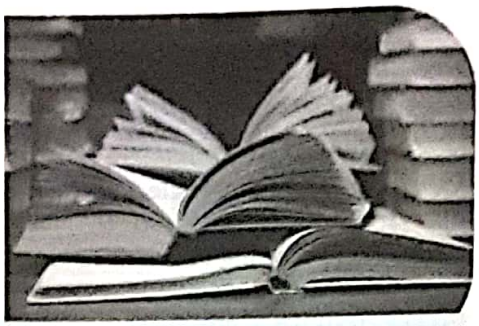
5 I'm Up and Down, and Round and Round

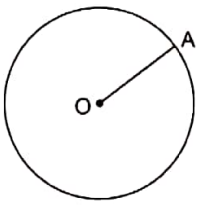


TOPICS COVERED

5.1, 5.2 Definitions, Symmetries of Circles, How many Circles?

CIRCLE AND ITS RELATED TERMS

- **Circle:** The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle and the fixed distance from the centre is called the radius of the circle. In the given figure, O is the centre and OA is the radius of the circle.





Terms related to circle:

(*i*) **Chord:** A line segment joining two point on the circle is called the chord of the circle. In fig (*i*), PQ is a chord of the circle.

(*ii*) **Diameter:** It is the longest chord of the circle, it passes through the centre of a circle and two times the radius.
AOB or AB is a diameter [fig (*i*)].

(*iii*) **Arc:** A part of a circle between two points on it is called an arc. Smaller part of the circle is called a minor arc, whereas larger part of circle is called a major arc. In fig (*ii*), PQ is the minor arc and PRQ is the major arc.

(*iv*) **Sector:** The region between arc and two radii is called sector [fig (*iii*)].

(*v*) Minor arc corresponds to minor sector and major arc corresponds to major sector [see fig (*iv*)].

(*vi*) **Segments:** The region between a chord and either of its arcs is called segment of a circle.
A circle has two segments.

- Major segment and Minor segment [see fig (*iii*)]

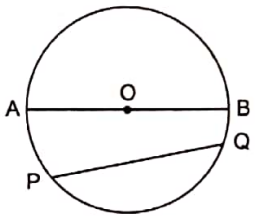


Fig. (*i*)

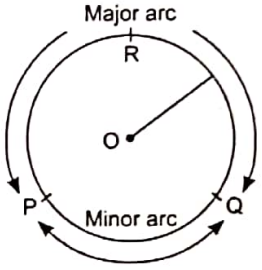


Fig. (*ii*)

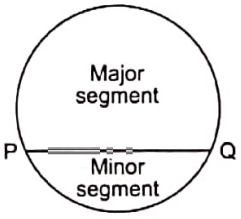


Fig. (*iii*)

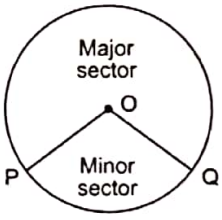
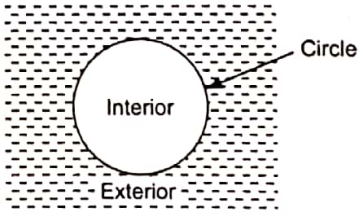


Fig. (*iv*)

- **Parts of the circle:**

A circle divides the plane on which it lies into three parts.



(*i*) inside the circle called interior.

(*ii*) outside the circle called exterior

(*iii*) the circle

Scanned with OKEN Scanner

A circle is one of the most symmetric geometric figures.

(i) **Rotational Symmetry:** It can be rotated about its centre by any angle and still looks exactly the same (e.g. 1°, 30°, 45°, or 0.001°) and it will look identical to its original position. This happens because every point on the circle is at the same distance (radius) from the centre.

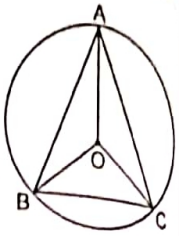
(ii) **Line (Reflectional) Symmetry:** A circle has infinite lines of symmetries.
- Every diameter of the circle is a line of symmetry.
- Folding the circle along any diameter divides it into two identical halves.

→ Any point that is equidistant from two given points A and B lies on the perpendicular bisector of AB.

→ An infinite number of circles can be drawn to pass through two given points A and B (say). These circles have their centres located anywhere along the perpendicular bisector of the line segment connecting the two points.

→ For three non-collinear points A, B and C, there is a unique circle passing through these three points.

Theorem: There is a unique circle passing through three non-collinear points.



→ The centre O of the circle passes through the vertices A, B, and C of $\triangle ABC$ is called the circumcentre of $\triangle ABC$. The circle is called the circumcircle and is said to circumscribe the triangle.

→ In an acute-angle triangle, circumcentre lies inside the triangle.

→ In an obtuse-angle triangle, circumcentre lies outside the triangle.

→ In a right-angle triangle, circumcentre lies on the hypotenuse of triangle.

COMPETENCY FOCUSSED QUESTIONS

Multiple Choice Questions

1 Mark

1. The fixed point in a circle is called

(a) Radius (b) Diameter
(c) center (d) Chord

Sol. (c) center

2. The distance from the center to any point on the circle is called

(a) Diameter (b) Radius
(c) Chord (d) Arc

Sol. (b) Radius

3. The region enclosed by a circle is called

(a) arc (b) segment
(c) interior (d) exterior

Sol. (c) interior

4. The centre of a circle passing through two given points lies on

(a) The line joining the points.
(b) The midpoint only
(c) The perpendicular bisector of the segment joining them.
(d) Any random point.

Sol. (c) The center must be equidistant from both the points, so it lies on the perpendicular bisector of the segment joining them.

5. If no restriction is given on radius, the number of circles passing through two distinct points is

(a) 1 (b) 2
(c) infinite (d) Can't be determined

Sol. (c) infinite

CONSTRUCTED RESPONSE TYPE QUESTIONS

Very Short Answer Type Questions

2 Marks

6. Find the radius of a circle passing through points A and B if the distance between A and B is 6 cm and the center lies on the perpendicular bisector at a distance 4 cm from midpoint of AB.

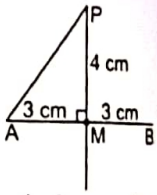
Sol. Let M be the midpoint of AB.

$\Rightarrow$ AM = 3 cm.

In right triangle PMA,

$PA = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25}$

$= 5$ cm



7. Is it possible to draw exactly one circle passing through two given points? Justify your answer.

Sol. No, it is not possible; as there are infinitely many points lies on the perpendicular bisector of line segment joining to given two points. Each point on the perpendicular bisector can be a center of circle passing through the two points.

Long Answer Type Questions

5 Marks

8. Draw $\triangle PQR$, with PQ = 6 cm, $\angle P = 110°$ and $\angle Q = 35°$. Construct its circumcircle. Is the circumcenter inside, outside or on the triangle?

Sol. We follow the following steps:

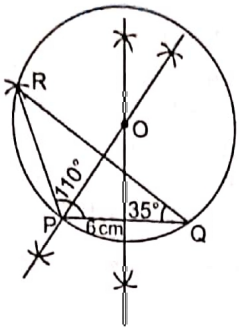
Step I: Draw a line segment PQ = 6 cm

Step II: At point P, use protractor to draw an angle of 110°. At Q, draw an angle of 35°. Mark the point of intersection as point R.

Step III: Construct perpendicular bisector of PQ and QR.

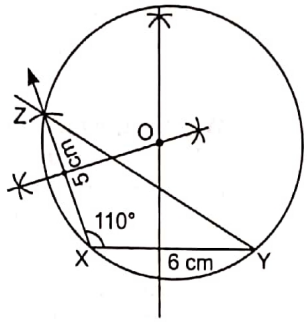
Scanned with OKEN Scanner

Step IV: Mark the point where the bisectors meet as O.
Step V: Place the compass at O and open it to P. Draw a circle.
Result: As $\angle P = 110° > 90°$, the circumcenter O lies outside the triangle.



9. Draw a ΔXYZ with XY=6cm, $\angle X = 110°$ and XZ = 5cm. Construct its circumcircle. Is the circumcenter inside or outside the triangle?

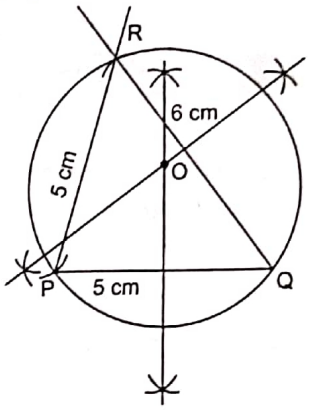
Sol. We follow the following steps
Step I: Draw line segment XY=6 cm.
Step II: At point X, construct an angle of 110°.
Step III: On this ray, mark a point Z such that XZ = 5 cm.
Step IV: Join YZ to form ΔXYZ.
Step V: Draw perpendicular bisectors of sides XY and XZ.



Let they intersect at point O.
Step VI: With center O and radius OX, draw a circle passing through X, Y, Z.
Observation:
As $\angle X = 110° > 90° \Rightarrow$ Circumcenter lies outside the triangle.

10. Draw ΔPQR with PQ=5cm, QR=6cm and RP = 5cm. Draw the circumcircle of ΔPQR. Let the center be O. Measure OP, OQ and OR.

Sol. We follow the following steps:
Step I: Draw PQ=5 cm.
Step II: With center P of radius 5 cm and center Q of radius 6 cm, draw arcs intersecting at R.
Step III: Join PR and QR to form ΔPQR.
Step IV: Draw perpendicular bisectors of any two sides (say PQ and QR). Let they bisect at O (Circumcenter).
Step V: With center O and radius OP, draw the circumcircle.
Measurement: OP=OQ=OR ≈ 3.6 cm (approx.)



PRACTICE QUESTIONS

11. In The longest chord of a circle is
(a) radius (b) arc
(c) diameter (d) radius

12. A line that intersects a circle at two distinct points is called
(a) radius (b) diameter
(c) secant (d) tangent

13. If the distance between two points is equal to the diameter of a circle, how many such circles can pass through them?
(a) Infinite (b) 2
(c) 1 (d) zero

14. Draw ΔXYZ with XY = 6 cm, YZ = 6 cm, and ZX = 6 cm. Construct its circumcircle let the circumcenter be O. Measure OX,OY and OZ.

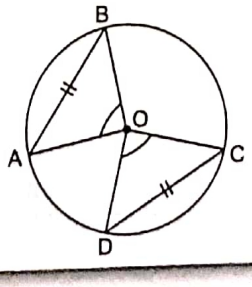
15. Draw ΔPQR with PQ=6 cm, $\angle P = 50°$ and $\angle Q = 60°$. Construct its circumcircle. Is the circumcircle lies inside or outside the triangle?

TOPICS COVERED

5.4, 5.5 Chords and The Angles They Subtend, Midpoints and Perpendicular Bisectors of chords

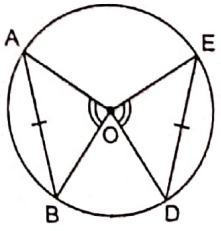
Theorem 2

Equal chords of a circle subtend equal angles at the centre of the circle.
AB = CD are equal chords of the circle centred at O.
$\therefore \angle AOB = \angle COD$.



Theorem 3

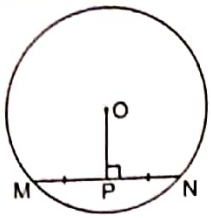
Chords of a circle that subtend equal angles at the centre are equal.
Let chords AB and DE are two chords subtend equal angle at O.
i.e. $\angle AOB = \angle DOE$, then AB = DE



Scanned with OKEN Scanner

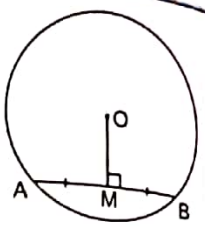
Theorem 4

The line joining the centre of a circle and the midpoint of the chord of a circle is perpendicular to the chord.
Let P is the midpoint of the chord, then OP ⊥ MN.



Theorem 5

The perpendicular from the centre of a circle to a chord of the circle bisects the chord.
If OM ⊥ AB (chord)
⇒ AM = MB



COMPETENCY FOCUSSED QUESTIONS

Multiple Choice Questions

1 Mark

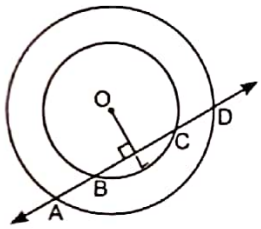
16. Given two concentric circles with centre O. A line cuts the circles at A, B, C, D respectively. If AB = 10 cm, then length CD is [Understanding]

(*a*) 5 cm (*b*) 10 cm
(*c*) 7.5 cm (*d*) 20 cm

Sol. (*b*) Draw OL ⊥ AD.

As OL ⊥ BC, so BL = LC ...(*i*)

Similarly, AL = LD ...(*ii*)



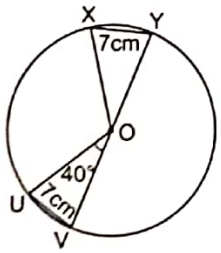
Subtracting (*i*) from (*ii*), we get

AL – BL = LD – LC

⇒ AB = CD

⇒ CD = 10 cm

17. Given below is a circle with centre O. [Understanding]



What is the measure of ∠XOY?

(*a*) 40° (*b*) 140° (*c*) 80° (*d*) 90°

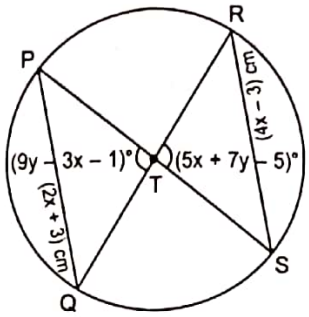
Sol. (*a*) Equal chords of a circle subtend equal angle at the centre.

Here XY = UV = 7 cm are chords of the circle

∠UOV = 40°, ∴ ∠XOY = 40°

18. In the figure below, PQ and RS are equal chords of a circle with centre T.

[Understanding and Application]



What is the measure of ∠PTQ ?

(*a*) 100° (*b*) 40° (*c*) 80° (*d*) 60°

Sol. (*c*) Since chords are of equal length.

∴ $2x + 3 = 4x - 3$

⇒ $6 = 2x \Rightarrow x = 3$ cm

Equal chords of a circle subtend equal angle at the centre.

∴ $9y - 3x - 1 = 5x + 7y - 5$

⇒ $2y = 8x - 4 = 24 - 4 = 20$

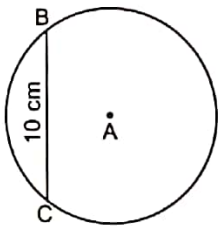
⇒ $y = 10$ cm

Now $\angle PTQ = 9y - 3x - 1$

$= 90 - 9 - 1 = 80°$

19. Given below is a circle with centre A and chord BC.

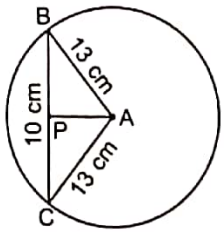
[Understanding and Application]



If radius of the chord is 13 cm, what is distance of the chord from the centre?

(*a*) 10 cm (*b*) 13 cm
(*c*) 12 cm (*d*) 5 cm

Sol. (*c*)



Construction: Draw AP ⊥ BC.

We know that the perpendicular drawn from the centre of a circle to the chords bisect the chord.

Scanned with OKEN Scanner

$\therefore$ BP = PC = $\frac{1}{2}$BC = 5 cm

In ΔAPB, we have

By Pythagoras Theorem.

$$AP = \sqrt{AB^2 - BP^2} = \sqrt{13^2 - 5^2}$$
$$= \sqrt{(13+5)(13-5)} = \sqrt{18 \times 8}$$
$$= \sqrt{144} = 12 \text{ cm}$$

$\therefore$ Distance of chord from the centre = 12 cm.

20. Two chords subtend angles 60° and 80° respectively at the center. Which chord is longer? [Conceptual]

(*a*) Chord with 60°
(*b*) Chord with 80°
(*c*) Both equal
(*d*) Cannot be determined

Sol. (*b*) Greater central angle $\Rightarrow$ longer chord.

21. If a chord passes through the centre of a circle, it is called: [Conceptual]

(*a*) radius (*b*) arc
(*c*) diameter (*d*) tangent

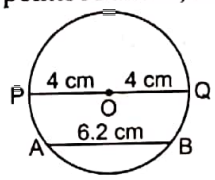
Sol. (*c*) diameter

CONSTRUCTED RESPONSE TYPE QUESTIONS

Very Short Answer Type Questions
2 Marks

22. Justify the statement: A circle of radius 4 cm can be drawn through two points A and B, such that AB = 6.2 cm. [Understanding]

Sol. It is true that a circle of radius 4 cm can be passed through two points A and B, where AB = 6.2 cm.

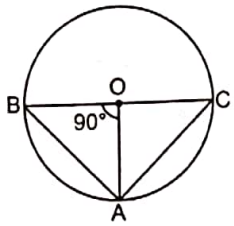


If we draw a circle of radius 4 cm, the length of longest chord, i.e. diameter = 8 cm.

Such diameter > AB = 6.2 cm. Hence, a chord of 6.2 cm can be drawn in a circle as shown in the figure.

23. In the given figure, AB = AC and O is the centre of the circle. If $\angle BOA = 90°$, determine $\angle AOC$.

[Conceptual and Application]



Sol. **Given:** A circle having centre O and AB = AC. Also, $\angle AOB = 90°$

To find $\angle AOC$

Proof: As we know that equal chords of a circle subtend equal angles at the centre of the circle

$\therefore$ $\angle AOB = \angle AOC$ (as AB = AC)

$\Rightarrow$ $\angle AOC = 90°$ (as $\angle AOB = 90°$)

24. In a circle with centre O, AB is chord. Prove that OA = OB. What can you conclude about ΔOAB?

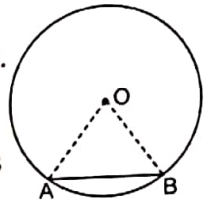
Sol. **Construction:** Join OA and OB.

As OA and OB are radii of the circle.

$\therefore$ OA = OB proved.

In ΔOAB,

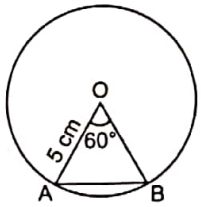
OA = OB $\Rightarrow$ ΔOAB is an isosceles triangle.



Short Answer Type Questions
3 Marks

25. In the given figure, chord AB subtends $\angle AOB$ equal to 60° at the centre O of the circle. If OA = 5 cm. then find the length of AB. [HOTS]

[Understanding and Application]



Sol. **Given:** $\angle AOB = 60°$, OA = 5 cm, where O is the centre of the circle.

To find: AB

Proof: In ΔAOB

$\angle AOB = 60°$ (Given)

OA = OB (Equal radii)

$\therefore$ $\angle OAB = \angle OBA$

(Angles opposite to equal sides OA and OB) ...(*i*)

In ΔAOB

$\angle OAB + \angle AOB + \angle OBA = 180°$

(Angle sum property of triangle)

$60° + \angle OAB + \angle OAB = 180°$

($\because$ $\angle OAB = \angle OBA$, using (*i*))

$\Rightarrow$ $2\angle OAB = 180° - 60° = 120°$

$\Rightarrow$ $\angle OAB = 60°$

$\Rightarrow$ $\angle OBA = 60°$

$\therefore$ ΔAOB is an equilateral triangle

Hence, OA = OB = AB

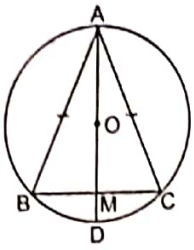
$\Rightarrow$ AB = 5 cm (as OA = 5 cm)

26. An isosceles triangle ΔABC is inscribed in a circle with diameter AD. If AB = AC, prove that AD is the angle bisector of $\angle BAC$.

Sol. **Given:** ΔABC is an isosceles triangle and AD is the diameter through A on BC. AB = AC.

