

MATHEMATICS
WORKSHEET_140926
CHAPTER 10 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS

CLASS : X

MAX. MARKS : 40

DURATION : 1½ hrs

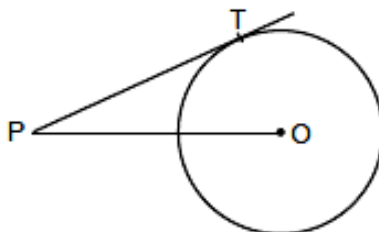
General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

SECTION – A

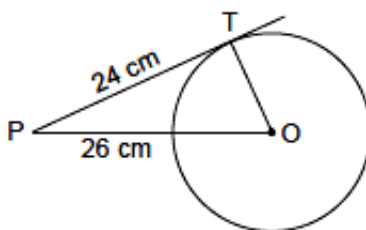
Questions 1 to 10 carry 1 mark each.

1. In the given below figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then the radius of the circle is



- (a) 25 cm (b) 26 cm (c) 24 cm (d) 10 cm

Ans: (d) $\because$ OT is radius and PT is tangent



$\therefore OT \perp PT$

Now, in $\triangle OTP$,

$$OP^2 = PT^2 + OT^2$$

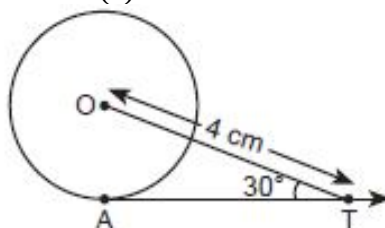
$$\Rightarrow 26^2 = 24^2 + OT^2$$

$$\Rightarrow 676 - 576 = OT^2$$

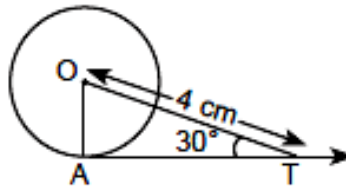
$$\Rightarrow 100 = OT^2 \Rightarrow 10 \text{ cm} = OT$$

2. In the below figure AT is a tangent to the circle with centre O such that $OT = 4 \text{ cm}$ and $\angle OTA = 30^\circ$. Then AT is equal to

- (a) 4 cm (b) 2 cm (c) $2\sqrt{3} \text{ cm}$ (d) $4\sqrt{3} \text{ cm}$



Ans: (c) $2\sqrt{3} \text{ cm}$

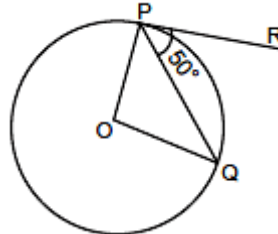


$\angle OAT = 90^\circ$ [$\because$ Tangent is perpendicular to the radius]

In right angled $\triangle OAT$,

$$\cos 30^\circ = \frac{AT}{OT} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4} \Rightarrow AT = 2\sqrt{3} \text{ cm}$$

3. In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to



- (a) 100° (b) 80° (c) 90° (d) 75°

Ans: (a) 100°

$OP \perp PR$ [$\because$ Tangent and radius are $\perp$ to each other at the point of contact]

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

$OP = OQ$ [Radii]

$$\therefore \angle OPQ = \angle OQP = 40^\circ$$

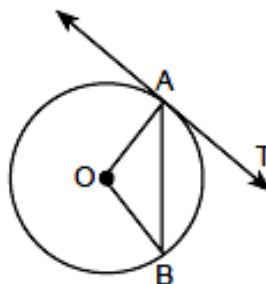
In $\triangle OPQ$,

$$\Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$\Rightarrow \angle POQ + 40^\circ + 40^\circ = 180^\circ$$

$$\angle POQ = 180^\circ - 80^\circ = 100^\circ.$$

4. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100^\circ$, then $\angle BAT$ is equal to



