

CIRCLES (Tangents to a Circle)

Worksheet 1 — Core Concepts — Answer Key & Marking Scheme

Name: _____	Roll No.: _____	Date: _____
Class: X Subject: Mathematics	Time: 90 minutes	Max. Marks: 50

Note for teachers: Marks for each step are shown in brackets. Any other correct method should be given full credit. Figures are not drawn to scale.

SECTION A — Multiple Choice Questions

1 mark each

Assertion-Reason (Q9-10): (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

1. A tangent is drawn from a point 17 cm away from the centre O of a circle of radius 8 cm. The length of the tangent is **[1]**

(a) 9 cm (b) 15 cm (c) 25 cm (d) $\sqrt{353}$ cm

Ans: (b) 15 cm — Tangent $\perp$ radius, so length = $\sqrt{(17^2 - 8^2)} = \sqrt{225} = 15$ cm.

2. The number of tangents that can be drawn to a circle from a point lying inside it is **[1]**

(a) 0 (b) 1
(c) 2 (d) infinitely many

Ans: (a) 0 — Every line through an interior point cuts the circle at two points, so none is a tangent.

3. PA and PB are tangents from an external point P to a circle with centre O. If $\angle APB = 70^\circ$, then $\angle OAB =$ **[1]**

(a) 20° (b) 35° (c) 55° (d) 70°

Ans: (b) 35° — $\angle PAB = \angle PBA = 55^\circ$; $\angle OAP = 90^\circ$ so $\angle OAB = 90^\circ - 55^\circ = 35^\circ$.

4. Two concentric circles have radii 10 cm and 6 cm. The length of the chord of the larger circle which touches the smaller circle is **[1]**

(a) 8 cm (b) 12 cm (c) 16 cm (d) 20 cm

Ans: (c) 16 cm — Half-chord = $\sqrt{(10^2 - 6^2)} = 8$ cm, so chord = 16 cm.

5. A quadrilateral ABCD circumscribes a circle. If AB = 7 cm, BC = 6 cm and CD = 5 cm, then AD = **[1]**

(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

Ans: (c) 6 cm — $AB + CD = BC + AD \Rightarrow AD = 7 + 5 - 6 = 6$ cm.

6. The distance between two parallel tangents of a circle of radius 4.5 cm is **[1]**

(a) 4.5 cm (b) 9 cm (c) 13.5 cm (d) 2.25 cm

Ans: (b) 9 cm — Parallel tangents lie at the two ends of a diameter: $2 \times 4.5 = 9$ cm.

7. TP and TQ are two tangents to a circle with centre O such that $\angle POQ = 130^\circ$. Then $\angle PTQ =$ **[1]**

(a) 40° (b) 50° (c) 60° (d) 65°

Ans: (b) 50° — $\angle PTQ + \angle POQ = 180^\circ \Rightarrow \angle PTQ = 50^\circ$.

8. ABC is a triangle right-angled at B with AB = 9 cm and BC = 12 cm. The radius of its inscribed circle is **[1]**

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

Ans: (b) 3 cm — $AC = 15$ cm; $r = (AB + BC - AC)/2 = (9 + 12 - 15)/2 = 3$ cm.

9. Assertion (A): The tangent at any point of a circle is perpendicular to the radius through the point of contact. **[1]**

Reason (R): Among all segments joining the centre to points of the tangent, the radius to the point of contact is the shortest.

Ans: (a) — Both true; the shortest-distance property is exactly how the theorem is proved $\Rightarrow$ (a).

10. Assertion (A): A line intersecting a circle at two distinct points is a tangent. [1]

Reason (R): A tangent meets the circle at exactly one point.

Ans: (d) — A is false (such a line is a secant); R is true $\Rightarrow$ (d).

SECTION B — Very Short Answer Questions

1 mark each

11. Fill in the blanks: A line that touches a circle at exactly one point is called a _____, and the common point is called the _____. [1]

Solution: Tangent; point of contact.

12. True or False: Two tangents can be drawn to a circle from a point lying on the circle. [1]

Solution: False — exactly one tangent can be drawn at a point on the circle.

13. What is the distance between two parallel tangents of a circle of radius 7 cm? [1]

Solution: 14 cm.

14. The length of the tangent from a point P to a circle of radius 5 cm is 12 cm. Find OP, where O is the centre. [1]

Solution: $OP = \sqrt{5^2 + 12^2} = 13$ cm.