Scanned with OKEN Scanner

To show: AD is the angle bisector of $\angle A$.
i.e. $\angle BAD = \angle CAD$

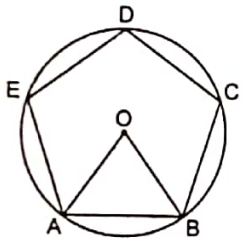


Proof: Diameter AD meets BC at M.
Now, $OM \perp BC \Rightarrow BM = MC$
Now in ΔABM and ΔAMC, we have

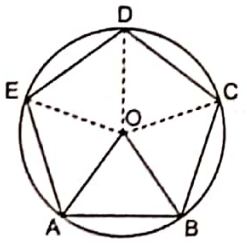
$AB = AC$ (Given)
$\angle ABM = \angle ACM$ [Angles of isosceles triangle]
$BM = MC$
$\therefore$ By SAS, $\Delta ABM \cong \Delta AMC$
$\Rightarrow \angle BAM = \angle CAM$ (By CPCT)
$\Rightarrow$ AM is the angle bisector of $\angle A$.
$\Rightarrow$ AD is the angle bisector of $\angle A$. proved

Long Answer Type Questions
5 Marks

27. In the given figure, O is the centre of a circle and A, B, C, D and E are points on the circle such that AB = BC = CD = DE = EA. Find the value of $\angle AOB$.
[HOTS] [Conceptual and Application]



Sol. **Given:** O is centre of circle and AB = BC = CD = DE = EA.
Construction: Join OC, OD, OE
To find: $\angle AOB$
Proof: A, B, C, D and E are the points which lie on the circle.

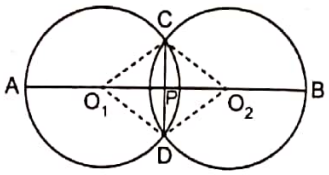


Also AB = BC = CD = DE = EA
All are the chords of the circle
As we know that equal chords subtend equal angle at the centre of circle.
$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOE = \angle AOE$ ...(*i*)
Also, $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$
(sum of angles at the centre of circle)
Using (*i*)
$\angle AOB + \angle AOB + \angle AOB + \angle AOB + \angle AOB = 360°$
$\Rightarrow 5\angle AOB = 360°$
$\therefore \angle AOB = 72°$

Marks Loss Alert
1. Students forget that equal chords in a circle subtend equal angles at the centre. They may lose marks.
2. The total angle around O is 360°. Students forget to divide it by 5 equal parts.

28. O_1 and O_2 are the centres of two congruent circles intersecting each other at points C and D. The line joining their centres intersects the circles in points A and B such that $AB > O_1O_2$. If CD = 6 cm and AB = 12 cm, determine the radius of either circle.
[HOTS] [Understanding and Application]

Sol. Let radius of each circle = r cm
$AB = 12$ cm
$\therefore O_1O_2 = 12 - 2r$



Now, CD is the common chord of the two circles and O_1O_2 is the line segment that joins the centres.
[Radii of congruent circles]
As we know that line joining the centres of two circles is perpendicular bisector of the common chord
$\therefore O_1O_2 \perp CD$ and O_1O_2 bisects CD

$$\therefore CP = \frac{1}{2} \times CD = 3 \text{ cm.}$$

and $$O_1P = \frac{1}{2}(O_1O_2) = \frac{1}{2}(12 - 2r)$$
$$= (6 - r) \text{ cm}$$

Now in right ΔCPO_1,

$$(O_1C)^2 = (O_1P)^2 + (PC)^2$$
$$\Rightarrow r^2 = (6 - r)^2 + (3)^2$$
$$\Rightarrow r^2 = 36 + r^2 - 12r + 9$$
$$\Rightarrow 12r = 45$$
$$\Rightarrow r = \frac{45}{12}$$
$$\Rightarrow r = 3.75 \text{ cm}$$

Scanned with OKEN Scanner

PRACTICE QUESTIONS

29. In a circle with centre O, chords AB and CD are of lengths 5 cm each and subtend angles $x°$ and $y°$ at the centre of the circle respectively. Then [Understanding]

(a) $x > y$ (b) $x = y$
(c) $x < y$ (d) cannot say

30. The perpendicular bisector of a chord [Understanding]

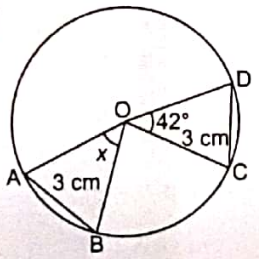
(a) passes through the centre
(b) is tangent to the circle
(c) lies outside the circle
(d) is parallel to radius

31. If a chord is very close to the centre of a circle, it will be [Understanding]

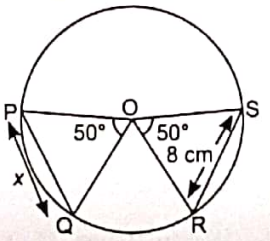
(a) very small (b) maximum length
(c) equal to radius (d) zero length

32. Find the value of x in the following figures, where 'O' is the centre of the circle. [Understanding and Application]

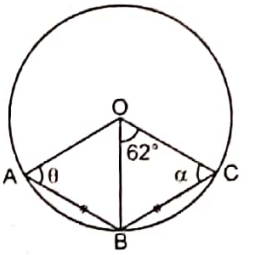
(i)



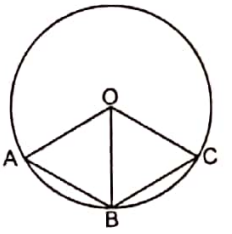
(ii)



33. Find the value of θ and α in the given figure, where 'O' is the centre of the circle. [Understanding and Application]



34. In the given figure, OABC is rhombus. Find ∠AOB and ∠BOC, where 'O' is the centre of circle. [Understanding and Application]

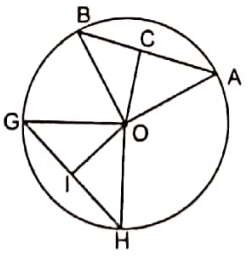


35. Prove that the line drawn through the centre of a circle to the mid-point of a chord is perpendicular to the chord. [Reasoning]

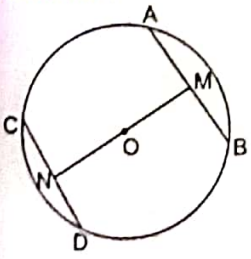
TOPICS COVERED 5.6 Distance of Chords from the centre

Theorem 6

Chords of a circle having the same length are all at the same distance from the centre of the circle.
In the given figure, if chords
AB = GH Then OC=OI



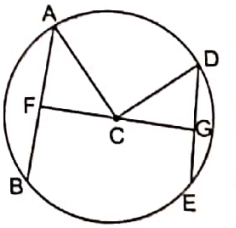
Theorem 7



Chords of a circle that are equidistant from the centre have equal length.
In the given circle, if OM = ON
⇒ AB = CD

Theorem 8

Let AB and DE be two chords of a circle with centre C. Suppose AB > DE. Then the distance from C to AB is less than the distance from C to DE.
In the given figure chord
AB > chord DE.
⇒ CF < CG



Note: The chord nearest to the centre is the chord containing the centre. Its distance from C is zero. So, the diameter is the longest chord of the circle.

Scanned with OKEN Scanner

COMPETENCY FOCUSSED QUESTIONS

Multiple Choice Questions

1 Mark

36. Given a circle with centre O and smallest chord AB is of length 3 cm, longest chord CD is of length 10 cm and chord PQ is of length 7 cm, then radius of the circle is [Understanding]

(a) 1.5 cm (b) 6 cm
(c) 5 cm (d) 3.5 cm

Sol. (c)

37. Given a chord AB of length 5 cm, of a circle with centre O. OL is perpendicular to chord AB and OL = 4 cm. OM is perpendicular to chord CD such that OM = 4 cm. Then CM is equal to [Understanding and Application]

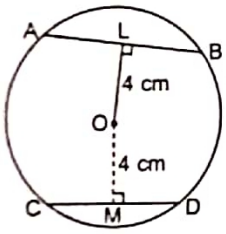
(a) 4 cm (b) 5 cm
(c) 2.5 cm (d) 3 cm

Sol. (c) Since OL = OM = 4 cm, so

AB = CD

(∵ Chords equidistant from the centre of a circle are equal in length)

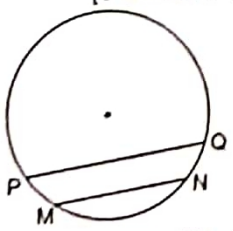
$\Rightarrow$ CD = 5 cm



Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so

$$CM = MD = \frac{1}{2}CD \Rightarrow CM = 2.5 \text{ cm}$$

38. In a circle of radius 13 cm. PQ and MN two chords are of length 24 cm and x cm respectively. The distance between the chords is 7 cm. What is the value of x? [Understanding and Application]

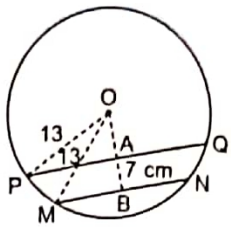


(a) 10 cm (b) 5 cm
(c) 11 cm (d) 6 cm

Conversion Ready (Subjective)

In a circle of radius 13 cm, two chords PQ and MN are drawn such that the length of chord PQ = 24 cm and the length of chord MN = x cm. The distance between the two chords is 7 cm. Find the perpendicular distance of chord PQ from the centre. Also find the distance of chord MN from the centre and length of chord MN.

Sol. (a)



AB = 7 cm, PQ = 24 cm, MN = x cm.

Construction: Join OA, OP and OM.

OA $\perp$ PQ and OB $\perp$ MN

$\Rightarrow$ PA = 12 cm and MB = $\frac{x}{2}$ cm

OM = OP = 13 cm (radii)

In ΔOBM, we have

$$OM^2 = OB^2 + MB^2$$

$$\Rightarrow 13^2 = (OA + 7)^2 + \left(\frac{x}{2}\right)^2 \quad ...(i)$$

In ΔOPA,

$$OP^2 = OA^2 + PA^2$$

$$\Rightarrow 13^2 = OA^2 + 12^2$$

$$\Rightarrow OA^2 = 13^2 - 12^2$$

$$= (13 + 12)(13 - 12) = 25$$

$$\Rightarrow OA = 5 \text{ cm}$$

Using OA = 5 cm in (i), we get

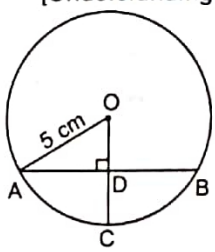
$$169 = (5 + 7)^2 + \left(\frac{x}{2}\right)^2$$

$$= 144 + \left(\frac{x}{2}\right)^2$$

$$\Rightarrow \left(\frac{x}{2}\right)^2 = 169 - 144 = 25$$

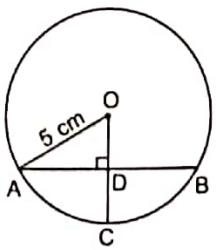
$$\Rightarrow \frac{x}{2} = 5 \Rightarrow x = 10 \text{ cm}$$

39. In figure, If OA = 5 cm, AB = 8 cm and OD is perpendicular to AB. Then CD is equal to [Understanding and Application]



(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

Sol. (a)



OD $\perp$ AB

$$\Rightarrow AD = \frac{1}{2}AB = 4 \text{ cm}$$

In ΔOAD,

$$OD^2 = OA^2 - AD^2 = 5^2 - 4^2$$

$$= (5 + 4)(5 - 4) = 9$$

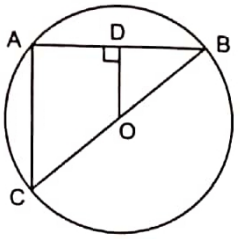
Scanned with OKEN Scanner

$\Rightarrow$ $OD = 3$ cm
$\because$ $OA = OC = 5$ cm (radii of circle)
$\therefore$ $CD = OC - OD = (5 - 3) = 2$ cm

Short Answer Type Questions
3 Marks

40. If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that CA = 2OD. [HOTS] [Conceptual and Analyzing]

Sol. Given: A circle of centre O, diameter BC and OD $\perp$ chord AB.
To prove: CA = 2OD



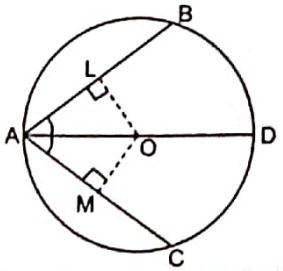
Proof: Since OD $\perp$ AB
$\therefore$ D is the mid-point of AB
(perpendicular drawn from the centre to a chord bisects the chord)
O is centre $\Rightarrow$ O is the mid-point of BC.
In ΔABC, O and D are the mid-points of BC and AB, respectively.
$\therefore$ OD || AC and $OD = \frac{1}{2}AC$ (mid-point theorem)
$\therefore$ CA = 2OD

Marks Loss Alert

1. **Forgetting the chord property:** Not stating that OD $\perp$ AB.
   $\Rightarrow$ D is the mid point of AB (the perpendicular drawn from a centre to the chord bisects the chord).
2. **Missing midpoint identification:** Not recognizing O is the midpoint of BC since BC is a diameter.
3. **Not applying Midpoint Theorem:** Failing to conclude OD||AC and $OD = \frac{1}{2}AC \Rightarrow CA = 2OD$

41. If two chords of a circle are equally inclined to the diameter passing through their point of intersection, prove that the chords are equal.
[Conceptual and Analyzing]

Sol.



Given: Two chords AB and AC of a circle are equally inclined to diameter AOD, i.e.
$\angle DAB = \angle DAC$
Construction: Draw OL $\perp$ AB and OM $\perp$ AC

Proof: In ΔOLA and ΔOMA

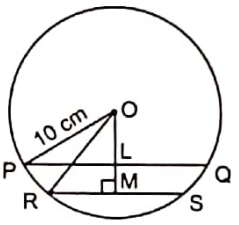
	$\angle OLA = \angle OMA$	(each 90°)
	$AO = AO$	(common)
	$\angle OAL = \angle OAM$	(given)
	$\Delta OLA \cong \Delta OMA$	(AAS rule)
$\Rightarrow$	$OL = OM$	(CPCT)
$\Rightarrow$	$AB = AC$	

(chords equidistant from the centre are equal)

Long Answer Type Questions
5 Marks

42. PQ and RS are two parallel chords of a circle on the same side of centre O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between the chords. [HOTS]
[Understanding and Application]

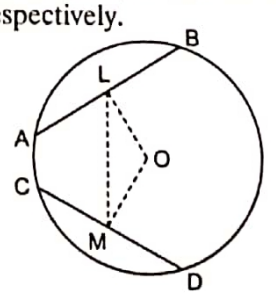
Sol. Given: A circle with centre O and two chords PQ and RS, such that PQ || RS
To find: LM
Construction: Draw OM $\perp$ RS which intersects PQ at L.



Proof: $\because$ OM $\perp$ RS
$\therefore$ OL $\perp$ PQ ($\because$ PQ || RS)
$\therefore$ $PL = \frac{1}{2}PQ$ and $RM = \frac{1}{2}RS$
Now, PL = 8 cm and RM = 6 cm
Let LM = x cm
OP = OR = 10 cm
In ΔOPL, $OL = \sqrt{(10)^2 - (8)^2}$ cm = 6 cm
Also, in ΔORM, $OM = \sqrt{(10)^2 - (6)^2}$ cm = 8 cm
$\therefore$ $x = OM - OL = 8$ cm $- 6$ cm
$\Rightarrow$ $x = 2$ cm
$\therefore$ Distance between the chords = LM = 2 cm

43. Prove that the line segment joining the mid-points of two equal chords of a circle make equal angles with the chords. [Conceptual and Analyzing]

Sol. Given: A circle C(O, r), AB and CD are two equal chords of a circle. L, M are the mid-points of AB and CD respectively.



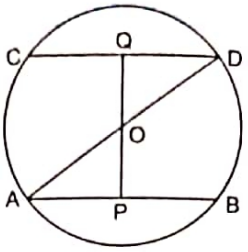
Scanned with OKEN Scanner

To Prove: (*i*) $\angle ALM = \angle CML$
(*ii*) $\angle BLM = \angle DML$.
Construction: LM, OL, OM are joined.
Proof: (*i*) $OL \perp AB$ and $OM \perp CD$.
(As the line joining the centre to the mid-point of the chord is perpendicular to the chord)
Now, $OL = OM$
[Equal chords are equidistant from the centre]
In ΔOLM, $OL = OM$ [Proved above]
$\Rightarrow \angle OLM = \angle OML$
[Angles opposite to equal sides are equal] ...(*i*)
$\angle OLA = \angle OMC$ [Each 90°]
$\Rightarrow \angle OLA - \angle OLM = \angle OMC - \angle OML$
[$\because \angle OLA = \angle OMC = 90°$]
$\Rightarrow \angle MLA = \angle LMC$ ...(*ii*)
Again from (*i*)
$\angle OLM + \angle OLB = \angle OML + \angle OMD$
[$\because \angle OLB = \angle OMD = 90°$]
$\Rightarrow \angle MLB = \angle LMD$.

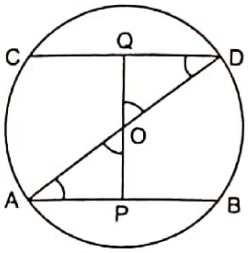
Marks Loss Alert

1. **Missing the perpendicular property:** Not stating that $OL \perp AB$ and $OM \perp CD$ since the line joining the centre to the midpoint of the chord is perpendicular to the chord.
2. **Ignoring equal distance theorem:** Forgetting that equal chords are equidistant from the centre $\Rightarrow OL = OM$, which need to prove $\angle OLM = \angle OML$.

44. In the given figure, AB || CD. AD is a diameter of the circle whose centre is O. Prove that AB = CD. [HOTS] [Conceptual and Analyzing]



Sol. Given: AB || CD, AOD is the diameter of circle, where O is the centre of circle.
To prove: AB = CD



Proof: In ΔDOQ and ΔAOP
$OD = OA$ (radii of circle)
$\angle DOQ = \angle AOP$
(vertically opposite angle)
$\angle QDO = \angle PAO$
[alternate angles as CD || AB (given)]
$\Rightarrow \Delta DOQ \cong \Delta AOP$ (by ASA)
$\Rightarrow OQ = OP$ (by CPCT)
$\Rightarrow CD = AB$ (chords equidistant from the centre of a circle are equal)

PRACTICE QUESTIONS

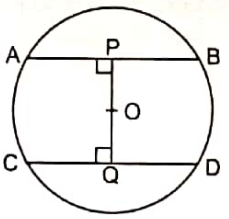
45. The radius of circle is 13 cm and the length of one of its chord is 10 cm. The distance of the chord from the centre is. [Understanding]
(*a*) 11.5 cm (*b*) 12 cm
(*c*) $\sqrt{69}$ cm (*d*) 23 cm

46. Distance of a chord AB of a circle from the centre is 12 cm and length of the chord AB is 10 cm. Find the diameter of the circle. [Understanding and Application]

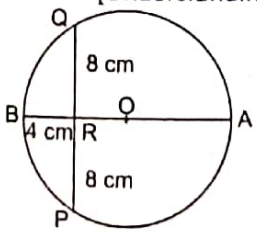
47. In a circle of radius 5 cm having centre O, OL is drawn perpendicular to the chord AB. If OL = 3 cm, find the length of AB. [Understanding and Application]

48. In a circle with centre O, chords AB and CD are of lengths 4 cm each and subtend angles $x°$ and $y°$ at the centre of the circle respectively. Determine the relation between $x°$ and $y°$. [HOTS]
[Understanding and Appl3ication]

49. In the given figure, O is the centre of the circle with radius 5 cm. $OP \perp AB$, $OQ \perp CD$, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ.
[Understanding and Application]



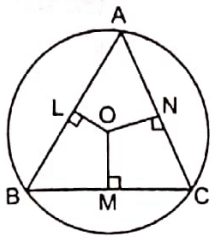
50. In the given figure, diameter AB of circle with centre O bisects the chord PQ. If PR = QR = 8 cm and RB = 4 cm, find the radius of the circle. [HOTS]
[Understanding and Application]



51. A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B. Give reason to support your answer. [HOTS]
[Understanding and Application]

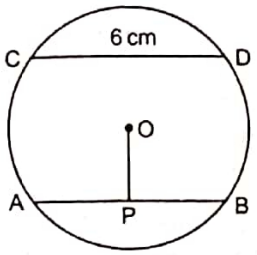
Scanned with OKEN Scanner

52. In figure, O is the centre of the circle, OM ⊥ BC, OL ⊥ AB, ON ⊥ AC and OM = ON = OL [Analyzing]



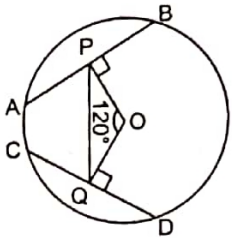
Prove that ΔABC is an equilateral triangle.

