Topic: Polynomials
Subject: Maths
Class: IX

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MATHEMATICS
WORKSHEET (SOLUTION)_020525
CHAPTER 02 POLYNOMIALS

Maximum Marks: 40
Time: 1.5 Hours

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **20** questions divided into five Sections A, B, C, D and E.
(iii) **Section A** contains **10 MCQs**, each worth **1 mark**. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** has one **5-mark** question, and **Section E** has two case study questions worth **4 marks** each.
(iv) There is no overall choice.
(v) Use of Calculators is not permitted.

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. $(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$ is equal to
(a) $(x-2y)(2y-3z)(3z-x)$ (b) $2(x-2y)(2y-3z)(3z-x)$
(c) $3(x-2y)(2y-3z)(3z-x)$ (d) $3(x-2y)(3z-x)$
Ans: (c) $3(x-2y)(2y-3z)(3z-x)$

2. $(x+1)$ is a factor of the polynomial
(a) x^3+x^2-x+1 (b) x^3+x^2+x+1
(c) $x^4+x^3+x^2+1$ (d) $x^4+3x^3+3x^2+x+1$
Ans: (b) x^3+x^2+x+1

3. If polynomial $p(x) = 3x^4 - 4x^3 - 3x - 1$ is divided by $(x-1)$, then remainder is
(a) 3 (b) -4 (c) -1 (d) $p(1)$
Ans: (d) $p(1)$

4. The coefficient of x in the expansion of $(x+3)^3$ is
(a) 1 (b) 9 (c) 18 (d) 27
Ans: (d) 27
$(x+3)^3 = x^3 + (3)^3 + 3 \times x \times 3(x+3)$
$= x^3 + 27 + 9x(x+3) = x^3 + 27 + 9x^2 + 27x$

5. Zeros of the polynomial $p(x) = (x-2)^2 - (x+2)^2$ are
(a) $2, -2$ (b) $2x$ (c) $0, -2$ (d) 0
Ans: (d) 0
$p(x) = (x-2)^2 - (x+2)^2 = x^2 + 4 - 4x - (x^2 + 4 + 4x)$
$= x^2 + 4 - 4x - x^2 - 4 - 4x = -8x$
Now, $p(x) = 0 \Rightarrow -8x = 0 \Rightarrow x = 0$

6. Factors of $x^2 + 11x + 18$ are
(a) $(x + 9)(x - 2)$ (b) $(x - 9)(x - 2)$
(c) $(x - 9)(x + 2)$ (d) $(x + 9)(x + 2)$
Ans: (d) $(x + 9)(x + 2)$

7. If $(2x + 5)$ is a factor of $2x^2 - k$, then value of k is
(a) 2 (b) – 1 (c) 25 (d) 25/2
Ans: (d) 25/2

8. Given a polynomial $p(t) = t^4 - t^3 + t^2 + 6$, then $p(-1)$ is
(a) 6 (b) 9 (c) 3 (d) – 1
Ans: (b) 9
$p(t) = t^4 - t^3 + t^2 + 6$
$\Rightarrow p(-1) = (-1)^4 - (-1)^3 + (-1)^2 + 6$
$= 1 - (-1) + 1 + 6 = 1 + 1 + 7 = 9$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. Assertion (A): The value of $(28)^3 + (-15)^3 + (-13)^3$ is 16380.
Reason (R): If $a + b + c = 0$, then $a^2 + b^2 + c^2 = 3abc$
Ans: (c) Assertion (A) is true but reason (R) is false.
Let $a = 28$, $b = -15$ and $c = -13$
We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
Here, $a + b + c = 28 - 15 - 13 = 0$
$(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = 16380$

10. Assertion (A): The factors of $x^6 - 64$ is $(x + 2)(x - 2)(x^4 + x^2 + 16)$.
Reason (R): $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
$x^6 - 64 = (x^2)^3 - (2^2)^3$
$\therefore x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$
$= (x^2 - 2^2)(x^4 + 4x^2 + 16)$
$= (x + 2)(x - 2)(x^4 + 4x^2 + 16)$

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Examine whether $x - 1$ is a factor of the following polynomials:
(i) $4x^3 + 3x^2 - 4x - 3$ (ii) $x^3 - 3x^2 - 9x + 5$
Ans: (i) Let $p(x) = 4x^3 + 3x^2 - 4x - 3$
$p(1) = 4(1)^3 + 3(1)^2 - 4(1) - 3 = 4 + 3 - 4 - 3 = 0$
Hence, $x - 1$ is a factor of the given polynomial.
(ii) Let $p(x) = x^3 - 3x^2 - 9x + 5$
$p(1) = 1^3 - 3(1)^2 - 9(1) + 5 = 1 - 3 - 9 + 5 = 6 - 12$
$\Rightarrow p(1) = -6 \neq 0$
Hence, $x - 1$ is not a factor of the given polynomial

