

MATHEMATICS
WORKSHEET_191026
CHAPTER 10 CIRCLES (ANSWERS)

SUBJECT: MATHEMATICS

CLASS : X

MAX. MARKS : 40

DURATION : 1½ hrs

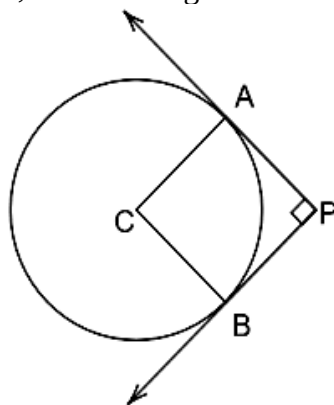
General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4cm. If $PA \perp PB$, then the length of each tangent is:



- (a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Ans. (b) 4 cm

$CA \perp AP$ and $CB \perp PB$ [As tangent to a circle is $\perp$ to radius]

Also, $\angle APB = 90^\circ$

Now, in quad. APBC, $\angle APB + \angle CAP + \angle CBP + \angle ACB = 360^\circ$

$$\Rightarrow 90^\circ + 90^\circ + 90^\circ + \angle ACB = 360^\circ \Rightarrow \angle ACB = 90^\circ$$

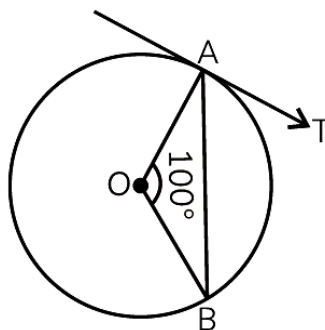
Now, each angle of quadrilateral APBC is 90° and $AP = PB$.

$\therefore$ APCB is a square.

$\therefore AP = BP = BC = CA = 4 \text{ cm}$

Thus, the length of each tangent is 4 cm.

2. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100^\circ$, then $\angle BAT$ is equal to:



- (a) 100° (b) 40° (c) 50° (d) 90°
 Ans. (c) 50°

Here, $\angle AOB = 100^\circ$

$\angle OAT = 90^\circ$ [As tangent at a point to a circle is perpendicular to the radius]

In $\triangle OAB$, $OA = OB$ [Radii of the circle]

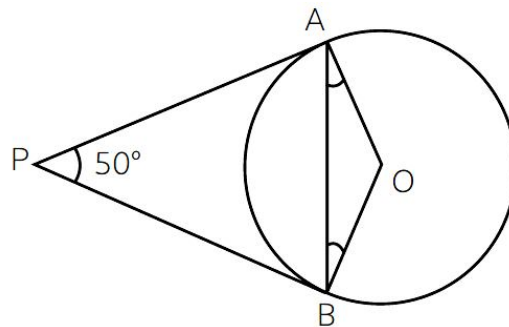
$\Rightarrow \angle OBA = \angle OAB$ [$\because$ angle opposite to equal sides are equal]

Since, sum of angles in a triangle is 180° .

$$\therefore \angle OBA = \angle OAB = (180^\circ - \angle AOB)/2 = (180^\circ - 100^\circ)/2 = 40^\circ$$

$$\text{Now, } \angle BAT = \angle OAT - \angle OAB = 90^\circ - 40^\circ = 50^\circ$$

3. In the figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is:



- (a) 25° (b) 30° (c) 40° (d) 50°
 Ans. (a) 25°

We know that the radius and tangent are perpendicular at their point of contact

$$\therefore \angle OBP = \angle OAP = 90^\circ$$

Now, In quadrilateral AOBP

$$\angle AOB + \angle OBP + \angle APB + \angle OAP = 360^\circ \text{ [Angle sum property of a quadrilateral]}$$

$$\Rightarrow \angle AOB + 90^\circ + 50^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow 230^\circ + \angle AOB = 360^\circ \Rightarrow \angle AOB = 130^\circ$$

