CHAPTER 2

EXPONENTS OF REAL NUMBERS

REVISION OF KEY CONCEPTS AND FORMULAE

1. Laws of exponents of real numbers: If a, b are positive real numbers and m, n are integers, then

(i) $a^m \times a^n = a^{m+n}$ (Product of powers) (ii) $a^{m_1} \times a^{m_2} \times a^{m_3} \times \ldots \times a^{m_n} = a^{m_1+m_2+m_3+\ldots+m_n}$

(iii) $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$ (Quotient of powers) (iv) $a^0 = 1,\ a \neq 0$

(v) $a^{-n} = \dfrac{1}{a^n}$ and $a^n = \dfrac{1}{a^{-n}}, a \neq 0$ (vi) $(a^m)^n = (a^n)^m = a^{mn}$ (Power of a power)

(vii) $\{(a^m)^n\}^p = a^{mnp} = \{(a^n)^m\}^p = \{(a^p)^m\}^n$ (viii) $(ab)^n = a^n b^n$ (Power of a product)

(ix) $(a_1 \times a_2 \times a_3 \times \ldots \times a_k)^n = a_1{}^n \times a_2{}^n \times a_3{}^n \times \ldots \times a_k{}^n$

(x) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ (Power of a quotient) (xi) $\left(\dfrac{a}{b}\right)^{-1} = \dfrac{b}{a}$

(xii) $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$ (xiii) $a^m = a^n \Rightarrow m = n$, where $a \neq 0, \pm 1$

(xiv) $a^n = b^n \Rightarrow \begin{cases} a = b, & \text{if } n \text{ is odd} \\ a = \pm b, & \text{if } n \text{ is even} \end{cases}$

2. If 'a' is a positive real number and n is a positive integer, then the principal n^{th} root of 'a' is the unique positive real number x such that $x^n = a$. The principal nth root of a is denoted by $a^{1/n}$ or $\sqrt[n]{a}$. Thus, $a^{1/n} = x$ or, $\sqrt[n]{a} = x \Leftrightarrow x^n = a$.

For example,

(i) $\sqrt[3]{\dfrac{8}{27}} = \dfrac{2}{3}$, because $\left(\dfrac{2}{3}\right)^3 = \dfrac{8}{27}$ (ii) $\sqrt[4]{\dfrac{16}{625}} = \dfrac{2}{5}$, because $\left(\dfrac{2}{5}\right)^4 = \dfrac{16}{625}$

REMARK *$\sqrt[n]{a}$ is called a radical or a radical expression. The sign '$\sqrt[n]{\ }$' is called the radical sign and the number under this sign i.e. 'a' is called the radicand. The number in the angular part of the sign i.e. 'n' is called the order of the radical.*

If $a > 0$, $b > 0$ and m, n, p are positive rational numbers, then the following are the laws of radicals:

(i) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ (ii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ (iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt[n]{a^m} = a^{m/n}$ (v) $\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}}$

Using these laws of radicals and the laws of integral exponents of real numbers, we obtain the following laws of rational exponents of positive real numbers.

If a, b are positive real numbers and m, n are rational numbers, then

(i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \div a^n = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$

(iv) $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$ (v) $(ab)^m = a^m b^m$ (vi) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(vii) $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ where m, n are positive integers

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *The value of* $(61^2 - 11^2)^{3/2}$ *is*

(a) 60 (b) 3600 (c) 216000 (d) 50^3

Ans. (c)

SOLUTION $(61^2 - 11^2)^{3/2} = \{(61+11)(61-11)\}^{3/2} = (72 \times 50)^{3/2} = (3600)^{3/2} = (60^2)^{3/2} = 60^{2 \times \frac{3}{2}} = 60^3 = 216000$

EXAMPLE 2 *The value of* $4^{2^{2^{1^{56}}}}$ *is*

(a) $(256)^{30}$ (b) 4^{120} (c) 4^8 (d) 2^8

Ans. (d)

SOLUTION We know that $1^{5^6} = 1$

$\therefore \quad 2^{1^{5^6}} = 2^1 = 2 \Rightarrow 2^{2^{1^{5^6}}} = 2^2 = 4 \Rightarrow 4^{2^{2^{1^{5^6}}}} = 4^4 = (2^2)^4 = 2^8$

EXAMPLE 3 *If* $a^b = 64, b \neq 1$, *then the sum of the greatest possible values of* $\frac{a}{b}$ *and* $\frac{b}{a}$ *is*

(a) $\frac{13}{4}$ (b) 7 (c) 67 (d) 4

Ans. (b)

SOLUTION We have, $a^b = 64, b \neq 1$

$\Rightarrow \quad a^b = 2^6$ or, $a^b = 4^3$ or, $a^b = 8^2$ or, $a^b = 64^1$

$\Rightarrow \quad (a = 2, b = 6)$ or, $(a = 4, b = 3)$ or, $(a = 8, b = 2)$ or, $(a = 64, b = 1)$

$\Rightarrow \quad (a = 2, b = 6)$ or, $(a = 4, b = 3)$ or, $(a = 8, b = 2)$ $[\because b \neq 1]$

$\Rightarrow \quad \frac{a}{b} = \frac{1}{3}, \frac{4}{3}, \frac{8}{2}$ and $\frac{b}{a} = 3, \frac{3}{4}, \frac{1}{4}$

$\Rightarrow$ Greatest value of $\frac{a}{b} = 4$, Greatest value of $\frac{b}{a} = 3$

$\therefore$ Required sum $= 4 + 3 = 7$.

EXAMPLE 4 $(65.61)^{1/8}$ *is equal to*

(a) $\frac{3}{\sqrt[4]{10}}$ (b) 0.3 (c) 0.03 (d) $\frac{3}{\sqrt{10}}$

Ans. (a)

SOLUTION $(65.61)^{1/8} = \left(\frac{6561}{100}\right)^{1/8} = \left(\frac{3^8}{10^2}\right)^{1/8} = \frac{(3^8)^{1/8}}{(10^2)^{1/8}} = \frac{3^{8 \times \frac{1}{8}}}{10^{2 \times \frac{1}{8}}} = \frac{3}{10^{1/4}} = \frac{3}{\sqrt[4]{10}}$

EXAMPLE 5 *If* $\{(27)^{2/3}-(81)^{1/2}\}^x=0$, *then which of the following cannot be the value of x?*

(a) -1 (b) 1 (c) 1/2 (d) 2

Ans. (a)

SOLUTION We find that $(27)^{2/3}-(81)^{1/2}=(3^3)^{2/3}-(9^2)^{1/2}=3^{3\times\frac{2}{3}}-9^{2\times\frac{1}{2}}=3^2-9^1=9-9=0$

Therefore, $\{(27)^{2/3}-(81)^{1/2}\}^x=0$, if $x>0$.

