6 LINES AND ANGLES

INTRODUCTION

In this chapter, we will study the properties of the angles formed when two lines intersect each other and also the properties of the angles formed when a line intersect two or more parallel lines at distinct points.

1 INTERSECTING LINES, ANGLES AND PAIR OF THE ANGLES

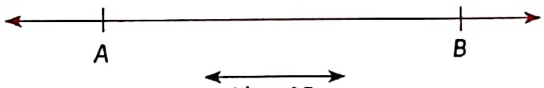
1.1 Basic Terms and Definitions

(1) **Point:** A point is an exact location and is represented by a fine dot made by a sharp pen on a sheet of paper. Thus, A is a point as shown in the adjoining figure.

(2) **Line:** Line is the collection of points which has only length, not breadth and thickness.

or

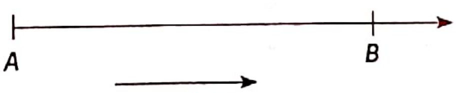
A line is a straight path that is endless in both directions. It is denoted by $\overline{AB}$ or $\overline{BA}$.



Line AB

A line has no end point.

(3) **Ray:** A part of a line with one end point is called a ray. The ray AB is denoted $\overline{AB}$.



Ray $\overline{AB}$

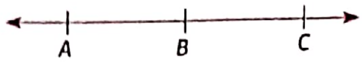
The ray $\overline{AB}$ has one end point namely A, called its initial point. Thus, a ray has no definite length.

(4) **Line Segment:** A part of a line with two end points is called a line segment. Line segment AB is denoted by $\overline{AB}$

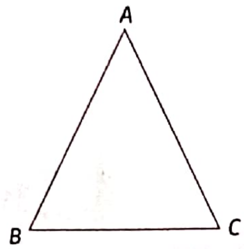
Line segment $\overline{AB}$

A line segment has a definite length, which can be measured. The line segment $\overline{AB}$ is the same thing as the line segment $\overline{BA}$.

(5) **Collinear Points:** If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear point. In the figure A, B and C are collinear points.



(6) **Non-collinear Points:** If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points. In the figure A, B and C are non-collinear points.



1.2 Distinction among a Line, a Ray and a Line Segment

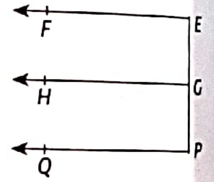
Line	Ray	Line Segment
A line has no end point.	A ray has only one end point.	A line segment has two end points.
A line does not have a definite length.	A ray does not have a definite length.	A line segment has a definite length.
We cannot draw a line on a paper. We can simply represent it two arrow head at both end.	We cannot drawn a ray on a paper. We can simply represent it an arrow head at one of the end.	A line segment has a definite length. it can be drawn on a paper.
Representation of a line $\overline{AB}$.	Representation of a ray $\overline{AB}$.	Representation of a line segment $\overline{AB}$.

In this chapter, we will not use these symbols and will denote the segment $\overline{AB}$, ray $\overline{AB}$, and line $\overline{AB}$ by the same symbol *AB*. Sometimes, we use small letter *l*, *m*, *n*, etc., to denote a line.

Example 1: In the given figure, write the name of

(i) line segments.

(ii) rays.



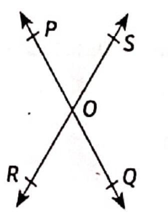
Hint: Use the property of line segment and a ray (i.e., a line segment has a definite length and a ray has only one end point).

Solution: Here, $\overline{GE}, \overline{GP}$ and $\overline{EP}$, are line segments and $\overline{EF}, \overline{GH}$ and $\overline{PQ}$ are rays.

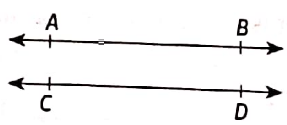
1.3 Intersecting Lines and Non-intersecting Lines

(1) Lines *PQ* and *RS* are intersecting lines because they intersect each other at *O*.

(2) Lines *AB* and *CD* are non-intersecting lines, i.e., parallel lines because they never intersect each other.



Intersecting lines



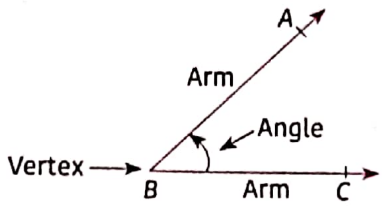
Non-intersecting (parallel lines)

- The lengths of the common perpendicular at different points on these parallel lines is the same and this equal length is called the distance between two parallel lines.
- If two lines are non-intersecting, then they will be parallel and vice-versa.

2 ANGLES AND THEIR TYPES

2.1 Angle

The figure formed by two rays with the same initial point, is called an angle.

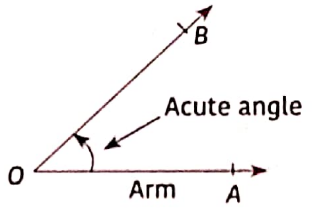


In the above figure, the common initial point B is known as the vertex of the angle and the rays ($\overrightarrow{BA}$ and $\overrightarrow{CA}$) forming the angle are called its arms.

2.2 Types of Angles

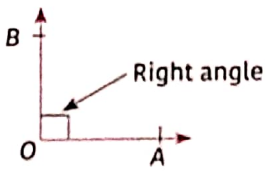
There are different types of angles such as acute angle, right angle, obtuse angle, straight angle, reflex angle and complete angle etc., which are discussed below:

(1) **Acute Angle:** An angle whose measure is more then 0° but less than 90°, is called an acute angle.