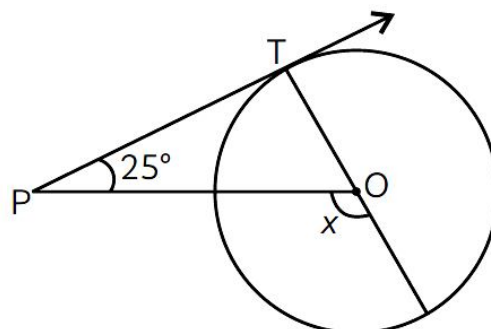
Now, In isosceles triangle AOB

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ \text{ [Angle sum property of a triangle]}$$

$$\Rightarrow 130^\circ + 2\angle OAB = 180^\circ [\because \angle OAB = \angle OBA]$$

$$\Rightarrow \angle OAB = 25^\circ$$

4. In the given figure, PT is a tangent at T to the circle with centre O. If $\angle TPO = 25^\circ$, then x is equal to:



- (a) 25° (b) 65° (c) 90° (d) 115°
 Ans. (d) 115°

Given, $\angle TPO = 25^\circ$

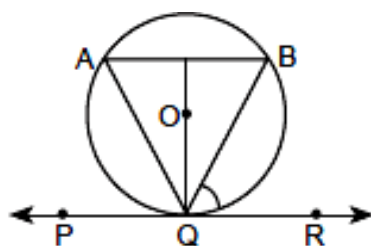
As we know, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle PTO = 90^\circ$$

Now, Exterior angle, $x = \angle PTO + \angle TPO$

$$\Rightarrow x = 90^\circ + 25^\circ \Rightarrow x = 115^\circ$$

5. In figure if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to



- (a) 20° (b) 40° (c) 35° (d) 45°

Ans. (b) 40°

$AB \parallel PR$

$\therefore \angle ABQ = \angle BQR$ [Alternate interior angles]

$\Rightarrow \angle ABQ = 70^\circ$

Also, $\angle BQR = \angle BAQ$ [Angles in alternate segment]

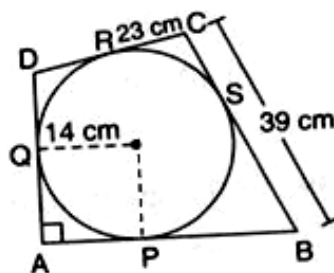
$\Rightarrow \angle BAQ = 70^\circ$

In $\triangle AQB$,

$\angle BAQ + \angle ABQ + \angle AQB = 180^\circ$

$\Rightarrow 70^\circ + 70^\circ + \angle AQB = 180^\circ \Rightarrow \angle AQB = 180^\circ - 140^\circ = 40^\circ$.

6. In the given figure, quadrilateral ABCD is circumscribed, touching the circle at P, Q, R and S such that $\angle DAB = 90^\circ$, If $CR = 23$ cm and $CB = 39$ cm and the radius of the circle is 14 cm, then the measure of AB is



- (a) 37 cm (b) 16 cm (c) 30 cm (d) 39 cm

Ans: (c) 30 cm

$\because$ Tangent is perpendicular to the radius through the point of contact.

$\angle OQA = \angle OPA = 90^\circ$ and $OQ = OP$ [Radii]

$\therefore$ OQAP is a square.

$\Rightarrow AP = 14$ cm

Now, $CR = CS = 23$ cm [Tangents from an external point to a circle are equal]

$\therefore BS = 39 - 23 = 16$ cm

And $BS = BP = 16$ cm [Tangents from an external point to a circle are equal]

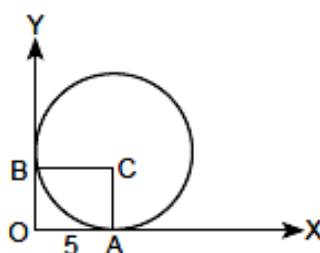
Now, $AB = AP + BP = 14 + 16 = 30$ cm

7. A circle touches x-axis at A and y-axis at B. If O is origin and $OA = 5$ units, then diameter of the circle is

- (a) 8 units (b) 10 units (c) $10\sqrt{2}$ units (d) $8\sqrt{2}$ units

Ans. (b) 10 units

$OA = OB \Rightarrow OB = 5$



$$AC = BC \text{ [Radii]}$$

$\Rightarrow$ OACB is a square.