EXAMPLE 6 *If* $7^{5x-8}\times5^{x+2}=30625$, *then* $x=$

(a) 4 (b) 3 (c) 2 (d) 1

Ans. (c)

SOLUTION We have,

$$7^{5x-8}\times5^{x+2}=30625$$

$\Rightarrow\quad 7^{5x-8}\times5^{x+2}=7^2\times5^4$

$\Rightarrow\quad 5x-8=2$ and $x+2=4$ [$\because$ 7 and 5 are relatively prime]

$\Rightarrow\quad 5x=10$ and $x=2\Rightarrow x=2$

EXAMPLE 7 *If* $P=a^1b^2c^3\ldots.y^{25}z^{26}$ *and* $Q=z^1y^2x^3\ldots.b^{25}a^{26}$, *where* $abcd\ldots xyz=\sqrt[54]{64}$, *then* $PQ=$

(a) 6 (b) 8 (c) 9 (d) 64

Ans. (b)

SOLUTION $PQ=a^{27}b^{27}c^{27}\ldots x^{27}y^{27}z^{27}$

$\Rightarrow\quad PQ=(abc\ldots xyz)^{27}=(\sqrt[54]{64})^{27}=\{(64)^{1/54}\}^{27}=(64)^{27/54}=(64)^{1/2}=(8^2)^{1/2}=8$

EXAMPLE 8 *Which is the greatest among* $81^{18}, 243^{15}, 27^{21}$ *and* 9^{38} ?

(a) 243^{15} (b) 27^{21} (c) 9^{38} (d) 81^{18}

Ans. (c)

SOLUTION Converting all exponents to the same base, we obtain

$$81^{18}=(3^4)^{18}=3^{4\times18}=3^{72};\ 27^{21}=(3^3)^{21}=3^{3\times21}=3^{63};\ 9^{38}=(3^2)^{38}=3^{2\times38}=3^{76}$$

and $(243)^{15}=(3^5)^{15}=3^{5\times15}=3^{75}$

Now, $63<72<75<76\Rightarrow3^{63}<3^{72}<3^{75}<3^{76}\Rightarrow27^{21}<81^{18}<(243)^{15}<9^{38}$

EXAMPLE 9 *Which of the following is the ascending order of* $(343)^3,(2401)^2$ *and* $(49)^5$?

(a) $49^5, 343^3, 2401^2$ (b) $2401^2, 343^3, 49^5$ (c) $343^3, 49^5, 2401^2$ (d) $49^5, 2401^2, 343^3$

Ans. (b)

SOLUTION Converting all exponents to the same base, we obtain

$$(343)^3=(7^3)^3=7^{3\times3}=7^9, (2401)^2=(7^4)^2=7^{4\times2}=7^8 \text{ and, } (49)^5=(7^2)^5=7^{2\times5}=7^{10}$$

Now, $8<9<10\Rightarrow7^8<7^9<7^{10}\Rightarrow(2401)^2<(343)^3<(49)^5$

EXAMPLE 10 *Which of the following is the ascending order of* $2^{1465}, 3^{1172}$ *and* 5^{879} ?

(a) $5^{879}, 2^{1465}, 3^{1172}$ (b) $2^{1465}, 3^{1172}, 5^{879}$ (c) $3^{1172}, 2^{1465}, 5^{879}$ (d) $3^{1172}, 5^{879}, 2^{1465}$

Ans. (b)

SOLUTION Expressing all numbers with the same exponent we obtain

$$2^{1465}=2^{5\times293}=(2^5)^{293}=32^{293};\ 3^{1172}=3^{4\times293}=(3^4)^{293}=81^{293} \text{ and } 5^{879}=5^{3\times293}=(5^3)^{293}=125^{293}$$

Now, $32<81<125\Rightarrow32^{293}<81^{293}<125^{293}\Rightarrow2^{1465}<3^{1172}<5^{879}$

EXAMPLE 11 *If* $a^x = b^y = c^z$ *and* $a^3 = b^2 c$, *then* $\frac{3}{x} - \frac{2}{y} =$

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) xyz (d) $\frac{1}{z}$

Ans. (d)

SOLUTION Let $a^x = b^y = c^z = k$. Then, $a = k^{1/x}, b = k^{1/y}$ and $c = k^{1/z}$.

$$\therefore \quad a^3 = b^2 c \Rightarrow (k^{1/x})^3 = (k^{1/y})^2 (k^{1/z}) \Rightarrow k^{\frac{3}{x}} = k^{\frac{2}{y}+\frac{1}{z}} \Rightarrow \frac{3}{x} = \frac{2}{y} + \frac{1}{z} \Rightarrow \frac{3}{x} - \frac{2}{y} = \frac{1}{z}$$

EXAMPLE 12 *If* $11^x = 3^y = 99^z = k$, *then* $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$

(a) $\frac{2}{z} - \frac{1}{y}$ (b) $\frac{2}{z} + \frac{1}{y}$ (c) $-\frac{1}{y}$ (d) 0

Ans. (a)

SOLUTION Let $11^x = 3^y = 99^z = k$. Then, $k^{1/x} = 11, k^{1/y} = 3$ and $k^{1/z} = 99$.

Now, $\quad 99 = 11 \times 3^2$

$$\Rightarrow \quad k^{1/z} = k^{1/x} \times (k^{1/y})^2 \Rightarrow k^{1/z} = k^{1/x + 2/y} \Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{2}{y} \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z} - \frac{1}{y} \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{z} - \frac{1}{y}$$

EXAMPLE 13 *If* $a = 2^{-2} - 2^{-3}, b = 2^{-3} - 2^{-4}$ *and* $c = 2^{-4} - 2^{-2}$, *then* $a^3 + b^3 + c^3 =$

(a) $-\frac{9}{1024}$ (b) $-\frac{9}{2048}$ (c) 0 (d) 1

Ans. (b)

SOLUTION We find that $a + b + c = 0$

$$\therefore \quad a^3 + b^3 + c^3 = 3abc$$
$$= 3(2^{-2} - 2^{-3})(2^{-3} - 2^{-4})(2^{-4} - 2^{-2})$$
$$= 3 \times 2^{-3}(2-1)2^{-4}(2-1)2^{-4}(1-2^2) = 3 \times 2^{-11} \times -3 = -\frac{9}{2^{11}} = -\frac{9}{2048}$$