In the above figure, $\angle AOB$ is an acute angle. i.e., $0° < \angle AOB < 90°$

(2) **Right Angle:** An angle whose measure is 90°, is called a right angle.

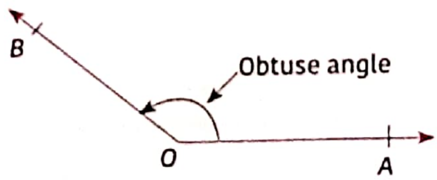


In the above figure, $\angle AOB$ is a right angle, i.e., $\angle AOB = 90°$ and $BO \perp OA$.

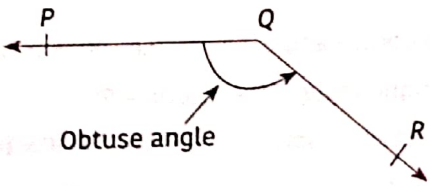
(3) **Obtuse Angle:** An angle whose measure is more than 90° but less than 180° is called an obtuse angle.

In the given figure, $\angle AOB$ and $\angle PQR$ are obtuse angles.

(i) $90° < \angle AOB < 180°$

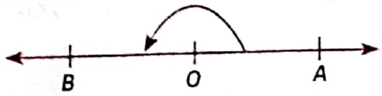


(ii) $90° < \angle PQR < 180°$



(4) **Straight Angle:** An angle whose measure is 180°, is called a straight angle.

In the given figure $\angle AOB = 180°$ is a straight angle.

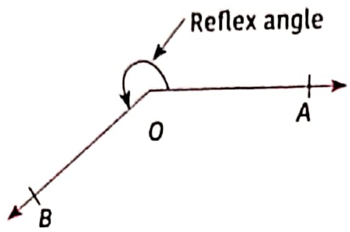


A straight angle has two right angles or an acute and an obtuse angle.

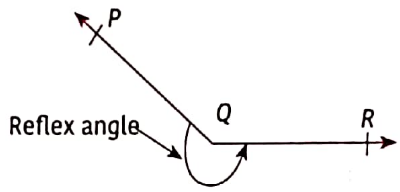
(5) **Reflex Angle:** An angle whose measure is more than 180° but less than 360° is called a reflex angle.

In the given figure, $\angle AOB$ and $\angle PQR$ are reflex angles.

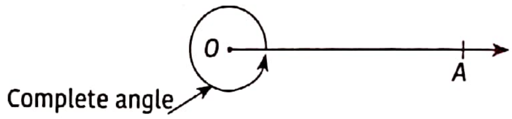
(i) $180^\circ <$ reflex $\angle AOB < 360^\circ$



(ii) $180^\circ <$ reflex $\angle PQR < 360^\circ$



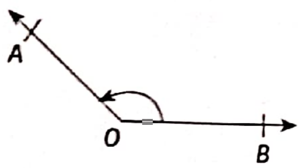
(6) **Complete Angle:** An angle whose measure is 360° is called a complete angle.



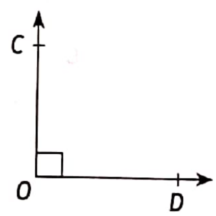
In the given figure $\angle AOA = 360^\circ$ is a complete angle.

Example 1: Which type of angle is formed in the given figures?

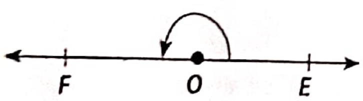
(i)



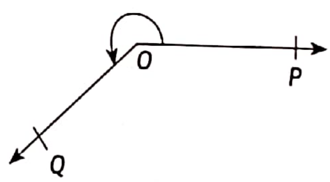
(ii)



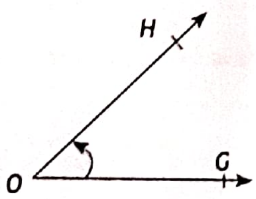
(iii)



(iv)



(v)



(vi)

Solution: (i) In the given figure, $\angle AOB$ is an obtuse angle (i.e., $90^\circ < \angle AOB < 180^\circ$)

(ii) In the given figure, $\angle COD$ is a right angle. (i.e., $\angle COD = 90^\circ$)

(iii) In the given figure, $\angle FOE$ is a straight angle. (i.e., $\angle FOE = 180^\circ$)

(iv) In the figure, $\angle POQ$ is a reflex angle. (i.e., $180^\circ <$ reflex $\angle POQ < 360^\circ$)

(v) In the given figure, $\angle HOG$ is an acute angle.(i.e., $0^\circ < \angle HOG < 90^\circ$)

(vi) In the given figure, $\angle POP$ is a complete angle. (i.e., $\angle POP = 360^\circ$)

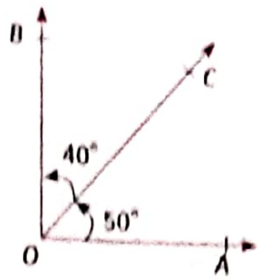
Example 2: In which angle the measure is more than 180° but less than 360° ?

Solution: In a reflex angle the measure is more than 180° but less than 360°.

2.3 Pair of Angles

Some of the pair of angles are complementary angles, supplementary angles, adjacent angles and linear pair of angles.

(1) **Complementary Angle:** Two angle are said to be complementary, if the sum of their measures is 90° in the adjoining figure ∠COB and ∠AOC are complementary angles. [∵ ∠COB + ∠AOC = 90°, i.e., 40° + 50° = 90°]



Example 3: If two angles are complements of each other, then what is the type of each angle?