- (a) 100° (b) 40° (c) 50° (d) 90°

Ans: (c) 50°

$$\angle AOB = 100^\circ$$

$$\angle OAB = \angle OBA \text{ } (\because \text{OA and OB are radii})$$

Now, in $\triangle AOB$,

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ \text{ (Angle sum property of } \triangle)$$

$$\Rightarrow 100^\circ + x + x = 180^\circ$$

$$[\text{Let } \angle OAB = \angle OBA = x]$$

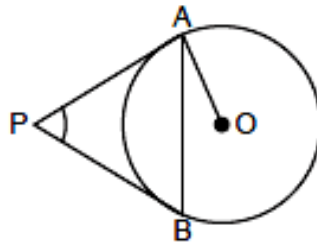
$$\Rightarrow 2x = 180^\circ - 100^\circ$$

$$\Rightarrow 2x = 80^\circ \Rightarrow x = 40^\circ$$

$$\text{Also, } \angle OAB + \angle BAT = 90^\circ \text{ } [\because \text{OA is radius and TA is tangent at A}]$$

$$\Rightarrow 40^\circ + \angle BAT = 90^\circ \Rightarrow \angle BAT = 50^\circ$$

5. In the figure PA and PB are tangents to the circle with centre O. If $\angle APB = 60^\circ$, then $\angle OAB$ is



- (a) 30° (b) 60° (c) 90° (d) 15°

Ans: (a) 30°

Given $\angle APB = 60^\circ$

$$\therefore \angle APB + \angle PAB + \angle PBA = 180^\circ$$

$$\Rightarrow \angle APB + x + x = 180^\circ [\because PA = PB \therefore \angle PAB = \angle PBA = x \text{ (say)}]$$

$$\Rightarrow 60^\circ + 2x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ$$

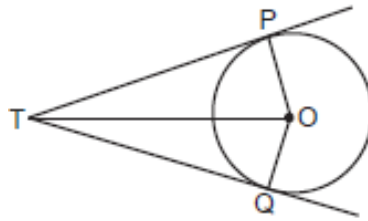
$$\Rightarrow 2x = 120^\circ \Rightarrow x = \frac{120^\circ}{2} = 60^\circ$$

$$\text{Also, } \angle OAP = 90^\circ \Rightarrow \angle OAB + \angle PAB = 90^\circ$$

$$\Rightarrow \angle OAB + 60^\circ = 90^\circ$$

$$\Rightarrow \angle OAB = 30^\circ$$

6. In the given figure, TP and TQ are two tangents to a circle with centre O, such that $\angle POQ = 110^\circ$. Then $\angle PTQ$ is equal to



- (a) 55° (b) 70° (c) 110° (d) 90°

Ans: (b) 70°

In quadrilateral POQT,

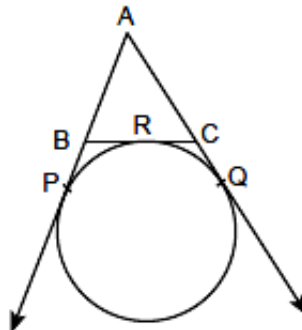
$$\angle PTQ + \angle TPO + \angle TQO + \angle POQ = 360^\circ$$

$$\Rightarrow \angle PTQ + 90^\circ + 90^\circ + 110^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ + 290^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$$

7. In figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is



- (a) 7.5 (b) 15 (c) 10 (d) 9

Ans: (a) 7.5

$$AP = AQ$$

$$\Rightarrow AB + BP = AC + CQ$$

$$\Rightarrow 5 + BP = 6 + CQ$$

$$BP = 1 + CQ$$

$$BP = 1 + CR$$

$$(\because CQ = CR)$$

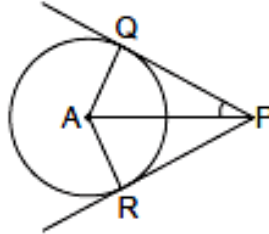
$$BP = 1 + (BC - BR)$$

$$BP = 1 + (4 - BP) (\because BR = BP)$$

$$2BP = 5 \Rightarrow BP = \frac{5}{2} = 2.5 \text{ cm}$$

$$\text{Now, } AP = AB + BP = 5 + 2.5 = 7.5 \text{ cm}$$

8. In figure, PQ and PR are tangents to a circle with centre A. If $\angle QPA = 27^\circ$, then $\angle QAR$ equals to



- (a) 63° (b) 153° (c) 126° (d) 117°

Ans: (c) 126°

$$\angle QPA = \angle RPA [\because \triangle AQP \cong \triangle ARP \text{ (RHS congruence rule)}]$$

$$\Rightarrow \angle RPA = 27^\circ$$

$$\therefore \angle QPR = \angle QPA + \angle RPA = 27^\circ + 27^\circ = 54^\circ$$

$$\text{Now, } \angle QAR + \angle AQP + \angle ARP + \angle QPR = 360^\circ$$

$$\Rightarrow \angle QAR = 90^\circ + 90^\circ + 54^\circ = 234^\circ$$

$$\Rightarrow \angle QAR = 360^\circ - 234^\circ = 126^\circ$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