EXAMPLE 14 *If* $a^{b^c} = 512$, *where a, b, c are positive integers, then the minimum possible value of abc is*

(a) 18 (b) 12 (c) 24 (d) 512

Ans. (b)

SOLUTION We have, $a^{b^c} = 512$ and $512 = 2^{9^1} = 2^{3^2} = 8^{3^1} = 512^{1^1}$. Therefore, we have the following possibilities:

$$a^{b^c} = 2^{9^1} \Rightarrow a = 2, b = 9, c = 1 \Rightarrow abc = 18; \quad a^{b^c} = 2^{3^2} \Rightarrow a = 2, b = 3, c = 2 \Rightarrow abc = 12$$
$$a^{b^c} = 8^{3^1} \Rightarrow a = 8, b = 3, c = 1 \Rightarrow abc = 24; \quad a^{b^c} = 512^{1^1} \Rightarrow a = 512, b = 1, c = 1 \Rightarrow abc = 512$$

Clearly, minimum value of abc is 12.

EXAMPLE 15 *If* $(1331)^{-x} = (225)^y$, *where x, y are integers, then the value of* $3xy$ *is*

(a) 0 (b) 1 (c) 3 (d) –3

Ans. (a)

SOLUTION We have,

$$(1331)^{-x} = (225)^y \Rightarrow (11^3)^{-x} = (15^2)^y \Rightarrow 11^{-3x} = 15^{2y}$$

This is possible only when $x = 0, y = 0$. Therefore, $3xy = 0$.

EXAMPLE 16 *Which of the following is not equal to* $\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6}$?

(a) $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$ (b) $1\div\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{1/6}$ (c) $\left(\frac{6}{5}\right)^{\frac{1}{30}}$ (d) $\left(\frac{5}{6}\right)^{-\frac{1}{30}}$

Ans. (a) [NCERT EXEMPLAR]

SOLUTION We find that $\left\{\left(\frac{5}{6}\right)^{1/5}\right\}^{-1/6}=\left(\frac{5}{6}\right)^{\frac{1}{5}\times-\frac{1}{6}}=\left(\frac{5}{6}\right)^{-\frac{1}{30}}$, $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}=\left(\frac{5}{6}\right)^{\frac{1}{30}}$,

$$1\div\left\{\left(\frac{5}{6}\right)^{\frac{1}{5}}\right\}^{\frac{1}{6}}=1\div\left(\frac{5}{6}\right)^{\frac{1}{5}\times\frac{1}{6}}=1\div\left(\frac{5}{6}\right)^{\frac{1}{30}}=\left(\frac{5}{6}\right)^{-\frac{1}{30}},\text{ and, }\left(\frac{6}{5}\right)^{1/30}=\left(\frac{5}{6}\right)^{-1/30}$$

Hence, option (a) is correct.

EXAMPLE 17 $\sqrt[4]{\sqrt[3]{2^2}}$ *equals*

(a) $2^{-1/6}$ (b) 2^{-6} (c) $2^{1/6}$ (d) 2^6

Ans. (c) [NCERT EXEMPLAR]

SOLUTION $\sqrt[4]{\sqrt[3]{2^2}}=\sqrt[4]{(2^2)^{1/3}}=(2^{2/3})^{1/4}=2^{\frac{2}{3}\times\frac{1}{4}}=2^{1/6}$

EXAMPLE 18 *Value of* $\sqrt[4]{(81)^{-2}}$ *is*

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{81}$

Ans. (a) [NCERT EXEMPLAR]

SOLUTION $\sqrt[4]{(81)^{-2}}=\sqrt[4]{(3^4)^{-2}}=\sqrt[4]{3^{4\times-2}}=(3^{-8})^{1/4}=3^{-8\times\frac{1}{4}}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

EXAMPLE 19 *The product* $\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32}$ *equals*

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt[12]{2}$ (d) $\sqrt[12]{32}$

Ans. (b) [NCERT EXEMPLAR]

SOLUTION $\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32}=2^{\frac{1}{3}}\times2^{\frac{1}{4}}\times(2^5)^{\frac{1}{12}}=2^{\frac{1}{3}}\times2^{\frac{1}{4}}\times2^{\frac{5}{12}}=2^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}}=2^1=2$

EXAMPLE 20 *Which of the following is equal to x?*

(a) $x^{\frac{12}{7}}-x^{-\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{1/3}}$ (c) $(\sqrt{x^3})^{2/3}$ (d) $x^{\frac{12}{7}}\times x^{\frac{7}{12}}$

Ans. (c) [NCERT EXEMPLAR]

SOLUTION We find that $x^{\frac{12}{7}}-x^{-\frac{5}{7}}=x^{\frac{12}{7}}-\frac{1}{x^{5/7}}=\frac{x^{\frac{12}{7}}\times x^{\frac{5}{7}}-1}{x^{5/7}}=\frac{x^{\frac{12}{7}+\frac{5}{7}}-1}{x^{5/7}}=\frac{x^{\frac{17}{7}}-1}{x^{5/7}}\neq x;$

$$\sqrt[12]{(x^4)^{1/3}}=\left(x^{4\times\frac{1}{3}}\right)^{\frac{1}{12}}=x^{4\times\frac{1}{3}\times\frac{1}{12}}=x^{1/9}\neq x;\ (\sqrt{x^3})^{2/3}=\left\{(x^3)^{\frac{1}{2}}\right\}^{\frac{2}{3}}=x^{\frac{3}{2}\times\frac{2}{3}}=x$$

and, $x^{\frac{12}{7}}\times x^{\frac{7}{12}}=x^{\frac{12}{7}+\frac{7}{12}}=x^{\frac{193}{12}}\neq x$

Hence, option (c) is correct.

