CHAPTER 1

NUMBER SYSTEM

REVISION OF KEY CONCEPTS AND FORMULAE

1. A number x is called a rational number, if it can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

2. A number x is called an irrational number, if it cannot be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

3. The decimal expansion of a rational number is either terminating or non-terminating recurring. Thus, we say that a number whose decimal expansion is either terminating or non-terminating recurring is a rational number.

4. In order to convert a pure recurring decimal of the form 0.22222 ... i.e. $0.\bar{2}$, 0.16161616... i.e. $0.\overline{16}$ etc, we may follow the following rule:

   Rule: Write a fraction (number) whose numerator is the recurring digits without the decimal point and denominator has as many 9's as there are different repeating digits.

   For example,

   $$0.\bar{2} = \frac{2}{9}, 0.\overline{16} = \frac{16}{99}, 0.\overline{247} = \frac{247}{999} \text{ etc.}$$

5. If the denominator of a rational number is of the form $2^m \times 5^n$, where m, n are non-negative integers, then its decimal representation is terminating and terminates after k places of decimals, where k is the larger of m and n.

6. If the denominator of a rational number is not expressible in the form $2^m \times 5^n$, where m, n are non-negative integers, then its decimal representation is non-terminating repeating.

7. The decimal expansion of an irrational number is non-terminating non-recurring. Thus, we say that a number whose decimal expansion is non-terminating non-recurring is an irrational number.

8. All the rational and irrational numbers taken together make up the collection of real numbers.

9. A real number is either rational or irrational.

10. There is a real number corresponding to every point on the number line. Also, corresponding to every real number there is a point on the number line.

11. If r is rational and s is irrational, then $r + s$, $r - s$, rs and $\frac{r}{s}$ are irrational numbers $s \neq 0$.

12. If n is a natural number other than a perfect square, then $\sqrt{n}$ is an irrational number.

13. For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line by using the following steps:
    (i) Obtain the positive real number x (say)
    (ii) Draw a line and mark a point A on it.
    (iii) Mark a point B on the line such that $AB = x$ units.
    (iv) From point B mark a distance of 1 unit and mark the new point as C.
    (v) Find the mid-point of AC and mark the point as O.
    (vi) Draw a circle with centre O and radius OC.
    (vii) Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Length BD is equal to $\sqrt{x}$.

14. If a, b are two distinct positive real numbers, then $\frac{a+b}{2}$ and $\sqrt{ab}$ are real numbers between a and b.

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *Every rational number is*
(a) *a natural number* (b) *an integer* (c) *a real number* (d) *a whole number*
Ans. (c)
[NCERT EXEMPLAR]

SOLUTION We observe $\frac{2}{3}$ is a rational number but it is neither a natural number nor an integer nor a whole number. But, it is a real number. In fact every rational number is a real number. Hence, option (c) is correct.

EXAMPLE 2 *Between two rational numbers*
(a) *there is no rational number* (b) *there is exactly one rational number*
(c) *there are infinitely many rational numbers*
(d) *there are only rational numbers and no irrational numbers.*
Ans. (c)
[NCERT EXEMPLAR]

SOLUTION Between two distinct rational numbers there are infinitely many rational as well as irrational numbers. In fact, this holds for any two distinct real numbers. Hence, option (c) is correct and remaining are false.

EXAMPLE 3 *The decimal representation of a rational number cannot be*
(a) *terminating* (b) *non-terminating*
(c) *non-terminating repeating* (d) *non-terminating non-repeating*
Ans. (d)

SOLUTION The decimal representation of rational number $\frac{2}{5}$ is 0.4, which is terminating. So, option (a) is not true. The decimal representation of $\frac{2}{3}$ is 0.66666... which is non-terminating repeating. So, option (b) and (c) are not true.
A rational number can not have non-terminating non-repeating decimal representation.
Hence, option (d) is correct.

EXAMPLE 4 *The product of any two irrational numbers is*
(a) *always an irrational number* (b) *always a rational number*
(c) *always an integer* (d) *sometimes rational, sometimes irrational.*

Ans. (d)

SOLUTION We find that the product of irrational numbers $\sqrt{3}$ and $\frac{2}{5}\sqrt{3}$ is $\frac{6}{5}$, which is a rational number. So, product of two irrational numbers need not be always an irrational number. The product of $\sqrt{3}$ and $2+\sqrt{3}$ is $2\sqrt{3}+3$, which is an irrational number. So, the product of two irrational numbers need not always be a rational number. The product of irrational numbers $\frac{1}{3}+\sqrt{2}$ and $\frac{1}{3}-\sqrt{2}$ is $-\frac{5}{3}$ which is not an integer. So, option (c) is incorrect. In fact, the product is sometimes rational and sometimes irrational.

