CHAPTER 17

HERON'S FORMULA

REVISION OF KEY CONCEPTS AND FORMULAE

1. Area of a triangle when its base and height are known is calculated by using the formula:

$$\text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

2. If b denote the base and p the perpendicular of a right triangle, then

$$\text{Area of the triangle} = \frac{1}{2} bp$$

3. For an isosceles right-angled triangle, each of whose equal side is a, Area $= \frac{a^2}{2}$.

4. For an equilateral triangle, each of whose side is a, we have

(i) Area $= \frac{\sqrt{3}}{4} a^2$ (ii) Altitude/height $= \frac{\sqrt{3}}{2} a$

5. If a, b, c denote the lengths of the sides of a triangle, then

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

6. Area of a quadrilateral can be calculated by dividing the quadrilateral into triangles and using Heron's formula for calculating area of each triangle.

7. For an isosceles triangle with base a and equal sides b: Area $= \frac{1}{4} a \sqrt{4b^2 - a^2}$

8. For a regular hexagon with side a: Area $= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$.

9. For an equilateral triangle whose altitude is p: Area $= \frac{p^2}{\sqrt{3}}$

10. For a rectangle, we have

(i) Area = Length × Breadth

(ii) Length $= \frac{\text{Area}}{\text{Breadth}}$ and, Breadth $= \frac{\text{Area}}{\text{Length}}$

(iii) Diagonal $= \sqrt{(\text{Length})^2 + (\text{Breadth})^2}$

(iv) Perimeter = 2 (Length + Breadth)

(v) Area of four walls of a room = 2 (Length + Breadth) × Height.

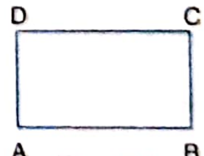

Fig. 17.1

11. For a square, we have

(i) Area $= (\text{Side})^2 = \frac{1}{2} (\text{Diagonal})^2$

(ii) Perimeter = 4 × Side

(iii) Diagonal $= \sqrt{2} \times$ Side

12. Area of a parallelogram = Base × Height $= AB \times h$

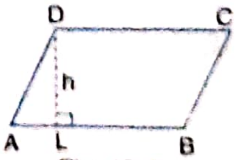

Fig. 17.2

13. Area of a rhombus $= \frac{1}{2} \times$ (Product of diagonals)

$= \frac{1}{2}(d_1 \times d_2)$

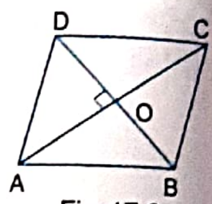

Fig. 17.3

Perimeter of a rhombus $= 2\sqrt{d_1^2 + d_2^2}$, where d_1, d_2 are lengths of diagonals.

14. Area of a trapezium $= \frac{1}{2}(AB + CD) \times h$ (see Fig. 17.4).

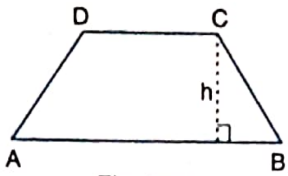

Fig. 17.4

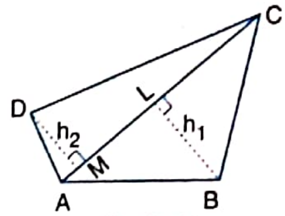

Fig. 17.5

15. Let $ABCD$ be a quadrilateral with diagonal AC and let $BL \perp AC$ and $DM \perp AC$. Then,

Area (quad. $ABCD$) $= \frac{1}{2} AC \times (BL + DM) = \frac{1}{2} AC \times (h_1 + h_2)$ (see Fig. 17.5).

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *The area of an equilateral triangle with side* $6\sqrt{3}$ *cm is*

(a) 27 cm^2 (b) $27\sqrt{3} \text{ cm}^2$ (c) $18\sqrt{3} \text{ cm}^2$ (d) $54\sqrt{3} \text{ cm}^2$

Ans. (b)

SOLUTION The area A of an equilateral triangle with side a unit is given by $A = \frac{\sqrt{3}}{4}a^2$.

Here, $a = 6\sqrt{3}$ cm

$$\therefore \quad A = \frac{\sqrt{3}}{4} \times (6\sqrt{3})^2 \text{ cm}^2 = 27\sqrt{3} \text{ cm}^2$$

EXAMPLE 2 *The area of an equilateral triangle with altitude* $2\sqrt{3}$ *cm is*

(a) $\frac{4}{\sqrt{3}} \text{cm}^2$ (b) $4\sqrt{3} \text{ cm}^2$ (c) 4 cm^2 (d) $\frac{8}{\sqrt{3}} \text{cm}^2$

Ans. (a)

SOLUTION The area of an equilateral triangle with altitude p is $\frac{p^2}{\sqrt{3}}$. Here, $p = 2\sqrt{3}$ cm. Therefore, area A is given by

$$A = \frac{p^2}{\sqrt{3}} = \frac{(2\sqrt{3})^2}{\sqrt{3}} = 4\sqrt{3} \text{ cm}^2$$