53. In the given figure, AB and CD are two chords equidistant from the centre O. OP is the perpendicular drawn from centre O to AB. If CD = 6 cm, find PB. [Understanding]



54. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre. Prove that $4q^2 = p^2 + 3r^2$. [Analyzing]

55. In the given figure, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If ∠POQ = 120°, find ∠APQ. [HOTS] [Understanding]



56. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the angle bisector of ∠BAC. [HOTS] [Analyzing]

57. In a circle of radius 18 cm, AB and AC are two chords such that AB = AC = 12 cm. Find the length of chord BC. [Understanding]

TOPICS COVERED

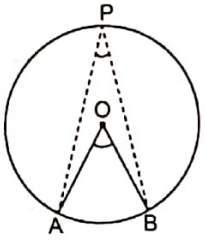
5.7, 5.8 Angle Subtended by an Arc, Concyclicity of Points

Theorem 9

The angle subtended by an arc at the centre of the circle is double the angle subtended by the arc at any point on the circle outside the arc.

In the given circle arc AB subtends angle ∠AOB at the centre O and ∠APB at P, then

∠AOB = 2∠APB.



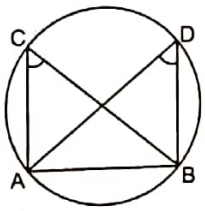
Concyclic Points

If three or more points that lies on the same circle's circumference are called concyclic points.

Theorem 10

If a line segment AB joining two points A, B subtends equal angles at other two points C, D that lie on the same side of AB, then the four points lie on a circle.

In the given figure line segment AB subtends equal angles ∠ACB and ∠ADB and these angles are in the same side of AB, then A, B, C, D lies on the circle.



Cyclic Quadrilateral

When the vertices of quadrilateral (4-gon) are concyclic, the quadrilateral is called a cyclic quadrilateral.

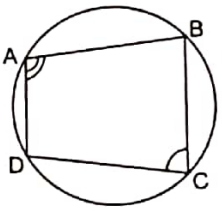
Theorem 11

The sum of two opposite angles of a cyclic quadrilateral is 180°.

In the given figure, ABCD is a cyclic quadrilateral.

∴ ∠A + ∠C = 180°

and ∠B + ∠D = 180°

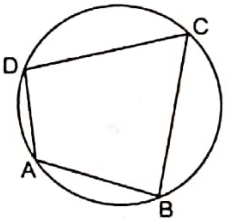


Theorem 12

If two opposite angles of a quadrilateral add upto 180°, then the vertices of the quadrilateral lie on a circle, i.e. they are concyclic.

In the given figure if ∠A + ∠C = 180°, ∠B + ∠D = 180°, then A, B, C and D are vertices of the quadrilateral lie on the circle.

i.e. ABCD is a cyclic quadrilateral.



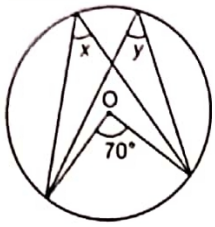
Scanned with OKEN Scanner

COMPETENCY FOCUSSED QUESTIONS

Multiple Choice Questions

1 Mark

58. **In the given figure, value of y is** [Understanding]

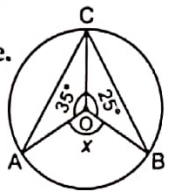


(a) 35° (b) 140°
(c) $70° + x$ (d) 70°

Sol. (a) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. So,

$$y = \frac{1}{2} \times 70° = 35°$$

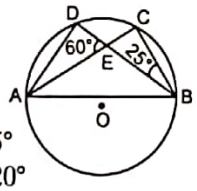
59. **In figure, O is the centre of the circle. The value of x is** [Understanding]



(a) 140° (b) 60°
(c) 120° (d) 300°

Sol. (d) We have $\angle AOC + \angle BOC + \angle AOB = 360°$
($\because$ Angle at the centre of a circle)
$\Rightarrow 35° + 25° + x = 360° \Rightarrow x = 300°$

60. **In the given figure, O is the centre of the circle, $\angle CBE = 25°$ and $\angle DEA = 60°$. The measure of $\angle ADB$ is** [Understanding]



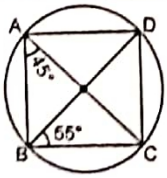
(a) 90° (b) 85°
(c) 95° (d) 120°

Conversion Ready (Subjective)

In the given figure, O is the centre of the circle, $\angle CBE = 25°$ and $\angle DEA = 60°$. Identify the type of angles CEB and DEA. Determine the measure of $\angle ADB$ with proper reason.

Sol. (c) We have $\angle DEA = \angle CEB = 60°$
(Vertically opposite angles)
Using angle sum property of triangle in ΔCEB, we have
$\angle CEB + \angle CBE + \angle ECB = 180°$
$\Rightarrow 60° + 25° + \angle ECB = 180°$
$\Rightarrow \angle ECB = 95°$
Now, $\angle ADB = \angle ACB$
($\because$ Angles in the same segment of a circle are equal)
$\Rightarrow \angle ADB = 95°$

61. **In the given figure, $\angle DBC = 55°$, $\angle BAC = 45°$ then $\angle BCD$ is** [Understanding]



(a) 45° (b) 55°
(c) 100° (d) 80°

Sol. (d) We have $\angle BAC = \angle BDC$
($\because$ Angles in the same segment of a circle are equal)
$\Rightarrow \angle BDC = 45°$
Using angle sum property of triangle in ΔBDC, we get
$\angle DBC + \angle BDC + \angle BCD = 180°$
$\Rightarrow 55° + 45° + \angle BCD = 180°$
$\Rightarrow \angle BCD = 80°$

62. **Four points lie on a circle if:** [Conceptual]
(a) All angles are equal
(b) Opposite angles of the quadrilateral sum to 180°
(c) Adjacent sides are equal
(d) Diagonals are equal

Sol. (b) Four points lie on a circle (i.e., form a cyclic quadrilateral) if and only if the sum of a pair of opposite angles is 180°. For ABCD,
$\angle A + \angle C = \angle 180°$ and $\angle B + \angle D = 180°$

63. **If $\angle A + \angle C = 180°$ in quadrilateral ABCD, then ABCD is:** [Conceptual]
(a) A parallelogram
(b) A trapezium
(c) A cyclic quadrilateral
(d) A rhombus

Sol. (c) In a quadrilateral, if the sum of a pair of opposite angles is 180°, then the quadrilateral is cyclic.

64. **Points A, B, C, and D are concyclic if:** [Conceptual]
(a) AB = CD
(b) $\angle ABC = \angle ADC$
(b) AC = BD
(d) AB ∥ CD

Sol. (b) Four points A, B, C, and D are concyclic (lie on the same circle) if a pair of angles subtended by the same chord are equal.
i.e. $\angle ABC = \angle ADC$
This means both angles lies on the same chord, and hence the points lie on a single circle.

65. **The shape of a clock dial is circular because:** [Understanding]
(a) It is easy to manufacture
(b) The hands rotate around a fixed centre
(c) It saves space
(d) It uses less material

Sol. (b) A clock dial is circular because the hands rotate around a fixed central point.
A circular shape allows smooth, even rotation in all directions, making it ideal for displaying time.

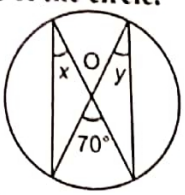
Scanned with OKEN Scanner

CONSTRUCTED RESPONSE TYPE QUESTIONS

Very Short Answer Type Questions

2 Marks

66. In the given figure, find the value of x and y where O is the centre of the circle. [Understanding]



Sol. $y = \frac{1}{2} \times 70° = 35°$

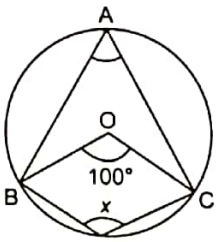
(Angle at the centre is double the angle subtended by the same arc at any point on the remaining part of the circle)

Also $\angle x = \angle y$

(Angles in the same segment are equal)

$\Rightarrow \quad x = 35°$

67. If $\angle BOC = 100°$, then find x from the given figure. [Understanding]



Sol. Here O is the centre of the circle

$\therefore \quad \angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 100° = 50°$

Also $\angle x + \angle BAC = 180°$

(sum of opposite angles of cyclic quadrilateral)

$\Rightarrow \angle x + 50° = 180° \Rightarrow x = 130°$

68. In a quadrilateral ABCD, $\angle A = 85°$ and $\angle C = 95°$. Show that ABCD is cyclic. [Understanding]

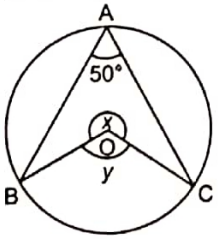
Sol. Given that $\angle A = 85°$ and $\angle C = 95°$

$\angle A + \angle C = 85° + 95° = 180°$

Since opposite angles are supplementary.

Hence, ABCD is a cyclic quadrilateral.

69. Find x in the given figure. [Understanding]



Sol. Here O is the centre of the circle

$\therefore \quad \angle BAC = \frac{1}{2}\angle y$

(By degree measure theorem)

$\Rightarrow \quad 50 = \frac{1}{2}\angle y$

$\Rightarrow \quad \angle y = 100°$

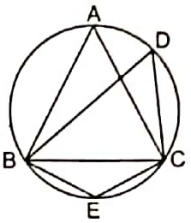
Also $\angle x + \angle y = 360°$

(Angle at the centre of a circle)

$\Rightarrow \quad \angle x + 100° = 360°$

$\Rightarrow \quad \angle x = 360° - 100° = 260°$

70. In the given figure, ΔABC is equilateral. Find $\angle BDC$ and $\angle BEC$. [Understanding]



Sol. $\angle BAC = 60°$

[$\because \Delta ABC$ is an equilateral triangle]

$\therefore \quad \angle BAC = \angle BDC$

[$\because$ Angles in the same segment of a circle are equal]

$\Rightarrow \quad \angle BDC = 60°$

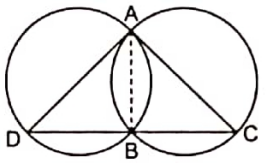
Now, $\square DBEC$ is a cyclic quadrilateral

$\therefore \angle BDC + \angle BEC = 180°$ [$\because$ Opposite angles of a cyclic quadrilateral are supplementary]

$60° + \angle BEC = 180° \Rightarrow \angle BEC = 180° - 60° = 120°$

71. Two circles intersect at two points A and B. AD and AC are diameters of the two circles. Prove that B lies on the line segment DC.

[Understanding and Analyzing]



Sol. **Given:** Two circles intersect at A and B. AD and AC are diameters.

To prove: B lies on DC

Construction: Join AB

Proof: AD is the diameter of a circle.

$\therefore \quad \angle ABD = 90°$ ...(i) (Angle in a semicircle)

AC is the diameter of another circle.

$\therefore \quad \angle ABC = 90°$ ...(ii) (Angle in a semicircle)

Adding (i) and (ii)

$\angle ABD + \angle ABC = 90° + 90° = 180°$

$\therefore$ These two angles from a linear pair angles.

$\Rightarrow$ DBC is a line. Hence point B lies on line segment DC.

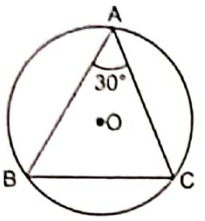
Short Answer Type Questions

3 Marks

72. In the given figure, ABC is a triangle in which $\angle BAC = 30°$. Show that BC is equal to the radius of the circumcircle of ΔABC, whose centre is O. [HOTS]

[Understanding and Application]

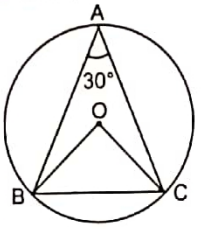
Scanned with OKEN Scanner



Sol. **Given:** A ΔABC having $\angle BAC = 30°$ and a circumcircle of ΔABC having centre O.

To prove: BC = radius of the circumcircle.

Construction: Join OB and OC.



Proof: $\angle BOC = 2\angle BAC$

(Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$\therefore \quad \angle BOC = 2 \times 30° = 60°$

Also, $\quad OB = OC$ (Radii of circle)

$\therefore \quad \angle OBC = \angle OCB$

(Angles opposite to equal sides are equal)

In ΔBOC,

$\angle BOC + \angle OBC + \angle OCB = 180°$

(Angle sum property of a triangle)

$\Rightarrow 60° + 2\angle OBC = 180°$

$\Rightarrow \quad 2\angle OBC = 180° - 60° = 120°$

$\Rightarrow \quad \angle OBC = \frac{120°}{2} = 60°$

$\therefore \quad \angle OCB = \angle OBC = 60°$

Since each angle of ΔBOC is 60°, therefore, ΔBOC is an equilateral triangle.

$\Rightarrow$ BC = OC = OB = radius of circumcircle of ΔABC

Hence proved

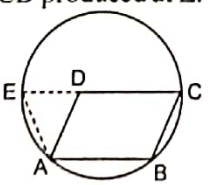
73. ABCD is a parallelogram. The circles through A, B and C intersect CD (produced, if necessary) at E. Prove that AE = AD. [HOTS]

[Understanding and Analysing]

Sol. **Given:** ABCD is a parallelogram. A circle passes through A, B and C intersect side CD produced at E.

To Prove: AE = AD

Construction: Join AE



Proof: ABCD is a ||gm

$\therefore \quad \angle ADC = \angle ABC$

[Opposite angles of parallelogram] ...(*i*)

$\angle ADC + \angle ADE = 180°$ ...(*ii*)

(Angles on straight line)

Also, $\angle ABC + \angle AEC = 180°$ ...(*iii*) [(Angles of cyclic quadrilateral ABCE by construction]

On equating (*ii*) and (*iii*)

$\angle ADC + \angle ADE = \angle ABC + \angle AEC$

$\Rightarrow \quad \angle ADE = \angle AEC$

(As $\angle ADC = \angle ABC$ opposite angles of ||gm)

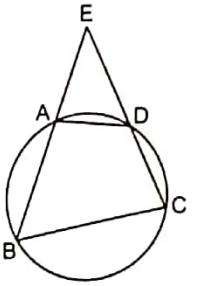
$\Rightarrow \quad AD = AE$

(Sides opposite to equal angles are equal)

74. ABCD is a cyclic quadrilateral. BA and CD produced meet at E. Prove that the triangles EBC and EDA are equiangular. [Understanding and Analysing]

Sol. **Given:** ABCD is a cyclic quadrilateral. BA and CD are produced to meet at E.

To Prove: Δs EBC and EDA are equiangular.



Proof: $\because$ ABCD is a cyclic quadrilateral.

$\therefore \angle BAD + \angle BCD = 180°$

[Sum of opposite angles of a cyclic quadrilateral] ...(*i*)

But $\angle BAD + \angle EAD = 180°$ [Linear pair] ...(*ii*)

From (*i*) and (*ii*),

$\angle BCD = \angle EAD$

Similarly, $\angle ABC = \angle EDA$

and $\angle BEC = \angle AED$ [Common]

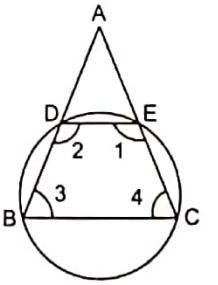
Hence, Δs EBC and EDA are equiangular.

75. ABC is an isosceles triangle in which AB = AC. A circle passing through B and C intersects AB and AC at D and E respectively. Prove that BC || DE.

[Understanding and Analysing]

Sol. **Given:** An isosceles triangle ABC, in which AB = AC and a circle through B and C intersecting AB and AC at D and E respectively.

To Prove: DE || BC.



Proof: In ΔABC, AB = AC $\Rightarrow \angle 3 = \angle 4$

[Angles opposite to equal sides are equal] ...(*i*)

Also, DBCE is a cyclic quadrilateral

$\Rightarrow \angle 2 + \angle 4 = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow \angle 2 + \angle 3 = 180°$ [From (*i*)] ...(*ii*)

But $\angle 2$ and $\angle 3$ are co-interior angles on the same side of transversal BD.

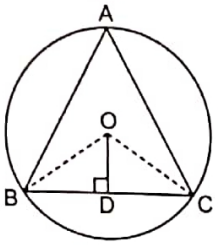
$\therefore$ DE || BC.

Scanned with OKEN Scanner

76. O is the circumcentre of the triangle ABC and OD is perpendicular to BC. Prove that $\angle BOD = \angle A$.
[Understanding and Analyzing]

Sol. Given: In triangle ABC, OD ⊥ BC where O is the circumcentre of ΔABC.

To prove: $\angle BOD = \angle A$



Construction: Join OB and OC

Proof: Here O is the centre of circle.

$\therefore \quad \angle BOC = 2\angle A \quad \ldots (i)$

(By degree measure theorem)

Also, in ΔBOD and ΔCOD

$OB = OC$ (radii of circle)

$OD = OD$ (common)

$\angle ODB = \angle ODC = 90°$ (OD ⊥ BC given)

$\Rightarrow \quad \Delta OBD \cong \Delta OCD$ (by RHS)

$\Rightarrow \quad \angle BOD = \angle COD$ (By CPCT) ...(*ii*)

$\Rightarrow \quad \angle BOC = \angle BOD + \angle COD$

$= \angle BOD + \angle BOD$ [Using (*ii*)]

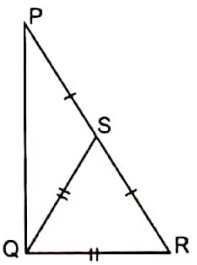
$\Rightarrow \quad \angle BOC = 2\angle BOD \quad \ldots (iii)$

Equating (*i*) and (*iii*)

$2\angle A = 2\angle BOD$

$\Rightarrow \quad \angle BOD = \angle A$

77. In ΔPQR, right-angled at Q. A point S is taken on the side PR such that PS = SR and QR = QS. Find the measure of ∠QRS. [HOTS]
[Understanding and Analysing]



Sol. In right ΔPQR, PS = SR

∴ S is the circumcentre of ΔPQR

[∵ Hypotenuse of a right Δ is diameter of its circumcircle]

∴ P, Q, R lie on circle with centre as S

$\therefore \quad SQ = SR$

Also, $SQ = QR$ (given)

⇒ ΔSQR is an equilateral Δ

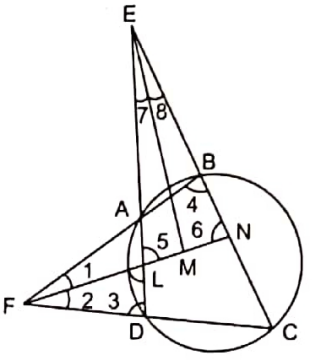
$\therefore \quad \angle QRS = 60°$.

Long Answer Type Questions
5 Marks

78. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral (provided they are not parallel) intersect at right angle. [HOTS] [Understanding and Analysing]

Sol. Given: ABCD is a cyclic quadrilateral whose opposite sides are produced to meet at E and F.

To Prove: Bisectors of ∠E and ∠F intersect at right angle.



Proof: In ΔFDL and ΔFBN,

$\angle 2 = \angle 1$

[∵ FN is the bisector of ∠F]

$\angle 3 = \angle 4$ [Exterior angle of cyclic quadrilateral is equal to interior opposite angle]

$\therefore \quad$ Third $\angle FLD =$ Third $\angle 6$

But $\angle FLD = \angle 5$ [Vertically opposite angles]

$\therefore \quad \angle 5 = \angle 6$

$\Rightarrow \quad EN = EL$

[Sides opposite to equal angles are equal]

Now in ΔELM and ΔENM

$EL = EN$ (Proved above)

$EM = EM$ (Common)

$\angle 7 = \angle 8$

(Given as EM is the bisector of ∠E)

$\therefore \quad \Delta ELM \cong \Delta ENM$ (SAS congruence rule)

$\therefore \quad \angle EML = \angle EMN$ (CPCT)

But $\angle EML + \angle EMN = 180°$ (Linear pair)

$\Rightarrow \quad \angle EML = \angle EMN = 90°$

Hence, EM ⊥ FM.

Hence, bisectors of ∠E and ∠F intersect at right angle.

