6.4.

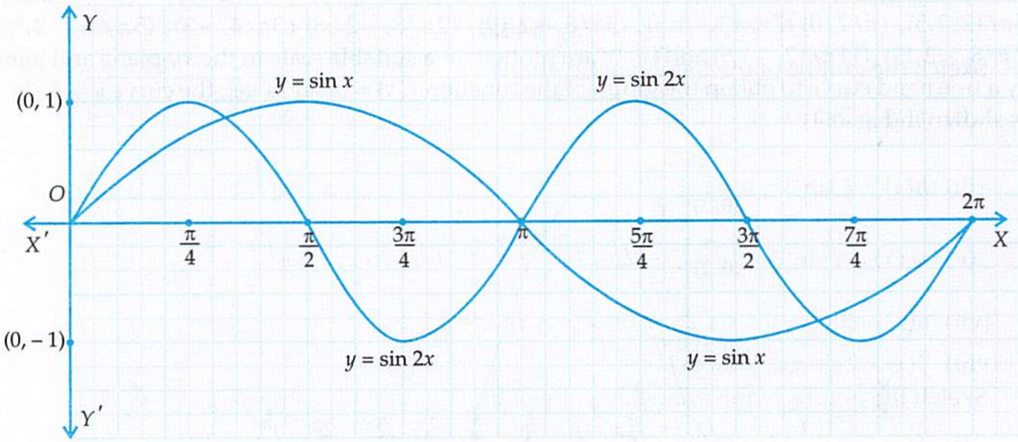

Fig. 6.4 Graph of $y = \sin x$ and $y = \sin 2x$ in $[0, 2\pi]$

EXAMPLE 3 *Sketch the graph of the function* $f(x) = 3 \sin\left(2x - \frac{\pi}{4}\right)$.

SOLUTION We know that if $f(x)$ is a periodic function with period T, then $f(ax + b)$ is periodic with period $\frac{T}{|a|}$. As $\sin x$ is periodic with period 2π. Therefore, $f(x) = 3 \sin\left(2x - \frac{\pi}{4}\right)$ is periodic with period $\frac{2\pi}{2}$ i.e. π. The values of $f(x)$ for different values of x are listed in the following table.

$2x - \frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
x	$\frac{\pi}{8}$	$\frac{5\pi}{24}$	$\frac{\pi}{4}$	$\frac{7\pi}{24}$	$\frac{3\pi}{8}$	$\frac{11\pi}{24}$	$\frac{\pi}{2}$	$\frac{13\pi}{24}$	$\frac{5\pi}{8}$	$\frac{17\pi}{24}$	$\frac{3\pi}{4}$	$\frac{19\pi}{24}$	$\frac{7\pi}{8}$	$\frac{23\pi}{24}$	π	$\frac{25\pi}{24}$	$\frac{9\pi}{8}$
$3\sin\left(2x - \frac{\pi}{4}\right)$	0	$\frac{3}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3\sqrt{3}}{2}$	3	$\frac{3\sqrt{3}}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$\frac{-3}{\sqrt{2}}$	$\frac{-3\sqrt{3}}{2}$	-3	$\frac{-3\sqrt{3}}{\sqrt{2}}$	$\frac{-3}{\sqrt{2}}$	$\frac{-3}{2}$	0

In the xy-plane, let us plot the points having x-coordinates given in the second row of the above table and y-coordinate as the corresponding value in the third row. By joining the points so obtained by a free hand curve, we obtain the curve $y = 3\sin\left(2x - \frac{\pi}{4}\right)$ i.e. the graph of the function $f(x) = 3 \sin\left(2x - \frac{\pi}{4}\right)$ as shown in Fig. 6.5.