$$\Rightarrow AC = OA = 5 \Rightarrow \text{Diameter} = 10 \text{ units.}$$

8. Two parallel lines touch the circle at points A and B respectively. If area of the circle is $25\pi \text{ cm}^2$, then AB is equal to

(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm

Ans. (c) 10 cm

Let radius of circle = R

$$\therefore \pi R^2 = 25\pi$$

$$\Rightarrow R = 5 \text{ cm}$$

$$\therefore \text{Distance between two parallel tangents} = \text{diameter} = 2 \times 5 = 10 \text{ cm.}$$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
 (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
 (c) Assertion (A) is true but reason (R) is false.
 (d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.

Reason (R): If the angle between two radii of a circle is 130° , then the angle between the tangents at the end points of radii at their point of intersection is 50° .

Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. **Assertion (A):** A circle can have infinitely many tangents.

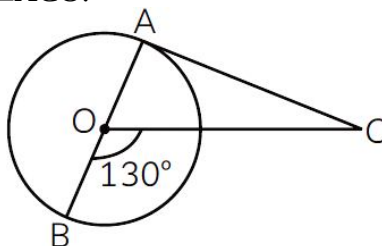
Reason (R): The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, then find $\angle ACO$.



Ans. Given, $\angle BOC = 130^\circ$

Since, AOB is the diameter of the circle.

$$\text{Then, } \angle AOB = 180^\circ$$

$$\Rightarrow \angle BOC + \angle AOC = 180^\circ$$

$$\Rightarrow 130^\circ + \angle AOC = 180^\circ$$

$$\Rightarrow \angle AOC = 50^\circ$$

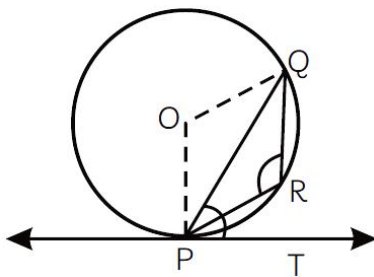
Now, $\angle OAC = 90^\circ$ [Since a tangent at any point on a circle is perpendicular to the radius]

$$\text{In } \triangle OAC, \angle AOC + \angle OAC + \angle ACO = 180^\circ$$

$$\angle ACO = 180^\circ - (90^\circ + 50^\circ)$$

$$= 180^\circ - 140^\circ = 40^\circ$$

12. In figure, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.



Ans. Given, $\angle QPT = 60^\circ$

Since, OP is the radius of the circle.

Now, $\angle OPT = 90^\circ$

$$\therefore \angle OPQ = \angle OPT - \angle QPT = 90^\circ - 60^\circ = 30^\circ$$

In $\triangle OPQ$, $OP = OQ$ [radii of circle]

$$\angle OQP = \angle POQ = 30^\circ [\because \text{Angles opposite to equal sides are equal}]$$

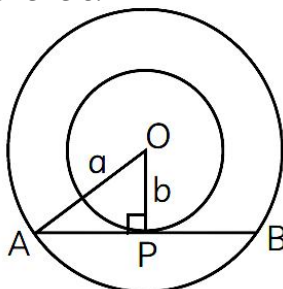
$$\therefore \angle POQ = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$\therefore \text{Reflex } \angle POQ = 360^\circ - 120^\circ = 240^\circ$$

We know that, angle subtended by an arc at the centre double the angle subtended by it on the remaining part of the circle.

$$\therefore \angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ = 240^\circ / 2 = 120^\circ$$

13. Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle.



Ans. Let O be the centre of the concentric circles and AB be the chord for bigger circle and tangent to the smaller circle.