⚠ Marks Loss Alert

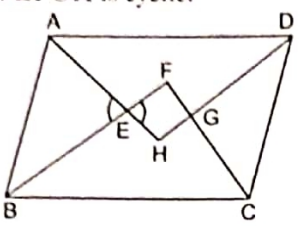
1. **Missing cyclic quadrilateral property:** Not using that the exterior angle of a cyclic quadrilateral equals the interior opposite angle to relate the required angles.
2. **Incorrect angle-bisector use:** Forgetting to use that FN and EM bisect ∠F and ∠E, which gives equal agles needed for triangle relations and congruence.
3. **Final conclusion error:** Not applying linear pair (180°) correctly to conclude each angle = 90° ⇒ Two bisectors are perpendicular.

79. Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic. [HOTS] [Understanding and Analysing]

Scanned with OKEN Scanner

Sol. **Given:** ABCD is a quadrilateral, AH, BF, CF and DH are the angle bisectors of internal angles A, B, C and D. These bisectors form a quadrilateral EFGH.

To Prove: EFGH is cyclic.



Proof: In ΔAEB,

$\angle EAB + \angle ABE + \angle AEB = 180°$

(Sum of angles of ΔABC)

$\Rightarrow \quad \angle AEB = 180° - (\angle EAB + \angle ABE)$ ...(*i*)

Also $\quad \angle AEB = \angle FEH$ ...(*ii*)

(Vertically opposite angles)

By equating (*i*) and (*ii*)

$\angle FEH = 180° - (\angle EAB + \angle ABE)$ ...(*iii*)

Similarly, in ΔGDC

$\angle FGH = 180° - (\angle GDC + \angle GCD)$ ...(*iv*)

Adding (*iii*) and (*iv*)

$\angle FEH + \angle FGH$

$= 360° - (\angle EAB + \angle ABE + \angle GDC + \angle GCD)$

$= 360° - \frac{1}{2}(\angle BAD + \angle ABC + \angle ADC + \angle BCD)$

(As AH, BF, CF and HD are bisectors of $\angle A$, $\angle B$, $\angle C$, $\angle D$)

$= 360° - \frac{1}{2} \times 360°$

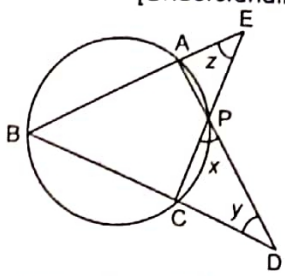
(Sum of angles of quadrilateral, ABCD)

$\angle FEH + \angle FGH = 360° - 180° = 180°$

$\Rightarrow$ FEHG is a cyclic quadrilateral.

(If the sum of opposite angles of quadrilateral is 180°, then it is cyclic)

80. **In the given figure, if $y = 32°$ and $z = 40°$, determine x. If $y + z = 90°$, prove that $x = 45°$.** [Understanding and Analysing]



Sol. **Given:** $y = 32°$ and $z = 40°$.

Proof: Let the line segments AD and CE cut each other at P.

Since, $\quad \angle APE = \angle CPD$

[Vertically opposite angles]

$\therefore \quad \angle APE = x$

Now, $\quad \angle BCP = \angle CDP + \angle CPD$

[Exterior angle]

and $\quad \angle PAB = \angle PEA + \angle APE$

[Exterior angle]

$\therefore \quad \angle BCP = x + y$ ...(*i*)

and $\quad \angle PAB = x + z$ ... (*ii*)

Since, ABCP is a cyclic quadrilateral

$\therefore \quad \angle BCP + \angle PAB = 180°$

$\Rightarrow \quad x + y + x + z = 180°$

or $\quad 2x + (y + z) = 180°$ ...(*iii*)

or $\quad 2x + (40° + 32°) = 180°$

or $\quad 2x = 180° - 72°$

$= 108°$ or $x = 54°$

Since, from (*iii*), we get

$2x + (y + z) = 180°$

and $\quad y + z = 90°$ [Given]

$\therefore \quad 2x + 90° = 180°$ or $2x = 90°$

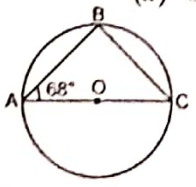
$\therefore \quad x = 45°$.

PRACTICE QUESTIONS

81. In the given figure, O is centre of the circle, $\angle BAO = 68°$, AC is diameter of the circle, then measure of $\angle BCO$ is [Understanding]

(*a*) 22° (*b*) 33°

(*c*) 44° (*d*) 68°



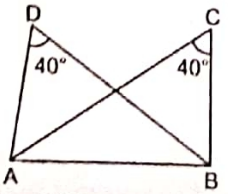
82. In the given figure, $\angle C = \angle D = 40°$, then the four points A, B, C, D [Understanding]

(*a*) concyclic

(*b*) are collinear

(*c*) A, B, D and A, B, C lie on the different circles.

(*d*) none of these



83. A chord of a circle is equal to the radius of the circle. Then the angle subtended by the chord at the point of major arc is [Understanding]

(*a*) 90° (*b*) 30°

(*c*) 150° (*d*) 60°

84. If four points lie on a circle, the quadrilateral formed is called: [Conceptual]

(*a*) Parallelogram

(*b*) Kite

(*c*) Cyclic quadrilateral

(*d*) Rectangle

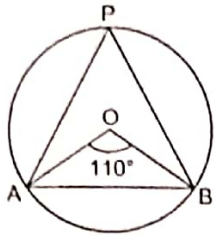
85. Which of the following is NOT circular in shape? [Conceptual]

(*a*) Coin (*b*) Wheel

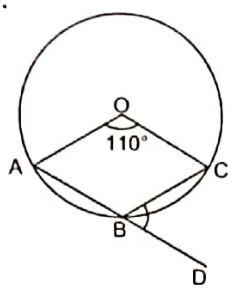
(*c*) Football field (*d*) CD

Scanned with OKEN Scanner

86. In the given figure, O is the centre of the circle and AB is a chord of the circle. If $\angle AOB = 110°$, find $\angle APB$. [Understanding]

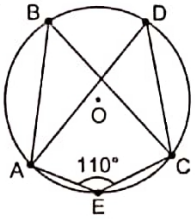


87. If O is the centre of the circle as shown in figure, find $\angle CBD$. [Understanding]

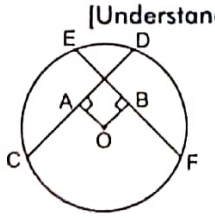


88. In the given figure, ABCE is a cyclic quadrilateral and O is the centre of circle. If $\angle AEC = 110°$, then find [Understanding and Application]

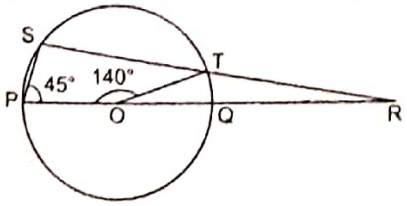
(*i*) $\angle ABC$ (*ii*) $\angle ADC$



89. In the given figure, OA and OB are respectively perpendiculars to chords CD and EF of a circle whose centre is O. If OA = OB, prove that $\overset{\frown}{EC} = \overset{\frown}{DF}$. [Understanding and Analyzing]

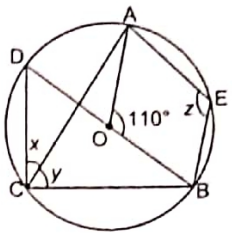


90. If O is centre of circle as shown in figure, find $\angle RQT$ and $\angle RTQ$ [Understanding]

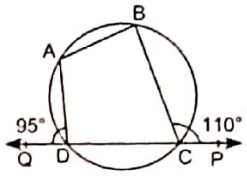


91. ABCD is a quadrilateral inscribed in a circle. CD is produced to any point F and the bisector of $\angle ABC$ intersects the circle at E. Prove that DE bisects $\angle ADF$. [Understanding and Analyzing]

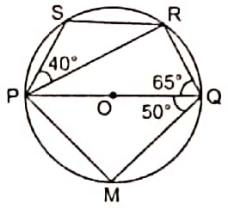
92. In the given figure, O is the centre of the circle and $\angle AOB = 110°$, find the value of x, y and z. [Understanding and Application]



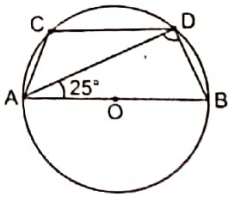
93. In the given figure, ABCD is a cyclic quadrilateral. Side CD is produced on both sides, such that $\angle BCP = 110°$ and $\angle ADQ = 95°$. Find the values of $\angle A$ and $\angle B$. [Understanding and Application]



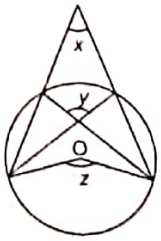
94. In the given figure, PQ is a diameter of a circle with centre O. If $\angle PQR = 65°$, $\angle SPR = 40°$, $\angle PQM = 50°$, find $\angle QPR$, $\angle PRS$ and $\angle QPM$. [Understanding and Analyzing]



95. In the given figure, AB is diameter of the circle with centre O and CD || AB. If $\angle DAB = 25°$, then find the measure of $\angle CAD$. [Understanding]

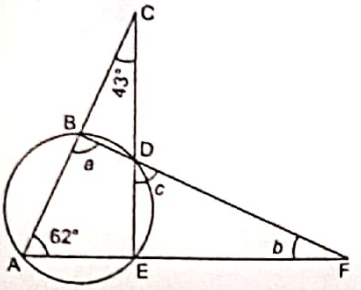


96. In the given figure, O is the centre of the circle. Prove that $\angle x + \angle y = \angle z$ [Understanding and Analyzing]

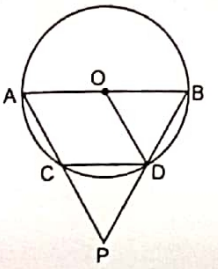


Scanned with OKEN Scanner

97. In the given figure, determine a, b and c. [Understanding and Application]



98. AB is a diameter of circle C(O, r). Chord CD is equal to radius OD. AC and BD produced intersect at P. Prove that $\angle APB = 60°$. [Understanding and Analyzing]



Integrated (Mixed) Questions

99. In a circle with centre O, chords AB and CD are of lengths 5 cm and 6 cm respectively and subtend angles $x°$ and $y°$ at the centre of the circle respectively. Then [Understanding]

Concept Applied: Angle subtended by chord at the centre

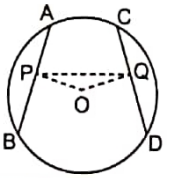
(a) $x = y$. (b) $x < y$
(c) $x > y$ (d) cannot say

100. Given a circle of radius 5 cm and centre O. OM is drawn perpendicular to the chord XY. If OM = 3 cm, then length of chord XY is [Understanding]

Concept Applied: Perpendicular drawn from centre to the chord bisect the chord

(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

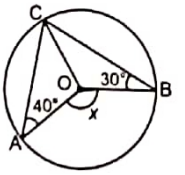
101. In the given figure, $\angle POQ = 150°$ and AB, CD are equal chords, OP $\perp$ AB and OQ $\perp$ CD. Then $\angle BPQ$ is [Understanding]



Concept Applied: Equal chords are equidistant from the centre and angle sum property

(a) 120° (b) 90° (c) 30° (d) 105°

102. In the given figure, O is the centre of the circle. The value of x is [Understanding]

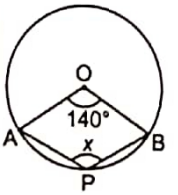


Concept Applied: Congruency of triangles and angle at the centre related property

(a) 140° (b) 70° (c) 290° (d) 210°

103. In the given figure, value of x is [Understanding]

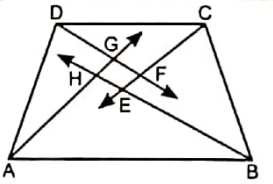
Concept Applied: Properties of Cyclic quadrilateral



(a) 140° (b) 70°
(c) 110° (d) 280°

104. In a quadrilateral ABCD, bisectors of angles A, B, C and D form another quadrilateral EFGH as shown [Understanding]

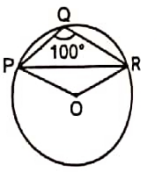
Concept Applied: Properties of Cyclic quadrilateral



If $\angle HEF = 70°$ and $\angle GHE = 80°$, then $\angle HGF$ and $\angle GFE$ are

(a) $\angle HGF = 80°$, $\angle GFE = 70°$
(b) $\angle HGF = 70°$, $\angle GFE = 80°$
(c) $\angle HGF = 110°$, $\angle GFE = 90°$
(d) $\angle HGF = 110°$, $\angle GFE = 100°$

105. In the given figure, the value of $\angle OPR$ is [Understanding]



Concept Applied: Properties of Cyclic quadrilateral

(a) 65° (b) 10° (c) 20° (d) 50°

106. Diagonals of a cyclic quadrilateral are the diameters of that circle, then quadrilateral is a [Conceptual]

Concept Applied: Properties of Cyclic quadrilateral

(a) parallelogram (b) square
(c) rectangle (d) trapezium

107. The radius of a circle is 10 cm and a chord of a circle is 12 cm in length. Find the distance of the chord from the centre of the circle. [Understanding]

Concept Applied: Perpendicular drawn from centre to the chord bisect the chord

108. Given a circle with centre O and the smallest chord AB = 3 cm, largest chord CD = 10 cm and chord PQ = 7 cm. Find the radius of the circle. [Understanding]

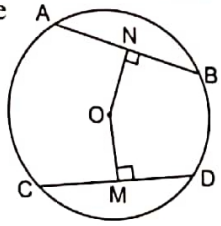
Concept Applied: Diameter is longest chord

Scanned with OKEN Scanner

109. Find the length of a chord which is at a distance of 4 cm from the centre of a circle whose radius is 5 cm. [Understanding]

Concept Applied: Perpendicular drawn from centre to the chord bisect the chord

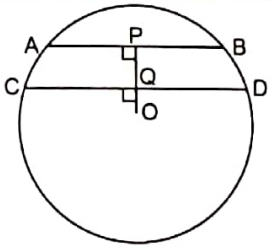
110. In the given figure, O in the centre of the circle. If OM < ON, show that CD > AB. [Understanding]



Concept Applied: Distance between two chords

111. In the given figure, O is the centre of the circle of radius 5 cm. OP ⊥ AB, OQ ⊥ CD, AB || CD, AB = 6 cm and CD = 8 cm. Determine PQ. [Understanding]

Concept Applied: Distance between two chords



112. Prove that the diameter of a circle that bisect a chord also bisects the angle subtended by the chord at the centre of the circle. [HOTS] [Understanding]

Concept Applied: Congruency of triangle

113. Prove that the circle drawn on any equal side of an isosceles triangle as diameter bisects the base. [Analyzing]

Concept Applied: Angle in semicircle and isosceles triangle properties.

114. In a cyclic quadrilateral PQRS, if ∠P − ∠R = 50°, find ∠P. [HOTS] [Understanding]

Concept Applied: Properties of Cyclic quadrilateral

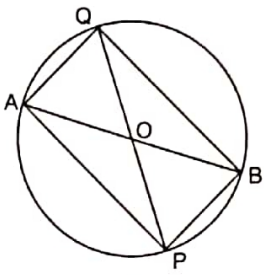
115. Draw ΔPQR with PQ = 6 cm, ∠P = 50° and ∠Q = 60°. Draw the circumcircle of ΔPQR. State whether the circumcentre lies inside or outside the triangle.

116. Draw ΔLMN with LM = 5.5 cm, ∠L = 80°, and LN = 4 cm. Construct the circumcircle and state the position of the circumcentre.

117. Draw ΔXYZ with XY = 7 cm; YZ = 8 cm, and ZX = 7 cm. Draw the circumcircle of ΔXYZ. Let the circumcentre be O. Measure OA, OB and OC.

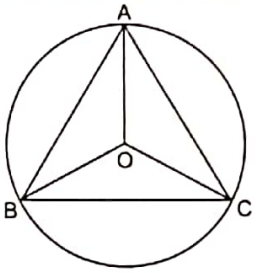
118. In the given figure, a diameter AB of a circle bisects a chord PQ and AQ || BP. Prove that chord PQ is also a diameter. What is the name given to quadrilateral AQBP.

[Analyzing] **Concept Applied:** Angles in semicircle, parallel lines and rectangle property.



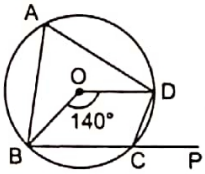
119. In the given figure, O is the centre of the circle, ∠AOB = ∠BOC = ∠COA and AO = $2\sqrt{3}$ cm. Find the perimeter of ΔABC. [Understanding and Application]

Concept Applied: Angle subtended by equal chord of circle and Pythagorous theorem



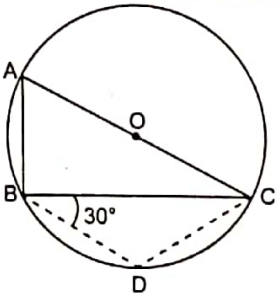
120. In the given figure, O is the centre of the circle. The angle subtended by arc BCD at the centre is 140°. BC is produced to P. Determine ∠BAD and ∠DCP. [Understanding]

Concept Applied: Angle at the centre is double that of the remaining part of circle and Properties of cyclic quadrilateral



121. In the given figure, BD = DC and ∠DBC = 30°. Find the measure of ∠BAC, if O is the centre of the circle. [Understanding]

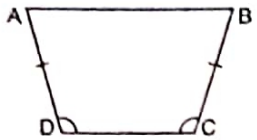
Concept Applied: Properties of Cyclic quadrilateral



122. In the given figure, ABCD is a quadrilateral in which AD = BC and ∠ADC = ∠BCD. Show that the points A, B, C and D lie on a circle. [Understanding and Analyzing]

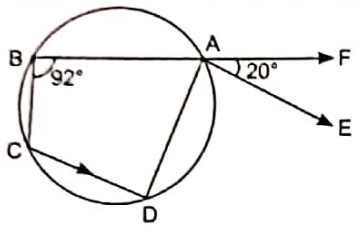
Concept Applied: Properties of Cyclic quadrilateral

Scanned with OKEN Scanner



123. In the given figure, ABCD is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced up to F. If $\angle ABC = 92°$, $\angle FAE = 20°$, find $\angle BCD$.
[Understanding]

Concept Applied: Properties of Cyclic quadrilateral

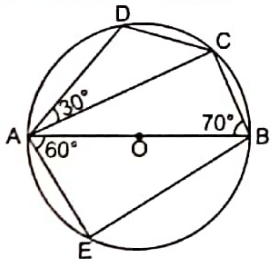


124. Two circles are drawn with sides AB and AC of a triangle ABC as diameters. The circles intersect at a point D. If AB = 5 cm, BD = 3 cm and AC = 6 cm, find BC.
[Understanding]

Concept Applied: Angles in semicircle and Pythagorous Theorem

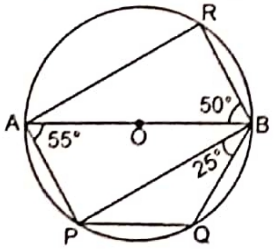
125. In the given figure, AB is a diameter of a circle with centre O. If $\angle ABC = 70°$, $\angle CAD = 30°$ and $\angle BAE = 60°$, find $\angle BAC$, $\angle ACD$ and $\angle ABE$.
[HOTS] [Understanding and Application]

Concept Applied: Properties of Cyclic quadrilateral and angle sum properties in a triangle



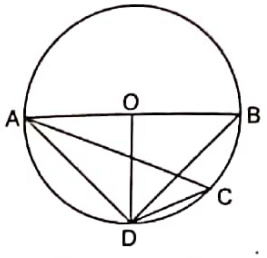
126. In the given figure, AB is a diameter of a circle with centre O. If $\angle PAB = 55°$, $\angle PBQ = 25°$ and $\angle ABR = 50°$, find $\angle PBA$, $\angle BPQ$ and $\angle BAR$.
[Understanding]

Concept Applied: Angles in a semicircle and angle sum property in a triangle



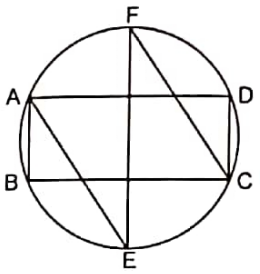
127. In the given figure, AB is a diameter of circle $C(O, r)$ and radius OD is perpendicular to AB. If C is any point on arc DB, find $\angle BAD$ and $\angle ACD$.
[Understanding and Application]

Concept Applied: Angle subtended by arc at centre is double the angle at other part of circle



128. The bisectors of the opposite angles A and C of a cyclic quadrilateral ABCD intersect the circle at the points E and F, respectively. Prove that EF is a diameter of the circle (See figure). [HOTS]
[Understanding and Analyzing]

Concept Applied: Properties of Cyclic quadrilateral

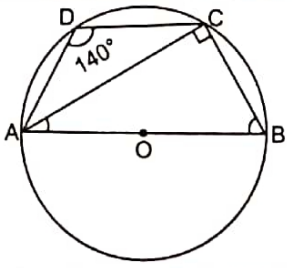


129. ABCD is a rectangle. The line through C perpendicular to AC meets AB produced at X and AD produced at Y. Prove that the points D, B, X, Y are concyclic.
[Understanding and Analyzing]

Concept Applied: Angle sum properties in a triangle

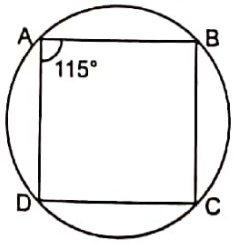
130. (*i*) In the given figure, ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B, C and D. If $\angle ADC = 140°$, find $\angle BAC$.
[Understanding and Analyzing]

Concept Applied: Properties of Cyclic quadrilateral



(*ii*) In the given figure, AB || DC. Determine $\angle DCB$ and $\angle ABC$.
[Understanding and Application]

Concept Applied: Properties of Cyclic quadrilateral



Scanned with OKEN Scanner

COMPETENCY FOCUSSED QUESTIONS

Assertion and Reason Questions [Reasoning]

COMMON INSTRUCTIONS

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

131. Assertion (A): There can be infinite numbers of equal chord's of a circle.
Reason (R): Circle has infinite points on it.