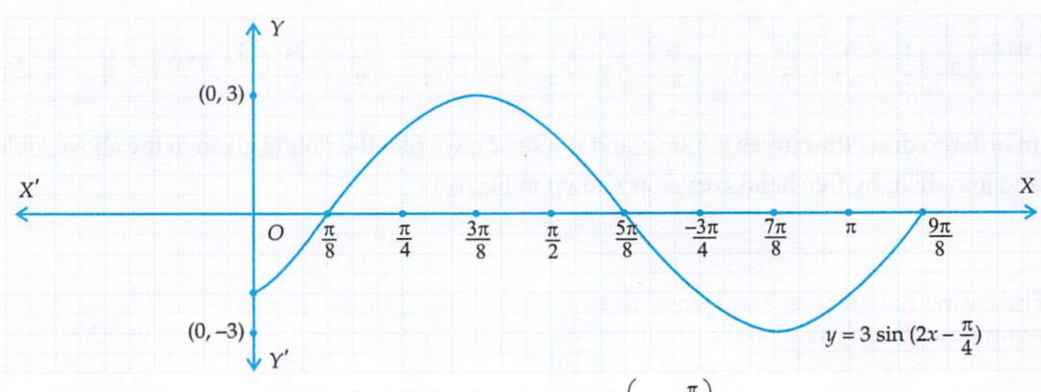

Fig. 6.5 Graph of $f(x) = 3 \sin\left(2x - \frac{\pi}{4}\right)$

EXERCISE 6.1

BASIC

1. Sketch the graphs of the following functions:

   (i) $f(x) = 2\sin x,\ 0 \le x \le \pi$

   (ii) $g(x) = 3\sin\left(x - \frac{\pi}{4}\right),\ 0 \le x \le \frac{5\pi}{4}$

   (iii) $h(x) = 2\sin 3x,\ 0 \le x \le \frac{2\pi}{3}$

   (iv) $\phi(x) = 2\sin\left(2x - \frac{\pi}{3}\right),\ 0 \le x \le \frac{7\pi}{5}$

   (v) $\psi(x) = 4\sin 3\left(x - \frac{\pi}{4}\right),\ 0 \le x \le 2\pi$

   (vi) $\theta(x) = \sin\left(\frac{x}{2} - \frac{\pi}{4}\right),\ 0 \le x \le 4\pi$

   (vii) $u(x) = \sin^2 x,\ 0 \le x \le 2\pi$ $\ v(x) = |\sin x|,\ 0 \le x \le 2\pi$

   (viii) $f(x) = 2\sin \pi x,\ 0 \le x \le 2.$

2. Sketch the graphs of the following pairs of functions on the same axes:

   (i) $f(x) = \sin x,\ g(x) = \sin\left(x + \frac{\pi}{4}\right)$

   (ii) $f(x) = \sin x,\ g(x) = \sin 2x$

   (iii) $f(x) = \sin 2x,\ g(x) = 2\sin x$

   (iv) $f(x) = \sin\frac{x}{2},\ g(x) = \sin x$

6.3 GRAPH OF COSINE FUNCTION

In earlier chapters, we have learnt that $f(x) = \cos x$ is a periodic function with period 2π. In order to draw the graph of $f(x) = \cos x$ it is sufficient to know its graph in $[0, 2\pi]$. The values of $f(x) = \cos x$ at various points in $[0, 2\pi]$ are given in the following table.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

On a suitable scale, let us plot the points $(0, 1), (\pi/6, \sqrt{3}/2), (\pi/4, 1/\sqrt{2}), (\pi/3, 1/2), (\pi/2, 0)$, $(2\pi/3, -1/2)$, $(3\pi/4, -1/\sqrt{2})$, $(5\pi/6, -\sqrt{3}/2)$, $(\pi, -1)$, $(7\pi/6, -\sqrt{3}/2)$, $(5\pi/4, -1/\sqrt{2})$, $(4\pi/3, -1/2), (3\pi/2, 0), (5\pi/3, 1/2), (7\pi/4, 1/\sqrt{2}), (11\pi/6, \sqrt{3}/2)$, and $(2\pi, 1)$ in the xy-plane. Now join these points by a free hand curve to obtain the graph of the function $f(x) = \cos x$ i.e. the curve $y = \cos x$ as shown in Fig. 6.6.

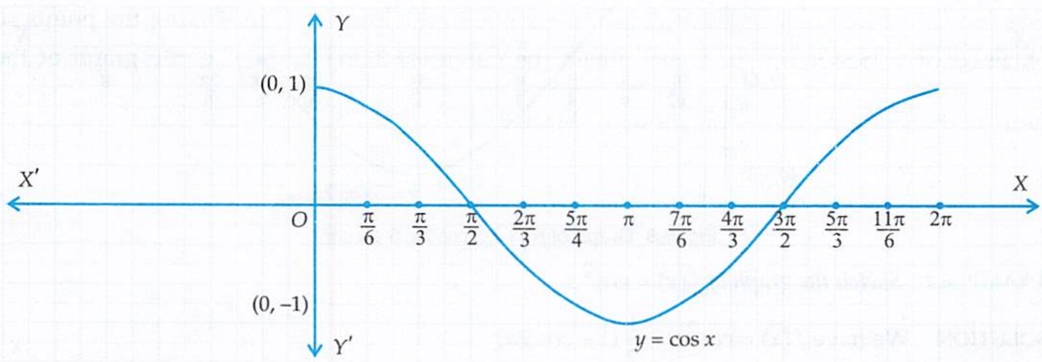

Fig. 6.6 Graph of $f(x) = \cos x,\ 0 \le x \le 2\pi$

The cosine function i.e. $f(x) = \cos x$ is an even function and the graph of an even function is symmetric about y-axis. So the graph of $f(x) = \cos x$ in $[-2\pi, 2\pi]$ is as shown in Fig. 6.7.