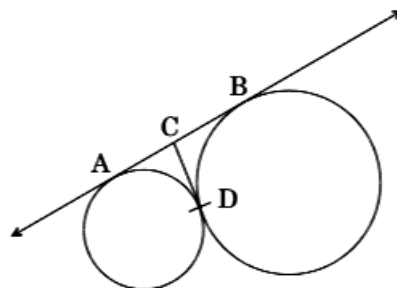
- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
 (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
 (c) Assertion (A) is true but reason (R) is false.
 (d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm then the radius of the circle is 7 cm.

Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** In the below figure, AB and CD are common tangents to circles which touch each other at D. If AB = 8 cm, then the length of CD is 4 cm



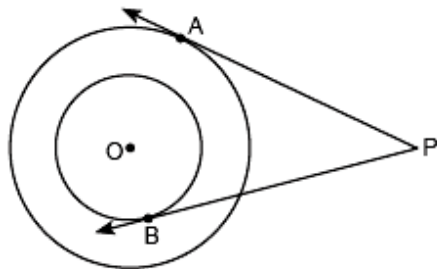
Reason (R): A tangent to a circle is perpendicular to the radius through the point of contact

Ans: (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

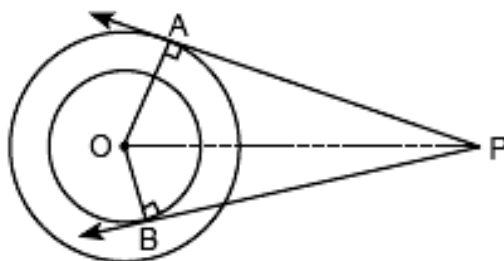
SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the below figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

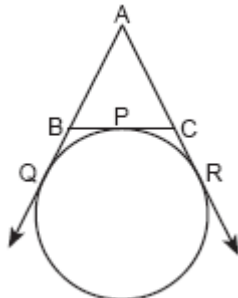


Ans: PA = 12 cm, OA = 5 cm, OB = 3 cm



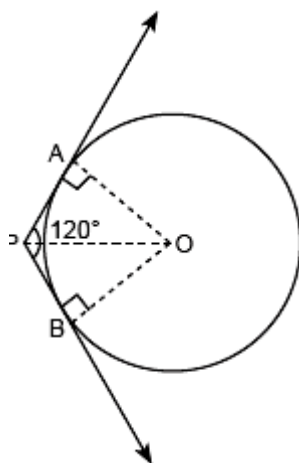
$$\begin{aligned} OP^2 &= OA^2 + AP^2 = OB^2 + BP^2 \\ \Rightarrow 25 + 144 &= 9 + BP^2 \\ \Rightarrow 169 - 9 &= BP^2 \\ \Rightarrow BP &= \sqrt{160} \text{ cm} = 12.65 \text{ cm. (Approx.)} \end{aligned}$$

12. In figure, a circle touches the side BC of $\triangle ABC$ at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of $\triangle ABC$.



Ans: AQ and AR are tangents from the same point
 $AQ = AR = 5 \text{ cm} \dots(i)$ [Tangents from the same external points are equal]
 BQ and BP are tangents from same point
 $\Rightarrow BQ = BP \dots(ii)$
 CP and CR are also tangents from the same point
 $\Rightarrow CP = CR \dots(iii)$
 In $\triangle ABC$, Perimeter of $\triangle ABC = AB + BC + AC = AB + BP + CP + AC$
 $AB + BQ + CR + AC = AQ + AR$ [From (ii) and (iii)]
 $= 5 \text{ cm} + 5 \text{ cm} = 10 \text{ cm}$ [From (i)]
 Perimeter of $\triangle ABC = 10 \text{ cm}$

13. Two tangents PA and PB are drawn to the circle with centre O, such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$.
 Ans: Consider $\triangle PAO$ and PBO



$PA = PB$ [Tangents to a circle, from a point outside it, are equal.]