Sol. (a)

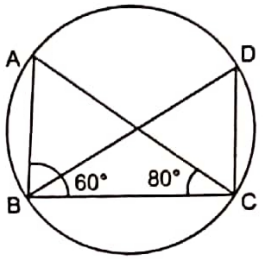
132. Assertion (A): The longest chord of a circle is a diameter of the circle.
Reason (R): Diameter of the circle always passes through the centre of a circle.

Sol. (a) because diameter is twice of the radius.

133. Assertion (A): The angle subtended by a diameter in a semicircle is 180°.
Reason (R): Angle in a semicircle is a right angle.

Sol. (b) because the angle subtended by an arc at the cube is double the angle subtended by it any point on the remaining part of the circle.

134. Assertion (A): In the given figure, $\angle ABC = 60°$ and $\angle ACB = 80°$. Then $\angle BDC = 40°$



Reason (R): Angles in the same segment of a circle are equal.

Sol. (a)

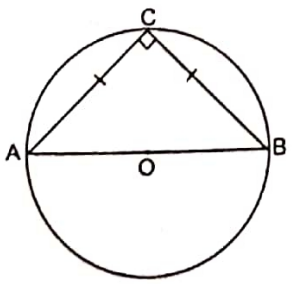
135. Assertion (A): Reactangle is a cyclic quadrilateral.
Reason (R): Diameter of the cyclic quadrilateral will not be the diameter of the circle.

Sol. (b) For a cyclic quadrilateral to be rectangle, the diagonal of the cyclic quadrilateral must be the diameter of the circle through the vertices of quadrilateral.

Case-Based Questions

136. Darts is a precision game that tests your accuracy and technique. The game involves throwing darts at a target board with a level of precision that will help you score better than your opponents. Three children were playing dart game to improve their concentration and threw dart on target board with radius 10 cm and centre O. Darts land on target board at positions A, B and C respectively. They drew lines to join AB, BC and CA and got a triangle shaped figure such that AC = BC.

[Understanding and Application]



Based on the above information answer the following questions:

(i) At what distance dart B is landed from the location of dart A?

(ii) What type of triangle is formed by all three darts on the target board?

(iii) (a) What is the measure of reflex $\angle AOB$?

OR

(iii) (b) State one property of the circle used to measure $\angle ACB$?

Sol. (i) Distance, AB = diameter of the circle
= 20 cm

(ii) Right angle isosceles triangle.

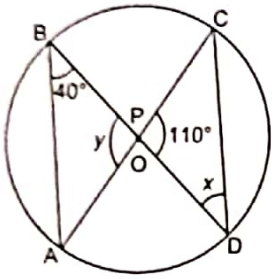
(iii) (a) Reflex $\angle AOB = 180°$

OR

(iii) (b) Angle in semicircle is a right angle.

Scanned with OKEN Scanner

137. Mr. Malhotra, Maths teacher entered in the class room and asked students whether they have revised the chapter circles. Students replied that they are ready for blackboard test. Mr. Malhotra drew a figure on blackboard and asked few questions. [Understanding and Application]



Based on the above information answer the following questions:

(*i*) What is point P called as?

(*ii*) What is the value of *y*?

(*iii*) (*a*) What is the measure of ∠C?

OR

(*iii*) (*b*) Why is ∠B = ∠C?

Sol. (*i*) Point of intersection

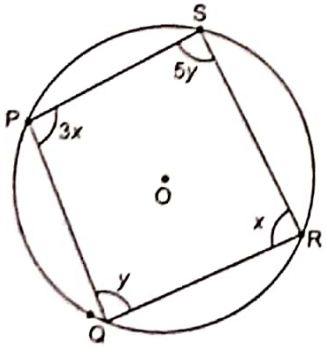
(*ii*) $y = 110°$ [vertically opposite angles]

(*iii*) (*a*) $\angle C = 40°$ [Angle on the same segments]

OR

(*iii*) (*b*) Because Angle on the same segment are equal.

138. In a residential colony, there was one circular ground. RWA members decided that there should be a skating area available in circular ground so that the children have opportunity to excel themselves in skating. For that purpose they chose four points P, Q, R and S and fenced the area as shown in figure. [Understanding and Application]



Based on the above information answer the following questions:

(*i*) What is the measure of ∠*x*?

(*ii*) What is the measure of ∠*y*?

(*iii*) (*a*) What is measure of ∠P?

OR

(*iii*) (*b*) What is measure of ∠S?

Sol. (*i*) $x + 3x = 180° \Rightarrow 4x = 180° \Rightarrow x = 45°$

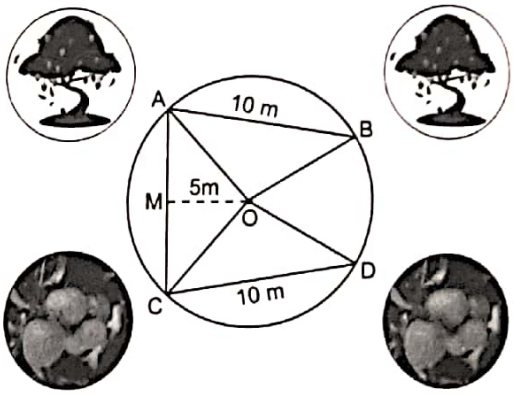
(*ii*) $y + 5y = 180° \Rightarrow 6y = 180° \Rightarrow y = 30°$

(*iii*) (*a*) $\angle P = 3x = 3 \times 45° = 135°$

OR

(*iii*) (*b*) $\angle S = 5y = 5 \times 30° = 150°$

139. A farmer has a circular garden as shown in the picture. He has different types of trees, plants and flower plants in his garden. In the garden, these are two mango tree A and B at a distance AB = 10 m. Similarly, the garden has two Litchi trees at the same distance of 10 m as shown at C and D. AB subtends ∠AOB = 80° at centre O. The perpendicular distance of AC from centre is 5 m and radius of garden is 13m. [DOE]
[Understanding and Application]



Based on the above information answer the following questions:

(*i*) What is the value of ∠COD?

(*ii*) What is the distance between mango tree A and Litchi tree C?

(*iii*) (*a*) If ∠BOD = 70° then show that ∠CAB = 75°?

OR

(*iii*) (*b*) What is the value of ∠OCD?

Sol. (*i*) ∠COD = 80° because equal chord subtend equal angle at the centre.

(*ii*) Given: OM = 5 m, AO = 13 m

By Pythagoras theorem

$$AO^2 = AM^2 + OM^2$$
$$13^2 = AM^2 + 5^2$$
$$AM = \sqrt{169 - 25}$$
$$= 12$$

So, $AC = 2\,AM = 2 \times 12$
$= 24$ m.

Scanned with OKEN Scanner

(*iii*) (*a*) In ΔAOB,

$\angle AOB = 80°$ (Given)

$OA = OB$

$= 13$ m (radius of circle)

$\Rightarrow \Delta AOB$ is an isosceles triangle

$\Rightarrow \angle OAB = \angle OBA = x$

[Angle opposite to equal side of triangle]

Now, $\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 2x + 80° = 180°$

$\Rightarrow 2x = 50°$ ...(1)

Again,

$\angle AOC + \angle AOB + \angle BOD + \angle COD = 360°$

$\Rightarrow \angle AOC + 80° + 70 + 80° = 360°$

$\Rightarrow \angle AOC = 130°$ ...(2)

Again, ΔAOC is an isosceles triangle because

$OA = OC = 13$ cm

(radius of circle)

$\therefore \angle OAC = \angle OCA = y$

(Angle opposite to equal sides)

So, $\angle OAC + \angle OCA + \angle AOC = 180°$

$\Rightarrow y + y + 130° = 180°$

$\Rightarrow y = 25°$ ...(3)

Therefore,

$\angle CAB = \angle OAB + \angle OAC$

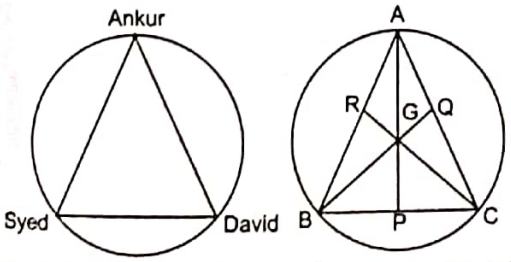
$= x + y$

$\Rightarrow \angle CAB = 50° + 25° = 75°$

OR

(*iii*) (*b*) In the same manner for obtaining eq. (1) in the part (*iii*) (*a*), we find, $\angle OCD = 50°$

140. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having toy telephone in hand to talk to each other as shown in the figure. AP, BQ and CR are the medians of triangle ABC. **[DOE] [Application]**



Based on the above information answer the following questions:

(*i*) What is the length of AG?

(*ii*) What is the length of AP?

(*iii*) (*a*) Find the measure of angle $\angle BGC$ and $\angle ABQ$.

OR

(*iii*) (*b*) Find length AB.

Sol. Tips: Use the following properties to solve the question

- The median intersect at a point which is the circumcentre of the circumscribed circle.
- Centroid and circumcentre of equilateral triangle coincides.
- Medians of triangle intersect each other in the ratio 2:1.
- Medians are perpendicular bisectors of the sides of triangle.
- Perpendicular drawn from the centre of a circle to the chord bisect the chord.

Given: Radius of circular park is 20 m

$\Rightarrow$ $AG = BG = CG = 20$m.

Also, all the three boy are sitting at equal distance from one-another.

$\therefore \Delta ABC$ is an equilateral triangle.

$\Rightarrow$ $AB = BC = CA$

(*i*) $AG = 20$m

(*ii*) Using Tip

$$\frac{AG}{GP} = \frac{2}{1}$$

$$\Rightarrow \frac{20}{GP} = \frac{2}{1} \Rightarrow GP = 10 \text{ m}$$

(*iii*) (*a*) ΔABC is an equilateral triangle

$\therefore \angle A = \angle B = \angle C = 60°$ (each)

median will be the angle bisector,

So, $\angle ABQ = \angle QBC = 30°$

In ΔPBG,

$\angle GBP + \angle BGP + \angle BPG = 180°$

$\Rightarrow \angle QBC + \angle BGP + 90° = 180°$

$\Rightarrow 30° + \angle BGP = 90°$

$\Rightarrow \angle BGP = 60°$

Similarly in ΔCGP, $\angle CGP = 60°$

$\therefore \angle BGC = \angle BGP + \angle CGP$

$= 60° + 60° = 120°$

OR

(*iii*) (*b*) In ΔBGP, GP is perpendicular bisector of BC.

$BG^2 = BP^2 + GP^2$

$\Rightarrow 20^2 = BP^2 + 10^2$

$\Rightarrow BP = \sqrt{300} = 10\sqrt{3}$

Hence, $BC = 2BP = 2 \times 10\sqrt{3}$

$= 20\sqrt{3}$ m

$\Rightarrow AB = BC = CA$

$= 20\sqrt{3}$ m

Scanned with OKEN Scanner

NCERT Zone

THINK AND REFLECT

Page 93

- **Jamuna has a circular piece of paper. She is trying to locate its centre. Amina gives her a suggestion. She follows the instructions and is thrilled to find that it works. Can you guess what Amina told her?**

Sol. Amina told Jamuna to fold the circular paper twice in different directions so that the boundaries overlap each time.

(*i*) Fold the circle so the boundary overlaps - the crease is a diameter (a line of reflection symmetry).

(*ii*) Unfold, then fold again in a different direction - this gives a second diameter.

(*iii*) The intersection of the two creases is the centre of the circle. This works because all diameters of a circle pass through the centre.

Page 94

1. **What are the rotational symmetries of a square? How many lines of reflection symmetry does it have? What about a regular pentagon? A regular hexagon?**

Sol. **Square**

A Square has:

Rotational symmetry: at 90°, 180°, 270°, and 360° of rotation.

Therefore, it has 4-fold rotational symmetry

Lines of reflection symmetry: 4

- 2 through opposite vertices
- 2 through midpoints of opposite sides

Regular Pentagon

A Regular Pentagon has:

5-fold rotational symmetry

Rotations: at 72°, 144°, 216°, 288°, 360° of rotation

Lines of reflection symmetry: 5

Regular Hexagon

A Regular Hexagon has:

6-fold rotational symmetry

Rotations: at 60°, 120°, 180°, 240°, 300°, 360° of rotation.

Lines of reflection symmetry: 6

- 3 through opposite vertices
- 3 through midpoints of opposite sides

2. **What is the length of the longest chord in a circle of radius 5 units? Is there a smallest chord?**

Sol. The longest chord = diameter = 2 × 5 units = 10 units. The diameter is always the longest chord (it passes through the centre, distance = 0).

There is no smallest chord. As a chord moves farther from the centre, it length gets shorter and approaches a single point (a tangent point). A chord length approaching closer and closer to 0 - but a true chord must have two distinct points, so there is no chord of length exactly zero; however, there is no positive minimum length either.

3. **The locus of points at a given distance from a given point is a circle. What can we say about the locus of points equidistant from two given points?**

(**Hint:** We know that any point that is equidistant from two given points A and B lies on the perpendicular bisector of AB. Does this make the perpendicular bisector the locus? For this, we have to show that all the points on the perpendicular bisector are equidistant from A and B.)

Sol. The locus of points equidistant from two given points A and B (say) is the perpendicular bisector of AB. In geometry this means every point on the perpendicular bisector is at the same distance from A and B.

If M is mid-point of AB, then the perpendicular bisector passes through M and makes an angle of 90° with AB. Hence MA = MB.

Verification

Any point P on the perpendicular bisector of AB satisfies PA = PB (by the properties of isosceles triangles SAS congruence: $\Delta PAM \cong \Delta PBM$ where M is the midpoint of AB). Conversely, if PA = PB, then P lies on the perpendicular bisector. Hence the locus is exactly the perpendicular bisector.

Page 95

1. **How many circles pass through two points on a plane?**

Sol. **Infinitely many.** The centre of any such circle must lie on the perpendicular bisector of the segment joining the two points. Every point on the perpendicular bisector serves as a valid centre, giving infinitely many circles.

2. **Are there circles of all possible radii passing through A and B? What is the radius of the smallest circle passing through A and B? What is the radius of the largest circle passing through A and B?**

Sol. Yes, there are circles of every radius $r \geq \frac{AB}{2}$ passing through A and B.

For smallest circle: Radius = $\frac{AB}{2}$, when the centre is at the midpoint of AB (AB is a diameter).

For largest circle: There is no largest circle, as the centre moves farther along the perpendicular bisector, the radius grows without bound.

3. **As you move away from segment AB along its perpendicular bisector, do the radii of the circles containing A and B increase or decrease?**

Sol. When the centre is at the midpoint of AB, radius = $\frac{AB}{2}$. As the centre moves farther up or down the bisector, the distance to A (and B) grows, so the radius increases.

Scanned with OKEN Scanner

4. **As you go along the perpendicular bisector, will the circle drawn from that point through A and B appear more curved or less curved?**

Sol. As we go along the perpendicular bisector, the distance increases, the circle drawn from that point though A and B becomes larger and its boundary appears flatter (less curve) near the chord AB.

5. **You are given two points A and B on a plane. How many squares can you draw on the same plane with A and B on the boundary?**

How many squares can you draw on the plane with A and B as the corners of the square?

Ans. **A and B anywhere on boundary:** Infinitely many squares (A and B can be on the same side, on adjacent sides, on opposite sides with different orientations, etc.).

A and B must be adjacent vertices of the square. Hence, Exactly 2 squares (one on each side of AB) are possible.

THINK, DRAW AND INFER

Page 98

1. **A, B and C are three collinear points. Can you find a point P such that PA = PB = PC? What can you say about the perpendicular bisectors of AB and BC? Draw and check. Can you show that for three collinear points A, B and C, the perpendicular bisector of AB and BC are parallel? Is it possible for a circle to pass through collinear points? Can you draw a line that cuts a given circle in three distinct points?**

Sol. **Part 1: Finding a point P such that PA = PB = PC**

For a point P to be equidistant from two points, say A and B, it must lie on the perpendicular bisector of the line segment AB.

- If PA = PB, then P must lie on the perpendicular bisector of AB.
- If PB = PC, then P must lie on the perpendicular bisector of BC.

For PA = PB = PC, point P must be the intersection of the perpendicular bisector of AB and the perpendicular bisector of BC.

Since A, B, and C are collinear, they lie on the same straight line. The line segments AB and BC are parts of this line.

The perpendicular bisector of AB is a line perpendicular to the line containing A, B, and C, passing through the midpoint of AB.

The perpendicular bisector of BC is a line perpendicular to the line containing A, B, and C, passing through the midpoint of BC.

Since both perpendicular bisectors are perpendicular to the same line (the line containing A, B, and C), they must be parallel to each other.

- If B is between A and C, the midpoint of AB and the midpoint of BC are distinct points. In this case, the two parallel perpendicular bisectors will never intersect. Therefore, there is no such point P that is equidistant from A, B, and C when A, B, and C are distinct collinear points.
- If A, B, and C are not distinct (e.g., A = B or B = C), the situation changes, but the problem implies distinct points.

Conclusion: For three distinct collinear points A, B, and C, it is not possible to find a point P such that PA = PB = PC.

Part 2: Properties of perpendicular bisectors of AB and BC

As explained above, if A, B, and C are distinct collinear points:

- The perpendicular bisector of AB is perpendicular to the line containing A, B, and C.
- The perpendicular bisector of BC is perpendicular to the line containing A, B, and C.

Therefore, the perpendicular bisectors of AB and BC are parallel to each other.

Part 3: Can a circle pass through collinear points?

A circle is defined as the set of all points in a plane that are equidistant from a fixed point (the center).

- **If the points are distinct and collinear:** If three distinct points lie on a straight line, it is impossible for a single circle to pass through all of them. This is because any two distinct points define a unique line, and a circle can intersect a line at a maximum of two points. If a circle passed through three distinct collinear points, it would intersect the line containing them at three distinct points, which is impossible.

Conclusion: No, it is not possible for a circle to pass through three distinct collinear points.

Part 4: Can a line cut a given circle in three distinct points?

A line is a straight path. A circle is a closed curve. When a line intersects a circle, it can do so in at most two distinct points.

- If the line does not touch the circle, there are zero intersection points.
- If the line touches the circle at exactly one point, it is a tangent line (one intersection point).
- If the line passes through the interior of the circle, it intersects the circle at two distinct points.

It is geometrically impossible for a straight line to intersect a circle at three distinct points.

Conclusion: No, we cannot draw a line that cuts a given circle in three distinct points.

Scanned with OKEN Scanner

2. The circumcircle of a given ΔABC is drawn. Can there be other triangles congruent to ΔABC that share the same circumcircle?