EXAMPLE 21 $\dfrac{8^{1/3}\times 16^{1/3}}{32^{-1/3}}$ *is equal to*

(a) 2 (b) 4 (c) 16 (d) 8

Ans. (c) [NCERT EXEMPLAR]

SOLUTION $\dfrac{8^{1/3}\times 16^{1/3}}{32^{-1/3}}=\dfrac{(2^3)^{1/3}\times(2^4)^{1/3}}{(2^5)^{-1/3}}=\dfrac{2^{3\times\frac{1}{3}+4\times\frac{1}{3}}}{2^{5\times-\frac{1}{3}}}=\dfrac{2^{1+\frac{4}{3}}}{2^{-\frac{5}{3}}}=2^{1+\frac{4}{3}+\frac{5}{3}}=2^4=16$

EXAMPLE 22 $\dfrac{9^{1/3}\times 27^{-1/2}}{3^{1/6}\times 3^{-2/3}}$ *is equal to*

(a) $3^{-1/3}$ (b) $3^{1/3}$ (c) 3 (d) $3^{2/3}$

Ans. (a)

SOLUTION $\dfrac{9^{1/3}\times 27^{-1/2}}{3^{1/6}\times 3^{-2/3}}=\dfrac{(3^2)^{1/3}\times(3^3)^{-1/2}}{3^{1/6}\times 3^{-2/3}}=\dfrac{3^{2/3}\times 3^{-3/2}}{3^{1/6}\times 3^{-2/3}}=\dfrac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}}=\dfrac{3^{-5/6}}{3^{-1/2}}=3^{-\frac{5}{6}+\frac{1}{2}}=3^{-\frac{1}{3}}$

EXAMPLE 23 *The value of* $\left\{9(64^{1/3}+125^{1/3})^3\right\}^{1/4}$ *is*

(a) 3 (b) 9 (c) $3\sqrt{3}$ (d) 81

Ans. (b)

SOLUTION $\left\{9(64^{1/3}+125^{1/3})^3\right\}^{1/4}=\left[9\left\{(4^3)^{1/3}+(5^3)^{1/3}\right\}^3\right]^{1/4}$

$$=\{9(4+5)^3\}^{1/4}=(9\times 9^3)^{1/4}=(9^4)^{1/4}=9^{4\times 1/4}=9^1=9$$

EXAMPLE 24 *If* $2^{5x}\div 2^x=\sqrt[5]{2^{20}}$, *then* $x=$

(a) 2 (b) 4 (c) 1 (d) *none of these*

Ans. (c)

SOLUTION We have,

$$2^{5x}\div 2^x=\sqrt[5]{2^{20}}\Rightarrow\frac{2^{5x}}{2^x}=(2^{20})^{1/5}\Rightarrow 2^{5x-x}=2^{20\times\frac{1}{5}}\Rightarrow 2^{4x}=2^4\Rightarrow 4x=4\Rightarrow x=1$$

EXAMPLE 25 *If* $\dfrac{2^{x-1}\times 4^{2x+1}}{8^{x-1}}=64$, *then x is equal to*

(a) 1 (b) 2 (c) -2 (d) 3

Ans. (a)

SOLUTION We have,

$$\frac{2^{x-1}\times 4^{2x+1}}{8^{x-1}}=64\Rightarrow\frac{2^{x-1}\times(2^2)^{2x+1}}{(2^3)^{x-1}}=64\Rightarrow\frac{2^{x-1}\times 2^{2(2x+1)}}{2^{3(x-1)}}=2^6\Rightarrow\frac{2^{x-1+4x+2}}{2^{3x-3}}=2^6$$

$$\Rightarrow\quad\frac{2^{5x+1}}{2^{3x-3}}=2^6\Rightarrow 2^{5x+1}\times 2^{-3x+3}=2^6\Rightarrow 2^{5x+1-3x+3}=2^6\Rightarrow 2x+4=6\Rightarrow 2x=2\Rightarrow x=1$$

EXAMPLE 26 *Which of the following is the greatest?*

(a) 7^2 (b) $(49)^{3/2}$ (c) $\left(\dfrac{1}{343}\right)^{-1/3}$ (d) $(2401)^{-1/4}$

Ans. (b)

SOLUTION We find that $(49)^{3/2}=(7^2)^{3/2}=7^{2\times 3/2}=7^3$ ...(i)

$$\left(\frac{1}{343}\right)^{-1/3}=\left(\frac{1}{7^3}\right)^{-1/3}=(7^{-3})^{-1/3}=7^{-3\times -\frac{1}{3}}=7 \quad ...(ii)$$

and $(2401)^{-1/4}=(7^4)^{-1/4}=7^{4\times -\frac{1}{4}}=7^{-1}=\frac{1}{7}$ ...(iii)

Clearly, $\frac{1}{7}<7<7^2<7^3 \Rightarrow (2401)^{-1/4}<\left(\frac{1}{343}\right)^{-1/3}<7^2<(49)^{3/2}$ [Using (i), (ii) and (iii)]

Hence, $(49)^{3/2}$ is the greatest.

EXAMPLE 27 *The value of* $\frac{(32)^{0.2}+(81)^{0.25}}{(256)^{0.5}-(121)^{0.5}}$ *is*

(a) 2 (b) 5 (c) 1 (d) 11

Ans. (c)

SOLUTION $\frac{(32)^{0.2}+(81)^{0.25}}{(256)^{0.5}-(121)^{0.5}}=\frac{(2^5)^{1/5}+(3^4)^{1/4}}{(2^8)^{1/2}-(11^2)^{1/2}}=\frac{2^{5\times\frac{1}{5}}+3^{4\times\frac{1}{4}}}{2^{8\times\frac{1}{2}}-11^{2\times\frac{1}{2}}}=\frac{2+3}{2^4-11}=\frac{5}{16-11}=\frac{5}{5}=1$

EXAMPLE 28 *If* $a=3$ *and* $b=2$, *then* $(3a-4b)^{b-a}\div(4a-3b)^{2b-a}$ *is equal to*

(a) 1 (b) 6 (c) 1/6 (d) 2/3

Ans. (c)

SOLUTION For $a = 3$, and $b = 2$, we find that

$$(3a-4b)^{b-a}\div(4a-3b)^{2b-a}=(9-8)^{-1}\div(12-6)^1=(1)^{-1}\div(6)^1=1\div 6=\frac{1}{6}$$

EXAMPLE 29 *If* $\sqrt{2^n}=1024$, *then* $3^{2(n/4-4)}$ *is equal to*

(a) 3 (b) 9 (c) 27 (d) 81

Ans. (b)

SOLUTION we have,

$$\sqrt{2^n}=1024 \Rightarrow (2^n)^{1/2}=2^{10} \Rightarrow 2^{n\times 1/2}=2^{10} \Rightarrow \frac{n}{2}=10 \Rightarrow n=20$$

$$\therefore \quad 3^{2\left(\frac{n}{4}-4\right)}=3^{2\left(\frac{20}{4}-4\right)}=3^{2(5-4)}=3^{2\times 1}=3^2=9$$