Solution: We know that if the sum of two angle is 90° then they are complementary angles. So each angle will be an acute angle

Example 4: Find the measure of an angle which is 24° more than its complementary angle.

Solution: Let the measure of the required angle be x. Then its complementary angle is (90° - x).

According to the question, difference between both angle = 24°

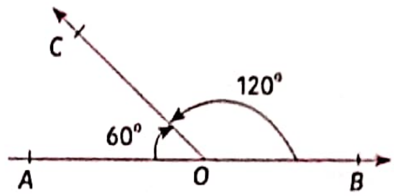
$\therefore\ x - (90° - x) = 24° \Rightarrow x - 90° + x = 24°$

$\Rightarrow 2x = 114° \Rightarrow x = \dfrac{114}{2} \Rightarrow x = 57°$ and $90° - x = 90 - 57° = 33°$

Hence the measure of required angle is 57°.

(2) **Supplementary Angle:** Two angles are said to be supplementary angles, if the sum of their measures is 180°.

In the given figure ∠BOC and ∠AOC are supplementary angles. [∴ ∠BOC + ∠AOC = 180°, i.e., 120° + 60° = 180°]



Example 5: If two supplementary angle are in line ratio 3 : 2, then the angles.

Hint: Firstly we write the ratio of angle in terms of variable and then use the property, sum of two supplementary angle is 180°.

Solution: Let the two supplementary angle be 3x and 2x.

We know that sum of two supplementary angle is 180°

$\therefore\ 3x + 2x = 180° \Rightarrow 5x = 180° \Rightarrow x = \left(\dfrac{180}{5}\right)^{\circ} = 36°$

So, one angle = 3x = 3 × 36° = 108° and its supplementary angle = 2x = 2 × 36° = 72°

Hence, two supplementary angle are 108° and 72°.

Example 6: Find the supplement angle of $\dfrac{3}{5}$ of a right angle.

Solution: Given, angle $= \dfrac{3}{5}$ of a right angle $= \dfrac{3}{5} \times 90° = 3 \times 18° = 54°$.

Supplement of 54° = An angle of measure (180° – 54°) = An angle of measure 126°. Hence the required angle is 126°.

Example 7: Find the measure of the angle, if six times its complements is 12° less than twice of its supplements angle.

Solution: Let the required angle be x°

Then its supplementary angle be (180 – x)° and its complementary angle be (90 – x)°

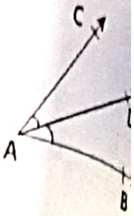
According to the question, $6(90° - x) = 2(180° - x) - 12° \Rightarrow 540° - 6x = 360° - 2x - 12°$

$\Rightarrow 540° - 360° + 12° = 4x \Rightarrow 4x = 192° \Rightarrow x = 48°$. Hence, the value of x is 48°.

(3) **Bisector of an Angle:** A ray which divides an angle into two equal parts is called bisector of an angle.

If ray AD is the bisector of $\angle BAC$,

then, $\angle BAD = \angle CAD = \frac{1}{2}\angle BAC$



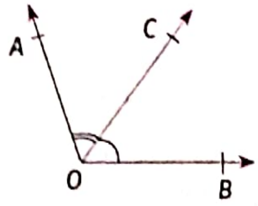
(4) **Adjacent Angle:** Two angle are called adjacent angle, if

(i) They have a common vertex.

(ii) They have a common arm.

(iii) Their non-common arms are on different sides of the common arm.

In the given figure $\angle AOC$ and $\angle COB$ are adjacent angle because these angle have a common vertex O, a common arm OC an non-common arms OA and OB are on different sides of the common arm OC.

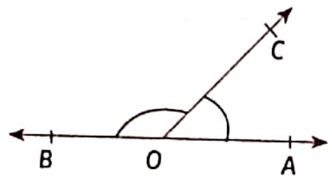


When two angle are adjacent, then their sum is always equal to the angle formed by the two non- common arms,

So, here $\angle AOB = \angle AOC + \angle COB$.

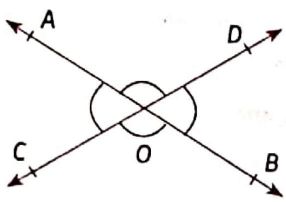
Here, $\angle AOB$ and $\angle AOC$ are not adjacent angle because their non-common arms OC and OB lie the same side of the common arm OA.

(5) **Linear Pair of Angle:** If the non-common arms of two adjacent angle form a line, then these angle are called linear pair o angles.



In the above figure $\angle AOC$ and $\angle BOC$ form a linear pair of angles.

(6) **Vertically Opposite Angles:** Two angles are called a pair of vertically opposite angles if their arms form two pairs opposite rays.

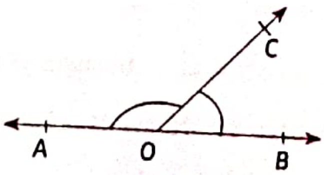


In other words, when two line intersect each other at a point, then there are two pairs of vertically opposite angles. In the above figure lines AB and CD intersect each other at O. So $\angle AOC$ and $\angle BOD$ are vertically opposite angles and $\angle AOD$ and $\angle BOC$ are also vertically opposite angles.

2.4 Relations between Pairs of Angles

There are some relations between these angles which are given in the form of axioms and theorems given below:

Axiom 1. If a ray stands on a line, then the sum of two adjacent angles formed is 180°, i.e. the sum of the linear pair is 180°



In the above figure, $\angle AOC + \angle BOC = 180°$

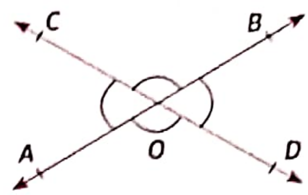
Whereas, $\angle AOC$ and $\angle BOC$ are linear pair of angles.