Let P be the point where AB meets smaller circle.

$$\therefore OA = a \text{ and } OP = b$$

Now, $\angle OPA = 90^\circ$ [As, tangent at any point is perpendicular to the radius]

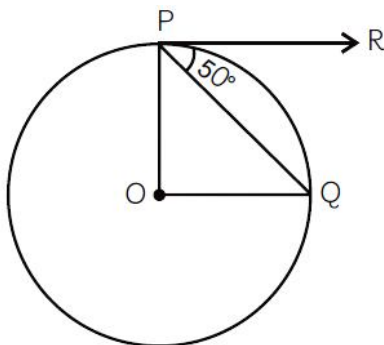
Now, in $\triangle OPA$, by Pythagoras theorem

$$OA^2 = OP^2 + AP^2 \Rightarrow AP = \sqrt{a^2 - b^2}$$

Now, $AB = 2AP$ [as perpendicular from centre to the chord bisects the chord]

$$\Rightarrow AB = 2 \sqrt{a^2 - b^2}$$

14. In the figure, if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find $\angle POQ$.



Ans. Given, $\angle RPQ = 50^\circ$

Now, $\angle OPR = 90^\circ$ [As tangent makes an angle of 90° with radius]

$$\Rightarrow \angle OPQ + \angle QPR = 90^\circ.$$

$$\Rightarrow \angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

In $\triangle OPQ$, $OP = OQ$ [Radii of a circle]

$$\Rightarrow \angle OQP = \angle OPQ = 40^\circ$$

$$\text{So, } \angle POQ = 180^\circ - (40^\circ + 40^\circ)$$

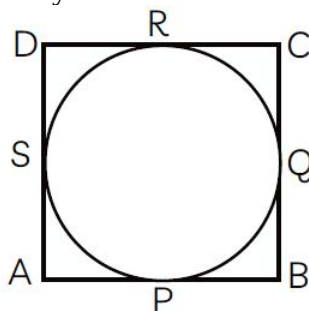
$$\Rightarrow \angle POQ = 100^\circ$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that the rectangle circumscribing a circle is a square.

Ans. Consider a rectangle ABCD circumscribing a circle such that it touches the sides AB, BC, CD and DA at P, Q, R and S respectively.



Now, we know lengths of tangents drawn from an external point to a circle are equal.

$$\therefore AP = AS, BP = BQ, CR = CQ \text{ and } DR = DS$$

Adding the above equations, we get

$$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + CB$$

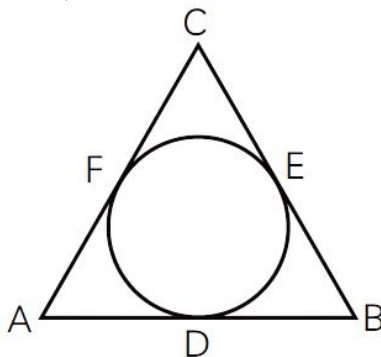
But $AB = CD$ and $AD = CB$ [Since, opposite sides of rectangle are equal]

$$\Rightarrow AB = AD$$

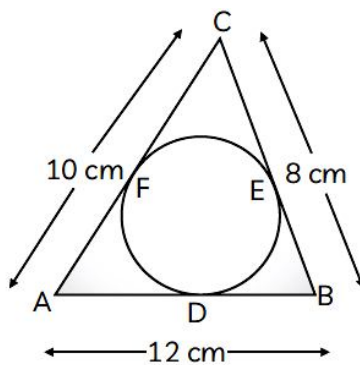
i.e., Adjacent sides of rectangle ABCD are equal.

Hence, ABCD is a square.

16. In the figure, a circle is inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the length of AD, BE and CF.



Ans. Given, A circle inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively.



Also, $AB = 12$ cm, $BC = 8$ cm and $CA = 10$ cm.