EXAMPLE 3 *The area of an equilateral triangle with perimeter* 12 *cm is*

(a) $16\sqrt{3} \text{ cm}^2$ (b) $8\sqrt{3} \text{ cm}^2$ (c) $4\sqrt{3} \text{ cm}^2$ (d) $6\sqrt{3} \text{ cm}^2$

Ans. (c)

SOLUTION Let the length of each side be a cm. Then,

Perimeter = 12 cm $\Rightarrow 3a = 12$ cm $\Rightarrow a = 4$ cm

$$\therefore \quad \text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 4^2 \text{ cm}^2 = 4\sqrt{3} \text{ cm}^2$$

EXAMPLE 4 *If the area of an equilateral triangle is* $81\sqrt{3}$ cm^2, *then its semi-perimeter is*

(a) 18 cm (b) 36 cm (c) 24 cm (d) 27 cm

Ans. (d)

SOLUTION Let a be the length of each side. Then,

$$\text{Area} = 81\sqrt{3}\ \text{cm}^2 \Rightarrow \frac{\sqrt{3}}{4} \times a^2 = 81\sqrt{3} \Rightarrow a^2 = 81 \times 4 \Rightarrow a = 18\ \text{cm}$$

$$\therefore \quad \text{Semi-perimeter} = \frac{3}{2}a = 27\ \text{cm}$$

EXAMPLE 5 *If each side of an equilateral triangle of area A is doubled, then the area of new triangle is*

(a) $2A$ (b) $4A$ (c) $8A$ (d) $6A$

Ans. (b)

SOLUTION Let a be the length of each side of the triangle. Then, $A = \frac{\sqrt{3}}{4}a^2$.

Let A' be the area of the triangle whose each side is $2a$. Then,

$$A' = \frac{\sqrt{3}}{4}(2a)^2 = 4\left(\frac{\sqrt{3}}{4}a^2\right) = 4A$$

EXAMPLE 6 *The area of an isosceles triangle with base 8 cm and each equal side is of length 6 cm, is*

(a) $4\sqrt{5}\ \text{cm}^2$ (b) $6\sqrt{5}\ \text{cm}^2$ (c) $8\sqrt{5}\ \text{cm}^2$ (d) $16\sqrt{5}\ \text{cm}^2$

Ans. (c)

SOLUTION The area A of an equilateral triangle with base a and each equal side b is given by

$$A = \frac{a}{4}\sqrt{4b^2 - a^2}.$$

Here, $a = 8$ cm and $b = 6$ cm.

$$\therefore \quad A = \frac{8}{4}\sqrt{4 \times 6^2 - 8^2} = 2\sqrt{144 - 64} = 8\sqrt{5}\ \text{cm}^2$$

EXAMPLE 7 *If the perimeter of an isosceles triangle is 32 cm and the ratio of the equal side to its base is 3 : 2, then area of the triangle is*

(a) $16\sqrt{2}\ \text{cm}^2$ (b) $20\sqrt{2}\ \text{cm}^2$ (c) $30\sqrt{2}\ \text{cm}^2$ (d) $32\sqrt{2}\ \text{cm}^2$

Ans. (d)

SOLUTION Let a cm be the base and each equal side be b cm. Then,

$$a + 2b = 32 \qquad \text{[Given]} \quad ...(i)$$

It is given that $\frac{b}{a} = \frac{3}{2} \Rightarrow 2b = 3a \Rightarrow b = \frac{3a}{2}$.

Putting $b = \frac{3a}{2}$ in (i), we obtain

$$a + 3a = 32 \Rightarrow a = 8\ \text{cm}$$

$$\therefore \quad b = 12\ \text{cm}$$

Let A be the area of the triangle. Then,

$$A = \frac{a}{4}\sqrt{4b^2 - a^2} = \frac{8}{4}\sqrt{4 \times 12^2 - 8^2}\ \text{cm}^2 = 2\sqrt{512}\ \text{cm}^2 = 32\sqrt{2}\ \text{cm}^2$$

EXAMPLE 8 *The perimeter of an isosceles right triangle having area 100* cm^2 *is*

(a) $20\sqrt{2}$ cm (b) $20(\sqrt{2}+1)$ cm (c) 10 cm (d) $(10+\sqrt{2})$ cm

Ans. (b)

SOLUTION Let a be the hypotenuse and b be the length of each remaining side of the triangle. Then,

$$a^2 = b^2 + b^2 \Rightarrow a^2 = 2b^2$$

It is given that the area of the triangle is 100 cm².