Axiom.2: If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line or two opposite rays.

The two axioms given together are called the linear pair of angles axiom.

2.5 **Theorem:** The vertical opposite angles are equal.

This theorem can be proved in the following way:



Given: Two lines AB and CD intersect each other at a point O.

To prove: The vertically opposite angles are equal i.e.,

(i) $\angle AOC = \angle BOD$

(ii) $\angle AOD = \angle BOC$

Proof:

(i) Here ray OC stands on line AB $\therefore \angle AOC + \angle BOC = 180^\circ$ [by linear pair axiom] ...(i)

Again ray OB stands on line CD $\therefore \angle BOC + \angle BOD = 180^\circ$ [by linear pair axiom] ...(ii)

From Eqs. (i) and (ii) we get $\angle AOC + \angle BOC = \angle BOC + \angle BOD$

On subtracting $\angle BOC$ from both sides we get $\angle AOC = \angle BOD$

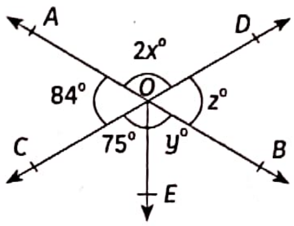
(ii) Similarly we can prove that $\angle AOD = \angle BOC$ **Hence proved.**

3 PROBLEMS BASED ON LINES AND ANGLES

3.1 Problems Based on lines and Angles

The linear pair axiom and the above theorem can be used to solve many problems based on lines and angles. Some of them are given below:

Example 1: In the given figure, lines AB and CD intersect each other at O. Find the values of x, y and z.



Solution: Here AOB and COD are straight lines. So, by linear pair axiom we have

$\angle AOC + \angle AOD = 180^\circ \Rightarrow 84^\circ + 2x = 180^\circ \Rightarrow 2x = 96^\circ \Rightarrow x^\circ = 48^\circ$

From figure $\angle COA = \angle BOD$ [vertically opposite angle]

$\Rightarrow 84^\circ = z$

Since, COD is straight line.

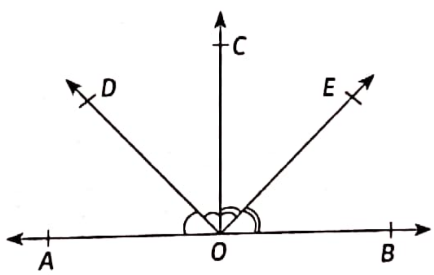
$\therefore \angle COE + \angle EOB + \angle DOB = 180^\circ \Rightarrow 75^\circ + y + z = 180^\circ \Rightarrow 75^\circ + y + 84^\circ = 180^\circ \Rightarrow y = 180^\circ - 159^\circ \Rightarrow y = 21^\circ$

Hence, $x = 48^\circ$, $y = 21^\circ$ and $z = 84^\circ$.

Example 2: Prove that the bisectors of the angles of a linear pair are at right angles.

Solution: Consider $\angle AOC$ and $\angle BOC$ form a linear pair of angles, OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$, respectively

To Prove: $\angle DOE = 90^\circ$



Proof: Here AOB is a straight line.

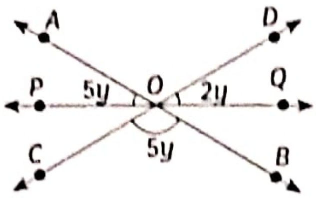
$\therefore \angle AOC + \angle BOC = 180^\circ$ [by linear pair axiom]

$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90^\circ$ [dividing both sides by 2]

$\Rightarrow \angle DOC + \angle COE = 90^\circ$ [Since OD is the bisector of $\angle AOC$ and OE is the bisector of $\angle BOC$]

$\angle DOE = 90^\circ$ **Hence proved.**

Example 3: In the given figure, AB, CD and PQ are three lines concurrent at O. If $\angle AOP = 5y$, $\angle QOD = 2y$ and $\angle BOC = 5y$ then find the value of y.



Solution: From the figure we have

$\angle AOP = \angle BOQ = 5y$ [Vertically opposite angle]

$\angle QOD = \angle COP = 2y$ [Vertically opposite angles] ...(i)

And $\angle BOC = \angle AOD = 5y$ [Vertically opposite angle] ...(ii)

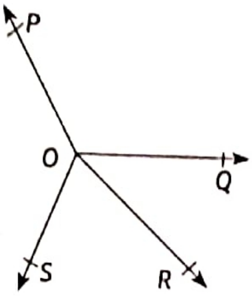
We know that sum of angle around a point is 360° ...(iii)

$\therefore \angle AOP + \angle BOQ + \angle QOD + \angle COP + \angle BOC + \angle AOD = 360°$

$\Rightarrow 5y + 5y + 2y + 2y + 5y + 5y = 360°$ [From eqs. (i) (ii) and (iii)]

$\Rightarrow 24y = 360° \quad \therefore y = \frac{360°}{24} = 15°$

Example 4: In the given figure OP, OQ, OR, and OS are four rays. Prove that $\angle POQ + \angle ROQ + \angle SOR + \angle POS = 360°$

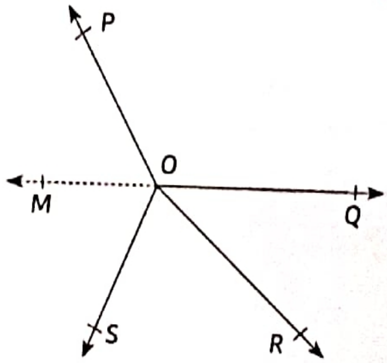


Hint: Firstly produce any of the rays backwards to a point and then use linear pair axiom

Solution: Let us produce a ray OQ backwards to a point M, then MOQ is a straight line.