15. PA and PB are tangents drawn from P to a circle. If $PA = 3x + 2$ and $PB = 5x - 6$, find x. [1]

Solution: $PA = PB \Rightarrow 3x + 2 = 5x - 6 \Rightarrow x = 4$.

16. Tangents PA and PB from P to a circle with centre O are inclined at 80° to each other. Find $\angle POA$. [1]

Solution: $\angle OPA = 40^\circ$, $\angle OAP = 90^\circ \Rightarrow \angle POA = 50^\circ$.

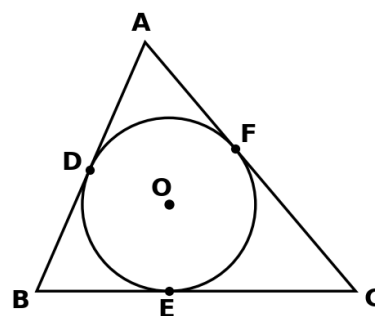
SECTION C — Short Answer Questions I

2 marks each

17. The length of the tangent from a point P to a circle is 12 cm and P is 37 cm from the centre. Find the radius of the circle. [2]

Solution: $OT \perp PT$, so $r = \sqrt{(37^2 - 12^2)} = \sqrt{(1369 - 144)} = \sqrt{1225} = 35$ cm (1).

18. In the figure, the circle inscribed in $\triangle ABC$ touches AB, BC and CA at D, E and F respectively. If $AB = 10$ cm, $BC = 9$ cm and $CA = 11$ cm, find AD, BE and CF. [2]



Solution: Let $AD = AF = x$, $BD = BE = y$, $CE = CF = z$ (tangents from a point). ($\frac{1}{2}$)
 $x + y = 10$, $y + z = 9$, $z + x = 11 \Rightarrow x + y + z = 15$. (1)
 $AD = 6$ cm, $BE = 4$ cm, $CF = 5$ cm. ($\frac{1}{2}$)

19. PT is a tangent at P to a circle with centre O and PQ is a chord such that $\angle QPT = 55^\circ$. Find $\angle POQ$. [2]

Solution: $\angle OPQ = 90^\circ - 55^\circ = 35^\circ$ (1). $OP = OQ \Rightarrow \angle OQP = 35^\circ$, so $\angle POQ = 180^\circ - 70^\circ = 110^\circ$ (1).

20. Two concentric circles have radii 13 cm and 5 cm. Find the length of the chord of the larger circle which touches the smaller circle. [2]

Solution: Half-chord = $\sqrt{(13^2 - 5^2)} = 12$ cm (1) (the tangent chord is bisected at the point of contact). Chord = 24 cm (1).

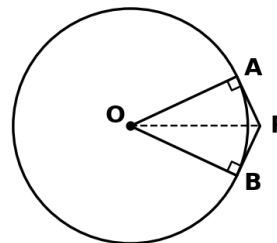
SECTION D — Short Answer Questions II

3 marks each

21. Prove that the lengths of the two tangents drawn from an external point to a circle are equal. [3]

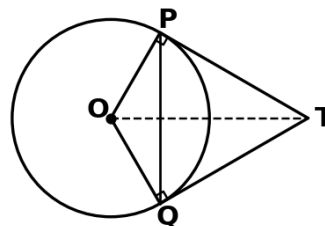
Solution: Let tangents AP, AQ touch the circle (centre O) at P, Q. Join OP, OQ, OA. ($\frac{1}{2}$)
 $OP \perp AP$ and $OQ \perp AQ$ (tangent $\perp$ radius). ($\frac{1}{2}$)
 In right Δ s OPA and OQA: $OP = OQ$ (radii), $OA = OA$ (common), $\angle OPA = \angle OQA = 90^\circ$. (1)
 So $\Delta OPA \cong \Delta OQA$ (RHS) ($\frac{1}{2}$) $\Rightarrow AP = AQ$ (CPCT). ($\frac{1}{2}$)

22. In the figure, PA and PB are tangents from P to a circle with centre O. If $\angle APB = 130^\circ$, find $\angle AOB$, $\angle OAB$ and $\angle PAB$. [3]



Solution: $\angle OAP = \angle OBP = 90^\circ$, so in quad. OAPB: $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 130^\circ = 50^\circ$. (1)
 $OA = OB \Rightarrow \angle OAB = (180^\circ - 50^\circ)/2 = 65^\circ$. (1)
 $\angle PAB = \angle OAP - \angle OAB = 90^\circ - 65^\circ = 25^\circ$. (1)

