

CIRCLES (Tangents to a Circle)

Worksheet 2 — Application & Reasoning — Answer Key & Marking Scheme

Name: _____	Roll No.: _____	Date: _____
Class: X Subject: Mathematics	Time: 90 minutes	Max. Marks: 50

Note for teachers: Marks for each step are shown in brackets. Any other correct method should be given full credit. Figures are not drawn to scale.

SECTION A — Multiple Choice Questions

1 mark each

Assertion-Reason (Q9-10): (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

1. The length of the tangent drawn from a point 25 cm from the centre of a circle of radius 7 cm is **[1]**
 (a) 18 cm (b) 26 cm (c) 24 cm (d) 32 cm

Ans: (c) 24 cm — $\sqrt{(25^2 - 7^2)} = \sqrt{576} = 24$ cm.

2. PA and PB are tangents from P to a circle with centre O. If $\angle AOB = 140^\circ$, then $\angle APB =$ **[1]**
 (a) 20° (b) 70° (c) 140° (d) 40°

Ans: (d) 40° — $\angle APB = 180^\circ - 140^\circ = 40^\circ$.

3. Two concentric circles have radii 17 cm and 8 cm. The chord of the larger circle that touches the smaller circle has length **[1]**
 (a) 30 cm (b) 15 cm (c) 25 cm (d) 34 cm

Ans: (a) 30 cm — $2 \times \sqrt{(17^2 - 8^2)} = 2 \times 15 = 30$ cm.

4. The incircle of $\triangle ABC$ touches BC, CA, AB at D, E, F. If AF = 4 cm, BD = 6 cm and CE = 8 cm, the perimeter of $\triangle ABC$ is **[1]**
 (a) 18 cm (b) 36 cm (c) 24 cm (d) 72 cm

Ans: (b) 36 cm — Perimeter = $2(AF + BD + CE) = 2 \times 18 = 36$ cm.

5. The sum of a pair of opposite sides of a quadrilateral circumscribing a circle is 15 cm. Its perimeter is **[1]**
 (a) 15 cm (b) 22.5 cm (c) 30 cm (d) 45 cm

Ans: (c) 30 cm — Both pairs have sum 15 cm, so perimeter = 30 cm.

6. PT is a tangent at T to a circle with centre O. If $\angle TPO = 25^\circ$, then $\angle POT =$ **[1]**
 (a) 25° (b) 75° (c) 90° (d) 65°

Ans: (d) 65° — $\angle PTO = 90^\circ$, so $\angle POT = 90^\circ - 25^\circ = 65^\circ$.

7. A chord PQ of a circle makes an angle of 65° with the tangent at P. The angle in the alternate segment is **[1]**
 (a) 65° (b) 25° (c) 115° (d) 130°

Ans: (a) 65° — Tangent-chord angle equals the angle in the alternate segment = 65° .

8. A parallelogram circumscribing a circle is a **[1]**
 (a) rectangle (b) rhombus (c) trapezium (d) kite

Ans: (b) rhombus — $AB + CD = AD + BC$ with $AB = CD$, $AD = BC$ gives $AB = AD$, so all sides are equal: a rhombus.

9. Assertion (A): If PA and PB are tangents from P to a circle with centre O, then $\angle APB + \angle AOB = 180^\circ$. **[1]**
 Reason (R): The radius through the point of contact is perpendicular to the tangent.

Ans: (a) — $\angle OAP = \angle OBP = 90^\circ$ (R) and the angle sum of a quadrilateral is 360° , which gives $A \Rightarrow (a)$.

10. Assertion (A): The lengths of the two tangents drawn from an external point to a circle are equal. [1]

Reason (R): A circle has infinitely many tangents.

Ans: (b) — Both are true, but R does not explain $A \Rightarrow (b)$.

SECTION B — Very Short Answer Questions

1 mark each

11. Two tangents drawn from a point P to a circle of radius 5 cm are inclined at 60° . Find the length of each tangent. [1]

Solution: OP bisects $\angle APB$, so $\angle OPA = 30^\circ$; $\tan 30^\circ = OA/PA \Rightarrow PA = 5\sqrt{3}$ cm.

12. True or False: A tangent can be drawn to a circle through any point lying inside it. [1]

Solution: False — no tangent can be drawn from an interior point.

13. Fill in the blank: The distance between two parallel tangents of a circle of radius r is _____. [1]

Solution: $2r$.