$OP = OP$ [Common]

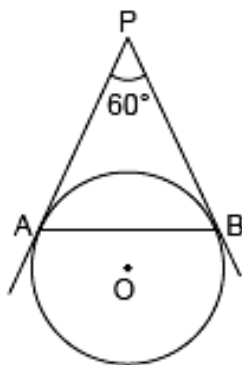
$\angle OAP = \angle OBP = 90^\circ$

$\therefore \triangle OAP \cong \triangle OBP$ [RHS]

$\therefore \angle OPA = \angle OPB = \frac{1}{2} \angle APB = \frac{1}{2} \times 120^\circ = 60^\circ$.

In right angled $\triangle OAP$, $\frac{AP}{OP} = \cos 60^\circ = \frac{1}{2} \Rightarrow OP = 2AP$.

14. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans: $AP = BP$ [tangents from external point P]

$\therefore \angle PAB = \angle PBA$ [Angles opposite to equal sides]

Now $\angle APB + \angle PAB + \angle PBA = 180^\circ$

$\Rightarrow 60^\circ + 2 \angle PAB = 180^\circ$

$\Rightarrow \angle PAB = 60^\circ$

$\therefore \triangle APB$ is an equilateral $\triangle$

$AB = AP = 5$ cm

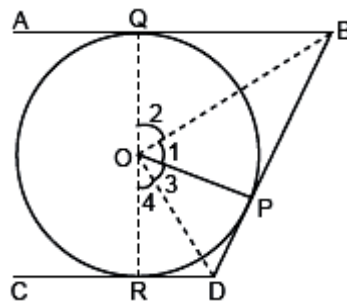
SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Ans: AB and CD are two tangents to a circle and $AB \parallel CD$.

Tangent BD intercepts an angle BOD at the centre.



$OP \perp BD$.

[A tangent at any point of a circle is perpendicular to the radius through the point of contact.]

In right angled Δ s OQB and OPB,

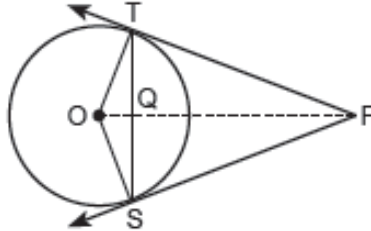
$$\angle 1 = \angle 2,$$

Similarly in right angled Δ s OPD and ORD

$$\angle 3 = \angle 4$$

$$\begin{aligned} \therefore \angle BOD &= \angle 1 + \angle 3 = \frac{1}{2} [2\angle 1 + 2\angle 3] = \frac{1}{2} (\angle 1 + \angle 1 + \angle 3 + \angle 3) \\ &= \frac{1}{2} (\angle 1 + \angle 2 + \angle 3 + \angle 4) = \frac{1}{2} (180^\circ) = 90^\circ. \end{aligned}$$

16. In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r . If $OP = 2r$, show that $\angle OTS = \angle OST = 30^\circ$.



Ans: In ΔOTS $OT = OS$ [radii]

$$\Rightarrow \angle OTS = \angle OST \dots (i)$$

$$\therefore \text{In right } \Delta OTP, \frac{OT}{OP} = \sin \angle TPO$$

$$\Rightarrow \frac{r}{2r} = \sin \angle TPO \Rightarrow \sin \angle TPO = \frac{1}{2} = \sin 30^\circ \Rightarrow \angle TPO = 30^\circ$$

Similarly $\angle OPS = 30^\circ$

$$\Rightarrow \angle TPS = 30^\circ + 30^\circ = 60^\circ$$

$$\text{Also } \angle TPS + \angle SOT = 180^\circ$$

$$\Rightarrow \angle SOT = 120^\circ$$

$$\text{In } \Delta SOT, \angle SOT + \angle OTS + \angle OST = 180^\circ$$

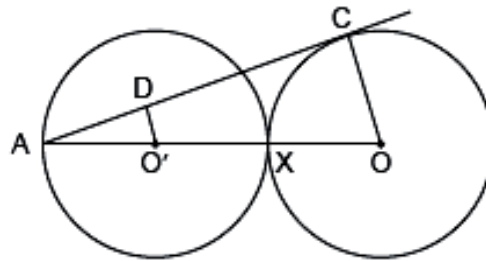
$$\Rightarrow 120^\circ + 2\angle OTS = 180^\circ$$