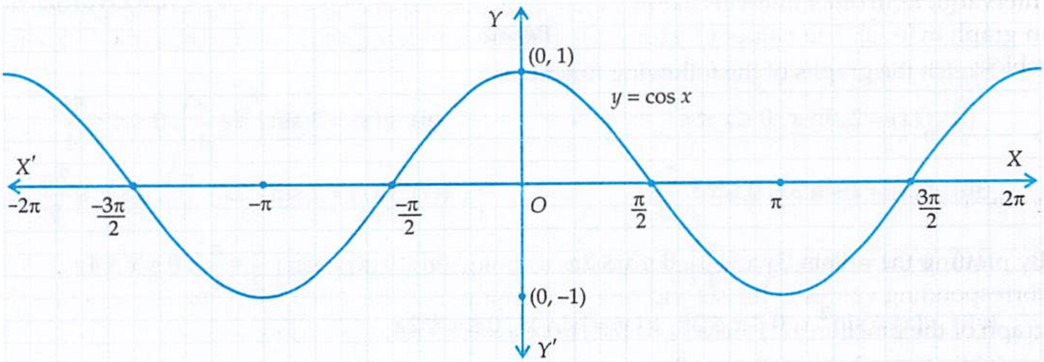

Fig. 6.7 Graph of $f(x) = \cos x, -2\pi \le x \le 2\pi$.

ILLUSTRATIVE EXAMPLES

BASED ON BASIC CONCEPTS (BASIC)

EXAMPLE 1 *Draw the graph of* $f(x) = 3\cos 2x$.

SOLUTION We know that $\cos x$ is a periodic function with period 2π. Therefore, $f(x) = 3\cos 2x$ is periodic with period π. So, it is sufficient to draw the graph of $f(x) = 3\cos 2x$ in the interval $[0, \pi]$. The values of $3\cos 2x$ for different values of x in $[0, \pi]$ are listed below.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$	π
$3\cos 2x$	3	$\frac{3\sqrt{3}}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$-\frac{3}{\sqrt{2}}$	$-\frac{3\sqrt{3}}{2}$	–3	$\frac{-3\sqrt{3}}{\sqrt{2}}$	$\frac{-3}{\sqrt{2}}$	$-\frac{3}{2}$	0	$\frac{3}{2}$	$\frac{3}{\sqrt{2}}$	$\frac{3\sqrt{3}}{2}$	3

Now, plot the points whose x-coordinates are points in the first row of the above table and the corresponding values in the second row as y-coordinates, By joining these points by a free hand curve, we obtain the graph of $f(x) = 3\cos 2x$ as shown in Fig. 6.8.

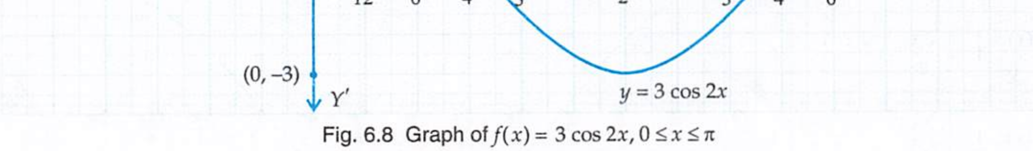

Fig. 6.8 Graph of $f(x) = 3\cos 2x, 0 \le x \le \pi$

EXAMPLE 2 *Sketch the graph of* $f(x) = \cos^2 x$.

SOLUTION We have, $f(x) = \cos^2 x = \frac{1}{2}(1 + \cos 2x)$

We know that $\cos x$ is a periodic function with period 2π. Therefore, $f(x) = \frac{1}{2} + \frac{1}{2}\cos 2x$ is also periodic function with period $\frac{2\pi}{2} = \pi$. So, it is sufficient to draw the graph of $f(x) = \cos^2 x$ in the interval $[0, \pi]$. In other intervals like $[\pi, 2\pi]$, $[-\pi, 0]$, $[2\pi, 3\pi]$ etc. the graph of $f(x)$ is identical to its in graph in $[0, \pi]$. The values of y (or $f(x)$) at some standard points are listed in the following table.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \cos^2 x$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

By plotting the points having x-coordinates as points in the first now and y coordinates as the corresponding values in the second row and joining them by a free hand curve, we obtain the graph of the function $f(x) = \cos^2 x$ or the curve $y = \cos^2 x$ as shown in Fig. 6.9.

