

Class 8 Maths: Chapter 5 — Tales by Dots and Lines

NCERT
Syllabus

Core Revision Notebook • Ganita Prakash Part-II • Quick Handwritten Notes

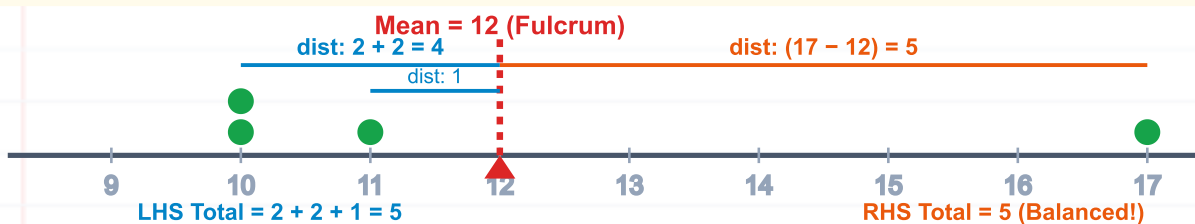
1. The Balancing Act: Mean as a Balance Point / Fulcrum

Last year we learned the *fair-share* idea of arithmetic mean. In Class 8, we visualize the mean as a physical **balance point (fulcrum)** on a dot plot!

$$\text{Arithmetic Mean } (\bar{x}) = (\text{Sum of all observations}) \div (\text{Total number of observations}) = (\sum x_i) \div n$$

The Balance Principle: The mean is the *unique point* where the **sum of distances to values on the left (LHS)** equals the **sum of distances to values on the right (RHS)**.

$$\sum (\text{Distances to left}) = \sum (\text{Distances to right}) \Rightarrow \sum (\bar{x} - x_L) = \sum (x_R - \bar{x})$$



Key Insight: Is mean always the midpoint of extremes? **No!** Only when dots are perfectly symmetrical. Otherwise, mean shifts so that the **sum of all left-side distances equals right-side distances**. Because shifting this line to the right increases all left distances while decreasing right distances, there is **strictly ONLY ONE** such center.

HANDWRITTEN WORKED EXAMPLE 1

Q: Given dots at 6, 7, 8. Check if Mean = 7 balances the plot.

Step 1: Distance to left = $|7 - 6| = 1$. Distance to right = $|8 - 7| = 1$.

Step 2: LHS = 1, RHS = 1. Since LHS = RHS, **Mean = 7** is the exact balancing center!

2. Dynamic Properties of Mean: What happens when data changes?

A. Adding / Removing Values

- **Add > Mean** Mean **increases**.
- **Add < Mean** Mean **decreases**.
- **Add = Mean** Mean stays **unchanged**.
- **Remove > Mean** Mean **decreases**.
- **Remove < Mean** Mean **increases**.
- **Remove = Mean** Mean remains **same**.

B. Uniform Algebraic Shifts

- **Add +k to all:** New Mean = $\bar{x} + k$. Relative positions don't change!
- **Subtract -k from all:** New Mean = $\bar{x} - k$.
- **Multiply each by m:** New Mean = $m \times \bar{x}$.
- **Divide each by d:** New Mean = $\bar{x} \div d$.

3. Mean and Median from Frequency Tables

When data items repeat, computing individually is tedious. We use **frequencies (f)**.

Weighted Mean = $(\sum f \times x) \div (\sum f)$ | **Median Position** = $(N + 1) \div 2$ (if N odd) or avg of $(N/2)$ and $(N/2 + 1)$ (if N even)

CLASSROOM EXAMPLE: FAMILY SIZE OF 36 STUDENTS

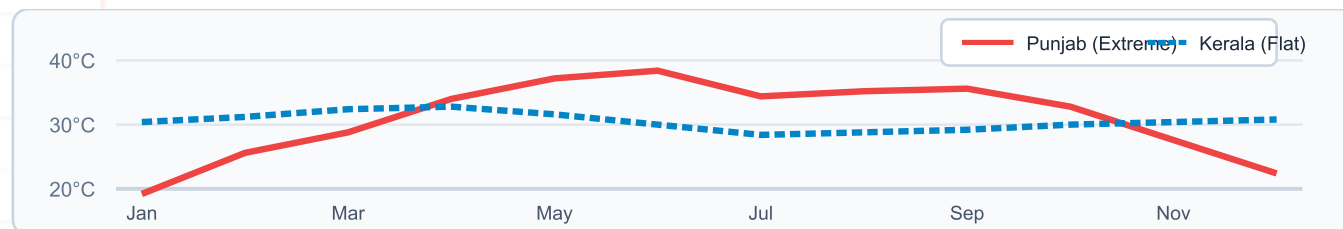
Let us compute both Mean and Median efficiently using cumulative count:

Family Size (x)	Frequency (f)	Product (f × x)	Cumulative Count (Positions)
3	3	9	1 to 3
4	11	44	4 to 14
5	9	45	15 to 23 (Contains 18th & 19th!)
6	7	42	24 to 30
7	3	21	31 to 33
8, 9, 10	1 + 1 + 1 = 3	8 + 9 + 10 = 27	34 to 36
Total	N = $\sum f = 36$	$\sum(f \times x) = 188$	Mean = $188/36 = 5.22$ Median = 5

How Median is found without expanding 36 numbers: Total N = 36 (even). The middle positions are $36/2 = 18$ th and 19th. Since positions 15 to 23 all hold the number 5, both the 18th and 19th numbers are 5. Average = 5.

4. Visualising Trends: Line Graphs & Infographics

A **Line Graph** connects sequential data points with straight line segments. While bar graphs compare distinct categories, line graphs are ideal for **continuous change over time** (temperature, growth, monthly rainfall, sleep patterns).



How to Read Line Slopes

- **Steep upward slope (/):** Rapid growth / increase in values.
- **Steep downward slope (\):** Rapid drop / decline.
- **Horizontal line (—):** Stable, steady rate (no change).

Activity & Time Strips

A strip of 48 blocks represents 24 hours (each block = **30 minutes**). Helps visualize personal schedules, sleep distributions, and screen time.

5. Digital Spreadsheets: Quick Formulas

A spreadsheet uses a grid of **Columns (letters A, B, C...)** and **Rows (numbers 1, 2, 3...)** intersecting at **Cells** (e.g., E5 = Column E, Row 5).

Task	Spreadsheet Syntax	What it Calculates
Sum of a row	=SUM(B3:G3)	Adds values from cell B3 through G3
Average of a column	=AVERAGE(D2:D25)	Calculates mean of all cells from D2 to D25
Range of cells	Start_Cell:End_Cell	Defines a rectangular block or line of cells

6. Chapter Practice Questions (Self-Assessment)

Solve these questions independently before checking the handwritten solutions on Page 5.

Q1. [Natural Numbers Mean] Calculate the mean of: (i) The first 50 natural numbers, (ii) The first 50 odd numbers, (iii) The first 50 multiples of 4. What patterns do you notice?

Q2. [The Balance Act / Missing Dot] A dot plot contains the values 4, 7, 8, 8, 9, 9, 9, 9, 9, 11, and one missing dot. If the arithmetic mean of all dots is exactly 9, find the exact position of the missing dot using the balance principle.

Q3. [Shoe-Height Uniform Shift] Shreyas measured the heights of all 24 students in his class and reported an average of 150.2 cm. Later, it was discovered that everyone wore shoes adding 1 cm to their height. Do we need to remeasure everyone? What is the true average height?

Q4. [Smudged Weight Record] Coach Balwan listed the weights of 10 wrestlers: 42, 40, 39, 33, 48, 38, 42, 35, 32, and one smudged weight w kg. The known mean is 39.2 kg. Find the smudged value w .

Q5. [Harvest Correction] A farmer calculated an average of 25.6 coconuts per tree across 15 trees. Later, one tree's yield was found to have been recorded as 3 coconuts higher than its actual yield. Find the corrected average.

Q6. [Median Stability Analysis] For the dataset: 8, 10, 19, 23, 26, 34, 40, 41, 41, 48, 51, 55, 70, 84, 91, 92:
(i) Find the median.
(ii) What single number can be inserted so the median remains unchanged?
(iii) What happens if two numbers, one $<$ median and one $>$ median, are inserted?

Q7. [Find the Unknown 'y'] The mean of 8 numbers: 8, 13, 10, 4, 5, 20, y , 10 is 10.375. Find the value of y .

Q8. [3×3 Mean Grid Magic] Fill a 3×3 square grid with 9 distinct numbers such that the average of every row, every column, and both diagonals is 10.