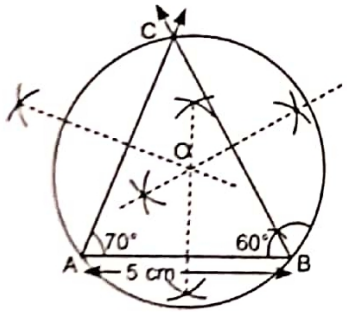
Sol. Yes. Given a circumcircle with a fixed radius, we can inscribe many triangles congruent to ΔABC in it. Specifically, by rotating ΔABC about the centre of the circumcircle by any angle, the rotated triangle is congruent to ΔABC and has the same circumcircle (since rotation preserves shape and size, and each vertex stays on the circle). So there are infinitely many such triangles.

EXERCISE 5.1

1. Draw ΔABC with AB = 5 cm, ∠A = 70° and ∠B = 60°. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

Sol. Steps of Construction:

1. Draw the line segment AB = 5 cm.
2. At point A, draw an angle of 70°.
3. At point B, draw an angle 60°.
4. Let the two rays meet at C. Then ΔABC is formed.
5. Draw the perpendicular bisector of AB.
6. Draw the perpendicular bisectors of BC and AC.
7. Let the three perpendicular bisectors meet at O. This point O is the circumcentre.
8. With centre O and radius OA, draw a circle passing through A, B and C. This is the circumcircle of ΔABC.



Now:

$\angle C = 180° - (70° + 60°) = 50°$

Since all the angles of the triangle are less than 90°, the triangle is acute-angled.

In an acute-angled triangle, the circumcentre lies inside the triangle.

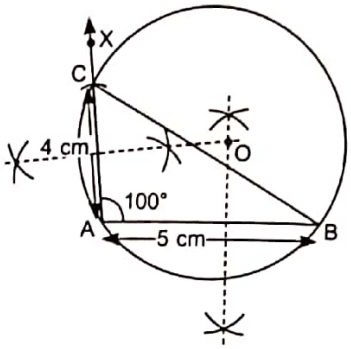
Therefore, the centre is inside the triangle.

2. Draw ΔABC with AB = 5 cm, ∠A = 100°, AC = 4 cm. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

Sol. Steps of Construction:

1. Draw the line segment AB = 5 cm.
2. At point A, draw an angle of 100°.
3. On the ray AX, mark point C such that AC = 4 cm.
4. Join C to B. Then ΔABC is formed.
5. Draw the perpendicular bisector of AB.
6. Draw the perpendicular bisector of AC and BC.
7. Let these bisectors meet at O. This point O is the circumcentre.
8. With centre O and radius OA, draw the circle through A, B and C.

This is the circumcircle.



Reasoning:

Since ∠A = 100°, the triangle is obtuse-angled.

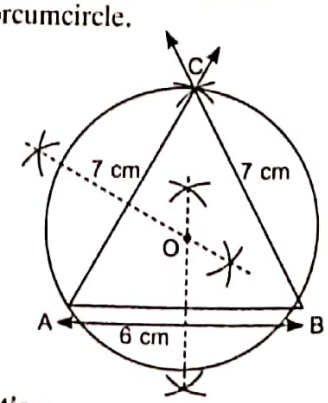
In an obtuse-angled triangle, the circumcentre lies outside the triangle.

Therefore, the centre is outside the triangle.

3. Draw ΔABC, with AB = 6 cm, BC = 7 cm and CA = 7 cm. Draw the circumcircle of ΔABC. Let the circumcentre be O. Measure OA, OB, OC.

Sol. Steps of Construction:

1. Draw AB = 6 cm.
2. With centre A and radius 7 cm, draw an arc.
3. With centre B and radius 7 cm, draw another arc cutting the first arc at C.
4. Join AC and BC. Then ΔABC is formed.
5. Draw the perpendicular bisector of AB.
6. Draw the perpendicular bisectors of AC and BC.
7. Let they intersect at O. This point O is the circumcentre.
8. With centre O and radius OA, draw the circumcircle.



Observation:

Since O is the circumcentre, it is equidistant from all three vertices.

Scanned with OKEN Scanner

Therefore, OA = OB = OC
For the triangle with sides 7 cm, 7 cm, 6 cm,
OA = OB = OC ≈ 4 cm
So, the three measurements are equal, each approximately 4 cm.

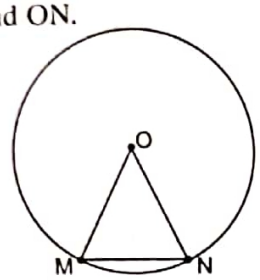
4. What is the least possible radius of a circle through two points A and B?

Sol. For all circles passing through two fixed points A and B, the centre lies on the perpendicular bisector of AB.
The smallest circle occurs when the centre is the midpoint of AB.
In that case, AB becomes the diameter of the circle.
Therefore, the least possible radius is AB/2
Hence, the least possible radius of a circle through A and B is half of AB.

EXERCISE 5.2

1. Show that the triangle formed by a chord and the centre of the circle is isosceles.

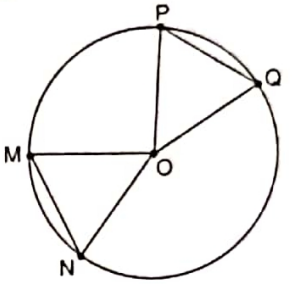
Sol. Let MN be a chord of a circle with centre O.
Join OM and ON.



In triangle OMN
OM = ON [Radii of the same circle]
Therefore, triangle OMN is an isosceles triangle.
Hence proved.

2. Show that if two such isosceles triangles (occurring in the previous question) have equal base length, they are congruent to each other.

Sol. Let MN and PQ be two equal chords of the same circle with centre O. Join OM, ON, OP and OQ.
Then triangles formed are ΔOMN and ΔOPQ.

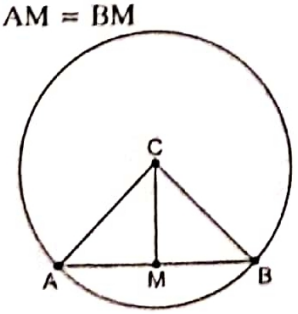


Now, in ΔOMN and ΔOPQ
OM = OP [Radii of the same circle]
ON = OQ [Radii of the same circle]
MN = PQ [Given]
So, by SSS congruence criterion
ΔOMN ≅ ΔOPQ
Hence, if two such isosceles triangles have equal base length, they are congruent to each other.
Hence proved.

EXERCISE 5.3

1. Can you explain why the converse to Theorem 4 is true, i.e., why does the perpendicular from the centre of a circle to a chord of the circle bisect the chord? (Hint: Use Fig. 5.12. You are told that ∠CMA = ∠CMB = 90°. You need to show that AM = BM.)

Sol. **Converse of Theorem 4:** The perpendicular from the centre of a circle to a chord of the circle bisects the chord.
Let AB be a chord of a circle with centre C, and let CM ⊥ AB.
To Prove: AM = BM



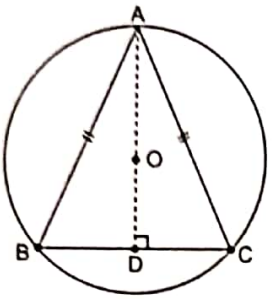
In ΔCMA and ΔCMB,
CA = CB [Radii of the same circle]
CM = CM [Common to both triangles]
∠CMA = ∠CMB [Each 90°, as CM ⊥ AB]
Therefore, by RHS congruence
ΔCMA ≅ ΔCMB
Hence, AM = BM [CPCT]
So, M is the midpoint of AB.
Therefore, the perpendicular from the centre of a circle to a chord bisects the chord.
Hence proved.

2. An isosceles triangle ABC is inscribed in a circle, with AB = AC. Show that the altitude from A to BC passes through the centre of the circle.

Sol. **Given:** ΔABC is inscribed in a circle with AB = AC.
Since, triangle ABC is isosceles with base BC and AB = AC.
Let AD be the altitude from A to BC.

Scanned with OKEN Scanner

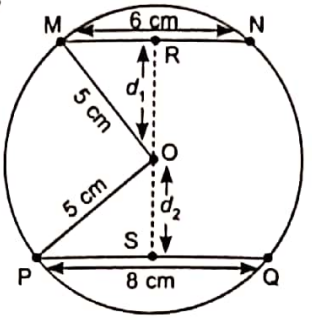
Then AD ⊥ BC



In ΔABD and ΔACD,
AB = AC [Given]
AD = AD [Common to both triangles]
∠ADB = ∠ADC [Each 90°, as AD ⊥ BC]
Therefore, by RHS congruence
ΔABD ≅ ΔACD
Hence, BD = DC [CPCT]
So, AD is bisector of BC.
Since AD is perpendicular to BC and also bisects BC.
Hence, AD is the perpendicular bisector of chord BC.
Therefore, AD passes through the centre of the circle.
Hence, the altitude from A to BC passes through the centre of the circle.
Hence proved.

3. Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre of a circle. If the radius of the circle is 5 cm, find the distance between the midpoints of the chords.

Sol. **Given:** Radius of the circle OM = OP = 5 cm
Lengths of the two chords are MN = 6 cm and PQ = 8 cm
The perpendicular from the centre O to a chords MN and PQ bisects the chords at R and S respectively.
So, half-lengths of chords are:
For the 6 cm chord: MR = 3 cm
For the 8 cm chord: PS = 4 cm
Let the distances of the chords from the centre be d_1 and d_2.



In triangle OMR, using Pythagoras theorem:
$OR^2 + MR^2 = OM^2$
$\Rightarrow d_1^2 + 3^2 = 5^2$
$\Rightarrow d_1^2 + 9 = 25$
$\Rightarrow d_1^2 = 16$
$\Rightarrow d_1 = 4$ cm
Similarly, in triangle OPS, using Pythagoras theorem:
$OS^2 + PS^2 = OP^2$
$\Rightarrow d_2^2 + 4^2 = 5^2$
$\Rightarrow d_2^2 + 16 = 25$
$\Rightarrow d_2^2 = 9$
$\Rightarrow d_2 = 3$ cm
Since the chords are on opposite sides of the centre and their centers lies on same line.
So, the distance between their midpoints = $d_1 + d_2$
= 4 + 3 = 7 cm
Therefore, the distance between the midpoints of the two chords is 7cm.

EXERCISE 5.4

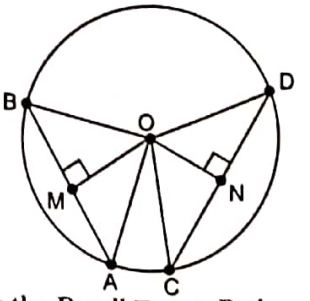
1. Use the Baudhāyana–Pythagoras theorem to show why Theorem 6 must be true.

Sol. **Theorem 6:** Chords of a circle having the same length are at the same distance from the centre.*
Let AB and CD be two equal chords of a circle with centre O.
Let OM perpendicular AB and ON perpendicular CD.
We know that the perpendicular from the centre to a chord bisects the chord.

So, $AM = MB = \frac{AB}{2}$

and $CN = ND = \frac{CD}{2}$

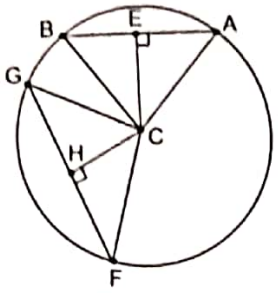
Since AB = CD, so, AM = CN
Also, OA = OC = radii of the circle.



Now apply the Baudhāyana–Pythagoras theorem in right triangles OMA and ONC:
In ΔOMA : $OA^2 = OM^2 + AM^2$
In ΔONC: $OC^2 = ON^2 + CN^2$
Since OA = OC and AM = CN
So, we have: $OM^2 + AM^2 = ON^2 + AM^2$
Therefore, $OM^2 = ON^2$
$\Rightarrow$ OM = ON
Hence, the equal chords AB and CD are equidistant from the centre.
Thus, Theorem 6 is proved.

Scanned with OKEN Scanner

2. Consider Fig. 5.15. If CE is perpendicular to AB, CH is perpendicular to GH, and CE = CH, show that AB = GF.



Sol. Given: CE perpendicular to AB, CH perpendicular to GF and CE = CH.
To Prove: AB = GF
Proof: Since CE is perpendicular to chord AB, the perpendicular from the centre to a chord bisects the chord.
Therefore, AE = EB
Similarly, since CH is perpendicular to chord GF.
So, GH = HF
In ΔCEA and ΔCHG,

	CA = CG	[Radii of the same circle]
	CE = CH	[Given]
	∠CEA = ∠CHG	[Each 90°]
	ΔCEA ≅ ΔCHG	[By RHS congruence]
So,	AE = GH	[CPCT]
⇒	2AE = 2GH	
⇒	AB = GF [∵ AB = 2AE and GF = 2 GH]	

Hence proved.

3. Solve the previous question using the Baudhāyana–Pythagoras theorem.

Sol. Given: CE perpendicular to AB, CH perpendicular to GF and CE = CH.
To Prove: AB = GF
Proof: Since CE is perpendicular to chord AB, it bisects AB.
So, E be the midpoint of AB.

$$\Rightarrow \quad AE = \frac{AB}{2}$$

Similarly, since CH is perpendicular to chord GF, it bisects GF.
So, H be the midpoint of GF.

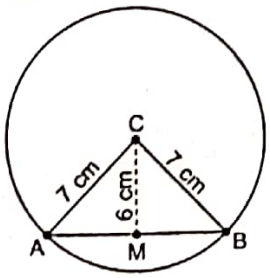
$$\Rightarrow \quad GH = \frac{GF}{2}$$

Now apply the Baudhāyana–Pythagoras theorem in right triangles ΔCEA and ΔCHG.
In ΔCEA: $CA^2 = CE^2 + AE^2$
In ΔCHG: $CG^2 = CH^2 + GH^2$
But, CA = CG [Radii of same circle]
Therefore, $CE^2 + AE^2 = CH^2 + GH^2$
$\Rightarrow \quad AE^2 = GH^2$ [∵ CE = CH Given]
Hence, AE = GH
So, $\frac{AB}{2} = \frac{GF}{2}$
Therefore, AB = GF
Hence proved.

EXERCISE 5.5

1. Find the length of the chord of a circle where the radius is 7 cm and perpendicular distance is 6 cm.

Sol. Let the chord be AB and C be the centre of the circle.
Let CM be the perpendicular from C to chord AB.
Given: Radius, CA = CB = 7
Perpendicular distance, CM = 6
We know that the perpendicular from the centre to a chord bisects the chord.
So, AM = MB



In right triangle CMA,

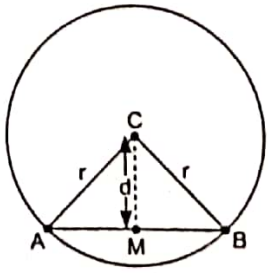
$$CA^2 = CM^2 + AM^2$$
$$\Rightarrow \quad 7^2 = 6^2 + AM^2$$
$$\Rightarrow \quad 49 = 36 + AM^2$$
$$\Rightarrow \quad AM^2 = 13$$
$$\Rightarrow \quad AM = \sqrt{13}$$

Therefore, chord AB = 2 × AM = $2\sqrt{13}$ cm

2. Explain why the following statement is true: If the perpendicular distance of a chord from the centre is d and the radius is r, then the chord length is $2\sqrt{r^2 - d^2}$.

Sol. Let AB be a chord of a circle with centre C.
Let CM be the perpendicular from C to AB.
Perpendicular distance from centre to chord = d,
Radius of circle = r
Since the perpendicular from the centre to a chord bisects the chord, so M is the midpoint of AB.

$$\text{So, } AM = MB = \frac{AB}{2}$$



Scanned with OKEN Scanner

In right triangle CMA, By the Baudhāyana–Pythagoras theorem:

$CA^2 = CM^2 + AM^2$

$\Rightarrow \quad r^2 = d^2 + AM^2$

$\Rightarrow \quad AM^2 = r^2 - d^2$

$\Rightarrow \quad AM = \sqrt{r^2 - d^2}$

But chord length AB = 2 × AM

So, $AB = 2\sqrt{r^2 - d^2}$

Hence, the chord length is $2\sqrt{r^2 - d^2}$.

*** 3. In a circle, if the distance of chord AB from the centre is twice the distance of another chord CD from the centre, then can we conclude that CD = 2 AB? Give reasons for your answer.** [Not for Assessment]

Sol. No, we cannot conclude that CD = 2AB.

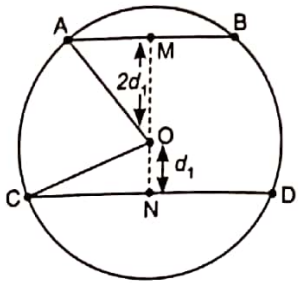
Reason: The length of a chord depends on the formula:

Chord length = $2\sqrt{r^2 - d^2}$

This relation is not directly proportional to the distance from the centre.

Let the distance of chord CD from the centre be d_1.

And the distance of chord AB from the centre is $2d_1$.



Now, $AB = 2\sqrt{r^2 - (2d_1)^2}$

$= 2\sqrt{r^2 - 4d_1^2}$

$CD = 2\sqrt{r^2 - d_1^2}$

Clearly, CD is not equal to 2AB in general.

EXERCISE [Page 107]

- **A circle with centre O is drawn, and A, B, C, D are points on the circle (see Fig. 5.19). Measure the angles subtended by arc AKB and arc CLD at the centre O. If the angle at the centre is less than 180°, it is a minor arc. If the angle at the centre is greater than 180°, it is a major arc. State whether arcs AKB and CLD are minor arcs or major arcs.**

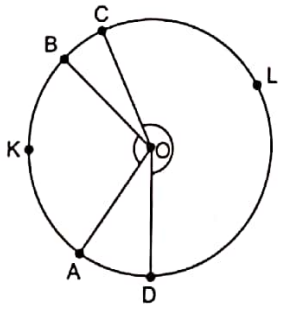


Fig. 5.19

Sol. An arc AKB is a minor arc as the central angle subtends is less than 180°, and arc CLD a major arc as the central angle is greater than 180°.

EXERCISE 5.6

1. In a circle with centre O, the central angle AOB is 60°. If the radius of the circle is 12 cm, what is the length of the chord AB?

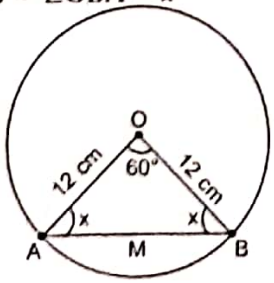
Sol. Given: Radius OA = OB = 12 cm

$\angle AOB = 60°$

In, Δ AOB: OA = OB [Radii of a circle]

So, $\angle OAB = \angle OBA$ [Angles opposite to equal sides of triangle]

Let $\angle OAB = \angle OBA = x$



In Δ, OAB,

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Angles sum property of the triangle]

$\Rightarrow 60° + x + x = 180°$ [Since $\angle AOB = 60°$]

$\Rightarrow 2x = 180° - 60° = 120°$

$\Rightarrow x = \frac{120°}{2} = 60°$

$\Rightarrow \angle OAB = \angle OBA = 60°$

Thus, Δ AOB is an equilateral triangle.

Therefore, AB = OA = 12 cm

Hence, the length of chord AB is 12 cm.

2. Let A and B be two points on circle with centre O.

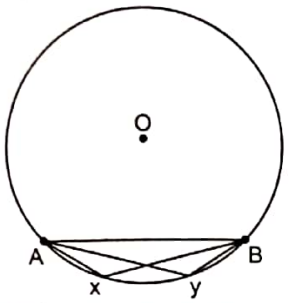
(*i*) Are there points X, Y on the circle, on the same side of AB, such that $\angle AXB$ is different from $\angle AYB$?

(*ii*) Is it true that if $\angle AXB = \angle AYB$, then X and Y lie on the same side of the circle?

Scanned with OKEN Scanner

(iii) **If $\angle AXB = \angle AYB$, and X and Y do not lie on the circle, does the circle through A, B and X also pass through Y?**

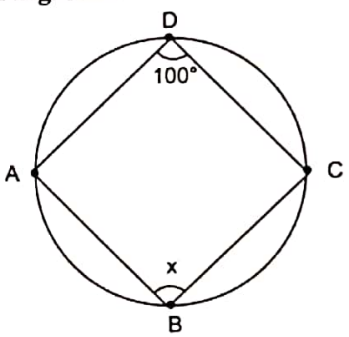
Sol. (i) No. Reason: If X and Y lie on the same side of chord AB, then they lie on the same arc AB. Angles subtended by the same chord in the same segment of a circle are equal. Therefore, $\angle AXB = \angle AYB$. So, there are no such points X and Y on the same side of AB for which the angles are different.