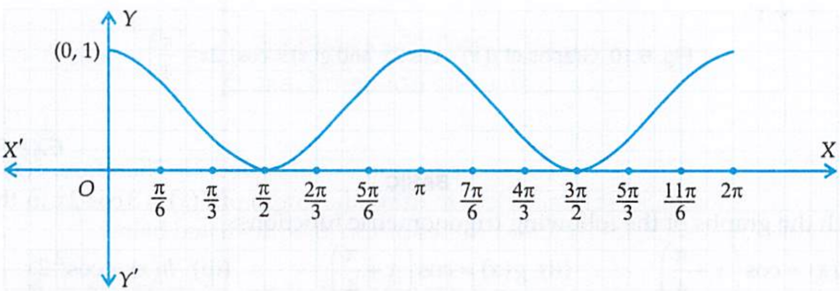

Fig. 6.9 Graph of $f(x) = \cos^2 x, 0 \le x \le 2\pi$

EXAMPLE 3 *Draw the graphs of* $f(x) = \cos 2x$ *and* $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$ *on the same axes and the same scale.*

SOLUTION We know that $\cos x$ is a periodic function with period 2π. Therefore, $f(x) = \cos 2x$ and $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$ are periodic functions with period π. So, to know about their graphs, it is sufficient to draw their graphs in an interval of length π. Let us choose the interval $[0, \pi]$. The values of $f(x) = \cos 2x$ and $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$ at various points in $[0, \pi]$ are listed below.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{8}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{8}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{5\pi}{8}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{8}$	$\frac{11\pi}{12}$	π
$f(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$\frac{-\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$g(x)$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	1	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	0	$\frac{1-\sqrt{3}}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{-1-\sqrt{3}}{2\sqrt{2}}$	-1	$-\frac{\sqrt{3}+1}{2\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1-\sqrt{3}}{2\sqrt{2}}$	0	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

By plotting the points having their x-coordinates as points in the first row of the above table and y-coordinates as the corresponding values of $f(x)$ in the second row, we obtain the graph of $f(x)$ i.e. the curve $y = \cos 2x$.

Similarly, we draw the graph of $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$ as shown in Fig. 6.10.

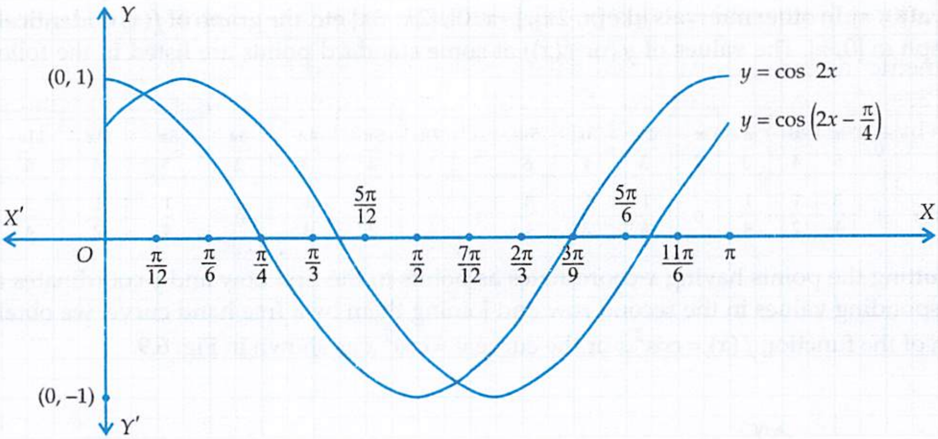

Fig. 6.10 Graphs of $f(x) = \cos 2x$ and $g(x) = \cos\left(2x - \frac{\pi}{4}\right)$

EXERCISE 6.2

BASIC

1. Sketch the graphs of the following trigonometric functions:

   (i) $f(x) = \cos\left(x - \frac{\pi}{4}\right)$ (ii) $g(x) = \cos\left(x + \frac{\pi}{4}\right)$ (iii) $h(x) = \cos^2 2x$

   (iv) $\phi(x) = 2\cos\left(x - \frac{\pi}{6}\right)$ (v) $\psi(x) = \cos 3x$ (vi) $u(x) = \cos^2 \frac{x}{2}$