EXAMPLE 5 *The decimal expansion of the number* $\sqrt{2}$ *is*

(a) *a finite decimal* (b) 1.41421

(c) *non-terminating recurring* (d) *non-terminating non-recurring*

Ans. (d) [NCERT EXEMPLAR]

SOLUTION $\sqrt{2}$ is an irrational number. So, its decimal expansion is non-terminating non-recurring. Hence, option (d) is correct.

EXAMPLE 6 *Which of the following rational numbers is equivalent to a decimal that terminates?*

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{8}$ (d) $\frac{5}{6}$

Ans. (c)

SOLUTION If the denominator of a rational number is not expressible in the form $2^m \times 5^n$, where m, n are non-negative integers, then its decimal representation is non-terminating and recurring. Therefore, $\frac{1}{3}, \frac{2}{3}$ and $\frac{5}{6} = \frac{5}{2\times 3}$ have non-terminating recurring decimal representation. Only $\frac{3}{8} = \frac{3}{2^3 \times 5^0}$ has terminating decimal representation.

EXAMPLE 7 *The representation of* $1.\overline{3}$ *in the form* $\frac{p}{q}$ *is*

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{5}{4}$ (d) *none of these*

Ans. (a)

SOLUTION $1.\overline{3} = 1 + 0.\overline{3} = 1 + \frac{3}{9} = 1 + \frac{1}{3} = \frac{4}{3}$ [Using rule given on page 1]

EXAMPLE 8 *When* $0.\overline{001}$ *is expressed in the form* $\frac{p}{q}$, *where p and q are integers not having any common factor except 1 then q is equal*

(a) 9 (b) 99 (c) 999 (d) 1000

Ans. (c)

SOLUTION Using rule given on page 1, we obtain $0.\overline{001} = \frac{1}{999}$.

Hence, $q = 999$.

EXAMPLE 9 *The value of* 0.9999 ... *when expressed as a fraction in the simplest form is*

(a) $\frac{1}{9}$ (b) $\frac{8}{9}$ (c) 1 (d) $\frac{10}{9}$

Ans. (c)

SOLUTION $0.9999\ldots = 0.\bar{9} = \frac{9}{9} = 1.$

ALITER Let $x = 0.9999\ldots$. Then,

$$10x = 9.9999\ldots$$

$$\Rightarrow \quad 10x - x = (9.9999\ldots) - (0.9999\ldots) \Rightarrow 9x = 9 \Rightarrow x = 1$$

EXAMPLE 10 *The value of* $2.\overline{36} + 0.\overline{23}$ *when expressed in the simplest form is*

(a) $\frac{257}{99}$ (b) $\frac{238}{99}$ (c) $\frac{247}{99}$ (d) $\frac{275}{99}$

Ans. (a)

SOLUTION $2.\overline{36} + 0.\overline{23} = 2 + 0.\overline{36} + 0.\overline{23} = 2 + \frac{36}{99} + \frac{23}{99} = 2 + \frac{59}{99} = \frac{257}{99}$

EXAMPLE 11 *The simplest form of* $0.12\bar{3}$ *is*

(a) $\frac{41}{330}$ (b) $\frac{37}{300}$ (c) $\frac{41}{333}$ (d) none of these

Ans. (b)

SOLUTION Let $x = 0.12\bar{3}$. Then,

$$100x = 12.\bar{3} \Rightarrow 100x = 12 + 0.\bar{3} \Rightarrow 100x = 12 + \frac{3}{9} \Rightarrow 100x = 12 + \frac{1}{3} \Rightarrow 100x = \frac{37}{3} \Rightarrow x = \frac{37}{300}$$

EXAMPLE 12 *The value of* $2.\overline{45} + 0.\overline{36}$ *in the simple form is*

(a) $\frac{67}{33}$ (b) $\frac{24}{11}$ (c) $\frac{31}{11}$ (d) $\frac{167}{110}$

Ans. (c)