14. Two tangents from P to a circle of radius 6 cm are perpendicular to each other. Find OP. [1]

Solution: OAPB is a square of side 6 cm, so $OP = 6\sqrt{2}$ cm.

15. If tangents PA and PB from P to a circle with centre O are inclined at 100° , find $\angle OAB$. [1]

Solution: $\angle PAB = 40^\circ$, $\angle OAP = 90^\circ \Rightarrow \angle OAB = 50^\circ$.

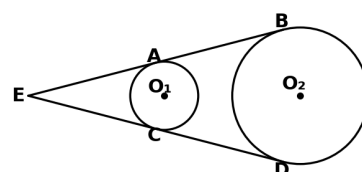
16. How many tangents can be drawn to a circle parallel to a given line? [1]

Solution: Exactly two.

SECTION C — Short Answer Questions I

2 marks each

17. In the figure, AB and CD are common tangents to two circles of unequal radii, meeting at E. Prove that $AB = CD$. [2]



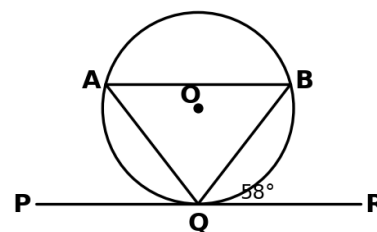
Solution: Tangents from E to the smaller circle: $EA = EC$. ($\frac{1}{2}$)

Tangents from E to the larger circle: $EB = ED$. ($\frac{1}{2}$)

$AB = EB - EA$ and $CD = ED - EC$. ($\frac{1}{2}$)

Since $EB = ED$ and $EA = EC$, $AB = CD$. ($\frac{1}{2}$)

18. In the figure, PQR is a tangent to a circle at Q and chord $AB \parallel PR$. If $\angle BQR = 58^\circ$, find $\angle AQB$. [2]



Solution: $\angle QAB = \angle BQR = 58^\circ$ (angles in alternate segments). ($\frac{1}{2}$)

$\angle ABQ = \angle BQR = 58^\circ$ (alternate interior angles, $AB \parallel PR$). ($\frac{1}{2}$)

$\angle AQB = 180^\circ - 58^\circ - 58^\circ = 64^\circ$. (1)

19. In isosceles $\triangle ABC$, $AB = AC = 17$ cm and $BC = 16$ cm. A circle touches AB, AC and BC at P, Q and R respectively. Find AP. [2]

Solution: $AP = AQ \Rightarrow BP = AB - AP = AC - AQ = CQ$. Also $BP = BR$ and $CQ = CR$, so $BR = CR = 8$ cm. (1)

$BP = BR = 8$ cm $\Rightarrow AP = 17 - 8 = 9$ cm. (1)

20. From a point A, 17 cm from the centre O of a circle of radius 8 cm, tangents AP and AQ are drawn. Find the area of quadrilateral OPAQ. [2]

Solution: $AP = \sqrt{17^2 - 8^2} = 15$ cm. (1)
 Area = $2 \times \frac{1}{2} \times OP \times AP = 8 \times 15 = 120$ cm². (1)

SECTION D — Short Answer Questions II

3 marks each

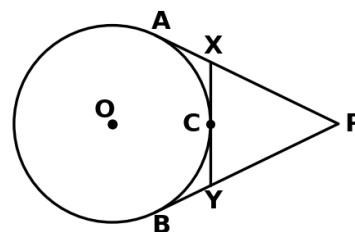
21. From an external point P, tangents PA and PB are drawn to a circle with centre O. Prove that OP is the perpendicular bisector of AB. [3]

Solution: Let OP meet AB at M. $PA = PB$ (tangents) and $\angle APM = \angle BPM$ (OP bisects $\angle APB$). $PM = PM$. (1)
 $\triangle APM \cong \triangle BPM$ (SAS) $\Rightarrow AM = BM$ and $\angle AMP = \angle BMP$. (1)
 $\angle AMP + \angle BMP = 180^\circ$ (linear pair) $\Rightarrow$ each is 90° . So $OP \perp AB$ and bisects it. (1)

22. A circle of radius 4 cm is inscribed in $\triangle ABC$ and touches BC at D, with $BD = 8$ cm and $DC = 6$ cm. If the area of $\triangle ABC$ is 84 cm², find AB and AC. [3]