   (vii) $f(x) = \cos \pi x$ (viii) $g(x) = \cos 2\pi x$

2. Sketch the graphs of the following curves on the same scale and the same axes:

   (i) $y = \cos x$ and $y = \cos\left(x - \frac{\pi}{4}\right)$ (ii) $y = \cos 2x$ and $y = \cos 2\left(x - \frac{\pi}{4}\right)$

   (iii) $y = \cos x$ and $y = \cos \frac{x}{2}$ (iv) $y = \cos^2 x$ and $y = \cos x$

6.4 GRAPH OF TANGENT FUNCTION

We have learnt in the earlier chapters that the tangent function i.e. $f(x) = \tan x$ is a periodic function with period π. So, it is sufficient to know the graph of $f(x) = \tan x$ over an interval of length π, in particular the interval $(-\pi/2, \pi/2)$. The values of $f(x) = \tan x$ at some standard values of x are listed in the following table. As the tangent function is an odd function so the values of $f(x) = \tan x$ at standard points in $(-\pi/2, 0)$ are negative of the corresponding values in $(0, \pi/2)$ and are also listed in the following table.

x	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$f(x) = \tan x$	$-\infty$	$-\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$-\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$	0	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$	∞

We also observe that $\tan x$ is an increasing function in $(0, \pi/2)$ and as $x \to \frac{\pi}{2}$ from the left the values of $f(x) = \tan x$ tend to infinity. So, the curve $y = \tan x$ gets closer and closer to the line

$x = \frac{\pi}{2}$ as $x \to \frac{\pi}{2}$ from the left but it never touches the line $x = \frac{\pi}{2}$. The graph of $f(x) = \tan x$ is symmetric in opposite quadrants as the function is an add function. By plotting the points $(-\pi/3, -\sqrt{3})$, $(-\pi/4, -1)$, $(-\pi/6, -1/\sqrt{3})$, $(0, 0)$, $(\pi/6, 1/\sqrt{3})$, $(\pi/4, 1)$, $(\pi/3, \sqrt{3})$ and joining them by a free hand curve, we obtain the sketch of the curve $y = \tan x$ as shown in Fig. 6.11.

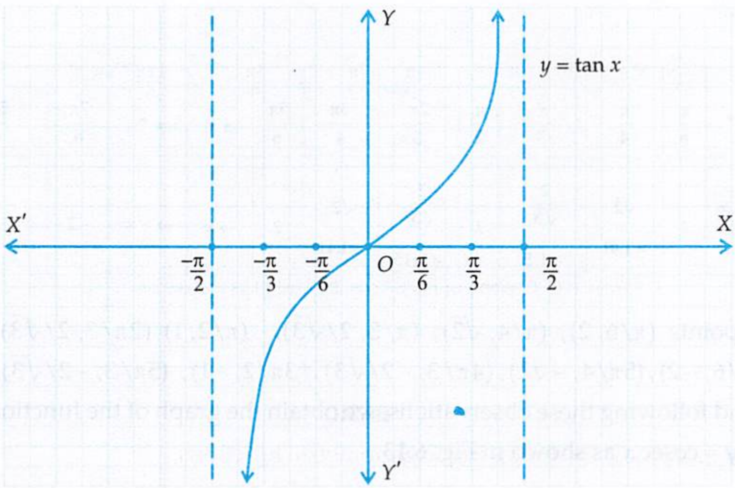

Fig. 6.11 Graph of $f(x) = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$

The function $f(x) = \tan x$ is a periodic function with period π. So, the graph of $f(x) = \tan x$ on $(\pi/2, 3\pi/2)$ and $(-3\pi/2, -\pi/2)$ is same as its graph on $(-\pi/2, \pi/2)$ as shown in Fig. 6.12.