$$\Rightarrow \angle OTS = 30^\circ \dots (ii)$$

From (i) and (ii)

$$\angle OTS = \angle OST = 30^\circ$$

17. In figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Ans: AC is tangent

$\therefore CO \perp AC$

Also $O'D \perp AC$

$\therefore O'D \parallel OC$

Now $OX = XO' = O'A$

$$\Rightarrow AO = 3AO' \Rightarrow \frac{AO'}{AO} = \frac{1}{3} \dots(i)$$

In $\triangle AO'D$, $\triangle AOC$

$\angle ADO = \angle ACO$ [each 90°]; $\angle A = \angle A$

$\therefore \triangle AO'D \sim \triangle AOC$

$$\therefore \frac{AO'}{AO} = \frac{DO'}{CO}$$

$$\Rightarrow \frac{DO'}{CO} = \frac{1}{3} \text{ [Using (i)]}$$

SECTION – D

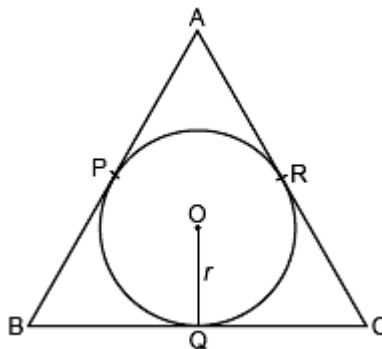
Questions 18 carry 5 marks.

- 18.** In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that

(i) $AB + CQ = AC + BQ$

(ii) $\text{Area}(\triangle ABC) = \frac{1}{2} (\text{perimeter of } \triangle ABC) \times r$

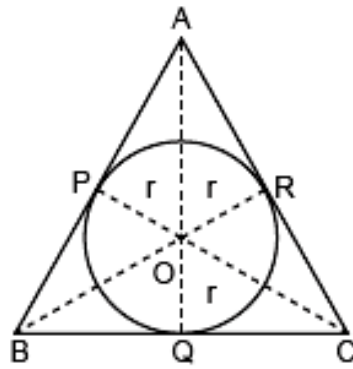


Ans:

(i) $AP = AR$ [Tangents from A] ...(i)

Similarly, $BP = BQ$...(ii)

$CR = CQ$...(iii)



Now,

$$\therefore AP = AR$$

$$\Rightarrow (AB - BP) = (AC - CR)$$

$$\Rightarrow AB + CR = AC + BP$$

$$\Rightarrow AB + CQ = AC + BQ \text{ [Using eq. (ii) and (iii)]}$$

$$\text{(ii) Let } AB = x, BC = y, AC = z$$

$$\therefore \text{Perimeter of } \triangle ABC = x + y + z \dots \text{(iv)}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} [\text{area of } \triangle AOB + \text{area of } \triangle BOC + \text{area of } \triangle AOC]$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} \times AB \times OP + \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AC \times OR$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} x \times r + \frac{1}{2} y \times r + \frac{1}{2} z \times r$$

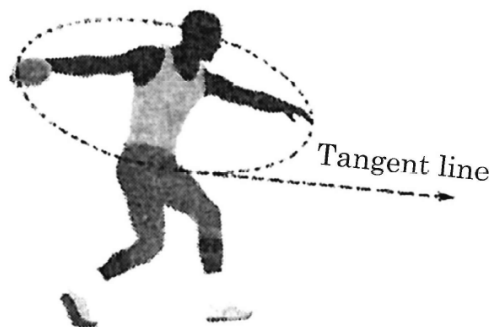
$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} (x + y + z) \times r$$

$$\Rightarrow \text{Area of } \triangle ABC = \frac{1}{2} (\text{Perimeter of } \triangle ABC) \times r \text{ [Using (iv)]}$$

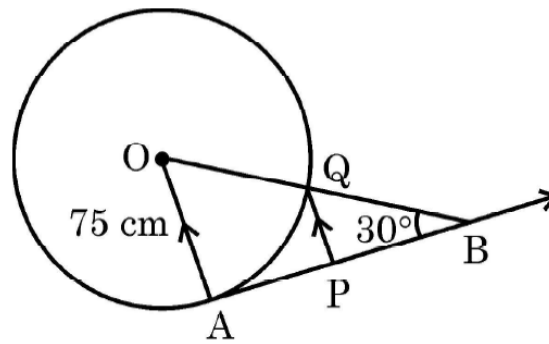
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

- 19.** The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and $\angle ABO = 30^\circ$. PQ is parallel to OA.