Now, OP is a ray on the line MOQ.

Then, by linear pair axiom, we have $\angle MOP + \angle POQ = 180°$ ...(i)



Similarly OS, is a ray on the line MOQ. Then by linear pair axiom, we have $\angle MOS + \angle SOQ = 180°$ ...(ii)

Also, $\angle SOR$ and $\angle ROQ$ are adjacent angles. $\therefore \angle SOQ = \angle SOR + \angle ROQ$ ...(iii)

On putting the value of $\angle SOQ$ from Eq (iii) in Eq (ii) we get $\angle MOS + \angle SOR + \angle ROQ = 180°$ ...(iv)

Now, on adding eqs (i) and(iv) we get

$\angle MOP + \angle POQ + \angle MOS + \angle SOR + \angle ROQ = 180° + 180°$

$\angle MOP + \angle MOS + \angle POQ + \angle SOR + \angle ROQ = 360°$ ...(v)

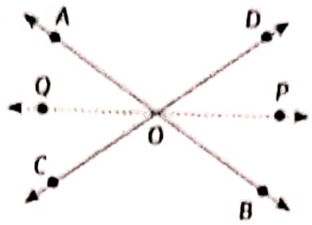
But $\angle MOP + \angle MOS = \angle POS$

Then, From eq (v) we get

$\angle POS + \angle POQ + \angle SOR + \angle ROQ = 360°$

Hence proved.

Example 8: In the given figure AB, CD are straight lines and OP, OQ are the bisectors of ∠BOD and ∠AOC respectively. Show that the rays OP and OQ are in the same line.



Solution: From the given figure we have ∠AOC = ∠BOD [vertically opposite angle]

$\Rightarrow \frac{1}{2}\angle AOC = \frac{1}{2}\angle BOD$ [dividing both sides by 2]

$\Rightarrow \angle COQ = \angle POD$ [Since OQ bisects ∠AOC and OP bisects ∠BOD] ...(i)

Now, CD is a straight line and OA stands on it

$\therefore \angle COA + \angle DOA = 180^\circ$ [by linear pair axiom]

$\Rightarrow \angle COQ + \angle QOA + \angle DOA = 180^\circ$

$\Rightarrow \angle POD + \angle QOA + \angle DOA = 180^\circ$ [From eq....(i)]

$\Rightarrow \angle QOA + \angle DOA + \angle POD = 180^\circ$

$\Rightarrow \angle QOP = 180^\circ$

Hence, QOP is a straight line. So rays OP and OQ are in the same line. **Hence proved.**

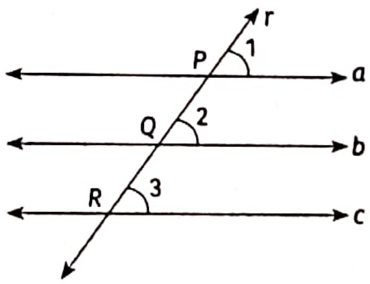
4 LINES PARALLEL TO THE SAME LINE

Theorem: Lines which are parallel to the same line are parallel to each other.

Given: Three lines *a*, *b* and c such *a* || c and *b* || c.

To prove: *a* || *b*

Construction: Draw a transversal *r* cutting *a*, *b* and c at point *P*, *Q* and *R* respectively.



Proof: Since parallel lines *a* and c are intersected by the transversal r at points P and R respectively.

$\therefore \angle 1 = \angle 3$ [corresponding angles axiom] ...(i)

Again parallel lines *b* and *c* are intersected by the transversal r at points Q and R respectively.

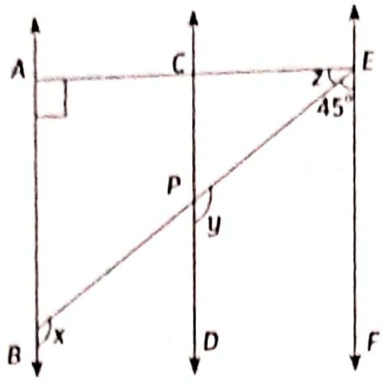
$\therefore \angle 2 = \angle 3$ [corresponding angles axiom] ...(ii)

From Eqs. (i) and (ii), we get $\angle 1 = \angle 2$.

But these are corresponding angles.

$\therefore a \parallel b$ [converse of corresponding angles axiom] **Hence proved.**

Example 1: In the given figure AB || CD and CD || EF Also, EA ⊥ AB if ∠BEF = 45° then find the values of x, y and z.



Solution: Given, CD || EF and EP is a transversal.

$\therefore \angle EPD + \angle FEP = 180°$

Since sum of interior angles on the same side of the transversal EP is 180° [∵ $\angle FEP = 45°$ Given]

$\Rightarrow y + 45° = 180°$

$\Rightarrow y = 180° - 45° \Rightarrow y = 135°$

Also given AB || CD and BP is a transversal.