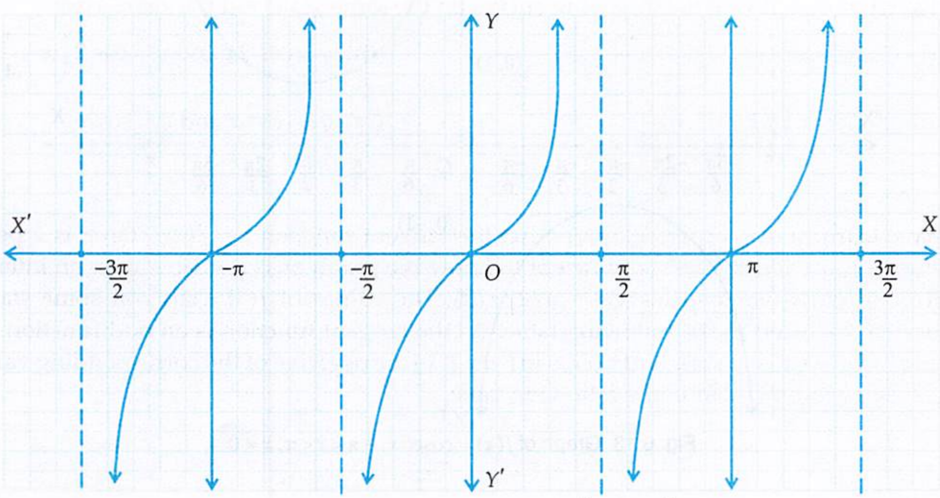

Fig. 6.12 Graph of $f(x) = \tan x, -\frac{3\pi}{2} < x < \frac{3\pi}{2}$

6.5 GRAPH OF COSECANT FUNCTION

In chapter 5, we have learnt that the cosecant function is the reciprocal of the sine function which is periodic with period 2π. So, $f(x) = \text{cosec}\, x$ is periodic with period 2π. Also, $f(x)$ is defined for all $x \in R - \{n\pi : n \in Z\}$. In order to know about the graph of $f(x) = \operatorname{cosec} x$, it is sufficient to draw it on an interval of length 2π. Let us choose $[0, 2\pi]$ as interval. The values of $f(x) = \operatorname{cosec} x$ at some standard points in $[0, 2\pi]$ are listed in the following table. We observe that when x is close to zero or π in $(0, \pi)$ the values of $f(x)$ tend to infinity. When $x \to \pi$ or $x \to 2\pi$ in $(\pi, 2\pi)$ the values of $f(x) \to -\infty$.

x	0^+	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π^-	π^+	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
$f(x) = \operatorname{cosec} x$	$\to \infty$	2	$\sqrt{2}$ $= 1.41$	$\frac{2}{\sqrt{3}}$ $= 1.5$	1	$\frac{2}{\sqrt{3}}$ $= 1.15$	$\sqrt{2}$ $= 1.41$	2	$\to \infty$	$\to -\infty$	-2	$-\sqrt{2}$ $= -1.41$	-1.15	-1

By plotting points $(\pi/6, 2)$, $(\pi/4, \sqrt{2})$, $(\pi/3, 2/\sqrt{3})$, $(\pi/2, 1)$ $(2\pi/3, 2/\sqrt{3})$, $(3\pi/4, \sqrt{2})$, $(5\pi/6, 2)$, $(7\pi/6, -2)$, $(5\pi/4, -\sqrt{2})$, $(4\pi/3, -2/\sqrt{3})$, $(3\pi/2, -1)$, $(5\pi/3, -2/\sqrt{3})$, $(7\pi/4, -\sqrt{2})$, $(11\pi/6, -2)$ and following these observations, we obtain the graph of the function $f(x) = \operatorname{cosec} x$ i.e. the curve $y = \operatorname{cosec} x$ as shown in Fig. 6.13.

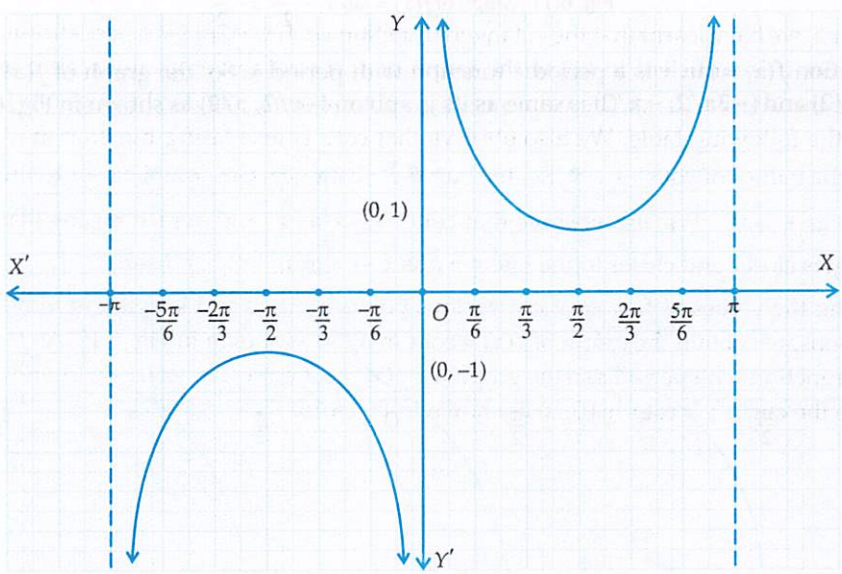

Fig. 6.13 Graph of $f(x) = \operatorname{cosec} x, -\pi < x < \pi, x \neq 0$

The function $f(x) = \operatorname{cosec} x$ is a periodic function with period 2π. So, the graph of $f(x) = \operatorname{cosec} x$ in the interval $[-2\pi, 2\pi]$ is as shown in Fig. 6.14.