Based on the above, information:

- (a) Find the length of AB. (1)
- (b) Find the length of OB. (1)
- (c) Find the length of AP. (2)

OR

- (c) Find the length of PQ. (2)

Ans: (a) $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{75}{AB} \Rightarrow AB = 75\sqrt{3} \text{ cm}$

(b) $\sin 30^\circ = \frac{1}{2} = \frac{75}{OB} \Rightarrow OB = 150 \text{ cm}$

(c) $QB = 150 - 75 = 75 \text{ cm}$

$\Rightarrow Q$ is mid point. of OB

Since $PQ \parallel AO$ therefore P is mid point of AB

Hence $AP = \frac{75\sqrt{3}}{2} \text{ cm.}$

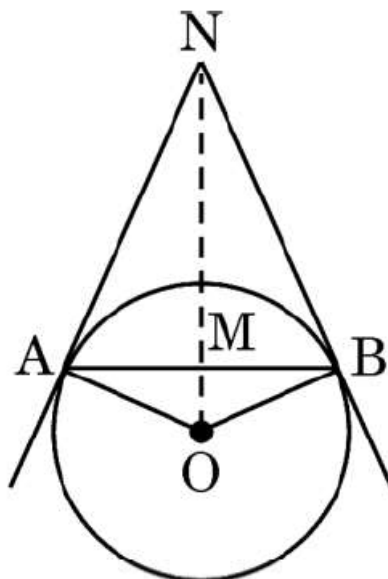
OR

$QB = 150 - 75 = 75 \text{ cm}$

Now, $\triangle BQP \sim \triangle BOA$

$\Rightarrow \frac{QB}{OB} = \frac{PQ}{OA} \Rightarrow \frac{1}{2} = \frac{PQ}{75} \Rightarrow PQ = \frac{75}{2} \text{ cm}$

- 20.** Circles play an important part in our life. When a circular object is hung on the wall with a cord at nail N, the cords NA and NB work like tangents. Observe the figure, given that $\angle ANO = 30^\circ$ and $OA = 5 \text{ cm}$.



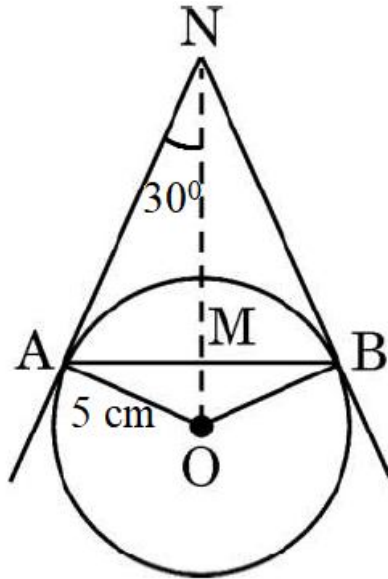
Based on the above, answer the following questions:

- (a) Find the distance AN.
- (b) Find the measure of $\angle AOB$.
- (c) Find the total length of cords NA, NB and the chord AB.

OR

(c) If $\angle ANO$ is 45° , then name the type of quadrilateral OANB.

Ans: (a) $\tan 30^\circ = \frac{5}{AN} \Rightarrow AN = 5\sqrt{3} \text{ cm}$



(b) $\angle BNO = 30^\circ \Rightarrow \angle BNA = 60^\circ$

$\therefore \angle AOB = 180^\circ - 60^\circ = 120^\circ$

(c) $AN = 5\sqrt{3}$ and in $\triangle ANB$, $\angle ANB = 60^\circ$ and $NA = NB$

$\therefore \angle NAB = \angle NBA = 60^\circ$ or $\triangle NAB$ is an equilateral triangle.

Hence, $AB = 5\sqrt{3} \text{ cm}$.

$AN + NB + AB = 3 \times 5\sqrt{3} = 15\sqrt{3} \text{ cm}$.

OR

(c) $\angle ANO = 45^\circ \Rightarrow \angle AOB = 90^\circ$

$\therefore$ Each angle of quad. AOBN is 90° .

Also, $OA = OB$.

$\therefore$ OANB is a square.

.....