So, $x = y$ [corresponding angles axiom]

$\therefore x = 135°$

Now, AB || CD and CD || EF

∴ AB || EF [by theorem 5]

Then, $\angle EAB + \angle FEA = 180°$ [Since sum of interior angles on the same side of the transversal EA is 180°]

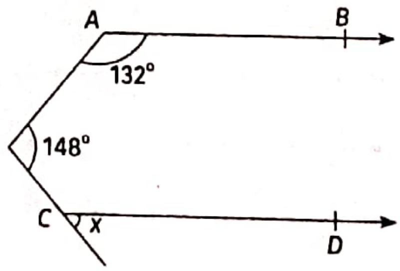
$\Rightarrow 90° + z + 45° = 180°$ [∵ $EA \perp AB \Rightarrow EAB = 90°$]

$\Rightarrow z + 135° = 180°$

$\Rightarrow z = 180° - 135°$

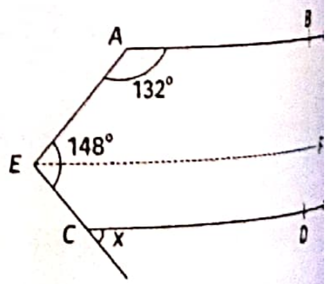
$\Rightarrow z = 45°$. Hence $x = 135°$, $y = 135°$ and $z = 45°$.

Example 2: In the given figure if AB||CD then find the value of x.



Solution: Draw a line EF || AB || CD.

Since AB || EF and AE is a transversal



$\therefore \angle BAE + \angle AEF = 180°$ [cointerior angles]

$\Rightarrow 132° + \angle AEF = 180°$

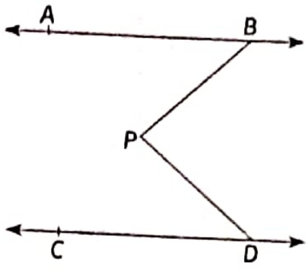
$\Rightarrow \angle AEF = 180° - 132° = 48°$

Now $\angle FEC = \angle AEC - \angle AEF = 148° - 48° = 100°$

Since EF || CD and EC is a transversal.

$\therefore \angle FEC = \angle x = 100°$ [by corresponding angles axiom]

Example 3: Lines AB and CD are parallel and P is any point between the two lines as shown in figure prove that $\angle ABP + \angle CDP = DPB$.

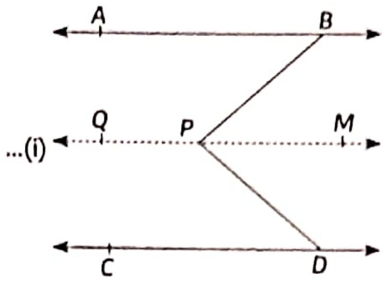


Solution:

Given: Two lines AB and CD are parallel and P is any point between them.

To prove: $\angle ABP + \angle CDP = \angle DPB$.

Construction: Through P draw $QPM \parallel AB \parallel CD$



Proof: Now $AB \parallel QPM$ and $CD \parallel QPM$

$\Rightarrow \angle ABP = \angle BPM$ [alternate interior angles] ...(i)

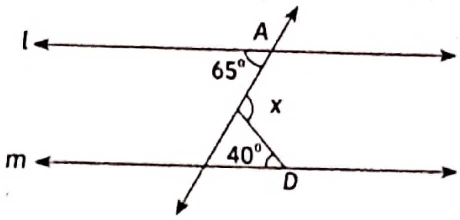
And $\angle CDP = \angle DPM$ [alternate interior angles] ...(ii)

On adding eq.. (i) and (ii), we get $\angle ABP + \angle CDP = \angle BPM + \angle DPM$

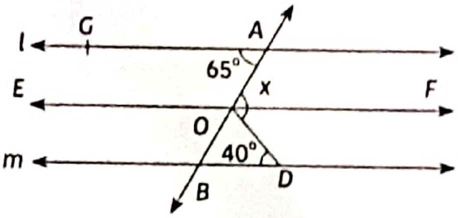
$\therefore \angle ABP + \angle CDP = \angle DPB$

Hence proved.

Example 4: In the given figure if $l \parallel m$ then find the value of x.



Solution: Draw a line EF such that $EF \parallel l \parallel m$

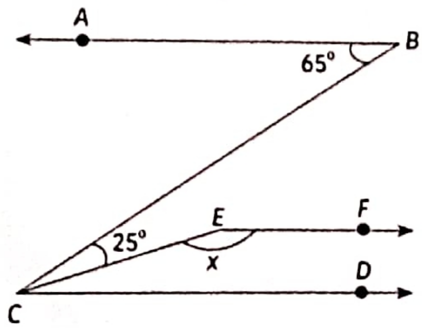


Then $\angle FOD = \angle BDO = 40^\circ$ [alternate interior angles]

and $\angle FOA = \angle GAO = 65^\circ$

Now, $\angle x = \angle FOD + \angle FOA = 40^\circ + 65^\circ = 105^\circ$

Example 5: In the figure what value of x will make $EF \parallel CD$, if $AB \parallel CD$?



Solution:

Given: $AB \parallel CD$ and BC is a transversal.

$\therefore \angle ABC = \angle BCD$ [alternate interior angles]

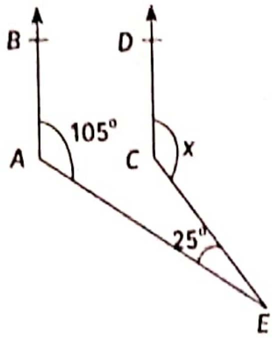
$\Rightarrow \angle ABC = \angle BCE + \angle ECD \Rightarrow 65^\circ = 25^\circ + \angle ECD \Rightarrow \angle ECD = 40^\circ$

For $EF \parallel CD$ the sum of $\angle FEC$ and $\angle ECD$ should be equal to 180°

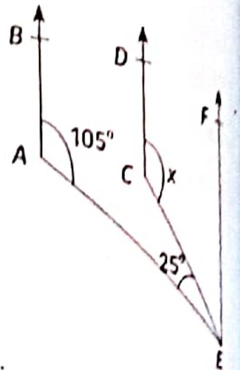
$\therefore x + \angle ECD = 180^\circ \Rightarrow x + 40^\circ = 180^\circ \Rightarrow x + 180^\circ - 40^\circ = 140^\circ$.