$\therefore$ $$\frac{1}{2}b \times b = 100 \Rightarrow b^2 = 200 \Rightarrow b = 10\sqrt{2} \text{ cm}$$

Putting $b^2 = 200$ in $a^2 = 2b^2$, we obtain

$$a^2 = 400 \Rightarrow a = 20 \text{ cm}$$

$\therefore$ $$\text{Perimeter} = a + 2b = (20 + 20\sqrt{2})\text{cm} = 20(\sqrt{2} + 1)\text{ cm}$$

EXAMPLE 9 *The area of a right angled triangle is 240 cm² and side other than hypotenuse is 30 cm, the perimeter of the triangle, is*

(a) 20 cm (b) 80 cm (c) 100 cm (d) 140 cm

Ans. (b)

SOLUTION Let the lengths of two sides, other than hypotenuse, of right triangle be a cm and b cm. It is given that $a = 30$ cm. Then,

$$\text{Area} = 240 \text{ cm}^2 \Rightarrow \frac{1}{2}ab = 240 \Rightarrow 30b = 480 \Rightarrow b = 16 \text{ cm}$$

Applying Pythagoras theorem, we obtain

$$\text{Hypotenuse} = \sqrt{a^2 + b^2} = \sqrt{30^2 + 16^2} = \sqrt{900 + 256} = \sqrt{1156} \text{ cm} = 34 \text{ cm}$$

$\therefore$ $$\text{Perimeter} = (30 + 16 + 34) \text{ cm} = 80 \text{ cm}.$$

EXAMPLE 10 *AD is a median of triangle ABC and area of* $\Delta ADC = 15$ *cm², then* $ar(\Delta ABC)$ *is*

(a) 15 cm² (b) 22.5 cm² (c) 30 cm² (d) 37.5 cm²

Ans. (c)

SOLUTION A median of a triangle divides it into two triangles of equal area. Therefore,

$$ar(\Delta ABC) = 2\, ar(\Delta ADC) = 2 \times 15 \text{ cm}^2 = 30 \text{ cm}^2$$

EXAMPLE 11 *If a kite in the shape of an isosceles triangle of base 8 cm and each equal side 6 cm is to be made, the area of paper required to make the kite, is*

(a) $10\sqrt{2}$ cm² (b) $8\sqrt{5}$ cm² (c) $\sqrt{5}$ cm² (d) 8 cm²

Ans. (b)

SOLUTION Here, $a = 8$ cm and $b = 6$ cm.

$\therefore$ $$\text{Area of kite} = \frac{a}{4}\sqrt{4b^2 - a^2} = \frac{8}{4}\sqrt{4 \times 36 - 64} = 2\sqrt{80} \text{ cm}^2 = 8\sqrt{5} \text{ cm}^2$$

EXAMPLE 12 *The perimeter of a right angled triangle is 72 cm and its area is 216 cm². The sum of the lengths of its perpendicular sides is*

(a) 36 cm (b) 32 cm (c) 42 cm (d) 50 cm

Ans. (c)

SOLUTION Let the lengths of base and perpendicular be a cm and b cm respectively. Then, its hypotenuse is of length $\sqrt{a^2 + b^2}$. It is given that the perimeter is 72 cm and area is 216 cm².

$\therefore$ $$a + b + \sqrt{a^2 + b^2} = 72 \quad \text{...(i)}$$

and, $$\frac{1}{2}ab = 216 \Rightarrow ab = 432 \quad \text{...(ii)}$$

Now, $$a + b + \sqrt{a^2 + b^2} = 72$$

$\Rightarrow \quad (a+b)+\sqrt{(a+b)^2-2ab}=72$

$\Rightarrow \quad (a+b)+\sqrt{(a+b)^2-2\times 432}=72$

$\Rightarrow \quad x+\sqrt{x^2-864}=72$

$\Rightarrow \quad \sqrt{x^2-864}=(72-x)$

$\Rightarrow \quad x^2-864=(72-x)^2$

$\Rightarrow \quad x^2-864=72^2-144x+x^2 \Rightarrow 144x=5184+864 \Rightarrow 144x=6048 \Rightarrow x=42$

$\Rightarrow \quad a+b=42$ cm.

EXAMPLE 13 *Area of a right-angled triangle is 6 cm² and its perimeter is 12 cm. Then length of its hypotenuse, is*

(a) 5 cm (b) 6 cm (c) 7 cm (d) 8 cm

Ans. (a)

SOLUTION Let the lengths of the perpendicular sides of the right-angled triangle be a cm and b cm. Then, its hypotenuse is of length $\sqrt{a^2+b^2}$ cm. It is given that the perimeter of the triangle is 12 cm and area is 6 cm².