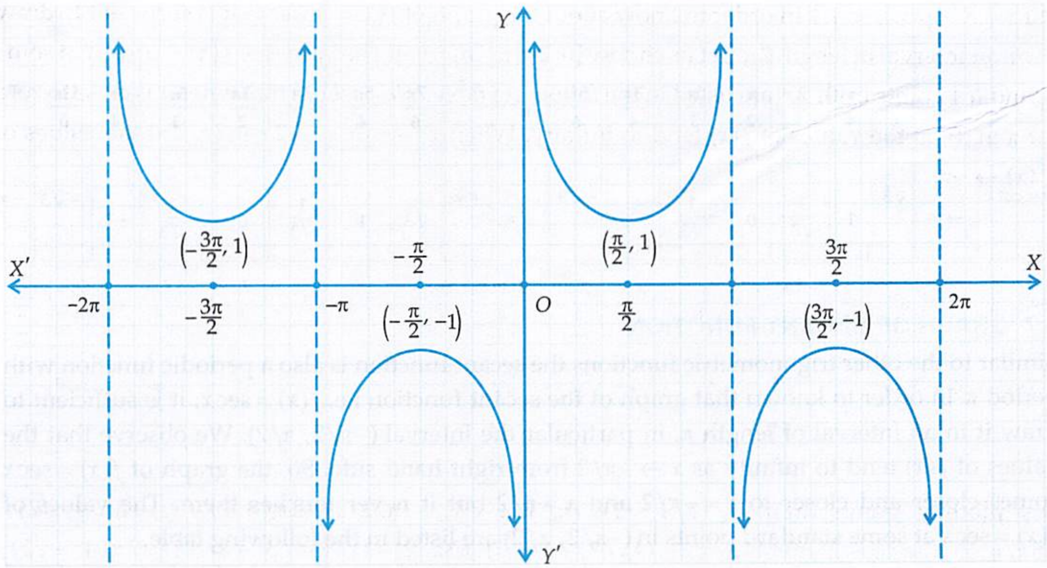

Fig. 6.14 Graph of $y = \operatorname{cosec} x, -2\pi < x < 2\pi$

6.6 GRAPH OF COTANGENT FUNCTIONS

In chapter 5, we have learnt that the cotangent function i.e. $f(x) = \cot x$ is a periodic function with period π. So, it is sufficient to know the graph of $f(x) = \cot x$ over an interval of length π, in particular the interval $(0, \pi)$. The values of $f(x) = \cot x$ at some standard values of x in $(0, \pi)$ are listed in the following table. We also observe that $\cot x$ is decreasing function in $(0, \pi)$ and as $x \to 0^+$, the values of $\cot x \to +\infty$. So, the curve $y = \cot x$ gets closer and closer to the line $x = 0$ i.e. y-axis as $x \to 0^+$. We also observe that $\cot x \to -\infty$ as $x \to \pi^-$ which means that the curve $y = \cot x$ gets closer and closer to the line $x = \pi$ as $x \to \pi$ from left hand side.

By plotting the values of $f(x) = \cot x$ at various points in $(0, \pi)$ and keeping in mind the above observations, we obtain the graph of $f(x) = \cot x$ in $(0, \pi)$ as shown in Fig. 6.15. As $f(x) = \cot x$ is periodic function with period π so the graphs of $f(x) = \cot x$ in $(-2\pi, -\pi)$, $(-\pi, 0)$ and $(\pi, 2\pi)$ are similar to the curve $y = \cot x$ in $(0, \pi)$ as shown in Fig. 6.15.