12. Using suitable identity, evaluate $(-32)^3 + (18)^3 + (14)^3$

Ans: Here, we find that

$a + b + c = -32 + 18 + 14 = -32 + 32 = 0$

Thus, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (-32)^3 + (18)^3 + (14)^3 = 3 \times (-32) \times 18 \times 14 = -24192$

13. Find the zeroes of the polynomial: $p(x) = (x-2)^2 - (x+2)^2$

Ans: Here, $p(x) = (x-2)^2 - (x+2)^2$

We know that, Zero of the polynomial $p(x) = 0$

$\Rightarrow (x-2)^2 - (x+2)^2 = 0$

Expanding using the identity, $a^2 - b^2 = (a-b)(a+b)$

$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$

$\Rightarrow 2x(-4) = 0 \Rightarrow -8x = 0$

Therefore, the zero of the polynomial = 0

14. Simplify: $(x + y + z)^2 - (x - y + z)^2$

Ans: Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We have,

$(x + y + z)^2 - (x - y - z)^2 = (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx) - (x^2 + y^2 + z^2 - 2xy + 2yz - 2zx)$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx - x^2 - y^2 - z^2 + 2xy - 2yz + 2zx$

$= 4xy + 4zx = 4x(y + z)$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. If $p(x) = x^2 - 4x + 3$, evaluate: $p(2) - p(-1) + p(½)$.

Ans: Given that, $p(x) = x^2 - 4x + 3$

$p(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = -4 + 3 = -1$

$p(-1) = (-1)^2 - 4(-1) + 3 = 1 + 4 + 3 = 8$

$p(½) = (½)^2 - 4(½) + 3 = ¼ - 2 + 3 = ¼ + 1 = 5/4$

Now, $p(2) - p(-1) + p(½) = -1 - 8 + (5/4)$

$= -9 + (5/4)$

$= (-36 + 5)/4 = -31/4$

16. Factorise the following: (i) $x^2 - \frac{y^2}{9}$ (ii) $2x^2 - 7x - 15$ (iii) $6x^2 + 5x - 6$

Ans: (i) $x^2 - \frac{y^2}{9} = x^2 - \left(\frac{y}{3}\right)^2 = \left(x + \frac{y}{3}\right)\left(x - \frac{y}{3}\right)$ $[x^2 - y^2 = (x + y)(x - y)]$

(ii) $2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$

$= 2x(x - 5) + 3(x - 5)$

$= (x - 5)(2x + 3)$

(iii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

17. If $2x + 3y = 13$ and $xy = 6$, find the value of $8x^3 + 27y^3$.
Ans: Given: $2x + 3y = 13$, $xy = 6$
Cubing $2x + 3y = 13$ both sides, we get
$(2x + 3y)^3 = (13)^3$
$\Rightarrow (2x)^3 + (3y)^3 + 3(2x)(3y)(2x + 3y) = 2197$
$\Rightarrow 8x^3 + 27y^3 + 18xy(2x + 3y) = 2197$
$\Rightarrow 8x^3 + 27y^3 + 18 \times 6 \times 13 = 2197$
$\Rightarrow 8x^3 + 27y^3 + 1404 = 2197$
$\Rightarrow 8x^3 + 27y^3 = 2197 - 1404 = 793$

OR

If $x + y + z = 8$ and $xy + yz + zx = 20$, find the value of $x^3 + y^3 + z^3 - 3xyz$.
Ans: We know, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
Squaring, $x + y + z = 8$ both sides, we get
$(x + y + z)^2 = (8)^2$
$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 64$
$\Rightarrow x^2 + y^2 + z^2 + 2 \times 20 = 64$
$\Rightarrow x^2 + y^2 + z^2 + 40 = 64$
$\Rightarrow x^2 + y^2 + z^2 = 24$
Now, $x^3 + y^3 + z^3 - 3xyz = (x + y + z)[x^2 + y^2 + z^2 - (xy + yz + zx)]$
$= 8(24 - 20) = 8 \times 4 = 32$

SECTION – D

Questions 18 carry 5 marks each.

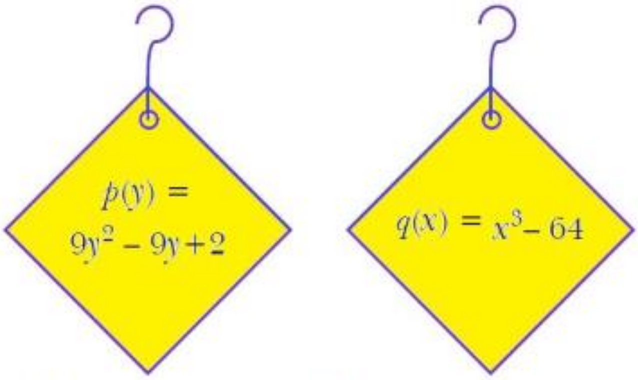
18. (a) If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a.
Ans: Let $p(x) = x^5 - 4a^2x^3 + 2x + 2a + 3$ and $g(x) = x + 2a$
$g(x) = 0 \Longrightarrow x + 2a = 0 \Longrightarrow x = -2a$
Therefore, zero of $g(x) = -2a$
According to the factor theorem,
If $g(x)$ is a factor of $p(x)$, then $p(-2a) = 0$
So, substituting the value of x in $p(x)$, we get,
$p(-2a) = (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$
$\Longrightarrow -32a^5 + 32a^5 - 2a + 3 = 0$
$\Longrightarrow -2a = -3 \Longrightarrow a = 3/2$