EXAMPLE 30 *If* $10^x=64$, *then* $10^{\frac{x}{2}+1}$ *is equal to*

(a) 18 (b) 42 (c) 80 (d) 81

Ans. (c)

SOLUTION We have, $10^x=64$

$$\therefore \quad 10^{\frac{x}{2}+1}=10^{\frac{x}{2}}\times 10^1=(10^x)^{1/2}\times 10=64^{1/2}\times 10=(8^2)^{1/2}\times 10=8\times 10=80$$

EXAMPLE 31 *If* $\frac{2^{m+n}}{2^{n-m}}=16, \frac{3^p}{3^n}=81$ *and* $a=2^{1/10}$, *then* $\frac{(a^{2m+n-p})^2}{(a^{m-2n+2p})^{-1}}=$

(a) 2 (b) 1/4 (c) 9 (d) 1/8

Ans. (a)

SOLUTION We have, $\frac{2^{m+n}}{2^{n-m}} = 16$ and $\frac{3^p}{3^n} = 81$

$\Rightarrow \quad 2^{(m+n)-(n-m)} = 2^4$ and $3^{p-n} = 3^4$

$\Rightarrow \quad 2^{2m} = 4$ and $3^{p-n} = 3^4$

$\Rightarrow \quad 2m = 4$ and $p - n = 4$

$\Rightarrow \quad m = 2$ and $p - n = 4$

$\therefore \quad \frac{(a^{2m+n-p})^2}{(a^{m-2n+2p})^{-1}} = a^{2(2m+n-p)} \times a^{m-2n+2p}$ $\left[\because \frac{1}{x^{-1}} = x\right]$

$= a^{4m+2n-2p+m-2n+2p} = a^{5m} = a^{10}$

$= (2^{1/10})^{10} + 2^{\frac{1}{10}\times 10} = 2^1 = 2$

EXAMPLE 32 *If* $2^{-m} \times \frac{1}{2^m} = \frac{1}{4}$, *then* $\frac{1}{14}\left\{(4m)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right\}$ *is equal to*

(a) $\frac{1}{2}$ (b) 2 (c) 4 (d) $-\frac{1}{4}$

Ans. (a)

SOLUTION We have,

$$2^{-m} \times \frac{1}{2^m} = \frac{1}{4} \Rightarrow 2^{-m} \times 2^{-m} = \frac{1}{2^2} \Rightarrow 2^{-m-m} = 2^{-2} \Rightarrow 2^{-2m} = 2^{-2} \Rightarrow -2m = -2 \Rightarrow m = 1$$

$$\therefore \quad \frac{1}{14}\left\{(4m)^{1/2} + \left(\frac{1}{5^m}\right)^{-1}\right\} = \frac{1}{14}\left\{4^{1/2} + \left(\frac{1}{5}\right)^{-1}\right\} = \frac{1}{14}(2+5) = \frac{1}{2}$$

EXAMPLE 33 $\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}}$ *is equal to*

(a) $\frac{5}{3}$ (b) $-\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

Ans. (b)

SOLUTION $\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}} = \frac{5^n \times 5^2 - 6 \times 5^n \times 5}{13 \times 5^n - 2 \times 5^n \times 5} = \frac{5^n(5^2 - 6 \times 5)}{5^n(13 - 2 \times 5)} = \frac{25 - 30}{13 - 10} = -\frac{5}{3}$

EXAMPLE 34 $\sqrt[6]{5} \times \sqrt[3]{2} \times \sqrt{3} \times \sqrt[12]{6}$ *is equal to*

(a) $\sqrt[12]{1749600}$ (b) $\sqrt[3]{2} \times \sqrt[12]{109350}$ (c) $\sqrt[12]{177960}$ (d) *both* (a) *and* (b)

Ans. (d)

SOLUTION $\sqrt[6]{5} \times \sqrt[3]{2} \times \sqrt{3} \times \sqrt[12]{6} = 5^{1/6} \times 2^{1/3} \times 3^{1/2} \times 6^{1/12} = 5^{2/12} \times 2^{4/12} \times 3^{6/12} \times 6^{1/12}$

$= (5^2)^{1/12} \times (2^4)^{1/12} \times (3^6)^{1/12} \times (6)^{1/12} = (5^2 \times 2^4 \times 3^6 \times 6)^{1/12}$

$= (25 \times 16 \times 729 \times 6)^{1/12} = (1749600)^{1/12} = \sqrt[12]{1749600}$

Again, $\sqrt[6]{5} \times \sqrt[3]{2} \times \sqrt{3} \times \sqrt[12]{6} = (5^2 \times 2^4 \times 3^6 \times 6)^{1/12} = 2^{4/12} \times (5^2 \times 3^6 \times 6)^{1/12} = 2^{1/3} \times (25 \times 729 \times 6)^{1/12}$

$= \sqrt[3]{2}\ \sqrt[12]{109350}$

EXAMPLE 35 $\sqrt{11\sqrt{11\sqrt{11 \ldots .4 \text{ terms}}}}$ *is equal to*

(a) $\sqrt[16]{11^5}$ (b) $\sqrt[16]{11}$ (c) $\sqrt[16]{11^{14}}$ (d) $\sqrt[16]{11^{15}}$

Ans. (d)

SOLUTION Let $x = \sqrt{11\sqrt{11\sqrt{11 \ldots .4 \text{ terms}}}}$. Then,

$$x = \sqrt{11\sqrt{11\sqrt{11\sqrt{11}}}} = \sqrt{11\sqrt{11\sqrt{11 \times 11^{1/2}}}} = \sqrt{11\sqrt{11\sqrt{11^{3/2}}}} = \sqrt{11\sqrt{11 \times (11^{3/2})^{1/2}}}$$

$$\Rightarrow x = \sqrt{11\sqrt{11 \times 11^{3/4}}} = \sqrt{11\sqrt{11^{7/4}}} = \sqrt{11 \times 11^{7/8}} = \sqrt{11^{1+\frac{7}{8}}} = \sqrt{11^{15/8}} = \left(11^{15/8}\right)^{1/2}$$

$$= (11)^{\frac{15}{8} \times \frac{1}{2}} = 11^{15/16} = \sqrt[16]{11^{15}}$$

EXAMPLE 36 *If* $\dfrac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$, *then the value of x is*

(a) 3 (b) – 3 (c) 1/3 (d) –1/3

Ans. (b)