$\therefore \quad \frac{1}{2}ab=6 \Rightarrow ab=12$ ...(i)

and, $a+b+\sqrt{a^2+b^2}=12$

$\Rightarrow \quad a+b+\sqrt{(a+b)^2-2ab}=12$

$\Rightarrow \quad x+\sqrt{x^2-2\times 12}=12$, where $x=a+b$

$\Rightarrow \quad \sqrt{x^2-24}=(12-x)$

$\Rightarrow \quad x^2-24=(12-x)^2 \Rightarrow x^2-24=144-24x+x^2 \Rightarrow 24x=168 \Rightarrow x=7 \Rightarrow a+b=7$ ...(ii)

$\therefore \quad \text{Hypotenuse}=\sqrt{a^2+b^2}=\sqrt{(a+b)^2-2ab}=\sqrt{7^2-2\times 12}=\sqrt{25}=5\text{ cm}.$

EXAMPLE 14 *In a triangle, the average of any two sides is 6 cm more than half of the third side. The area of the triangle is*

(a) $64\sqrt{3}$ cm² (b) $48\sqrt{3}$ cm² (c) $72\sqrt{3}$ cm² (d) $36\sqrt{3}$ cm²

Ans. (d)

SOLUTION Let the lengths of three sides of the triangle be a, b, c (in cms). It is given that

$$\frac{a+b}{2}=\frac{c}{2}+6,\ \frac{b+c}{2}=\frac{a}{2}+6 \text{ and } \frac{c+a}{2}=\frac{b}{2}+6$$

$\Rightarrow \quad a+b-c=12,\ b+c-a=12 \text{ and } c+a-b=12$

$\Rightarrow \quad (a+b+c)-2c=12,\ (b+c+a)-2a=12 \text{ and } (c+a+b)-2b=12$

$\Rightarrow \quad 2s-2c=12,\ 2s-2a=12 \text{ and } 2s-2b=12$, where $2s=a+b+c$

$\Rightarrow \quad s-c=6,\ s-a=6 \text{ and } s-b=6$

$\Rightarrow \quad (s-a)+(s-b)+(s-c)=6+6+6 \Rightarrow 3s-(a+b+c)=18 \Rightarrow 3s-2s=18 \Rightarrow s=18$

So, area A of the triangle is given by

$$A=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18\times 6\times 6\times 6}\text{ cm}^2=36\sqrt{3}\text{ cm}^2$$

EXAMPLE 15 *If the sum of any two sides of a triangle exceeds the third by 6 cm, the area of the triangle is*

(a) $12\sqrt{3}$ cm² (b) $18\sqrt{3}$ cm² (c) $15\sqrt{3}$ cm² (d) $9\sqrt{3}$ cm²

Ans. (d)

SOLUTION Let s be the semi-perimeter of a triangle of sides a cm, b cm and c cm. Then, $2s = a + b + c$. It is given that

$$a + b = c + 6, b + c = a + 6 \text{ and } c + a = b + 6$$
$$\Rightarrow \quad a + b - c = 6, b + c - a = 6 \text{ and } c + a - b = 6$$
$$\Rightarrow \quad (a + b + c) - 2c = 6, (a + b + c) - 2a = 6 \text{ and } (a + b + c) - 2b = 6$$
$$\Rightarrow \quad 2s - 2c = 6, 2s - 2a = 6 \text{ and } 2s - 2b = 6 \Rightarrow s - a = 3, s - b = 3 \text{ and } s - c = 3$$

Adding these three, we obtain

$$3s - (a + b + c) = 9 \Rightarrow 3s - 2s = 9 \Rightarrow s = 9$$

So, area A of the triangle is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9 \times 3 \times 3 \times 3} \text{ cm}^2 = 9\sqrt{3} \text{ cm}^2$$

EXAMPLE 16 *The area of a right triangle is 28 cm². If one of its perpendicular sides exceeds the other by 10 cm, then the length of the longest of the perpendicular is*

(a) 16 cm (b) 14 cm (c) $6\sqrt{5}$ cm (d) 18 cm

Ans. (b)

SOLUTION Let the perpendicular sides be of length x cm and $(x + 10)$ cm respectively. Then,

$$\text{Area} = 28 \text{ cm}^2 \Rightarrow \frac{1}{2}x(x + 10) = 28 \Rightarrow x^2 + 10x - 56 = 0 \Rightarrow (x + 14)(x - 4) = 0 \Rightarrow x = 4$$

Thus, the length of the longest perpendicular side is $(x + 10)$ cm = 14 cm.

EXAMPLE 17 *Each side of a triangle is multiplied with the sum of the squares of the other two sides. If the sum of all such possible results is 6 times the product of the sides, then the triangle must be*

(a) *equilateral* (b) *isosceles* (c) *scalene* (d) *right angled*

Ans. (a)

SOLUTION Let a, b, c be the lengths of the sides of the triangle. According to the question

$$a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2) = 6abc$$
$$\Rightarrow \quad a(b - c)^2 + b(c - a)^2 + c(a - b)^2 = 0$$
$$\Rightarrow \quad a(b - c) = 0, b(c - a) = 0, c(a - b) = 0 \quad [\because a(b - c)^2 \geq 0, b(-a)^2 \geq 0 \text{ and } c(a - b)^2 \geq 0]$$
$$\Rightarrow \quad b = c, c = a \text{ and } a = b \Rightarrow a = b = c$$

So, triangle ABC is equilateral.