(ii) No, this is not always true.
Reason: Equal angles can also be subtended by the same chord AB at points on opposite arcs.
Let AB be the diameter of the circle. Let point X lies on one of the semicircle, so $\angle AXY = 90°$.
If point Y lies on the opposite semicircle, $\angle AYB = 90°$.
So, even if $\angle AXB = \angle AYB$, then X and Y need not lie on the same side of AB.
Therefore, the statement is false.

(iii) NO, Reason: Let us call the circle C_1 (centred at O). Point A and B lies on the circle. Points X and Y lies somewhere else in the plane (not on C_1) but $\angle AXB = \angle AYB$.
Any three non-collinear points determine a unique circle. Let us call this unique circle C_2, that pass through A, B and X.
For Y to lie on C_2, it must satisfy specific geometric conditions relative to X.
Case I: If X and Y are on the same side of AB, the statement is true.
By the converse of inscribed angle theorem, if two angles subtended by the same line segment (AB) at two different points (X and Y) on the same side of the line segment are equal, then the four points A, B, X and Y are concyclic, in this scenario, the circle C_2 pass through A, B and X will absolutely pass through Y.
Case II: If X and Y lie on the opposite side of AB.
Let the point X anywhere and the point Y be the reflecting point (mirror image) X across the line AB.
As reflection preserves distances and angles.
$\Delta AXB \cong AYB, \Rightarrow \angle AXB = \angle AYB$ (by CPCT).
However, because Y is the mirror image on the opposite side of the chord, it will not lie on the circle C_2 unless $\angle AXB = 90°$. If $\angle AXB \neq 90°$, the circle passing through A, B and X will away from Y, leaving completely outside (or inside) of it.

3. Find x in Fig. 5.26.



Sol. In the figure, A, D, C and B are points on the same circle.
So, quadrilateral ADCB is a cyclic quadrilateral.
$\angle ADC = 100°$ [Given]
In a cyclic quadrilateral, opposite angles are supplementary.
Therefore, $\angle ABC + \angle ADC = 180°$
$\Rightarrow x + 100° = 180°$
$\Rightarrow x = 80°$

EXERCISE [Page 113]

- **A cyclic quadrilateral has angles measuring $\angle A = 80°$, $\angle B = 110°$, $\angle C = 100°$, and $\angle D = 70°$. Can such a quadrilateral be drawn? Explain why or why not.**

Sol. Check opposite angle sums:

$\angle A + \angle C = 80° + 100° = 180°$

$\angle B + \angle D = 110° + 70° = 180°$

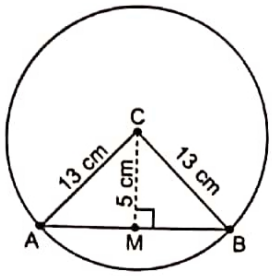
Both pairs of opposite angles sum to 180°. By Theorem 12, such a quadrilateral can be drawn and it is indeed a cyclic quadrilateral.

Scanned with OKEN Scanner

END-OF-CHAPTER EXERCISES

1. In a circle, a chord is 5 cm away from the centre. If the radius of the circle is 13 cm, what is the length of the chord?

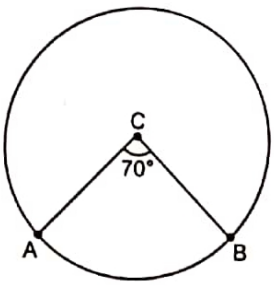
Sol. Let AB be the chord and C be the centre of the circle.
Let CM be the perpendicular from C to AB.
CM = 5 cm
CA = 13 cm [Given]
Since the perpendicular from the centre to a chord bisects the chord, so, AM = MB.



In right angle triangle CMA,
$CA^2 = CM^2 + AM^2$
$\Rightarrow 13^2 = 5^2 + AM^2$
$\Rightarrow 169 = 25 + AM^2$
$\Rightarrow AM^2 = 144$
$\Rightarrow AM = 12$ cm
Therefore, AB = 2 × AM = 2 × 12 cm = 24 cm
Hence, the length of the chord is 24 cm.

2. An arc of a circle subtends an angle of 70° at the centre. What is the measure of the angle subtended by the arc at a point on the circle?

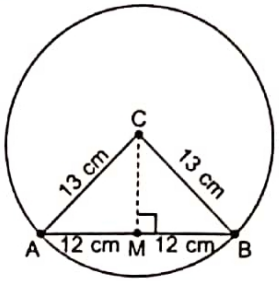
Sol.



We know:
Angle subtended by an arc at the centre
= 2 × angle subtended by the same arc at a point on the circle
Angle at the centre = 70°
So, angle at a point on the circle
$= \frac{70°}{2} = 35°$
Hence, the required angle is 35°.

3. The diameter of a circle is 26 cm. A chord of length 24 cm is drawn in the circle. Find the distance from the centre of the circle to the chord.

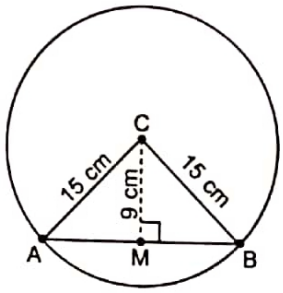
Sol. Diameter = 26 cm $\Rightarrow$ radius = $\frac{26}{2}$ cm = 13 cm
Chord AB length = 24 cm
$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ × 24 = 12 cm
Let CM be the perpendicular distance from the centre C to the chord AB.
Then M is the midpoint of the chord.
Half of chord AM = 12 cm



In right angle triangle CMA,
$CA^2 = CM^2 + AM^2$
$\Rightarrow 13^2 = CM^2 + 12^2$
$\Rightarrow 169 = CM^2 + 144$
$\Rightarrow CM^2 = 25$
$\Rightarrow CM = 5$ cm
Hence, the distance from the centre to the chord is 5 cm.

4. A circle has a radius of 15 cm. A chord is drawn. The distance from the centre of the circle to the chord is 9 cm. What is the length of the chord?

Sol. Let AB be the chord and C be the centre of the circle.
Let CM be the perpendicular from C to AB.
Since the perpendicular from the centre to a chord bisects the chord, AM = MB



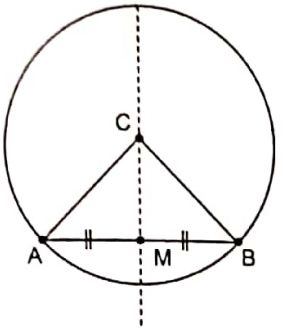
In right angle triangle CMA,
$CA^2 = CM^2 + AM^2$
$\Rightarrow 15^2 = 9^2 + AM^2$
$\Rightarrow 225 = 81 + AM^2$
$\Rightarrow AM^2 = 144$
$\Rightarrow AM = 12$ cm
Therefore, AB = 2 × AM = 24 cm
Hence, the length of the chord is 24 cm.

Scanned with OKEN Scanner

5. Prove that the perpendicular bisector of a chord passes through the centre of the circle.

Sol. Let AB be a chord of a circle with centre C and M be the midpoint of AB.

We have to show that CM is perpendicular to AB, which means the perpendicular bisector of AB passes through C.



So, in triangles CMA and CMB:

CA = CB [Radii of the same circle]

AM = MB [M is the midpoint of AB]

CM = CM [Common]

Therefore, by SSS congruence

$\Delta CMA \cong \Delta CMB$

Hence, $\angle CMA = \angle CMB$ [By CPCT]

But these two angles form a linear pair, so $\angle CMA + \angle CMB = 180°$

Since they are equal, $\angle CMA = \angle CMB = 90°$

Therefore, CM is perpendicular to AB

So, the line through C and M is the perpendicular bisector of chord AB. Hence proved.

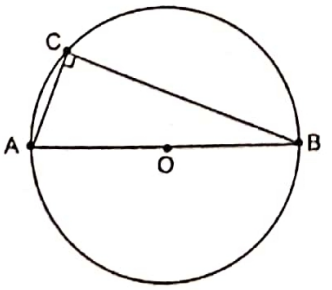
6. The diameter of a circle is AB. Point C is on the circumference. What is the measure of the $\angle ACB$? Explain your reasoning.

Sol. AB is a diameter.

The angle subtended by a diameter at any point on the circle is 90°.

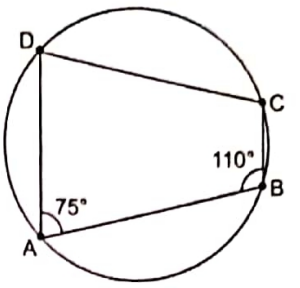
Therefore, $\angle ACB = 90°$

Hence, the measure of $\angle ACB$ is 90°.



7. ABCD is a cyclic quadrilateral inscribed in a circle.
If $\angle A$ measures 75°, what is the measure of $\angle C$?
If $\angle B$ measures 110°, what is the measure of $\angle D$?

Sol. In a cyclic quadrilateral, opposite angles are supplementary.



So, $\angle A + \angle C = 180°$

$\Rightarrow 75° + \angle C = 180°$

$\Rightarrow \angle C = 105°$

Also, $\angle B + \angle D = 180°$

$\Rightarrow 110° + \angle D = 180°$

$\Rightarrow \angle D = 70°$

Hence, $\angle C = 105°$ and $\angle D = 70°$.

8. Quadrilateral PQRS is inscribed in a circle. If $\angle P = (2x + 10)°$ and $\angle R = (3x - 20)°$, find the value of x and the measures of $\angle P$ and $\angle R$.

Sol. Since PQRS is a cyclic quadrilateral, opposite angles are supplementary.

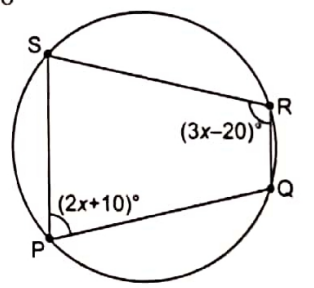
So, $\angle P + \angle R = 180°$

$\Rightarrow (2x + 10)° + (3x - 20)° = 180°$

$\Rightarrow 5x - 10° = 180°$

$\Rightarrow 5x = 190°$

$\Rightarrow x = 38°$



Now, $\angle P = 2x + 10° = 2(38°) + 10° = 86°$

$\angle R = 3x - 20° = 3(38°) - 20° = 94°$

Hence, $x = 38°$, $\angle P = 86°$ and $\angle R = 94°$.

9. The distance of a chord of length 16 cm from the centre of a circle is 6 cm. Find the radius of the circle.

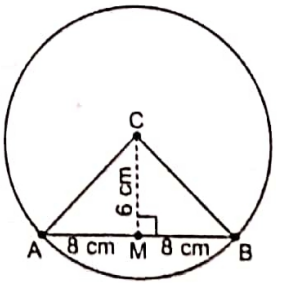
Sol. Chord length AB = 16 cm

Half chord $= \frac{1}{2}AB = \frac{1}{2} \times 16$ cm

$\Rightarrow AM = MB = 8$ cm

Distance from centre to chord CM = 6 cm

Let r be the radius.



Scanned with OKEN Scanner

In the right angle triangle CAM,
$CA^2 = AM^2 + CM^2$
$\Rightarrow r^2 = 8^2 + 6^2 \Rightarrow r^2 = 64 + 36$
$\Rightarrow r^2 = 100 \Rightarrow r = 10$ cm
Hence, the radius of the circle is 10 cm.

10. A cyclic quadrilateral has sides 5, 5, 12, 12 units. Find its area.

Sol. Since the cyclic quadrilateral has sides 5, 5, 12, 12, units, its semiperimeter is

$$s = \frac{5+5+12+12}{2} = \frac{34}{2} = 17 \text{ units}$$

For a cyclic quadrilateral, area is given by Brahmagupta's formula:

Area $= \sqrt{[(s-a)(s-b)(s-c)(s-d)]}$

Substituting the values, we have:

Area $= \sqrt{[(17-5)(17-5)(17-12)(17-12)]}$

$= \sqrt{[(12)(12)(5)(5)]} = 12 \times 5 = 60$ sq units

Hence, the area of the cyclic quadrilateral is 60 square units.

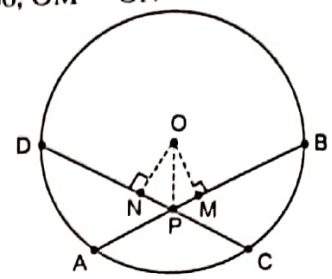
***11. Consider a cyclic quadrilateral. Without drawing its circumcircle, how can we find out whether the centre of the circumcircle lies inside the quadrilateral or outside? What is the best way of finding out?**

[Not for Assessment]

Sol. In a cyclic quadrilateral, we know that opposite angles are supplementary (i.e. their sum is 180°).
To find whether the centre of the circumcircle (circumcentre) lies inside or outside, we can observe the angles of the quadrilateral:
If all angles are less than or equal to 90°, then the circumcentre lies inside the quadrilateral.
If one of the angles is greater than 90° (obtuse), then the circumcentre lies outside the quadrilateral.
Best way: Check the angles of the quadrilateral. This method is easy and does not require drawing the circumcircle.
If any angle is a right angle (90°) then the circumference lies on the midpoint of the side opposite that right angle. This side is the diameter of the circumcircle.
So, by just looking at whether the quadrilateral has an obtuse angle or not, we can decide the position of the circumcentre

***12. When two chords intersect, each of them is divided into two line segments. Show that if the intersecting chords are of equal length, then the line segments of one chord are equal to the corresponding line segments of the other chord.** [Not for Assessment]

Sol. Let chords AB and CD intersect at point P inside the circle, and O be the centre.
From the figure, ON ⊥ CD and OM ⊥ AB.
Given: AB = CD ...(*i*)
To Prove: PB = PD and AP = CP.
Equal chords ⇒ equal distances from centre
So, the perpendicular distances from the centre are equal. So, OM = ON

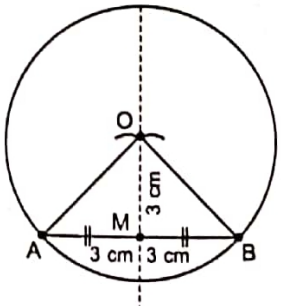


In ΔOPM and ΔOPN, we have
OP = OP [Common]
OM = ON [Proved above]
∠OMP = ∠ONP = 90° [Perpendiculars]
So, by RHS congruency
ΔOPM ≅ ΔOPN
So, PM = PN [CPCT] ...(*ii*)
Now, OM ⊥ AB, so AM = MB [Perpendicular from the centre bisect the chord]
Similarly, ON ⊥ CD, so CN = ND
Since, AB = CD [From (*i*)]
Therefore, BM = DN ...(*iii*)
$[\because BM = \frac{1}{2} AB \text{ and } DN = \frac{1}{2} CD]$
Adding (*ii*) and (*iii*), we get
PM + BM = PN + DN
⇒ PB = PD ...(*iv*)
Subtracting (*iv*) from (*i*), we get
AB – PB = CD – PD ⇒ AP = CP
Hence proved.

***13. Draw a circle in which a chord of 6 cm length stands at a distance of 3 cm from the centre.**
(Hint: Is it a circumcircle of a suitable triangle?)

[Not for Assessment]

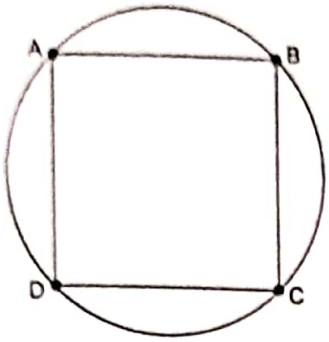
Sol. Chord length = 6 cm Distance from centre to chord = 3 cm
Steps of Construction:
Step I. Draw a line segment AB = 6 cm.
Step II. Find the midpoint M of AB.
Step III. Draw a perpendicular line to AB at M.
Step IV. On this perpendicular, mark a point O such that OM = 3 cm.
Step V. Join OA and OB.
Step VI. With centre O and radius OA, draw a circle.



Observation: Yes it is a circumcircle of the triangle

*14. **Show that rectangle is the only parallelogram that can be inscribed in a circle.** [Not for Assessment]

Sol. Let ABCD be a parallelogram inscribed in a circle.



Since it is a cyclic quadrilateral:

$\angle A + \angle C = 180°$

But in a parallelogram, opposite angles are equal:

$\angle A = \angle C$

Therefore, $\angle C + \angle C = 180°$

$\Rightarrow \quad 2\angle C = 180° \quad \Rightarrow \quad \angle C = 90°$

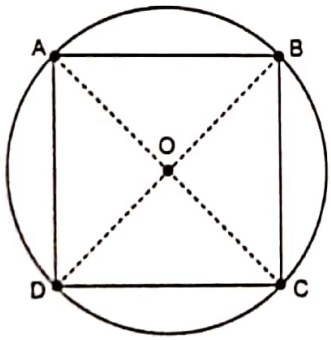
Similarly, all the angles are 90°.

Hence, the parallelogram is a rectangle.

Therefore, a rectangle is the only parallelogram that can be inscribed in a circle.

*15. **Show that if a rectangle is inscribed in a circle, then the point of intersection of its diagonals must lie at the centre of the circle.** [Not for Assessment]

Sol. Let ABCD be a rectangle inscribed in a circle and its diagonals AC and BD intersect at O.



In a rectangle, diagonals are equal and bisect each other.

$$AC = BD \text{ and } \frac{1}{2}AC = \frac{1}{2}BD$$

So, $OA = OC$ and $OB = OD$

Therefore, $OA = OB = OC = OD$

This means O is equidistant from all four vertices A, B, C and D.

The point equidistant from all points on a circle is the centre.

Hence, the point of intersection of the diagonals is the centre of the circle.

*16. **Consider all chords of a circle of a fixed length. What is the shape formed by the midpoints of all these chords?** [Not for Assessment]

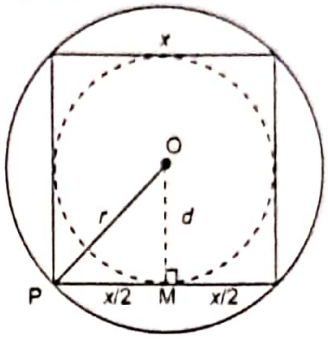
Sol. Let each chord have fixed length x.

Let the radius of the circle be r.

The perpendicular drawn from the centre to a chord bisects the chord.

So, for every chord, half chord $= \frac{x}{2} = OM$

Let d be the distance of the midpoint of the chord from the centre.



Using Pythagoras theorem in ΔOMP:

$$r^2 = d^2 + \left(\frac{x}{2}\right)^2$$

So, $$d^2 = r^2 - \left(\frac{x}{2}\right)^2$$

Since r and x are fixed, d is also fixed.

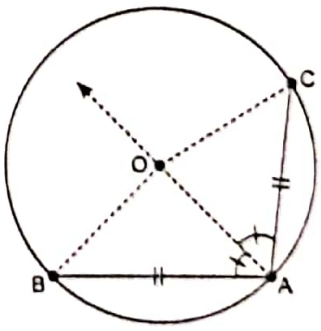
Therefore, every midpoint is at the same distance from the centre.

Hence, the midpoints form a circle with the same centre as the original circle.

*17. **In a circle with centre O, chords AB and AC are congruent. Explain why this statement is true: "The centre of the circle lies on the angle bisector of $\angle BAC$".** [Not for Assessment]

Sol. Given: AB = AC

Join OA, OB and OC.



In ΔAOB and ΔAOC, we have

OB = OC [Radii of circle]

OA = OA [Common]

AB = AC [Given]

Therefore, by SSS congruence

$\Delta OBA \cong \Delta OCA$

So, $\angle BAO = \angle OAC$

Hence, AO bisects $\angle BAC$.

So, the centre O lies on the angle bisector of $\angle BAC$.

Scanned with OKEN Scanner

18. Two parallel chords of lengths 10 cm and 24 cm are on the same side of the centre of a circle. The distance between the chords is 7 cm. Find the radius of the circle.