SOLUTION We have, $\dfrac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$

$$\Rightarrow \frac{3^{5x} \times (3^4)^2 \times 3^8}{3^{2x}} = 3^7 \Rightarrow 3^{5x} \times 3^8 \times 3^8 = 3^{2x} \times 3^7 \Rightarrow 3^{5x+16} = 3^{2x+7} \Rightarrow 5x + 16 = 2x + 7 \Rightarrow 3x - 9 \Rightarrow x - 3$$

EXAMPLE 37 *If* $\sqrt[x]{3} \times \sqrt[y]{5} = 10125$, *then* $12xy =$

(a) 1 (b) 1/3 (c) 2 (d) 1/2

Ans. (a)

SOLUTION We have, $\sqrt[x]{3} \times \sqrt[y]{5} = 10125$

$\Rightarrow \quad 3^{1/x} \times 5^{1/y} = 3^4 \times 5^3$

$\Rightarrow \quad \dfrac{1}{x} = 4$ and $\dfrac{1}{y} = 3$ [$\because$ 3 and 5 are relatively prime]

$\Rightarrow \quad x = \dfrac{1}{4}$ and $y = \dfrac{1}{3} \Rightarrow 12xy = 12 \times \dfrac{1}{4} \times \dfrac{1}{3} = 1.$

EXAMPLE 38 *The value of* $\sqrt{3^2\sqrt{9^2\sqrt{81^2\sqrt{16^{16}}}}}$ *is equal to*

(a) 6×2^4 (b) $3^3 \times 2$ (c) $6^3 \times 2^3$ (d) $6^3 \times 2$

Ans. (d)

SOLUTION $\sqrt{3^2\sqrt{9^2\sqrt{81^2\sqrt{16^{16}}}}}$

$= \sqrt{3^2\sqrt{9^2\sqrt{81^2 \times 16^8}}}$ $\left[\because \sqrt{16^{16}} = (16^{16})^{1/2} = 16^{16 \times 1/2} = 16^8\right]$

$= \sqrt{3^2\sqrt{9^2 \times 81 \times 16^4}}$ $\left[\because \sqrt{81^2 \times 16^8} = (81^2 \times 16^8)^{1/2} = 81^{2 \times \frac{1}{2}} \times 16^{8 \times \frac{1}{2}} = 81 \times 16^4\right]$

$= \sqrt{3^2 \times 9 \times 9 \times 16^2}$ $\left[\because \sqrt{9^2 \times 81 \times 16^4} = (9^2 \times 81 \times 16^4)^{1/2} = 9^{2 \times \frac{1}{2}} \times (9^2)^{\frac{1}{2}} \times 16^{4 \times \frac{1}{2}} = 9 \times 9 \times 16^2\right]$

$= \sqrt{3^2 \times 3^2 \times 3^2 \times 16^2}$

$= (3^6 \times 16^2)^{1/2} = 3^{6\times\frac{1}{2}} \times 16^{2\times\frac{1}{2}} = 3^3 \times 16 = 3^3 \times 2^3 \times 2 = (3\times 2)^3 \times 2 = 6^3 \times 2$

EXAMPLE 39 *The value of* $\sqrt{\sqrt[x]{2^x \sqrt[x^2]{3^{x^3} \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}}}}}$ *is*

(a) 18 (b) 24 (c) 36 (d) 54

Ans. (a)

SOLUTION We find that $\sqrt[x^4]{9^{x^{10}}} = \left(9^{x^{10}}\right)^{\frac{1}{x^4}} = 9^{x^{10}\times\frac{1}{x^4}} = 9x^6$

$\therefore \quad \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}} = \sqrt[x^3]{6^{x^6} \times 9^{x^6}} = \sqrt[x^3]{(6\times 9)^{x^6}} = \left\{(6\times 9)^{x^6}\right\}^{\frac{1}{x^3}} = (54)^{x^6\times\frac{1}{x^3}} = (54)^{x^3}$

$\Rightarrow \quad \sqrt[x^2]{3^{x^3} \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}}} = \sqrt[x^2]{3^{x^3}(54)^{x^3}} = \sqrt[x^2]{(3\times 54)^{x^3}} = \{(162)^{x^3}\}^{\frac{1}{x^2}} = (162)^x$

$\Rightarrow \quad \sqrt[x]{2^x \sqrt[x^2]{3^{x^3} \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}}}} = \sqrt[x]{2^x (162)^x} = \{(2\times 162)^x\}^{\frac{1}{x}} = 324$

Hence, required value $= \sqrt{324} = 18$.

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 40 *Statement-1 (Assertion):* $\left[\left\{\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}}\right\}^{a+b}\right]^{\frac{1}{(a+b)^2}} = x^{-1}$

Statement-2 (Reason): $(a^m)^n = a^{mn}$, *$a > 0$ and m, n are rational numbers.*

Ans. (a)

SOLUTION Statement-2 is true as it is one of the laws of exponents. Using statement-2, we find that

$$\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}} = (x^{-(a^2-b^2)})^{\frac{1}{a-b}} = x^{-(a^2-b^2)\times\frac{1}{a-b}} = x^{-(a+b)}$$

$$\therefore \quad \left\{\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}}\right\}^{a+b} = \{x^{-(a+b)}\}^{(a+b)} = x^{-(a+b)(a+b)} = x^{-(a+b)^2}$$

$$\therefore \quad \left[\left\{\left(\frac{1}{x^{a^2-b^2}}\right)^{\frac{1}{a-b}}\right\}^{a+b}\right]^{\frac{1}{(a+b)^2}} = \{x^{-(a+b)^2}\}^{\frac{1}{(a+b)^2}} = x^{-(a+b)^2\times\frac{1}{(a+b)^2}} = x^{-1}$$

So, statement-1 is also true. Also, statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 41 *Statement-1 (Assertion):* $\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}}}}} \ldots \infty = \frac{9}{8}$.

Statement-2 (Reason): *For any positive real number x:* $\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}} \ldots \infty = x$.

Ans. (c)

SOLUTION Let $y = \sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}} \ldots \infty$. Then

$$y^2 = x\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}} \ldots \infty$$

$$\Rightarrow \quad y^2 = xy \Rightarrow y^2 - xy = 0 \Rightarrow y(y-x) = 0 \Rightarrow y - x = 0 \Rightarrow y = x. \quad [\because y \neq 0]$$

So, statement-2 is true. Using statement-2, we find that

$$\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}\sqrt{\frac{81}{64}}}}} \ldots \infty = \frac{81}{64}$$

So, statement-1 is not true. Hence, option (c) is correct.