SOLUTION $2.\overline{45} + 0.\overline{36} = 2 + 0.\overline{45} + 0.\overline{36} = 2 + \frac{45}{99} + \frac{36}{99} = 2 + \frac{5}{11} + \frac{4}{11} = \frac{31}{11}$

EXAMPLE 13 *If the number* $x = 1.242424\ldots$ *is expressed in the simplest form* $\frac{p}{q}$*, then* $p + q$ *equals*

(a) 41 (b) 53 (c) 72 (d) 74

Ans. (d)

SOLUTION We have,

$$x = 1.242424\ldots$$

$$\Rightarrow \quad x = 1.\overline{24} = 1 + 0.\overline{24} = 1 + \frac{24}{99} = 1 + \frac{8}{33} = \frac{41}{33}$$

$$\therefore \quad p = 41 \text{ and } q = 33 \Rightarrow p + q = 41 + 33 = 74$$

EXAMPLE 14 *A rational number between* $\sqrt{2}$ *and* $\sqrt{3}$ *is*

(a) $\frac{\sqrt{2} + \sqrt{3}}{2}$ (b) $\frac{\sqrt{2} \cdot \sqrt{3}}{2}$ (c) 1.5 (d) 1.8

Ans. (c) [NCERT EXEMPLAR]

SOLUTION We know that $\sqrt{2} = 1.41421356\ldots$ and $\sqrt{3} = 1.732050807\ldots$

We find that $\frac{\sqrt{2}+\sqrt{3}}{2}$ and $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$ are irrational numbers. So, options (a) and (b) are incorrect. Clearly, 1.5 is a rational number between $\sqrt{2}$ and $\sqrt{3}$.

EXAMPLE 15 *Which of the following is equivalent to* $0.5\overline{782}$?

(a) $\frac{5770}{9990}$ (b) $\frac{5772}{9990}$ (c) $\frac{5777}{9990}$ (d) $\frac{5782}{9990}$

Ans. (c)

SOLUTION Let $x = 0.5\overline{782}$

Then, $10x = 5.\overline{782}. \Rightarrow 10x = 5 + \frac{782}{999} \Rightarrow 10x = \frac{5777}{999} \Rightarrow x = \frac{5777}{9990}$

EXAMPLE 16 *An irrational number between* $\frac{1}{7}$ *and* $\frac{2}{7}$ *is*

(a) $\frac{1}{2}\left(\frac{1}{7}+\frac{2}{7}\right)$ (b) $\frac{1}{7}\times\frac{2}{7}$ (c) $\sqrt{\frac{1}{7}\times\frac{2}{7}}$ (d) *none of these*

Ans. (c)

SOLUTION We know that $\frac{a+b}{2}$ and $\sqrt{ab}$ are real numbers between any two real numbers. So, $\frac{1}{2}\left(\frac{1}{7}+\frac{2}{7}\right)$ and $\sqrt{\frac{1}{7}\times\frac{2}{7}}$ are real numbers between $\frac{1}{7}$ and $\frac{2}{7}$.

We observe that $\frac{1}{2}\left(\frac{1}{7}+\frac{2}{7}\right)$ and $\frac{1}{7}\times\frac{2}{7}$ are rational numbers.

So, options (a) and (b) are incorrect. Clearly, $\sqrt{\frac{1}{7}\times\frac{2}{7}} = \frac{\sqrt{2}}{7}$ is an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$. So, option (c) is incorrect.

EXAMPLE 17 *An irrational number between 5 and 6 is*

(a) $\frac{1}{2}(5+6)$ (b) $\sqrt{5+6}$ (c) $\sqrt{5\times 6}$ (d) *none of these*

Ans. (c)

SOLUTION We observe that $\frac{1}{2}(5+6)$ is a rational number between 5 and 6. So, option (a) is in correct.

$\sqrt{5+6} = \sqrt{11} = 3.3166247\ldots$ is an irrational number not lying between 5 and 6.

$\sqrt{5\times 6}$ is an irrational number lying between $\sqrt{5}$ and $\sqrt{6}$. Hence, option (c) is correct.

EXAMPLE 18 *An irrational number between* $\sqrt{2}$ and $\sqrt{3}$ *is*

(a) $\sqrt{2}+\sqrt{3}$ (b) $\sqrt{2}\times\sqrt{3}$ (c) $\sqrt{\sqrt{2+3}}$ (d) $\sqrt{\sqrt{2}\times\sqrt{3}}$

Ans. (d)

SOLUTION $\sqrt{ab}$ is a real number between two real numbers a and b. Therefore, $\sqrt{\sqrt{2}\times\sqrt{3}}$ is an irrational number between $\sqrt{2}$ and $\sqrt{3}$.