23. In the figure, TP and TQ are tangents from T to a circle with centre O. Prove that $\angle PTQ = 2\angle OPQ$. [3]



Solution: $TP = TQ \Rightarrow \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \angle PTQ) = 90^\circ - \frac{1}{2}\angle PTQ$. (1)
 $OP \perp TP \Rightarrow \angle OPQ = 90^\circ - \angle TPQ$. (1)
 $\angle OPQ = 90^\circ - (90^\circ - \frac{1}{2}\angle PTQ) = \frac{1}{2}\angle PTQ \Rightarrow \angle PTQ = 2\angle OPQ$. (1)

24. ABC is a right triangle, right-angled at B, with AB = 15 cm and BC = 20 cm. A circle is inscribed in it. Find the radius of the circle. [3]

Solution: $AC = \sqrt{15^2 + 20^2} = 25$ cm. ($\frac{1}{2}$)
 Let the circle touch AB, BC, CA at P, Q, R. OPBQ is a square, so $BP = BQ = r$. (1)
 $AR = AP = 15 - r$, $CR = CQ = 20 - r$. ($\frac{1}{2}$)
 $AC = AR + CR \Rightarrow 25 = 35 - 2r \Rightarrow r = 5$ cm. (1)

SECTION E — Long Answer Questions

5 marks each

25. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. [5]

Solution: Given: circle, centre O; tangent AB at P. To prove: $OP \perp AB$. (1)
 Take any point Q $\neq$ P on AB; join OQ. Q lies on the tangent, so it is outside the circle. (1)
 Let OQ meet the circle at R. Then $OR < OQ$. (1)
 $OP = OR$ (radii) $\Rightarrow OP < OQ$. (1)
 So OP is shorter than every other segment from O to AB, i.e. the shortest distance, and the shortest distance from a point to a line is the perpendicular. Hence $OP \perp AB$. (1)

26. (a) Prove that if a quadrilateral ABCD circumscribes a circle, then $AB + CD = AD + BC$. (3) [5]
 (b) A circle with centre O touches the sides AB, BC, CD, DA of quadrilateral ABCD at P, Q, R, S respectively, $\angle B = 90^\circ$, $AB = 27$ cm, $AD = 20$ cm and $DS = 5$ cm. Find the radius of the circle. (2)

Solution: (a) $AP = AS$, $BP = BQ$, $CR = CQ$, $DR = DS$ (tangents from a point). (1)

$AB + CD = (AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)$ (1) $= (AS + DS) + (BQ + CQ) = AD + BC$. (1)

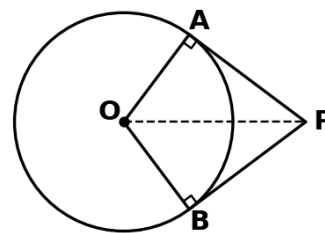
(b) $AS = AD - DS = 15$ cm, so $AP = 15$ cm and $BP = 27 - 15 = 12$ cm. (1)

$OP \perp AB$, $OQ \perp BC$, $\angle B = 90^\circ \Rightarrow OPBQ$ is a square, so $r = BP = 12$ cm. (1)

SECTION F — Case Study Based Question

4 marks

27. A circular pond has centre O and radius 15 m. A water pump is installed at a point P outside the pond, 25 m from O. Two straight pipes PA and PB run from the pump and touch the pond at A and B, as shown. [4]



(i) Find the length of the pipe PA. [1]

(ii) Give a reason why $PA = PB$. [1]

(iii) Find the area of quadrilateral OAPB. [2]

OR

(iii) Show that O, A, P and B lie on one circle and find the radius of that circle. [2]

Solution:

(i) $OA \perp PA$, so $PA = \sqrt{(25^2 - 15^2)} = 20$ m. (1)

(ii) Lengths of the two tangents drawn from an external point to a circle are equal. (1)

(iii) $\text{Area} = 2 \times (\frac{1}{2} \times OA \times PA) = 2 \times \frac{1}{2} \times 15 \times 20 = 300 \text{ m}^2$. (2)

OR (iii) $\angle OAP = \angle OBP = 90^\circ$, so A and B lie on the circle with OP as diameter (angle in a semicircle). (1) Radius = $\frac{1}{2} \times OP = 12.5$ m. (1)

— End of Worksheet —