Q9. [Algebraic Proof] Is the average of any two even numbers always even? Prove or disprove using algebra.

Q10. [Weight Changes] In a health club of adults, the mean weight was 65.3 kg and the median was 67 kg. This month, one person lost 2 kg while two other members each gained 1 kg. What happens to the mean? Can we definitively determine the new median?

7. Detailed Step-by-Step Solutions & Answers

Ans 1: (i) Sum of first n natural numbers = $n(n+1)/2$. For $n=50$: Sum = $50 \times 51 / 2 = 1275$. Mean = $1275 \div 50 = \mathbf{25.5}$. (Exactly halfway between 1 and 50).

(ii) First 50 odd numbers (1, 3, 5, ... 99): Sum = $n^2 = 50^2 = 2500$. Mean = $2500 \div 50 = \mathbf{50}$. (Mean equals n).

(iii) Multiples of 4: $4 \times (1, 2, \dots 50)$. By multiplication property: New Mean = $4 \times 25.5 = \mathbf{102}$.

Ans 2: Mean = 9. Let unknown dot be x .

• Dots to the left of 9: at 4 (dist = 5), at 7 (dist = 2), at 8 (dist = 1, two dots $\Rightarrow 2 \times 1 = 2$).

Sum of known LHS distances = $5 + 2 + 2 = \mathbf{9}$.

• Dots to the right of 9: at 11 (dist = 2, one dot $\Rightarrow 2$).

Known RHS distances = $\mathbf{2}$.

To balance: RHS needs $(9 - 2) = 7$ units of distance. Therefore, missing dot must be at $9 + 7 = \mathbf{16}$!

Ans 3: (i) No need to remeasure! Since every student's recorded height was inflated by exactly 1 cm, subtracting 1 cm from all values shifts the mean down by 1 cm.

(ii) Correct Average = $150.2 \text{ cm} - 1.0 \text{ cm} = \mathbf{149.2 \text{ cm}}$ (Option d).

Ans 4: Total weight = $10 \times 39.2 = 392 \text{ kg}$. Known sum = $42 + 40 + 39 + 33 + 48 + 38 + 42 + 35 + 32 = 349 \text{ kg}$.

Smudged weight $w = 392 - 349 = \mathbf{43 \text{ kg}}$.

Ans 5: Initial recorded total coconuts = $15 \times 25.6 = 384$.

Corrected total = $384 - 3 = 381$ coconuts.

Corrected average = $381 \div 15 = \mathbf{25.4 \text{ coconuts per tree}}$.

Ans 6: (i) Total count = 16 (even). 8th value = 41, 9th value = 41. Median = $(41 + 41)/2 = \mathbf{41}$.

(ii) Inserting $\mathbf{41}$ keeps median exactly 41 (new count 17, 9th value is 41).

(iii) Inserting any two numbers such that one is ≤ 41 and the other is ≥ 41 keeps the median unchanged at 41.

Ans 7: Total Sum = $8 \times 10.375 = 83$.

Sum of given 7 numbers = $8 + 13 + 10 + 4 + 5 + 20 + 10 = 70$.

Value of $y = 83 - 70 = \mathbf{13}$.

Ans 8: For each row/column/diagonal of 3 numbers to have an average of 10, each triplet must sum to $3 \times 10 = 30$.

The center cell must be 10. By centering a 3×3 magic square with common sum 30:

Row 1: [8, 15, 7] | Row 2: [9, 10, 11] | Row 3: [13, 5, 12] (All rows, columns, and diagonals sum to 30, so mean is 10!).

Ans 9: Not always true! Let two even numbers be $2m$ and $2n$. Average = $(2m + 2n)/2 = m + n$. If one number is 2 and the other is 4: Avg = $(2+4)/2 = 3$ (which is **ODD**). The statement holds only if m and n have the same parity.

Ans 10: • **Mean:** Net change in total sum = $-2 \text{ kg} + 1 \text{ kg} + 1 \text{ kg} = \mathbf{0 \text{ kg}}$. Since total sum is unchanged, the **mean remains exactly 65.3 kg!**

• **Median:** Cannot be definitively known without the exact positions of these 3 individuals relative to 67 kg. (If they are far from the median, median stays 67 kg; if someone crosses the boundary, median might shift slightly).

✓ Chapter 5 "Tales by Dots and Lines" Completely Mastered!

Keep practicing dot plots & line graphs! ☀