EXAMPLE 19 *Which of the following is an irrational number?*

(a) $\frac{\sqrt{12}}{\sqrt{3}}$ (b) $\frac{\sqrt{18}}{\sqrt{2}}$ (c) $\frac{\sqrt{42}}{\sqrt{7}}$ (d) $\frac{\sqrt{45}}{\sqrt{5}}$

Ans. (c)

SOLUTION Using the result $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, we obtain

$$\frac{\sqrt{12}}{\sqrt{3}} = \frac{\sqrt{42}}{\sqrt{7}} = \sqrt{4} = 2, \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3, \frac{\sqrt{42}}{\sqrt{7}} = \sqrt{\frac{42}{7}} = \sqrt{6} \text{ and } \frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3.$$

Clearly, $\frac{\sqrt{42}}{\sqrt{7}} = \sqrt{6}$ is an irrational number.

EXAMPLE 20 *On a number line, $\frac{3}{\sqrt{18}}$ is halfway located between 0 and $\sqrt{a}$. What is the value of a?*

(a) 2 (b) 4 (c) 6 (d) 8

Ans. (a)

SOLUTION A number located halfway between 0 and $\sqrt{a}$ on the number line is $\frac{0+\sqrt{a}}{2} = \frac{1}{2}\sqrt{a}$.

$$\therefore \quad \frac{1}{2}\sqrt{a} = \frac{3}{\sqrt{18}} \Rightarrow \frac{1}{2}\sqrt{a} = \frac{3}{3\sqrt{2}} \Rightarrow \frac{1}{2}\sqrt{a} = \frac{1}{\sqrt{2}} \Rightarrow \sqrt{a} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow a = 2$$

EXAMPLE 21 *Which of the following statements is true?*

(a) $\frac{\sqrt{2}}{3}$ *is a rational number*

(b) *There are infinitely many integers between any two integers*

(c) *Number of rational numbers between 15 and 18 is finite.*

(d) *There are numbers which cannot be written in the form $\frac{p}{q}, q \neq 0, p, q$ both are integers.*

Ans. (d)

SOLUTION $\frac{\sqrt{2}}{3} = \frac{1}{3} \times \sqrt{2}$ is an irrational number. So, statement in option (*a*) is not true.
There is no integer between integers 7 and 8. So, statement is option (b) is not true.
There are infinitely many rational numbers between any two distinct integers. So, statement in option (c) is not true.

$\sqrt{\frac{3}{2}}$ can be written as $\frac{\sqrt{3}}{\sqrt{2}}$ where $\sqrt{3}$ and $\sqrt{2}$ are not integers. So, statement in option (d) is true.

EXAMPLE 22 *Which of the following statements is true?*

(a) *The square of an irrational number is always rational*

(b) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers

(c) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}, q \neq 0$ and so it is a rational number.

(d) If x is rational and y is an irrational number then xy is not necessarily irrational number.

Ans. (d)

SOLUTION Consider irrational number $x = \sqrt{\sqrt{3}}$. We find that $x^2 = \sqrt{3}$ is also an irrational number. So, statement in option (a) many not be true.

$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$ is a rational number. So, statement in option (b) is not true.

$\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \frac{\sqrt{5}}{1}$ is an irrational number. So, statement in option (c) is not true.

We find that 0 is rational and $\sqrt{3}$ is an irrational number. But, $0 \times \sqrt{3} = 0$ is rational. Hence, statement in option (d) is true.