EXAMPLE 18 *In a scalene triangle, one side exceeds the other two sides by 4 cm and 5 cm respectively and the perimeter of the triangle is 36 cm. The area of triangle is*

(a) 63 cm² (b) $9\sqrt{10}$ cm² (c) $18\sqrt{10}$ cm² (d) $12\sqrt{21}$ cm²

Ans. (d)

SOLUTION Let the sides of the triangle be a, $a - 4$ and $a - 5$. It is given that perimeter of the triangle is 36 cm.

$$\therefore \quad a + a - 4 + a - 5 = 36 \Rightarrow 3a = 45 \Rightarrow a = 15$$

Thus, the lengths of the sides are 15 cm, 11 cm and 10 cm.

We have, $2s = 36 \Rightarrow s = 18$

The area of the triangle is given by

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18(18-15)(18-11)(18-10)} = 12\sqrt{21} \text{ cm}^2$$

EXAMPLE 19 *The sides of a triangle are 45 cm, 60 cm and 75 cm. The length of the altitude drawn to the longest side from its opposite vertex is*

(a) 27 cm (b) 21 cm (c) 39 cm (d) 36 cm

Ans. (d)

SOLUTION Let s be the semi-perimeter of the triangle. Then,

$$2s = (45 + 60 + 75) \text{ cm} \Rightarrow s = 90$$

Let A be the area of the triangle. Then,

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{90(90-45)(90-60)(90-75)}$$

$$\Rightarrow A = \sqrt{90 \times 45 \times 30 \times 15} = \sqrt{2 \times 3^2 \times 5 \times 3^2 \times 5 \times 3 \times 2 \times 5 \times 3 \times 5} = 2 \times 3^3 \times 5^2 \text{ cm}^2 = 1350 \text{ cm}^2$$

Also, $A = \frac{1}{2}$ (75 × Altitude drawn to the longest side)

$\Rightarrow$ $1350 = \frac{1}{2} \times 75 \times$ Altitude drawn to the longest side

$\Rightarrow$ Altitude drawn to the longest side = 36 cm.

CASE STUDY BASED

EXAMPLE 20 *To beautify parks in a city, city municipal corporation decided to make triangular flower beds in parks as shown in Fig. 17.6. The dimensions of a triangular flower bed are 75 m × 80 m × 85 m. Based on this information answer the following questions:*

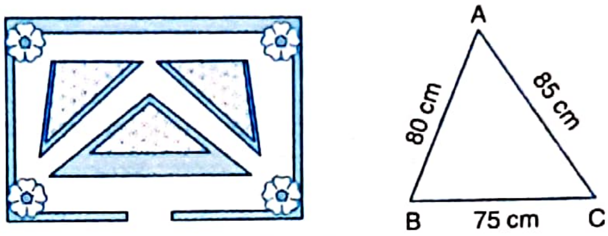

Fig. 17.6

(i) *If each triangular flower bed is to be fenced with two parallel wires one below the other than the length of the wire used is*

(a) 120 m (b) 240 m (c) 260 m (d) 480 m

(ii) *The area of a flower bed is*

(a) $300\sqrt{42}$ m² (b) $300\sqrt{21}$ m² (c) $600\sqrt{21}$ m² (d) $400\sqrt{21}$ m²

(iii) *If each triangular bed is an equilateral triangle of side 60 m, then its area is*

(a) $900\sqrt{3}$ m² (b) $600\sqrt{3}$ m² (c) $1200\sqrt{3}$ m² (d) $400\sqrt{3}$ m²

(iv) *The area of an isosceles triangle with base 'a' and equal sides 'b' is given by*

(a) $\frac{a}{4}\sqrt{4b^2 - a^2}$ (b) $\frac{b}{4}\sqrt{4a^2 - b^2}$ (c) $\frac{a}{2}\sqrt{2b^2 - a^2}$ (d) $\frac{b}{2}\sqrt{4a^2 - b^2}$

(v) *If each triangular bed is in the form of an isosceles triangle with base 60 m and equal sides of length 40 m each, then area of a flower bed is*

(a) $150\sqrt{7}$ m² (b) $75\sqrt{7}$ m² (c) $300\sqrt{7}$ m² (d) $200\sqrt{7}$ m²

SOLUTION (i) Ans. (d): Length of wire = 2 (Perimeter of the triangular field)

$$= 2(75 + 80 + 85) \text{ m} = 480 \text{ m}$$

(ii) Ans. (c): We have, $2s = 240 \Rightarrow s = 120$

Let Δ be the area of a flower bed. Then,

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{120(120-75)(120-80)(120-85)} = 600\sqrt{21} \text{ m}^2$$

(iii) Ans. (a): Using: Area $= \frac{\sqrt{3}}{4}(\text{Side})^2$, we obtain

$$\text{Area of a flower bed} = \frac{\sqrt{3}}{4} \times (60)^2 \text{ m}^2 = 900\sqrt{3} \text{ m}^2$$

(iv) Ans. (a)