Since, the lengths of tangents drawn from an external point to a circle are equal, therefore

$$AD = AF = x \text{ (say)}$$

$$BD = BE = y \text{ (say)}$$

$$CE = CF = z \text{ (say)}$$

$$\text{Then, } AD + BD = AB$$

$$\Rightarrow x + y = 12 \quad \dots(i)$$

$$\text{Also, } BE + EC = BC$$

$$\Rightarrow y + z = 8 \quad \dots(ii)$$

$$\text{and } CF + AF = AC$$

$$\Rightarrow z + x = 10 \quad \dots(iii)$$

Adding equations (i), (ii) and (iii), we get

$$2(x + y + z) = 30$$

$$\Rightarrow x + y + z = 15 \quad \dots(iv)$$

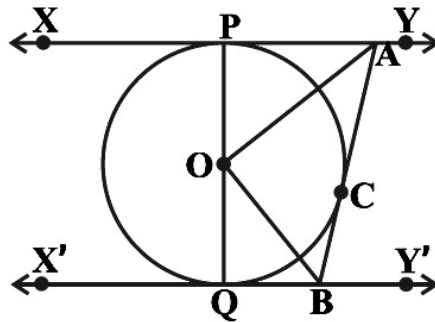
Subtracting eq. (i) from eq. (iv), we get $z = 3$

Subtracting eq. (ii) from eq. (iv), we get $x = 7$

And, subtracting eq (iii) from eq (iv), we get $y = 5$

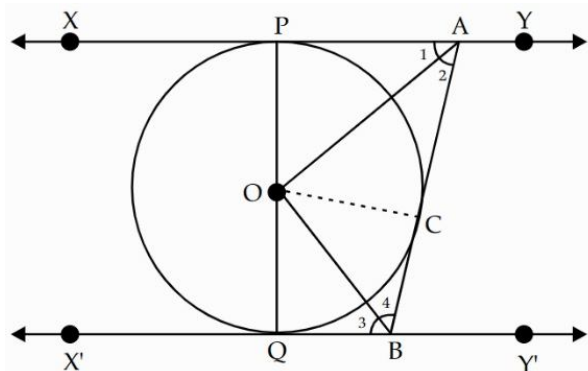
Hence, the lengths of AD, BE and CF are 7 cm, 5 cm and 3 cm, respectively.

17. In the below figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



Ans: Join OC. Since, the tangents drawn to a circle from an external point are equal.

$$\therefore AP = AC$$



In $\triangle PAO$ and $\triangle AOC$, we have:

$$AO = AO \text{ [Common]}$$

$$OP = OC \text{ [Radii of the same circle]}$$

$$AP = AC$$

$$\Rightarrow \triangle PAO \cong \triangle AOC \text{ [SSS Congruency]}$$

$$\therefore \angle PAO = \angle CAO = \angle 1$$

$$\angle PAC = 2 \angle 1 \quad \dots(1)$$

$$\text{Similarly } \angle CBQ = 2 \angle 2 \quad \dots(2)$$

Again, we know that sum of internal angles on the same side of a transversal is 180° .

$$\therefore \angle PAC + \angle CBQ = 180^\circ$$

$$\Rightarrow 2 \angle 1 + 2 \angle 2 = 180^\circ \text{ [From (1) and (2)]}$$

$$\Rightarrow \angle 1 + \angle 2 = 180^\circ/2 = 90^\circ \quad \dots(3)$$

Also $\angle 1 + \angle 2 + \angle AOB = 180^\circ$ [Sum of angles of a triangle]

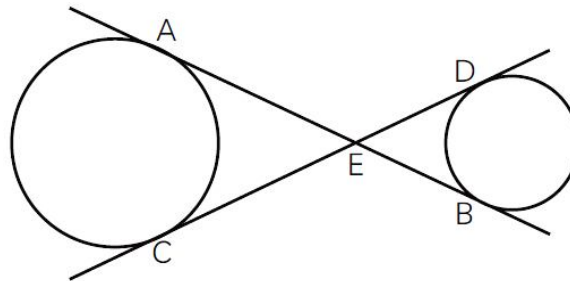
$$\Rightarrow 90^\circ + \angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ \Rightarrow \angle AOB = 90^\circ.$$