CASE STUDY BASED

EXAMPLE 23 *Ravish and Aarushi dedcided to visit world book fair which is organised every year. During their visit Aarushi was fascinated by the cover page of a book with π / e written on it. π and e are mathematical constants. In Euclidean geometry π is defined as the ratio of a circle's circumference to its diameter. It is also referred to as Archimede's constant. The constant e is known as Euler's number and it is the limiting value of $\left(1 + \frac{1}{n}\right)^n$ as n approches infinity. Using the knowledge of rational and irrational numbers answer the following questions.*

(i) *π represents*
(a) *an integer* (b) *a rational number*
(c) *an irrational number* (d) *a natural number*

(ii) *e represents*
(a) *a natural number* (b) *an integer*
(c) *a rational number* (d) *an irrational number*

(iii) *The product of any two irrational numbers is*
(a) *always an irrational number* (b) *not necessarily an irrational number*
(c) *never an irrational number* (d) *always an integer*

(iv) *A rational number between $\sqrt{2}$ and $\sqrt{3}$ is*
(a) $\frac{\sqrt{3} - \sqrt{2}}{2}$ (b) $\frac{\sqrt{3} + \sqrt{2}}{2}$ (c) $1.\bar{6}$ (d) $0.\bar{2} + 0.\bar{3}$

(v) *Which of the following is true?*
(a) $\pi = \frac{22}{7}$ (b) $e = 2.71$
(c) *π and e are irrational numbers*
(d) *The sum of two irrational numbers is an irrational number.*

SOLUTION (i) Ans. (c): π is an irrational number its value is 3.1415926538 ...

(ii) Ans. (d): e is an irrational number and its value is 2.71828182845 ...

(iii) Ans. (b): The product of irrational numbers $2 + \sqrt{3}$ and $2 - \sqrt{3}$ is 1, which is rational. However, the product of rational numbers $(2 + \sqrt{3})$ and $\sqrt{3}$ is $2\sqrt{3} + 3$ which is an irrational number. Hence, the product of two irrational numbers is sometimes rational and sometimes irrational.

(iv) Ans. (c): $\frac{\sqrt{3} - \sqrt{2}}{2}$ and $\frac{\sqrt{3} + \sqrt{2}}{2}$ are irrational numbers. So, options (a) and (b) are not correct.

We find that $0.\bar{2} + 0.\bar{3} = 0.\bar{5}$, which does not lie between $\sqrt{2}$ and $\sqrt{3}$.

We have $\sqrt{2} = 1.412...$ and $\sqrt{3} = 1.736...$

Clearly, $\sqrt{2} < 1.\bar{6} < \sqrt{3}$. Hence, option (c) is correct.

(v) Ans. (c): Values of π and e given in options (a) and (b) respectively are rational numbers whereas both π and e are irrational numbers. The sum of two irrational numbers is not necessarily an irrational number. Hence, option (c) is true.

EXAMPLE 24 *Aarushi and Avni are playing with match-sticks by making different geometrical and other figures. Avni kept one match-stick horizontally and then two match-sticks vertically as shown in Figure 1.1 and then asks Aarushi to join the open ends of horizontally and vertically placed strings by a thread. Avni's eleder sister Mira comes and ask them to find the length of the thread if each matchstick is of unit length.*

Fig. 1.1

Aarushi replies that the length of the thread can be found by using Pythagoras Theorem and it is equal to $\sqrt{1^2 + 2^2} = \sqrt{4+1} = \sqrt{5}$ *units using your knowledge about numbers, answer the following questions.*

(i) $\sqrt{5}$ *is*
   (a) *a rational number* (b) *an irrational number*
   (c) *an integer* (d) *a whole number*

(ii) *The decimal representation of an irrational number is*
   (a) *terminating* (b) *non-terminating recurring*
   (c) *non-terminating non-recurring* (d) *not possible*

(iii) *The decimal representation of a rational number cannot be*
   (a) *terminating* (b) *non-terminating*
   (c) *non-terminating repeating* (d) *non-terminating non-repeating*

(iv) *the sum of any two irrational numbers is*
   (a) *always an irrational number* (b) *always a rational number*
   (c) *always an integer* (d) *sometimes rational, sometimes irrational*

SOLUTION (i) Ans. (b): $\sqrt{5}$ is an irrational number, because the square root of a prime number is an irrational number.

(ii) Ans. (c)

(iii) Ans. (d)

(iv) Ans. (a).

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 25 *Statement-1 (Assertion):* 0.7 *and* $0.00323232\ldots$ *are rational numbers.*
Statement-2 (Reason): If the decimal expansion of a real number is either terminating or non-terminating recurring it is a rational number.

Ans. (a)

SOLUTION Statement-2, being the definition of a rational number, is true. Statement-1 is also true and statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 26 *Statement-1 (Assertion):* *π is an irrational number.*
Statement-2 (Reason): *Euler's constant e is an irrational number.*

Ans. (b)

SOLUTION π and e both are irrational numbers. It is not correct to say that π is irrational because e is irrational. Thus, both the statements are true and statement-2 is not a correct explanation for statement-1. So, options (b) is correct.