(v) Ans. (c): We have, $a = 60$ m and $b = 40$ m

$\therefore$ Area of flower bed $= \frac{a}{4}\sqrt{4b^2 - a^2} = \frac{60}{4}\sqrt{4 \times 1600 - 60^2}\ \text{m}^2 = 300\sqrt{7}\ \text{m}^2$

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 21 *Statement-1 (Assertion):* *The area of an equilateral triangle with each side a is* $\Delta = \frac{\sqrt{3}}{4}a^2$ *sq. units.*

Statement-2 (Reason): *The area of a triangle with perimeter 2s and sides a, b, c is given by* $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

Ans. (a)

SOLUTION Statement-2 is the standard Heron's formula. So, statement-2 is true. For an equilateral triangle, we have,

$$a = b = c \text{ and } s = \frac{3a}{2} \text{ sq. units}$$

$$\therefore \quad \Delta = \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)} = \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$$

Thus, statement-1 is also true. Clearly, statement-1 is a direct consequence of statement-2. Hence, option (a) is correct.

EXAMPLE 22 *Statement-1 (Assertion):* *The altitude p of an equilateral triangle having each side a is given by* $p = \frac{\sqrt{3}}{2}a$.

Statement-2 (Reason): *The area* Δ *of an equilateral triangle having each side a is given by* $\Delta = \frac{\sqrt{3}}{4}a^2$.

Ans. (b)

SOLUTION Let ABC be an equilateral triangle such that $AB = AC = BC = a$. Draw $AL \perp BC$. In triangle ALB, we obtain

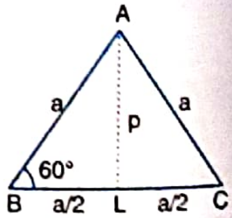

Fig. 17.7

$$AB^2 = AL^2 + BL^2 \Rightarrow a^2 = p^2 + \frac{a^2}{4} \Rightarrow p^2 = \frac{3a^2}{4} \Rightarrow p = \frac{\sqrt{3}a}{2}$$

So, statement-1 is true.

$$\Delta = \frac{1}{2}\text{ Base} \times \text{Height} = \frac{1}{2}(BC \times p) = \frac{1}{2}\left(a \times \frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$$

So, statement-2 is also true.

Thus, both the statements are true. Hence, option (b) is correct.

EXAMPLE 23 *Statement-1 (Assertion):* *The area of an equilateral triangle the length of whose each side is positive integer, is an irrational number.*

Statement-2 (Reason): *The area of an equilateral triangle having each side equal to a is* $\frac{\sqrt{3}}{4}a^2$.

Ans. (a)

SOLUTION Statement-2 is true. If a is a positive integer, then so is a^2 and hence $\frac{a^2}{4}$ is a rational number. Consequently $\frac{\sqrt{3}}{4}a^2$ is an irrational number. Thus, statement-1 is true and statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 24 *Statement-1 (Assertion):* *The area of a given triangle and the area of a triangle obtained by doubling its sides are in the ratio* 1 : 2.

Statement-2 (Reason): *If a, b, c are lengths of the sides of a triangle with semi-perimeter s, then its area* Δ *is given by* $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

Ans. (d)

SOLUTION Statement-2 is true. Let s' be the semi-perimeter of triangle of sides $2a, 2b, 2c$ and Δ' be its area. Then,

$$s' = \frac{2a+2b+2c}{2} = a+b+c = 2s$$

and, $$\Delta' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)} = 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta \Rightarrow \frac{\Delta}{\Delta'} = \frac{1}{4}$$

So, statement-1 is not true. Hence, option (d) is correct.

EXAMPLE 25 *Statement-1 (Assertion):* *The area of an isosceles triangle each of whose equal side is 13 cm and whose base is 24 cm is 60 cm*^{2}.

Statement-2 (Reason): *The area of an isosceles triangle having base a and each equal side b is* $\frac{b}{4}\sqrt{4a^2-b^2}$.

Ans. (c)

SOLUTION Statement-2 is not true, because the area of an isosceles triangle having base a and each equal side b is $\frac{a}{4}\sqrt{4b^2-a^2}$. Putting $a = 24$ and $b = 13$, we obtain

$$\text{Area} = \frac{24}{4}\sqrt{4\times 13^2 - 24^2} = 6\sqrt{676-576} = 6\times 10 = 60 \text{ cm}^2$$

So, statement-1 is true. Hence, option (c) is correct.

EXAMPLE 26 *Statement-1 (Assertion):* *The area of the isosceles triangle is* $\frac{5}{4}\sqrt{11}$ *cm*^{2}*, if the perimeter is 11 cm and the base is 5 cm.*

Statement-2 (Reason): *The area of the equilateral triangle is* $20\sqrt{3}$ *cm*^{2} *whose each side is 8 cm.*

Ans. (c)

SOLUTION We have base $(a) = 5$ cm. Let the length of each equal side be b cm. Then,

$$\text{Perimeter} = 11 \text{ cm} \Rightarrow a+b+a = 11 \Rightarrow 2b+5 = 11 \Rightarrow 2b = 6 \Rightarrow b = 3 \text{ cm}$$

$$\therefore \quad \text{Area} = \frac{a}{4}\sqrt{4b^2-a^2} = \frac{5}{4}\sqrt{4\times 9 - 25} = \frac{5}{4}\sqrt{11}\text{ cm}^2$$

So, statement-1 is true.