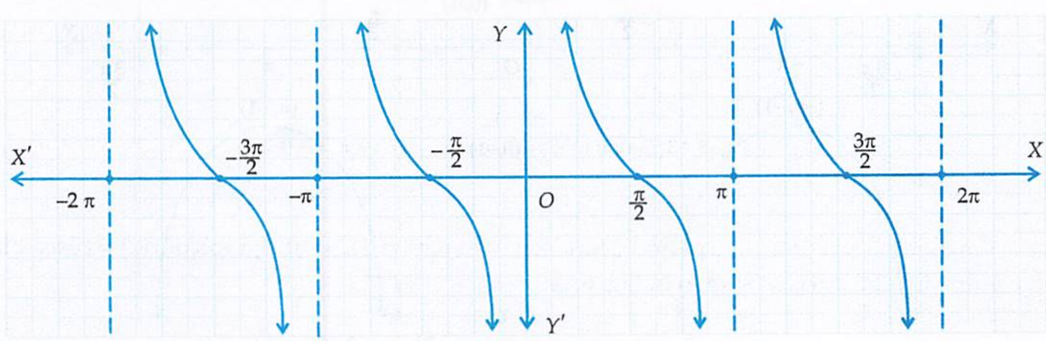

Fig. 6.15 Graph of $y = \cot x, -2\pi < x < 2\pi$

x	0^+	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π^-	π^+	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi^-$
$y = \cot x$	$\to \infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$	$+\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$

6.7 GRAPH OF SECANT FUNCTION

Similar to the other trigonometric functions the secant function is also a periodic function with period π. In order to known that graph of the secant function i.e. $f(x) = \sec x$, it is sufficient to draw it in an interval of length π, in particular the interval $(-\pi/2, \pi/2)$. We observe that the values of $f(x)$ tend to infinity as $x \to -\pi/2$ from right hand side. So, the graph of $f(x) = \sec x$ comes closer and closer to $x = -\pi/2$ and $x = \pi/2$ but it never touches them. The values of $f(x) = \sec x$ at some standard points in $(-\pi/2, \pi/2)$ are listed in the following table.

x	$-\frac{\pi^+}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi^-}{2}$	$\frac{\pi^+}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$f(x) = \sec x$	$\to \infty$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	$-\infty$	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1

By plotting the points given by the above table and joining them by a free hand curve. We obtain the graph of $f(x) = \sec x$ i.e. the curve $y = \sec x$ as shown in Fig. 6.16.

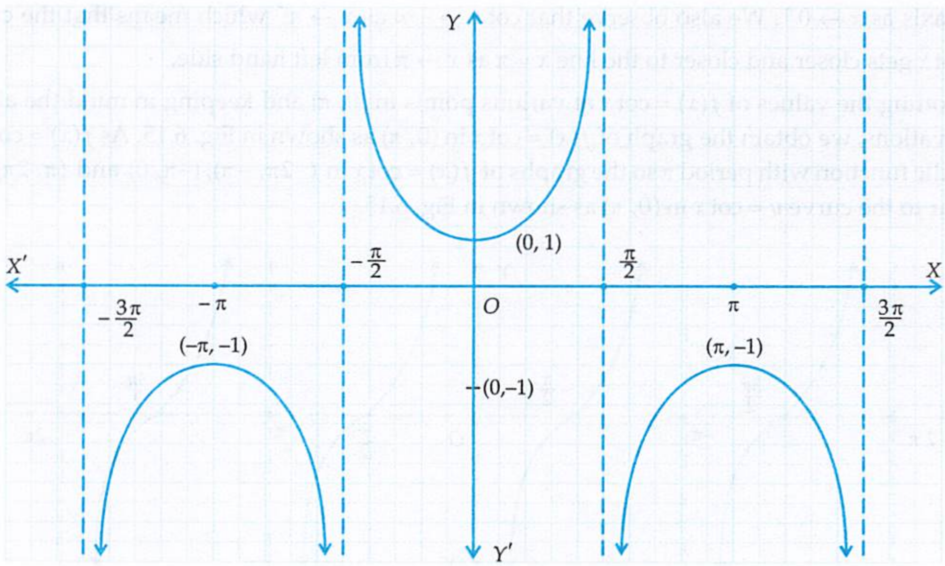

Fig. 6.16 Graph of $y = \sec x, -\frac{3\pi}{2} < x < \frac{3\pi}{2}$

EXERCISE 6.2

BASIC

Sketch the graphs of the following functions:

1. $f(x) = 2 \text{ cosec } \pi x$
2. $f(x) = 3 \sec x$
3. $f(x) = \cot 2x$
4. $f(x) = 2 \sec \pi x$
5. $f(x) = \tan^2 x$
6. $f(x) = \cot^2 x$
7. $f(x) = \cot \frac{\pi x}{2}$
8. $f(x) = \sec^2 x$
9. $f(x) = \text{cosec}^2 x$
10. $f(x) = \tan 2x$