(b) Find the value of a and b so that $x + 1$ and $x - 1$ are factors of $x^4 + ax^3 + 2x^2 - 3x + b$.
Ans: Let $f(x) = x^4 + ax^3 + 2x^2 - 3x + b$ be the given polynomial and $g(x) = x + 1$, $h(x) = x - 1$
If $g(x)$ is a factor of $f(x)$, then by factor theorem, $f(-1) = 0$
$\Rightarrow (-1)^4 + a(-1)^3 + 2(-1)^2 - 3(-1) + b = 0$
$\Rightarrow 1 - a + 2 + 3 + b = 0$
$\Rightarrow -a + b = -6$ ...(i)
If $h(x)$ be a factor of $f(x)$, then, again by factor theorem, $f(1) = 0$
$\Rightarrow 1^4 + a(1)^4 + 2(1)^2 - 3(1) + b = 0$
$\Rightarrow 1 + a + 2 + 3 + b = 0$
$\Rightarrow a + b = 0$ ...(ii)
Adding (i) and (ii), we get
$2b = -6$ or $b = -3$
From (ii), we have $a - 3 = 0 \Rightarrow a = 3$
Hence, required value of a and b are 3 and -3 respectively.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

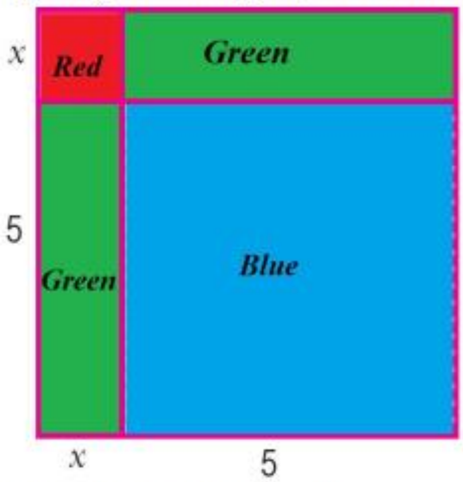
19. A school organised a mathematics exhibition in the school permises. Children of all classes made various models and games to depict the use of mathematics in daily life. To make the decoration more attractive, they made hangings related to mathematics one of the students made two hangings with polynomials written on them.



(a) Find the factors of polynomial q(x) [1]
(b) Find the factors of polynomial p(y) [1]
(c) Find the value of value of p(–2). [1]
(d) Find the zeroes of the polynomial $x^2 - 81$ [1]

Ans: (*a*) $q(x) = x^3 - 64 = x^3 - 4^3$
$= (x - 4)(x^2 + 4x + 16)$
(*b*) $p(y) = 9y^2 - 9y + 2$
$= 9y^2 - 6y - 3y + 2 = 3y(3y - 2) - 1(3y - 2) = (3y - 2)(3y - 1)$
(*c*) $p(y) = 9y^2 - 9y + 2$
$p(-2) = 9(-2)^2 - 9(-2) + 2$
$= 36 + 18 + 2 = 56$
(*d*) For zeroes of polynomial
$x^2 - 81 = 0$
$\Rightarrow (x - 9)(x + 9) = 0$
$\Rightarrow x = -9, 9$ are zeros of the polynomial.

20. Mahesh formed a square using four pieces of origami, as shown in the figure.



Based on above information answer the following questions.
(i) (a) Write the trinomial which describes the area of the given square. [1]
(b) If area of the square is given by the polynomial $x^2 - 10x + 25$; then what will be the side of the square? [1]

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(ii) (a) If $p(y) = y^2 - 2y + 1$, then find the value of $p(y) + p(-y)$. [1]
(b) What is the degree of the trinomial $x^3 + 2x^2 + 3x + 4$? [1]

Ans: (*i*) (*a*) Area of the given square = $(\text{Side})^2$
$= (x + 5)^2 = (x + 5)(x + 5) = x^2 + 10x + 25$
(*b*) Area of square $= x^2 - 10x + 25$
Area of square $= (x - 5)^2$
$\because$ Area of square $= (\text{side})^2$
$\therefore$ Side of square $= (x - 5)$
(*ii*) (*a*) $p(y) = y^2 - 2y + 1$
Also, $p(-y) = (-y)^2 - 2(-y) + 1 = y^2 + 2y + 1$
$\therefore p(y) + p(-y) = (y^2 - 2y + 1) + (y^2 + 2y + 1) = 2y^2 + 2$
(*b*) Here highest power of variable x is 3.
$\therefore$ Degree of the polynomial is 3.