The area of the equilateral triangle whose each side is 8 cm is

$$A = \frac{\sqrt{3}}{4}\times 8^2 \text{ cm}^2 = 16\sqrt{3}\text{ cm}^2$$

So, statement-2 is not true. Hence, option (c) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. The sides of a triangle are 16 cm, 30 cm, 34 cm. Its area is
   (a) 225 cm^2 (b) 240 cm^2 (c) $225\sqrt{2} \text{ cm}^2$ (d) 450 cm^2
2. The base of an isosceles right triangle is 30 cm. Its area is
   (a) 225 cm^2 (b) $225\sqrt{3} \text{ cm}^2$ (c) $225\sqrt{2} \text{ cm}^2$ (d) 450 cm^2
3. The sides of a triangle are 7 cm, 9 cm and 14 cm. Its area is
   (a) $12\sqrt{5} \text{ cm}^2$ (b) $12\sqrt{3} \text{ cm}^2$ (c) $24\sqrt{5} \text{ cm}^2$ (d) 63 cm^2
4. The sides of a triangular field are 325 m, 300 m and 125 m. Its area is
   (a) 18750 m^2 (b) 37500 m^2 (c) 97500 m^2 (d) 48750 m^2
5. The sides of a triangle are 50 cm, 78 cm and 112 cm. The smallest altitude is
   (a) 20 cm (b) 30 cm (c) 40 cm (d) 50 cm
6. The sides of a triangle are 11 m, 60 m and 61 m. The altitude to the smallest side is
   (a) 11 m (b) 66 m (c) 50 m (d) 60 m
7. The sides of a triangle are 11 cm, 15 cm and 16 cm. The altitude to the largest side is
   (a) $30\sqrt{7}$ cm (b) $\frac{15\sqrt{7}}{2}$ cm (c) $\frac{15\sqrt{7}}{4}$ cm (d) 30 cm
8. The base and hypotenuse of a right triangle are respectively 5 cm and 13 cm long. Its area is
   (a) 25 cm^2 (b) 28 cm^2 (c) 30 cm^2 (d) 40 cm^2
9. The length of each side of an equilateral triangle of area $4\sqrt{3} \text{ cm}^2$, is
   (a) 4 cm (b) $\frac{4}{\sqrt{3}}$ cm (c) $\frac{\sqrt{3}}{4}$ cm (d) 3 cm
10. If an isosceles right triangle has area 8 cm^2, then the length of its hypotenuse is
    (a) $\sqrt{32}$ cm (b) $\sqrt{48}$ cm (c) $\sqrt{24}$ cm (d) 4 cm
    [NCERT EXEMPLAR]
11. The perimeter of an equilateral triangle is 60 m. The area is
    (a) $10\sqrt{3} \text{ m}^2$ (b) $15\sqrt{3} \text{ m}^2$ (c) $20\sqrt{3} \text{ m}^2$ (d) $100\sqrt{3} \text{ m}^2$
    [NCERT EXEMPLAR]
12. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is
    (a) $\sqrt{15} \text{ cm}^2$ (b) $\frac{\sqrt{15}}{2} \text{ cm}^2$ (c) $2\sqrt{15} \text{ cm}^2$ (d) $4\sqrt{15} \text{ cm}^2$
    [NCERT EXEMPLAR]
13. The length of each side of an equilateral triangle having an area of $9\sqrt{3} \text{ cm}^2$ is
    (a) 8 cm (b) 36 cm (c) 4 cm (d) 6 cm
    [NCERT EXEMPLAR]
14. If the area of an equilateral triangle is $16\sqrt{3} \text{ cm}^2$, then its perimeter is
    (a) 48 cm (b) 24 cm (c) 12 cm (d) 36 cm
    [NCERT EXEMPLAR]
15. The sides of a triangle are 35 cm, 54 cm and 61 cm respectively. The length of its longest altitude is
    (a) $16\sqrt{5}$ cm (b) $10\sqrt{5}$ cm (c) $24\sqrt{5}$ cm (d) 28 cm
    [NCERT EXEMPLAR]

16. The sides of a triangle are 56 cm, 60 cm and 52 cm. Area of the triangle is
(a) 1322 cm^2 (b) 1311 cm^2 (c) 1344 cm^2 (d) 1392 cm^2
[NCERT EXEMPLAR]

17. The edges of a triangular board are 6 cm, 8 cm and 10 cm long. The cost of painting it at the rate of 9 paise per cm^2 is
(a) ₹ 2 (b) ₹ 2.16 (c) ₹ 2.48 (d) ₹ 3
[NCERT EXEMPLAR]