EXAMPLE 42 *Statement-1 (Assertion):* $\{(a^{-1}+b^{-1})(a^{-1}-b^{-1})\} \div \left\{\left(\frac{1}{a^{-1}}-\frac{1}{b^{-1}}\right)\left(\frac{1}{a^{-1}}+\frac{1}{b^{-1}}\right)\right\} = 1$.

Statement-2 (Reason): *For any* $a \neq 0, a^{-m} = \frac{1}{a^m}$ *and* $a^m = \frac{1}{a^{-m}}$.

Ans. (d)

SOLUTION We observe that statement-2 is true.

Now, $\{(a^{-1}+b^{-1})(a^{-1}-b^{-1})\} \div \left\{\left(\frac{1}{a^{-1}}-\frac{1}{b^{-1}}\right)\left(\frac{1}{a^{-1}}+\frac{1}{b^{-1}}\right)\right\}$

$$= \left\{\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a}-\frac{1}{b}\right)\right\} \div \{(a-b)(a+b)\}$$

$$= \left\{\left(\frac{a+b}{ab}\right)\left(\frac{b-a}{ab}\right)\right\} \div \{(a-b)(a+b)\} = -\frac{(a-b)(a+b)}{a^2b^2} \div (a-b)(a+b) = -\frac{1}{a^2b^2}$$

So, statement-1 is not true. hence, option (d) is correct.

EXAMPLE 43 *Statement-1 (Assertion): If* $(16)^{2x+3} = (64)^{x+3}$, *then* $4^{2x-2} = 256$.

Statement-2 (Reason): If $a \neq 0, \pm 1$, *then* $a^m = a^n \Rightarrow m = n$ *and* $(a^m)^n = a^{mn}$.

Ans. (a)

SOLUTION Clearly, statement-2 is true as it is one of the laws of exponents.

Now, $(16)^{2x+3} = (64)^{x+3} \Rightarrow (2^4)^{2x+3} = (2^6)^{x+3} \Rightarrow 2^{4(2x+3)} = 2^{6(x+3)} \Rightarrow 4(2x+3) = 6(x+3)$

$\Rightarrow \quad 8x + 12 = 6x + 18 \Rightarrow 2x = 6 \Rightarrow x = 3$

$4^{2x-2} = 4^{6-2} = 4^4 = 256$

So, statement-1 is also true. Also, statement-2 is a correct explanation for statement-1. Hence option (a) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. The value of $\{2-3(2-3)^3\}^3$, is
   (a) 5 (b) 125 (c) 1/5 (d) -125
2. The value of $x-y^{x-y}$ when $x=2$ and $y=-2$, is
   (a) 18 (b) -18 (c) 14 (d) -14
3. The product of the square root of x with the cube root of x, is
   (a) cube root of the square root of x (b) sixth root of the fifth power of x
   (c) fifth root of the sixth power of x (d) sixth root of x
4. The seventh root of x divided by the eighth root of x, is
   (a) x (b) $\sqrt{x}$ (c) $\sqrt[56]{x}$ (d) $\frac{1}{\sqrt[56]{x}}$
5. The square root of 64 divided by the cube root of 64, is
   (a) 64 (b) 2 (c) $\frac{1}{2}$ (d) $64^{2/3}$
6. The value of $\left\{(23+2^2)^{2/3}+(140-19)^{1/2}\right\}^2$ is
   (a) 400 (b) 324 (c) 289 (d) 196
7. When simplified $(x^{-1}+y^{-1})^{-1}$ is equal to
   (a) xy (b) $x+y$ (c) $\frac{xy}{x+y}$ (d) $\frac{x+y}{xy}$
8. If $8^{x+1}=64$, what is the value of 3^{2x+1}?
   (a) 1 (b) 3 (c) 9 (d) 27
9. If $(2^3)^2=4^x$, then $3^x=$
   (a) 3 (b) 6 (c) 9 (d) 27
10. If $x^{-2}=64$, then $x^{1/3}+x^0=$
    (a) 2 (b) 3 (c) 3/2 (d) 2/3
11. When simplified $\left(-\frac{1}{27}\right)^{-2/3}$, is
    (a) 9 (b) -9 (c) $\frac{1}{9}$ (d) $-\frac{1}{9}$
    [NCERT EXEMPLAR]
12. Which one of the following is not equal to $(\sqrt[3]{8})^{-1/2}$?
    (a) $(\sqrt[3]{2})^{-1/2}$ (b) $8^{-1/6}$ (c) $\frac{1}{(\sqrt[3]{8})^{1/2}}$ (d) $\frac{1}{\sqrt{2}}$

13. Which one of the following is not equal to $\left(\frac{100}{9}\right)^{-3/2}$?

(a) $\left(\frac{9}{100}\right)^{3/2}$ (b) $\frac{1}{\left(\frac{100}{9}\right)^{3/2}}$ (c) $\frac{3}{10}\times\frac{3}{10}\times\frac{3}{10}$ (d) $\sqrt{\frac{100}{9}\times\frac{100}{9}\times\frac{100}{9}}$

14. If a, b, c are positive real numbers, then $\sqrt{a^{-1}b}\times\sqrt{b^{-1}c}\times\sqrt{c^{-1}a}$ is equal to

(a) 1 (b) abc (c) $\sqrt{abc}$ (d) $\frac{1}{abc}$

15. If $\left(\frac{2}{3}\right)^x\left(\frac{3}{2}\right)^{2x}=\frac{81}{16}$, then $x=$

(a) 2 (b) 3 (c) 4 (d) 1

16. The value of $\left\{8^{-4/3}\div 2^{-2}\right\}^{1/2}$, is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4

17. If a, b, c are positive real numbers, then $\sqrt[5]{3125a^{10}b^5c^{10}}$ is equal to

(a) $5a^2bc^2$ (b) $25ab^2c$ (c) $5a^3bc^3$ (d) $125a^2bc^2$

18. If a, m, n are positive integers, then $\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn}$ is equal to

(a) a^{mn} (b) a (c) $a^{m/n}$ (d) 1

19. If $x=2$ and $y=4$, then $\left(\frac{x}{y}\right)^{x-y}+\left(\frac{y}{x}\right)^{y-x}=$

(a) 4 (b) 8 (c) 12 (d) 2

20. The value of m for which $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4}=7^m$, is

(a) $-\frac{1}{3}$ (b) $\frac{1}{4}$ (c) -3 (d) 2

21. The value of $(0.00243)^{3/5}+(0.0256)^{3/4}$ is

(a) 0.083 (b) 0.073 (c) 0.081 (d) 0.091

22. $(256)^{0.16}\times(256)^{0.09}=$

(a) 4 (b) 16 (c) 64 (d) 256.25

[NCERT EXEMPLAR]