EXAMPLE 27 *Statement-1 (Assertion):* $\frac{13}{20}, \frac{14}{20}$ *and* $\frac{15}{20}$ *are three rational numbers between* $\frac{1}{2}$ *and* $\frac{4}{5}$
Statement-2 (Reason): *A rational number between two rational numbers p and q is* $\frac{1}{2}(p+q)$.

Ans. (a)

SOLUTION Statement-2 is true. Using statement-2, a rational number between $\frac{1}{2}$ and $\frac{4}{5}$ is $\frac{1}{2}\left(\frac{1}{2}+\frac{4}{5}\right)=\frac{13}{20}$. But, $\frac{4}{5}=\frac{16}{20}$. So, rational numbers between $\frac{13}{20}$ and $\frac{16}{20}$ are $\frac{14}{20}$ and $\frac{15}{20}$.

Hence, $\frac{13}{20}, \frac{14}{20}$ and $\frac{15}{20}$ are rational numbers between $\frac{1}{2}$ and $\frac{4}{5}$. Hence, statement-1 is true and statement-2 is correct explanation for statement-1. So, option (a) is correct.

EXAMPLE 28 *Statement-1 (Assertion):* *If x and y are rational and irrational numbers respectively, then $x + y$ is an irrational number.*
Statement-2 (Reason): *If x and y are two irrational numbers, then $x + y$ is an irrational number.*

Ans. (c)

SOLUTION If possible let $x + y$ be a rational number equal to z, when x is rational and y is irrational. Then,

$$z = x + y \Rightarrow z - x = y \Rightarrow y \text{ is rational} \qquad [\because z - x \text{ rational}]$$

This is a contradiction to fact that y is irrational. Hence, $x + y$ is irrational. So, statement-2 is true. $x = \sqrt{2}$ and $y = 3 - \sqrt{2}$ are irrational numbers but $x + y = 3$ is a rational number. So, statement-2 need not be true. Thus, statement-1 is true and statement-2 is not true. Hence, option (c) is correct.

EXAMPLE 29 *Statement-1 (Assertion):* *There are infinitely many rational numbers between any two integers.*
Statement-2 (Reason): *The square of an irrational number is always a rational number.*

Ans. (c)

SOLUTION Between any two integers a and b, there is a rational number $\frac{a+b}{2}$. Between a and $\frac{a+b}{2}$, there is a rational number $\frac{a+\frac{a+b}{2}}{2}$. Continuing in this manner, we can find infinitely many rational numbers between a and b. So, statement-1 is true.

Let $x = \sqrt{\sqrt{3}}$, be an irrational number, then $x^2 = \sqrt{3}$ is an irrational number. So, square of an irrational number is not necessarily a. rational number. Thus, statement-2 is not true.
Hence, option (c) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. Which of the following is true about $x = 0.\bar{3}$?
   (a) x is a rational number, because x can be expressed in the form $\frac{p}{q}$, by solving the equation $10x = 3 + x$.
   (b) x is a rational number because x can be expressed in the form $\frac{p}{q}$ by solving the equation $10x = 3 - x$.
   (c) x is an irrational number because x can be expressed in the form $\frac{p}{q}$ by solving the equation $10x = 3 + x$.
   (d) x is an irrational number because x can be expressed in the form $\frac{p}{q}$ by solving the equation $10x = 3 - x$