18. The area of an equilateral triangle with side $2\sqrt{3}$ cm is
(a) 5.196 cm^2 (b) 0.866 cm^2 (c) 3.496 cm^2 (d) 1.732 cm^2
[NCERT EXEMPLAR]

19. If the area of a regular hexagon is $54\sqrt{3} \text{ cm}^2$, then the length of its each side is
(a) 3 cm (b) $2\sqrt{3}$ cm (c) 6 cm (d) $6\sqrt{3}$ cm

20. If the length of each edge of a regular tetrahedron is 'a', then its surface area is
(a) $\sqrt{3}\, a^2$ sq. units (b) $3\sqrt{2}\, a^2$ sq. units (c) $2\sqrt{3}\, a^2$ sq. units (d) $\sqrt{6}\, a^2$ sq. units

21. If the area of an isosceles right triangle is 8 cm^2, what is the perimeter of the triangle?
(a) $8 + \sqrt{2} \text{ cm}^2$ (b) $8 + 4\sqrt{2} \text{ cm}^2$ (c) $4 + 8\sqrt{2} \text{ cm}^2$ (d) $12\sqrt{2} \text{ cm}^2$

22. The lengths of the sides of ΔABC are consecutive integers. It ΔABC has the same perimeter as an equilateral triangle with a side of length 9 cm, what is the length of the shortest side of ΔABC?
(a) 4 (b) 6 (c) 8 (d) 10

23. In Figure 17.8, the ratio of AD to DC is 3 to 2. If the area of ΔABC is 40 cm^2, what is the area of ΔBDC?
(a) 16 cm^2 (b) 24 cm^2 (c) 30 cm^2 (d) 36 cm^2

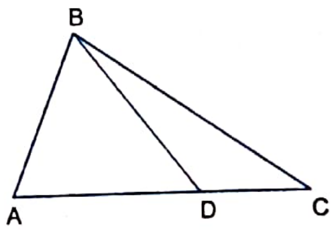

Fig. 17.8

24. If the length of a median of an equilateral triangle is x cm, then its area, is
(a) x^2 (b) $\frac{\sqrt{3}}{2}x^2$ (c) $\frac{x^2}{\sqrt{3}}$ (d) $\frac{x^2}{2}$

25. If every side of a triangle is doubled, then increase in the area of the triangle, is
(a) $100\sqrt{2}\%$ (b) 200% (c) 300% (d) 400%

26. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then area of the triangle is
(a) $24\sqrt{2} \text{ cm}^2$ (b) $24\sqrt{3} \text{ cm}^2$ (c) $48\sqrt{3} \text{ cm}^2$ (d) $64\sqrt{3} \text{ cm}^2$

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

27. Statement-1 (Assertion): The area Δ of an isosceles triangle with base a and each equal side b is given by $\Delta = \frac{a}{4}\sqrt{4b^2 - a^2}$.

Statement-2 (Reason): The area Δ of a triangle with semi-perimeter s and sides a, b and c is given by $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

28. Statement-1 (Assertion): The altitude p of an equilateral triangle having each side a is given by $p = \frac{\sqrt{3}}{2}a$.

Statement-2 (Reason): If p is the altitude of an equilateral triangle, then its area A is given by $\Delta = \frac{p^2}{\sqrt{3}}$.

29. Statement-1 (Assertion): The area of an equilateral triangle the length of whose altitude is 6 cm, is $12\sqrt{3}\ \text{cm}^2$.

Statement-2 (Reason): The area of an equilateral triangle with altitude p is $\Delta = \frac{p^2}{\sqrt{3}}$.

30. Statement-1 (Assertion): If the area of an equilateral triangle is $36\sqrt{3}\ \text{cm}^2$, then its perimeter is 36 cm.

Statement-2 (Reason): If the perimeter of an equilateral triangle is 72 cm, then its altitude is $8\sqrt{3}$ cm.

31. Statement-1 (Assertion): The area of an isosceles triangle having base 24 cm and each of the equal sides equal to 13 cm is 60 cm^2.

Statement-2 (Reason): The area of an isosceles triangle with base a and each equal side b is $\frac{b}{4}\sqrt{4a^2 - b^2}$.

32. Statement-1 (Assertion): If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm^2.

Statement-2 (Reason): The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm respectively. The area of the parallelogram is 30 cm^2.

ANSWERS

1. (b)	2. (b)	3. (a)	4. (a)	5. (b)	6. (d)	7. (c)
8. (c)	9. (a)	10. (a)	11. (d)	12. (a)	13. (d)	14. (b)
15. (c)	16. (c)	17. (b)	18. (a)	19. (c)	20. (c)	21. (b)
22. (c)	23. (a)	24. (c)	25. (c)	26. (d)	27. (a)	28. (b)
29. (a)	30. (c)	31. (c)	32. (c)			