Solution: Let the circle touch AB at F and AC at E; $AF = AE = x$. So $AB = x + 8$, $AC = x + 6$, $BC = 14$. (1)
 Area = $\frac{1}{2} \times r \times \text{perimeter} \Rightarrow 84 = \frac{1}{2} \times 4 \times (2x + 28)$. (1)
 $84 = 4x + 56 \Rightarrow x = 7$. $AB = 15$ cm, $AC = 13$ cm. (1)

23. In the figure, PA and PB are tangents from P to a circle, with $PA = 15$ cm. A tangent XY touches the circle at C, a point on the minor arc AB, meeting PA at X and PB at Y. Find the perimeter of $\triangle PXY$. [3]



Solution: $XA = XC$ and $YB = YC$ (tangents from X and Y). (1)
 Perimeter = $PX + XY + PY = PX + (XC + CY) + PY = (PX + XA) + (PY + YB)$ (1)
 $= PA + PB = 15 + 15 = 30$ cm. (1)

24. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. [3]

Solution: Let PA, PB be tangents from P and O the centre. $OA \perp PA$ and $OB \perp PB$, so $\angle OAP = \angle OBP = 90^\circ$. (1)
 In quadrilateral OAPB: $\angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$. (1)
 $\angle AOB + \angle APB = 360^\circ - 180^\circ = 180^\circ$. (1)

SECTION E — Long Answer Questions

5 marks each

25. In a right triangle ABC, right-angled at B, a circle is drawn with AB as diameter, intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC. [5]

Solution: $CB \perp AB$ at the end B of the diameter, so CB is a tangent at B. Let the tangent at P meet BC at Q. (1)
 $QP = QB$ (tangents from Q). (1)
 $\angle APB = 90^\circ$ (angle in a semicircle) $\Rightarrow \angle BPC = 90^\circ$. (1)
 $\angle QPC = 90^\circ - \angle QPB$ and $\angle QCP = 90^\circ - \angle QBP$; since $\angle QPB = \angle QBP$ ($QP = QB$), $\angle QPC = \angle QCP$. (1)
 So $QC = QP = QB$, i.e. Q is the midpoint of BC. (1)

26. A circle touches the side BC of $\triangle ABC$ at P and touches AB and AC, when produced, at Q and R respectively. Prove that $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$). [5]

Solution: Tangents from A: $AQ = AR$. (1)
 Tangents from B: $BQ = BP$. Tangents from C: $CR = CP$. (1)
 Perimeter = $AB + BC + AC = AB + (BP + PC) + AC$ (1)
 $= (AB + BQ) + (AC + CR) = AQ + AR$ (1) = $2AQ$.
 Hence $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$). (1)

SECTION F — Case Study Based Question

4 marks

- 27.** A triangular park ABC has sides $AB = 13$ m, $BC = 14$ m and $CA = 15$ m. A circular fountain inside the park touches the sides AB, BC and CA at P, Q and R respectively. The area of the park is 84 m^2 .

[4]

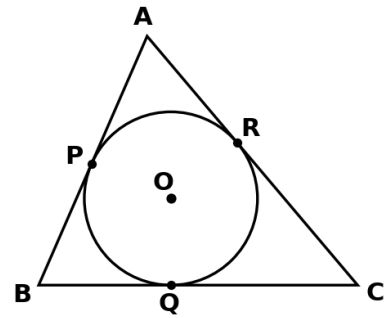
(i) Find the length AP. **[1]**

(ii) State why $BP = BQ$. **[1]**

(iii) Find the radius of the fountain. **[2]**

OR

(iii) If the radius of the fountain is 4 m, find the area of $\triangle OBC$, where O is its centre. **[2]**

**Solution:**

(i) Let $AP = AR = x$, $BP = BQ = y$, $CQ = CR = z$: $x + y = 13$, $y + z = 14$, $z + x = 15 \Rightarrow x + y + z = 21$, so $AP = 21 - 14 = 7$ m. (1)

(ii) Tangents drawn from an external point (B) to a circle are equal. (1)

(iii) $\text{Area} = r \times s$, with $s = 21$ m: $84 = r \times 21 \Rightarrow r = 4$ m. (2)

OR (iii) $OQ \perp BC$, so area of $\triangle OBC = \frac{1}{2} \times BC \times OQ$ (1) $= \frac{1}{2} \times 14 \times 4 = 28 \text{ m}^2$. (1)

— End of Worksheet —