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that the lengths of tangents drawn from an external point to a circle are equal. (4)
 (b) In the given figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD. (1)



Ans. (a) Given, To prove, Construction and figure of 2 marks

Proof of 2 marks

(b) We know that lengths of tangents drawn from an external point to a circle is equal.

$$\therefore EB = ED \text{ and } EA = EC$$

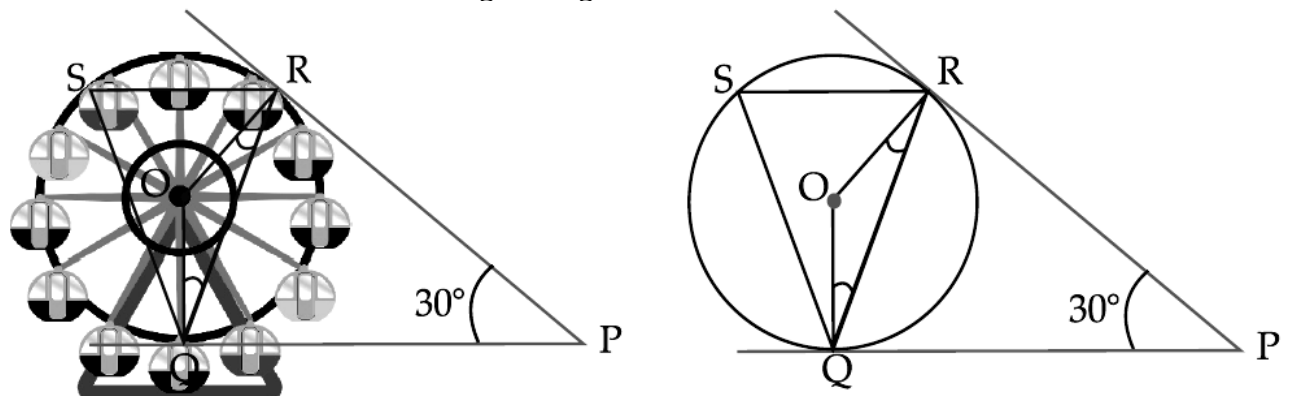
$$\text{On adding, we get } EA + EB = EC + ED \Rightarrow AB = CD$$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Monika came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.



- (a) In the given figure, find $\angle ROQ$. (2)
 (b) Find the measure of $\angle RQP$. (2)

OR

- (b) Find measure of $\angle RSQ$. Also, find the sum of $\angle ORP$ and $\angle OQP$. (2)

Ans. (a) $\angle ORP = 90^\circ = \angle OQP$

[$\because$ radius of circle is perpendicular to tangent]

$$\therefore \angle ROQ + \angle ORP + \angle OQP + \angle QPR = 360^\circ$$

$$\Rightarrow \angle ROQ + 90^\circ + 90^\circ + 30^\circ = 360^\circ$$

$$\Rightarrow \angle ROQ + 210^\circ = 360^\circ$$

$$\Rightarrow \angle ROQ = 360^\circ - 210^\circ$$

$$\Rightarrow \angle ROQ = 150^\circ.$$

(b) In $\triangle OQR$, $\angle OQR = \angle ORQ$

$$\angle ROQ = 150^\circ$$

$$\text{and } \angle ROQ + \angle OQR + \angle ORQ = 180^\circ$$

$$\Rightarrow 150^\circ + 2 \angle ORQ = 180^\circ$$

$$\Rightarrow 2 \angle ORQ = 30^\circ$$

$$\Rightarrow \angle ORQ = 15^\circ$$

$$\therefore \angle OQR = \angle ORQ = 15^\circ$$

$$\text{Now } \angle RQP = \angle OQP - \angle OQR = 90^\circ - 15^\circ = 75^\circ.$$