Hence, for $x = 140^\circ$, EF will be parallel to CD.

Example 6: In the given figure if AB || CD, then find the value of x



Solution: From point E, draw EF || AB || CD. Now EF || CD and CE is the transversal.



$\therefore \angle DCE + \angle CEF = 180^\circ$ [co-interior angles]

$\Rightarrow x + \angle CEF = 180^\circ \Rightarrow \angle CEF = (180^\circ - x)$

Angle EF || AB and AE is the transversal.

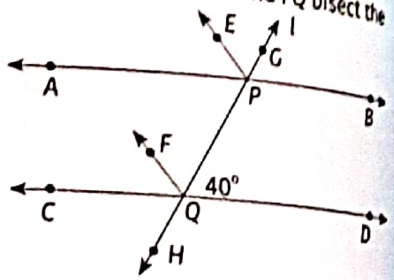
$\therefore \angle BAE + \angle AEF = 180^\circ$ [co-interior angles]

$\Rightarrow 105^\circ + \angle AEC + \angle CEF = 180^\circ$ [$\because \angle AEF = \angle AEC + \angle CEF$]

$\Rightarrow 105^\circ + 25^\circ + (180^\circ - x) = 180^\circ \Rightarrow x = 130^\circ$. Hence, the value of x is 130°.

Example 7: A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Solution: Consider two lines AB and CD are parallel and intersected by transversal l at P and Q respectively. EP and FQ bisect the corresponding angles $\angle APG$ and $\angle CQP$, respectively.



To prove: EP || FQ

Proof : Given AB || CD.

$\therefore \angle APG = \angle CQP \Rightarrow \frac{1}{2}(\angle APG) = \frac{1}{2}(\angle CQP) \Rightarrow \angle EPG = \angle FQP$

But these are corresponding angles on line EP and FQ.

$\therefore$ EP || FQ [converse of corresponding angles axiom] **Hence proved.**

Chapter at Glance

(1) A line is a straight path that is endless in both directions. It is denoted by $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$.

(2) If three or more than three point lies on the same line then they are called collinear points. Otherwise, they are called non-collinear point.

(3) Two lines are said to be intersecting lines. If they intersect each other at one point.

(4) Angle is formed in a figure by two rays with the same initial point, then following types of angles are discussed below

(i) **Acute angles:** An angle whose measure is less than 90° but more than 0°.

(ii) **Obtuse angles:** An angles whose measure is more than 90° but less than 180°.

(iii) **Right angles:** An angles whose measure is 90°.

(iv) **Straight angles:** An angles whose measure is 180°.

(v) **Reflex angles:** An angles whose measure is more than 180° but less than 360°.

(vi) **Complete angles:** An angles whose measure is 360°.

(5) Pair of Angles

(i) **Complementary angles:** Two angles are said to be complementary. If the sum of their measure is 90°.

(ii) **Supplementary angles:** Two angles are said to be supplementary. If the sum of their measure is 180°.

(iii) **Bisector of a angles:** A ray which divides the angles into two equal parts. Is called the bisector of an angles.

(iv) **Adjacent angles:** Two angles are called adjacent angles, if

(a) They have a common vertex.

(b) Their non-common arms are on different sides of the common arm.

(v) **Linear pair of angles:-** If the non- common arms of two adjacent angles form a line, then these angles are called linear pair of angles.

(vi) **Vertically opposite angles:-** Two angles are said to be pair of vertically opposite angles, if their arms form two pairs of opposite rays.

(6) Axiom and theorems of pair of angles.

Axiom (i):- If a ray stand on a line, then the sum of the adjacent angles so formed is 180°, i.e., the sum of the linear pair of angles is 180° (converse is also true)

(7) Lines which are parallel to the same line, are parallel to each other.

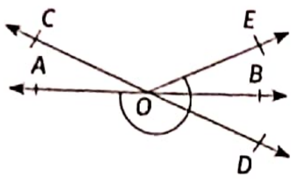
CHAPTER PRACTICE

MULTIPLE CHOICE QUESTIONS

1. If an angle is such that six times its complement is 12° less than twice its supplement, then the value of angle is

(a) 38° (b) 48°
(c) 58° (d) 68°

2. In the given figure, lines *AB* and *CD* intersect at *O* if ∠*AOC* + ∠*BOE* = 80° and ∠*BOD* = 30° then ∠*BOE* equals to



(a) 30° (b) 40°
(c) 50° (d) 60°

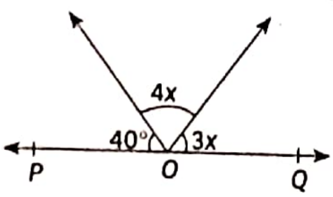
3. If angles with measures *x* and *y* form a complementary pair then which of the following measure of angles will form a supplementary pair?