2. If the some of the rational numbers between 7 and 11 are written in the form $\frac{m}{6}$, then integer values of m lie between
   (a) 42 and 60 (b) 42 and 66 (c) 42 and 77 (d) 48 and 60
3. Which of the following statements is / are correct?
   (i) Every integer is a rational number
   (ii) Every rational number is an integer
   (iii) A real number is either rational or irrational number
   (iv) Every whole number is a natural number.
   (a) (ii) (b) (iii) (c) (i) and (iii) (d) all of these
4. The decimal expansion of a rational number is
   (a) terminating or non-terminating non-repeating
   (b) terminating or non-terminating repeating
   (c) terminating and repeating
   (d) none of these
5. A number is an irrational if and only if its decimal representation is
   (a) non-terminating (b) non-terminating and repeating
   (c) non-terminating and non-repeating (d) terminating
6. Which of the following is an irrational number?
   (a) 0.13 (b) $0.13\overline{15}$ (c) $0.\overline{1315}$ (d) 0.301323100523 ...
7. The value of $0.\bar{4}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is
   (a) $\frac{4}{9}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$
8. The simplest form of $1.\bar{6}$ is
   (a) $\frac{8}{5}$ (b) $\frac{5}{3}$ (c) $\frac{833}{500}$ (d) $\frac{7}{6}$
9. Which of the following is a rational number?
   (a) π (b) $\sqrt{5}$
   (c) 0.101001000100001 ... (d) 0.835835835 ...
10. How many digits are there in the repeating block of digits in the decimal expansion of $\frac{17}{7}$?
    (a) 16 (b) 6 (c) 26 (d) 7
11. The decimal expansion that a rational number cannot have is
    (a) 0.25 (b) $0.25\overline{28}$ (c) $0.\overline{2528}$ (d) 0.5030030003 ...

12. Which of the following statements is true?

(a) π and $\frac{22}{7}$ are both rationals (b) π and $\frac{22}{7}$ are both irrationals

(c) π is rational and $\frac{22}{7}$ is irrational (d) π is irrational and $\frac{22}{7}$ is rational

13. Which of the following numbers is irrational?

(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{1250}}{\sqrt{8}}$ (c) $\sqrt{8}$ (d) $\frac{\sqrt{24}}{\sqrt{6}}$

14. Which of the following is a rational number?

(a) $\sqrt{3}+1$ (b) π (c) $2\sqrt{3}$ (d) 0

15. A rational number between -3 and 3 is

(a) 0 (b) -4.3 (c) -3.4 (d) 1.101100110001 ...

16. Which of the following is an irrational number?

(a) 3.14 (b) 3.141414 ...

(c) 3.1444444 ... (d) 3.14114111411114 ...

17. The product of a non-zero rational number with an irrational number is

(a) irrational number (b) rational number (c) whole number (d) natural number

18. The sum of $0.\bar{2}$ and $0.\bar{5}$ is

(a) $\frac{7}{10}$ (b) $\frac{7}{9}$ (c) $\frac{7}{99}$ (d) $\frac{3}{10}$

19. The value of $0.\bar{2} \times 0.\bar{5}$ is

(a) 1 (b) $\frac{10}{9}$ (c) $\frac{10}{81}$ (d) $\frac{10}{99}$

20. There is a number x sum that x^2 is irrational but x^4 is rational. Then x can be

(a) $\sqrt{5}$ (b) $\sqrt{2}$ (c) $\sqrt[3]{2}$ (d) $\sqrt[4]{5}$

21. Which one of the following is a correct statement?

(a) Decimal expansion of a rational number is terminating
(b) Decimal expansion of a rational number is non-terminating
(c) Decimal expansion of an irrational number is terminating
(d) Decimal expansion of an irrational number is non-terminating and non-repeating

22. Which one of the following statements is true?

(a) The sum of two irrational numbers is always an irrational number
(b) The sum of two irrational numbers is always a rational number
(c) The sum of two irrational numbers may be a rational number or an irrational number
(d) The sum of two irrational numbers is always an integer

23. Which of the following is a correct statement?

(a) Sum of two irrational numbers is always irrational
(b) Sum of a rational and irrational number is always an irrational number
(c) Square of an irrational number is always a rational number
(d) Sum of two rational numbers can never be an integer

24. Which of the following statements is true?

(a) Product of two irrational numbers is always irrational
(b) Product of a non-zero rational and an irrational number is always irrational
(c) Sum of two irrational numbers can never be irrational
(d) Sum of an integer and a rational number can never be an integer

25. Which of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{4}{5}$ (c) $\sqrt{7}$ (d) $\sqrt{81}$

26. Which of the following is irrational?

(a) 0.14 (b) $0.14\overline{16}$ (c) $0.\overline{1416}$ (d) 0.1014001400014..