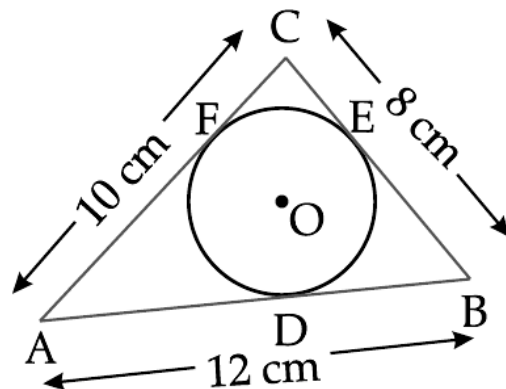
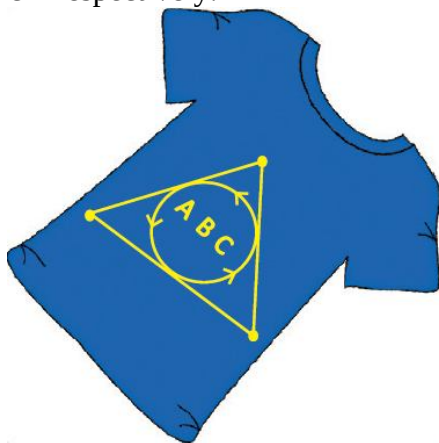
OR

$$(b) \angle RSQ = \frac{1}{2} \angle ROQ = 75^\circ. \text{ (Angle subtended at the centre is double)}$$

From the figure, $\angle ORP = \angle OQP = 90^\circ$ (Radius is perpendicular to tangent)

$$\therefore \angle ORP + \angle OQP = 90^\circ + 90^\circ = 180^\circ.$$

20. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo is designed in different geometry and different colours according to the theme. In given figure, a circle with centre O is inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



- (a) Find the length of AD and BE. (2)

OR

If the radius of the circle is 4 cm, find the area of $\triangle OAB$.

- (b) Find the perimeter of $\triangle ABC$. (1)

- (c) Find the length of CF. (1)

Ans. (a) Let AD be x cm, then DB = $(12 - x)$ cm

$\therefore AD = AF$, $CF = CE$, $DB = BE$ [Tangents to a circle from external points]

$$\therefore AF = x \text{ cm,}$$

$$\text{then } CF = (10 - x) \text{ cm}$$

$$BE = (12 - x) \text{ cm,}$$

$$\text{then } CE = 8 - (12 - x) = (x - 4) \text{ cm}$$

$$\text{Now } CF = CE$$

$$\Rightarrow 10 - x = x - 4$$

$$\Rightarrow 2x = 14$$

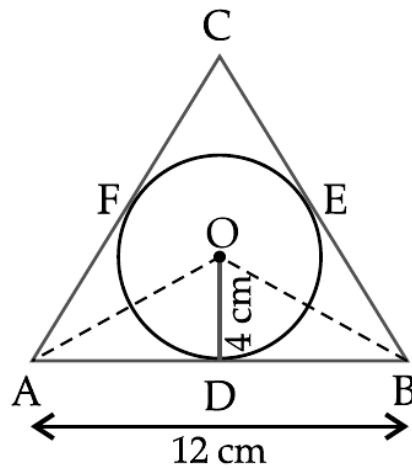
$$\Rightarrow x = 7. \dots(i)$$

Hence, $AD = 7\text{ cm}$.

Since, $BE = (12 - x)\text{ cm} = (12 - 7)\text{ cm}$ [From (i)]
 $= 5\text{ cm}$

OR

Radius, $OD = 4\text{ cm}$ and $AB = 12\text{ cm}$



Then, area of $\triangle OAB = \frac{1}{2} \times OD \times AB$

$$= \frac{1}{2} \times 4 \times 12$$

$$= 24\text{ cm}^2.$$

(b) Perimeter of $\triangle ABC = AB + BC + CA$

$$= (12 + 8 + 10)\text{ cm} = 30\text{ cm}.$$

(c) From question (a), $CF = (10 - x)\text{ cm}$

$$= (10 - 7)\text{ cm} = 3\text{ cm}.$$

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