23. If $10^{2y}=25$, then 10^{-y} equals

(a) $-\frac{1}{5}$ (b) $\frac{1}{50}$ (c) $\frac{1}{625}$ (d) $\frac{1}{5}$

24. If $9^{x+2}=240+9^x$, then $x=$

(a) 0.5 (b) 0.2 (c) 0.4 (d) 0.1

25. If x is a positive real number and $x^2 = 2$, then $x^3 =$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) 4

26. If $\frac{x}{x^{1.5}} = 8x^{-1}$ and $x > 0$, then $x =$

(a) $\frac{\sqrt{2}}{4}$ (b) $2\sqrt{2}$ (c) 4 (d) 64

27. If $g = t^{2/3} + 4t^{-1/2}$, what is the value of g when $t = 64$?

(a) $\frac{31}{2}$ (b) $\frac{33}{2}$ (c) 16 (d) $\frac{257}{16}$

28. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals

(a) $5\sqrt{5}$ (b) $\sqrt{5}$ (c) $25\sqrt{5}$ (d) 125

29. When simplified $(256)^{-(4^{-3/2})}$, is

(a) 8 (b) $\frac{1}{8}$ (c) 2 (d) $\frac{1}{2}$

30. If $\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$, then $x =$

(a) 2 (b) 3 (c) 5 (d) 4

31. The value of $64^{-1/3}(64^{1/3} - 64^{2/3})$, is

(a) 1 (b) $\frac{1}{3}$ (c) -3 (d) -2

32. If $\sqrt{5^x} = 125$, then $5^{\sqrt[x]{64}} =$

(a) 25 (b) $\frac{1}{125}$ (c) 625 (d) $\frac{1}{5}$

33. If $(16)^{2x+3} = (64)^{x+3}$, then $4^{2x-2} =$

(a) 64 (b) 256 (c) 32 (d) 512

34. If $\sqrt{5^x} = 125$, then $5^{\sqrt[x]{64}} =$

(a) 25 (b) $\frac{1}{125}$ (c) 625 (d) $\frac{1}{5}$

35. If a, b, c are positive integers such that $a^{b^c} = 256$ then the maximum possible value of abc is

(a) 12 (b) 16 (c) 32 (d) 256

36. If a, b, c are positive integers such that $a^{b^c} = 6561$, then the least possible value of abc is

(a) 24 (b) 36 (c) 162 (d) none of these

37. If $2^x = 3^y = 6^z$, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$

(a) $\frac{2}{x}$ (b) $\frac{2}{y}$ (c) $\frac{2}{z}$ (d) 1

38. If $x^y = y^z = z^x$ and $xz = y^2$, then which of the following is correct?

(a) $z = \dfrac{2xy}{x+y}$ (b) $y = \dfrac{x-z}{x+z}$ (c) $x = \dfrac{y-z}{yz}$ (d) $xyz = \dfrac{x-z+y}{x+z-y}$

39. If $6^{x-y} = 36$ and $3^{x+y} = 729$, then $x^2 - y^2 =$

(a) 12 (b) 4 (c) 24 (d) 8

40. Which is the greatest among $3^{198}, 27^{64}, 9^{100}$ and 81^{49}?

(a) 9^{100} (b) 81^{49} (c) 27^{64} (d) 3^{198}

41. If $\sqrt[x]{3} \times \sqrt[y]{5} = 10125$, then $12xy =$

(a) 1 (b) 1/3 (c) 2 (d) 1/2

42. If $0 < y < x$, which statement must be true?

(a) $\sqrt{x} - \sqrt{y} = \sqrt{x-y}$ (b) $\sqrt{x} + \sqrt{x} = \sqrt{2x}$ (c) $x\sqrt{y} = y\sqrt{x}$ (d) $\sqrt{xy} = \sqrt{x}\sqrt{y}$

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

43. Statement-1 (Assertion): $\left[\left\{\left(\dfrac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^{-1/3}$.

Statement-2 (Reason): $\left((a^m)^n\right)^s = a^{mns}, a > 0$.

44. Statement-1 (Assertion): If $a^x = b^y = c^z = abc$, then $xy + yz + zx = xyz$.

Statement-2 (Reason): If $a^n = k$, then $a = k^{1/n}$.

45. Statement-1 (Assertion): $\sqrt{7\sqrt{7\sqrt{7\sqrt{7}}}} = \sqrt[16]{7^{15}}$.

Statement-2 (Reason): $\sqrt{a\sqrt{a\sqrt{a \ldots\ldots}}}$ n terms $= a^{\frac{2^n = 1}{2^n}}$.

46. Statement-1 (Assertion): $\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}} \ldots\ldots\ldots \infty = 5\sqrt{5}$.

Statement-2 (Reason): $\sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}} \ldots\ldots\ldots \infty = x,\ x > 0$.

47. Statement-1 (Assertion): $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+}}}} \ldots\ldots\ldots \infty = 3$.

Statement-2 (Reason): $\sqrt{x+\sqrt{x+\sqrt{x+}}} \ldots\ldots\ldots \infty = x,\ x > 0$.

48. Statement-1 (Assertion): If m, n are positive integers, then for any positive real number a, $\left\{\sqrt[m]{\sqrt[n]{a}}\right\}^{mn} = a$.

Statement-2 (Reason): If m, n, p are rational numbers and a is any positive real number, then $\left((a^m)^n\right)^p = a^{mnp}$.

ANSWERS

1. (b)	2. (d)	3. (b)	4. (c)	5. (b)	6. (a)	7. (c)
8. (d)	9. (d)	10. (c)	11. (a)	12. (a)	13. (d)	14. (a)
15. (c)	16. (a)	17. (a)	18. (b)	19. (b)	20. (a)	21. (d)
22. (a)	23. (d)	24. (a)	25. (b)	26. (d)	27. (b)	28. (c)
29. (d)	30. (c)	31. (c)	32. (a)	33. (b)	34. (a)	35. (d)
36. (d)	37. (b)	38. (a)	39. (a)	40. (a)	41. (a)	42. (d)
43. (a)	44. (a)	45. (a)	46. (d)	47. (c)	48. (a)	