27. Which of the following is rational?

(a) $\sqrt{3}$ (b) π (c) $\frac{4}{0}$ (d) $\frac{0}{4}$

28. The number 0.318564318564318564......... is:
(a) a natural number (b) an integer
(c) a rational number (d) an irrational number

29. If n is a natural number, then $\sqrt{n}$ is
(a) always a natural number
(b) always a rational number
(c) always an irrational number
(d) sometimes a natural number and sometimes an irrational number

30. Which of the following numbers can be represented as non-terminating, repeating decimals?

(a) $\frac{39}{24}$ (b) $\frac{3}{16}$ (c) $\frac{3}{11}$ (d) $\frac{137}{25}$

31. Every point on a number line represents
(a) a unique real number (b) a natural number
(c) a rational number (d) an irrational number

32. Which of the following is irrational?

(a) 0.15 (b) 0.01516 (c) $0.\overline{1516}$ (d) 0.5015001500015..

33. The number $1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(a) $\frac{14}{9}$ (b) $\frac{14}{11}$ (c) $\frac{14}{13}$ (d) $\frac{14}{15}$

34. The number $0.\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(a) $\frac{33}{100}$ (b) $\frac{3}{10}$ (c) $\frac{1}{3}$ (d) $\frac{3}{100}$

35. $0.3\overline{2}$ when expressed in the form $\frac{p}{q}$ (p, q are integers $q \neq 0$), is

(a) $\frac{8}{25}$ (b) $\frac{29}{90}$ (c) $\frac{32}{99}$ (d) $\frac{32}{199}$

36. $23.\overline{43}$ when expressed in the form $\frac{p}{q}$ (p, q are integers $q \neq 0$), is

(a) $\frac{2320}{99}$ (b) $\frac{2343}{100}$ (c) $\frac{2343}{999}$ (d) $\frac{2320}{199}$

37. $0.\overline{001}$ when expressed in the form $\frac{p}{q}$ (p, q are integers, $q \neq 0$), is

(a) $\frac{1}{1000}$ (b) $\frac{1}{100}$ (c) $\frac{1}{1999}$ (d) $\frac{1}{999}$

38. The value of $0.\overline{23} + 0.\overline{22}$ is

(a) $0.\overline{45}$ (b) $0.\overline{43}$ (c) $0.\overline{54}$ (d) 0.45

39. An irrational number between 2 and 2.5 is

(a) $\sqrt{11}$ (b) $\sqrt{5}$ (c) $\sqrt{22.5}$ (d) $\sqrt{12.5}$

40. The number of consecutive zeros in $2^3 \times 3^4 \times 5^4 \times 7$, is

(a) 3 (b) 2 (c) 4 (d) 5

41. The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal, is

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) 3 (d) 30

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

42. Statement-1 (Assertion): $\sqrt{2}$ is an irrational number.
Statement-2 (Reason): The decimal expansion of $\sqrt{2}$ is non-terminating non-recurring.

43. Statement-1 (Assertion): The sum of any two irrational numbers is an irrational number.
Statement-2 (Reason): There are two irrational numbers whose sum is a rational number.

44. Statement-1 (Assertion): The product of any two irrational numbers is an irrational number.
Statement-2 (Reason): There are two irrational numbers whose product is not an irrational number.

45. Statement-1 (Assertion): $\sqrt{3}$ is an irrational number.
Statement-2 (Reason): The square root of a positive integer which is not a perfect square is an irrational number.

46. Statement-1 (Assertion): $\sqrt{2}$ is an irrational number.
Statement-2 (Reason): The sum of a rational number and an irrational number is an irrational number.

47. Statement-1 (Assertion): There are two rational numbers whose sum and product both are rationals.
Statement-2 (Reason): There are numbers which cannot be written in the form $\frac{p}{q}, q \neq 0, p, q$ both are integers.

48. Statement-1 (Assertion): The decimal representation of $\frac{3}{8}$ is terminating.

Statement-2 (Reason): If the denominator of a rational number is of the form $2^m \times 5^n$, where m, n are non-negative integers, then its decimal representation is terminating.

ANSWERS

1. (a)	2. (b)	3. (c)	4. (b)	5. (c)	6. (d)	7. (a)
8. (b)	9. (d)	10. (b)	11. (d)	12. (d)	13. (c)	14. (d)
15. (a)	16. (d)	17. (a)	18. (b)	19. (c)	20. (d)	21. (d)
22. (c)	23. (b)	24. (b)	25. (c)	26. (d)	27. (d)	28. (c)
29. (d)	30. (c)	31. (a)	32. (d)	33. (b)	34. (c)	35. (b)
36. (a)	37. (d)	38. (a)	39. (b)	40. (a)	41. (b)	42. (a)
43. (d)	44. (d)	45. (a)	46. (b)	47. (b)	48. (a)	