Sol. Let the radius of the circle be r. So, OA = OC = r.
Length of chord AB = 10 cm:

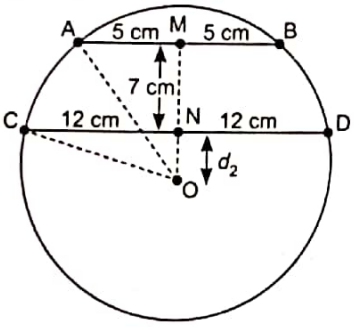
$$\frac{1}{2}AB = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$$

Let its distance from centre OM = d_1
For chord CD, length = 24 cm:

$$\frac{1}{2}CD = \frac{1}{2} \times 24 \text{ cm} = 12 \text{ cm}$$

Let its distance from centre ON = d_2
Since the longer chord is closer to the centre:

$$d_1 - d_2 = 7 \quad \ldots(i)$$



In ΔOAM, we have

$$OA^2 = OM^2 + AM^2$$
$$\Rightarrow \quad r^2 = d_1^2 + 5^2$$
$$\Rightarrow \quad r^2 = d_1^2 + 25 \quad \ldots(ii)$$

In triangle ΔOCN, we have

$$OC^2 = ON^2 + CN^2$$
$$\Rightarrow \quad r^2 = d_2^2 + 12^2$$
$$\Rightarrow \quad r^2 = d_2^2 + 144 \quad \ldots(iii)$$

From (*ii*) and (*iii*), we have

$$d_1^2 + 25 = d_2^2 + 144$$
$$\Rightarrow \quad d_1^2 - d_2^2 = 119$$
$$\Rightarrow \quad (d_1 - d_2)(d_1 + d_2) = 119$$
$$\Rightarrow \quad 7(d_1 + d_2) = 119 \text{ [Since } d_1 - d_2 = 7\text{]}$$
$$\Rightarrow \quad d_1 + d_2 = 17 \quad \ldots(iv)$$

Now solving equations (*i*) and (*iv*), we have

$$2d_1 = 24 \Rightarrow d_1 = 12$$
Then $d_2 = 17 - 12 = 5$
Now, $r^2 = d_2^2 + 5^2$
$= 12^2 + 5^2$
$= 144 + 25$
$= 169$
$\Rightarrow \quad r = 13$ cm

Therefore, the radius of the circle is 13 cm.

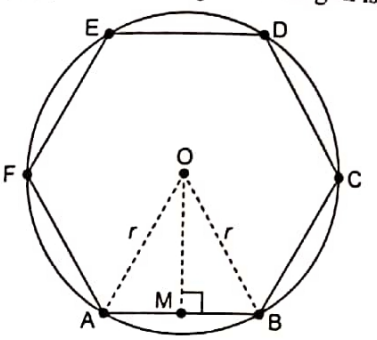
***19. A regular hexagon is inscribed in a circle of radius r. Find the length of the sides of the hexagon and the distance of each side from the centre of the circle.**

[Not for Assessment]

Sol. A regular hexagon divides the circle into 6 equal parts.

Each central angle $= \frac{360°}{6} = 60°$

Let AB be one side of the hexagon and O be the centre.
Then OA = OB = r and ∠AOB = 60°
So, ΔAOB is an equilateral triangle.
Therefore, AB = r
Hence, each side of the regular hexagon is r.



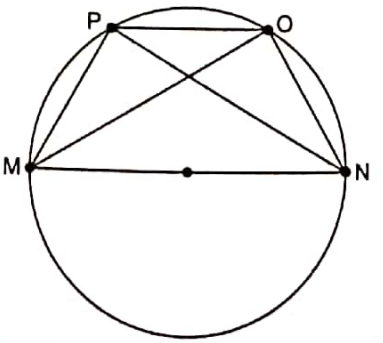
Now to find the distance of each side from the centre.
Let OM be perpendicular to AB.
Since ΔAOB is an equilateral triangle AM $= \frac{AB}{2}$
$= \frac{r}{2}$
In right triangle OMA:

$$OA^2 = OM^2 + AM^2$$
$$\Rightarrow \quad r^2 = OM^2 + \left(\frac{r}{2}\right)^2$$
$$\Rightarrow \quad OM^2 = r^2 - \frac{r^2}{4}$$
$$\Rightarrow \quad OM^2 = \frac{3r^2}{4}$$
$$\Rightarrow \quad OM = \left(\frac{\sqrt{3}}{2}\right)r$$

Therefore, side length of hexagon = r
Distance of each side from centre $= \left(\frac{\sqrt{3}}{2}\right)r$.

20. A quadrilateral MNOP is inscribed in a circle. If MN is a diameter, what can you say about ∠MOP and ∠MNP? Explain your reasoning.

Sol. Given that MNOP quadrilateral in which MN is a diameter. Then ∠MOP and ∠MNP lies in the same segment of the chord MP.

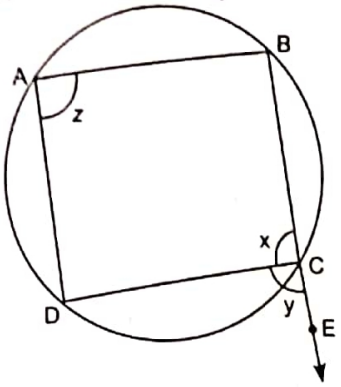


We know that angles in the same segment of a circle are equal.
∴ ∠MOP = ∠MNP

Scanned with OKEN Scanner

21. Let ABCD be a cyclic quadrilateral. Explain why the exterior angle at any vertex is equal to the interior opposite angle (e.g., ∠CDE = ∠ABC, where E is a point on the extension of side CD).

Sol. Given ABCD is a cyclic quadrilateral inscribed in a circle. Extend side BC to meet point let us call E.
Let ∠BCD = x, ∠DCE = y also ∠BAD = z



To prove $y = z$.
Now ABCD is a cyclic quadrilateral.
∴ Sum of opposite angles = 180°
∴ ∠BAD + ∠BCD = 180°
i.e. $z + x = 180°$ ...(i)
Also as BC extended to E
∴ ∠BCD and ∠DCE are linear pair.
i.e. ∠BCD + ∠DCE = 180°
$\Rightarrow x + y = 180°$ ...(ii)
From (i) and (ii), we have
$z + x = x + y \Rightarrow z = y$
i.e. ∠BAD = ∠DCE
i.e. Exterior angle at vertex C is equal to interior opposite angle. Hence Proved

***22. There is no chord of a circle that is longer than its diameter. How do you justify this statement?**
[Not for Assessment]

Sol. A chord is a line segment joining two points on a circle.
The longest chord passes through the centre of the circle. Such a chord is called the diameter.
For any other chord, its perpendicular distance from the centre is greater than 0.
If radius = r and distance of chord from centre = d, then:

$$\text{Chord length} = 2\sqrt{(r^2 - d^2)}$$

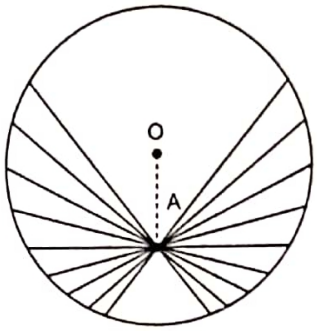
Since $d > 0$ for any chord not passing through the centre,

$$r^2 - d^2 < r^2$$

So, $2\sqrt{(r^2 - d^2)} < 2r$
But diameter = $2r$.
Therefore, every chord other than the diameter is shorter than the diameter.
Hence, no chord of a circle is longer than its diameter.

***23. Let A be any point within a given circle with centre O. Show that the shortest chord of the circle that passes through point A is the one that is perpendicular to OA.**
[Not for Assessment]

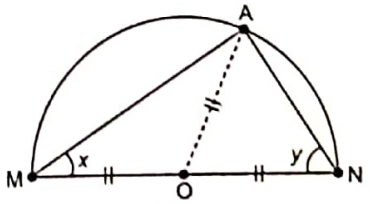
Sol. Let the circle have centre O and A be a point inside it.
Any chord passing through A will have some perpendicular distance from the centre O.
We know that the farther a chord is from the centre, the shorter it is.
So, the shortest chord through A will be the chord whose distance from centre is maximum.
Among all lines passing through A, the perpendicular distance from centre to the line is greatest when the line is perpendicular to OA.



Therefore, the chord through A that is perpendicular to OA is farthest from the centre.
Hence, it is the shortest chord passing through A.
Therefore, the shortest chord through A is the one perpendicular to OA.
Hence proved.

24. How would you use the following figure to justify the statement that the angle in a semicircle is 90°?

Sol. Let the diameter be MN and O be the centre of the circle.



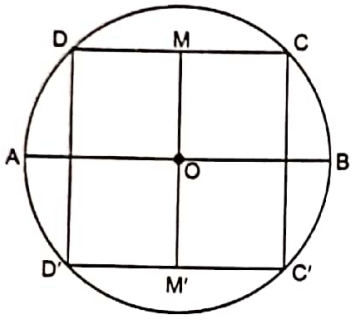
Point A lies on the semicircle.
Join OA.
Since O is the centre:
OA = OM = ON (Radii of a circle)
So, ΔAOM is isosceles and ΔAON is isosceles.
Let: ∠AMO = x and ∠ANO = y
Then: ∠MAO = x and ∠OAN = y
[Since ∠AMO = ∠MAO and ∠ANO = ∠OAN]
Therefore, ∠MAN = $x + y$
Now, in triangle AMN:
In ΔAMN, ∠MAN + ∠ANM + ∠AMN = 180°
$\Rightarrow (x + y) + y + x = 180° \Rightarrow 2(x + y) = 180°$
$\Rightarrow x + y = 90° = \angle MAN$
$\therefore \angle MAN = 90°$
Hence, the angle in a semicircle is 90°.

Scanned with OKEN Scanner

*25. **In a circle, two chords CC′ and DD′ are drawn perpendicular to a diameter AB. Prove that the segment MM′ joining the midpoints of the chords CD and C′D′ is perpendicular to AB.**

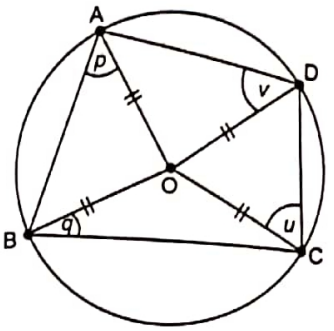
[Not for Assessment]

Sol. As AB is the diameter of the circle.
Since chords CC′ and DD′ are perpendicular to AB, the points C and C′ are symmetric about AB.



Similarly, D and D′ are also symmetric about AB.
Now, since M is the midpoint of CD and M′ is the midpoint of C′D′.
Since C corresponds to C′ and D corresponds to D′ by reflection in AB, the midpoint of CD corresponds to the midpoint of C′D′.
Therefore, M and M′ are symmetric about AB.
The line joining two points that are symmetric about a line is perpendicular to that line.
Hence, $MM' \perp AB$
Therefore, the segment joining the midpoints of CD and C′D′ is perpendicular to AB.
Hence proved.

*26. **How would you use the following figure to justify the statement that the sum of the opposite angles of a cyclic quadrilateral is 180°?** [Not for Assessment]



Sol. Let ABCD be a cyclic quadrilateral with centre O.
Join OA, OB, OC and OD.
Since OA, OB, OC and OD are radii of the same circle:

$$OA = OB = OC = OD$$

So the triangles formed with the centre are isosceles triangles.
In ΔOAB, $OA = OB \Rightarrow \angle OAB = \angle OBA = p$
In ΔOBC, $OB = OC \Rightarrow \angle OBC = \angle OCB = q$
In ΔOCD, $OC = OD \Rightarrow \angle OCD = \angle ODC = u$
In ΔOAD, $OA = OD \Rightarrow \angle ODA = \angle OAD = v$
$\Rightarrow \angle A = p + v$, $\angle B = p + q$, $\angle C = q + u$,
$\Rightarrow \angle D = u + v$
Since sum of angles in any quadrilateral is always 360°.
Here $\angle A + \angle B + \angle C + \angle D = 360°$
$\Rightarrow (p + v) + (p + q) + (q + u) + (u + v) = 360°$
$\Rightarrow 2(p + q + u + v) = 360°$
$\Rightarrow p + q + u + v = 180°$ ...(*i*)
Now sum of opposite angles
$\angle A + \angle C = p + v + q + u = 180°$ [Using (*i*)]
also $\angle B + \angle D = p + q + u + v = 180°$ [Using (*i*)]
As sum of opposite angle is 180°
$\Rightarrow$ ABCD is a cyclic quadrilateral.

Scanned with OKEN Scanner

Topper's Tips

(✓) Do's

- Understand **why circles are used in real life** (wheels, gears, domes, roundabouts).
- Relate **radius and diameter** to practical measurements.
- Use correct terminology when explaining applications.

- Remember:
  - **Centre**: fixed point inside the circle.
  - **Radius**: segment joining centre to a point on the circle.
  - **Diameter**: longest chord passing through the centre.
- Use relation:
  Diameter = 2 × Radius
- Label diagrams clearly (O for centre, A, B on circumference).
- Use the theorem:
  Angle at centre = 2 × angle at the circumference (same arc).
- Identify the **same arc** carefully.

- Write theorem name while proving.
- Remember theorem:
  Perpendicular from centre to a chord bisects the chord.
- Use **right triangle reasoning** when proving.

- Mark the **right angle symbol** in diagrams.

- Learn the key results:
  - **Equal chords are equidistant from the centre.**
  - **Chords equidistant from the centre are equal.**
- Draw perpendicular from centre to chord to measure distance.
- Remember theorem:
  Angles in the same segment of a circle are equal.
- Identify the **same arc** correctly.
- Use clear arc markings in the diagram.
- Remember properties:
  - **Opposite angles of cyclic quadrilateral are supplementary.**
  - **Exterior angle equals interior opposite angle.**
- Check if four points lie on the **same circle.**
- Use **angle sum = 180°** to prove cyclicity.

(✗) Don'ts

- Don't confuse **circular objects with spherical ones.**

- Don't ignore **centre and radius** when explaining real-life structures.
- Don't give examples that are **not perfectly circular** unless stated approximate.
- Don't treat **radius and diameter as the same.**

- Don't draw **unclear or unlabeled diagrams** in proofs.
- Don't forget that **diameter is also a chord.**

- Don't compare angles formed by **different arcs.**

- Don't assume equality without mentioning the **relevant theorem.**
- Don't forget to show **points lie on the circle.**
- Don't assume midpoint without proof.

- Don't forget to state **"OD ⊥ AB"** before using the theorem.
- Don't mix this with **perpendicular bisector theorem of triangles.**
- Don't measure distance along the chord itself.

- Don't assume chords are equal without showing **equal perpendicular distances.**
- Don't compare angles formed by **different arcs.**

- Don't forget that **points must lie on the circle.**
- Don't skip diagram justification.
- Don't assume quadrilateral is cyclic without proof.

- Don't mix **parallelogram properties with cyclic quadrilateral properties.**
- Don't forget to write **∠A + ∠C = 180°** condition.

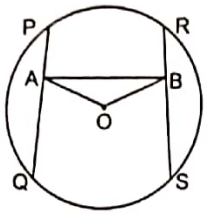
Scanned with OKEN Scanner

CHAPTER END ASSESSMENT

A comprehensive set of questions to assess conceptual understanding, application, and reasoning.

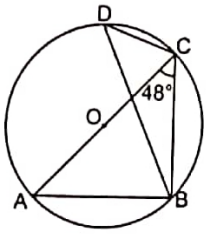
[Time Allowed: 40 Minutes] [Maximum Marks: 20]

1. In figure, PQ and RS are two equal chords of a circle with centre O. OA and OB are perpendicular on chords PQ and RS respectively of $\angle PAB = 75°$, then $\angle AOB$ is equal to



(a) 15°
(b) 30°
(c) 120°
(d) 150°

2. In figure, O is centre of the circle, if measure of $\angle ACB = 48°$, then measure of $\angle BDC$ is

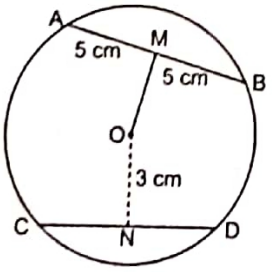


(a) 48°
(b) 42°
(c) 90°
(d) 30°

3. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at point of minor arc.

4. Two congruent circles have centres O_1 and O_2. Arc AB of a circle, with centre O_1, subtends an angle of 100° at the centre and an arc A′B′ of the circle, with centre O_2, subtends an angle of 25°. Then find arc (AB): arc (A′B′)

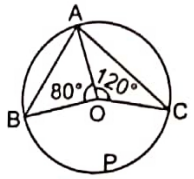
5. In the given figure, O is the centre of the circle. AB and CD are two chords of the circle.
Also, OM ⊥ AB and ON ⊥ CD. If OM = ON = 3 cm and AM = BM = 5 cm, find the chord CD.



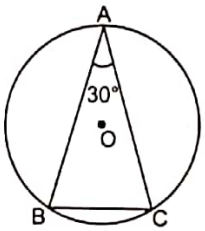
6. AB and CD are two parallel chords of a circle whose centre is O and radius is 20 cm. If AB = 24 cm and CD = 32 cm, find the distance between AB and CD, if its given that they lie.
(i) on opposite sides of centre O.
(ii) on the same side of centre O.

7. Prove that an isosceles trapezium is always cyclic and its diagonal are equal.

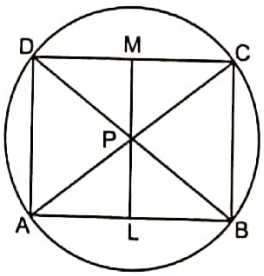
8. In the given figure, A, B, C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 80° and 120° respectively. Determine $\angle BAC$ and the degree measure of arc BPC.



9. In the given figure, ABC is a triangle in which $\angle BAC = 30°$. Show that the length of BC is equal to the radius of the circumcircle of ΔABC, whose centre is O.

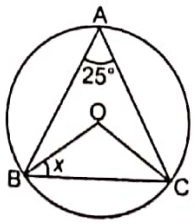


10. In the given figure, $\angle BPC = 90°$, LM ⊥ CD. If AB is 10 cm, find BL.



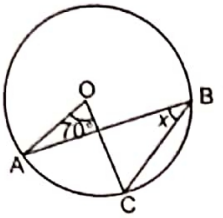
11. ABCD is a parallelogram. The circle through A, B and C intersects CD produced at E. If AB = 10 cm, BC = 8 cm, CE = 14 cm, find AE.

12. Prove that the angle subtended by an arc at the centre of a circle is double the angle subtended by it on any point on the remaining part of the circle.
Use above result to find the following:
(i) O is the centre of the circle and $\angle BAC = 25°$, find $\angle x$.

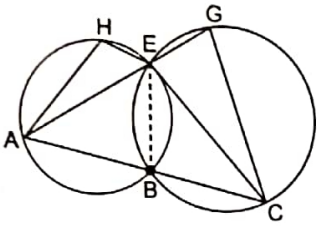


Scanned with OKEN Scanner

(*ii*) Find x in the figure where O is the centre of the circle.

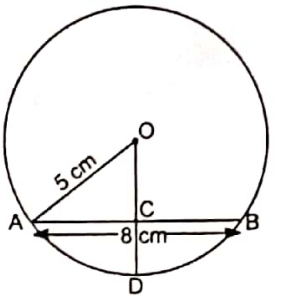


13. In the given figure, ABC, AEG and HEC are straight lines. Prove that $\angle AHE$ and $\angle EGC$ are supplementary.



14. **Assertion (A):** If the sum of pairs of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.
Reason (R): The vertices lies on the circumference of circle.

15. **PAPER FOLDING**



Teaching Mathematics includes wonder of patterns, shapes, visual points and possibilities that could be explored in classrooms but most fascinating has been the use of paper folding. It enthusiases, calms, inspires and delights teachers and pupils both. Mrs Anupama was explaining same concept of circles in classroom when she drew the following figure and labelled the figure. She told them that OA = 5 cm, AB = 8 cm and OC ⊥ AB.

Now above the following questions

(*i*) What is length of AC?

(*ii*) Find the length of OC.

(*iii*) (*a*) What is the diameter of circle?

OR

(*iii*) (*b*) What is the measure of $\angle ACO$?

ANSWERS

1. (*d*) **2.** (*b*) **3.** 150° **4.** 4 : 1 **5.** 10 cm

6. 28 cm; 4 cm **8.** 80°, 160° **10.** 5 cm **11.** 8 cm **12.** (*i*) 65° (*ii*) 35°

14. (*a*)

15. (*i*) 4 cm (*ii*) 3 cm (*iii*) (*a*) 10 cm **OR** (*iii*) (*b*) 90°

Scan me for Solutions

Scanned with OKEN Scanner