(a) $(x + 47°), (y + 43°)$ (b) $(x - 23°), (y + 23°)$
(c) $(x - 43°), (y - 47°)$ (d) No such pair is possible

4. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is

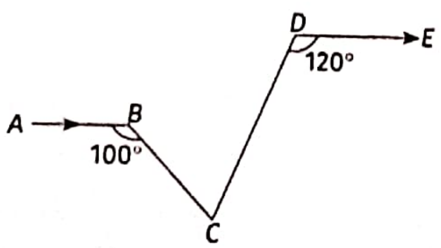
(a) $37\frac{1}{2}^\circ$ (b) $52\frac{1}{2}^\circ$
(c) $72\frac{1}{2}^\circ$ (d) 75°

5. In the figure, *POQ* is line. The value of *x* is



(a) 20° (b) 25°
(c) 30° (d) 35°

6. In the given figure, if *AB* || *DE*, then the value of ∠*BCD* is

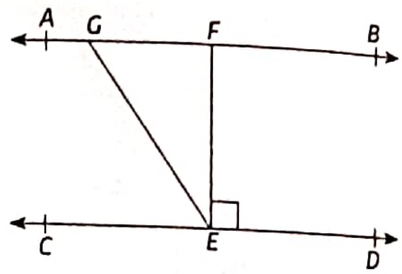


(a) 30° (b) 40°
(c) 50° (d) 70°

7. How many pairs of adjacent angles are formed when two lines intersect at a point?

(a) 3
(b) 4
(c) 6
(d) 8

8. If *AB* || *CD*, *EF* ⊥ *CD* and ∠*GED* = 135° as per the figure given below. Find ∠*AGE*.



(a) 110°
(b) 120°
(c) 128°
(d) 135°

1 MARK QUESTIONS

1. If two angles of a triangle are complementary, then what type of triangles will be formed?

2. If the ratio between two complementary angles are 2 : 3, then find the angles.

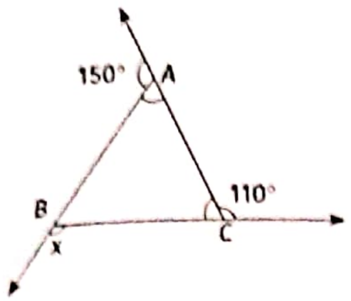
3. Find the measure of the angle which is complement of itself.

4. If two supplementary angles are in the ratio 13 : 5, then find the angles.

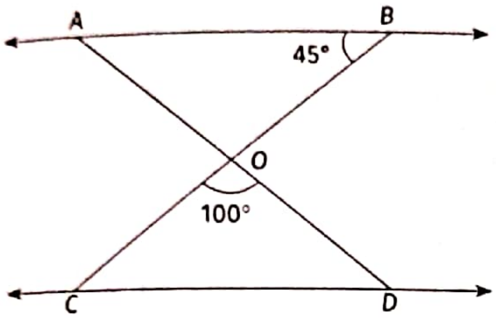
5. Two lines *l* and *m* are perpendicular to the same line *n*. Are *l* and *m* perpendicular to each other? Give reason for your answer.

6. An exterior angle is drawn to a triangle. If this exterior angle is acute, then what type of triangle will be formed?

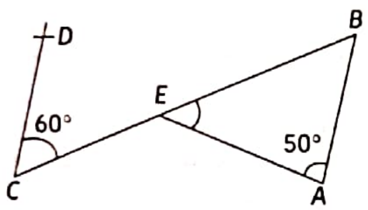
7. Find the value of x from the given figure.



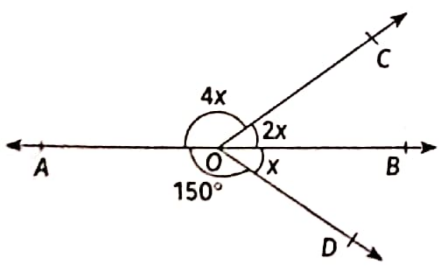
8. In the given figure AB || CD. If ∠ABO = 45° and ∠COD = 100°, then find ∠ODC.



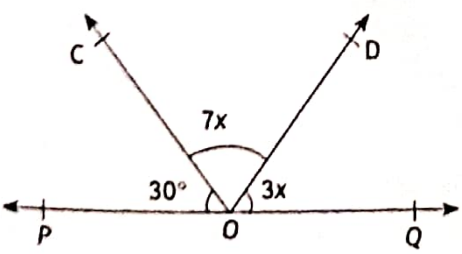
9. In the given figure, AB || CD, ∠EAB = 50°. If ∠ ECD = 60°, then find ∠AEB.



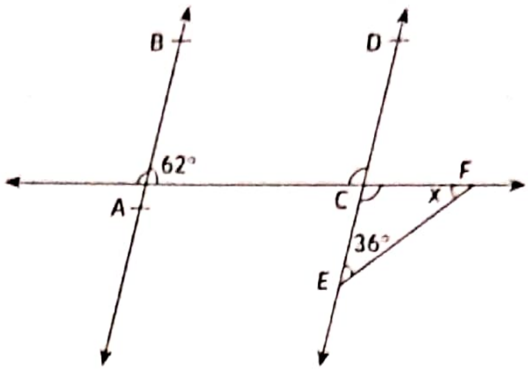
10. In the given figure, find the value of x



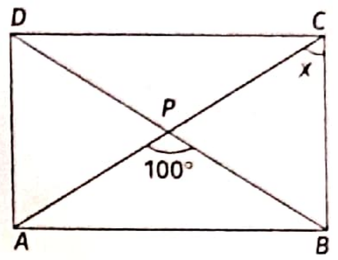
11. In the given figure, if POQ is a line, then find the value of x.



12. In the given figure, if AB || ED then, find the value of x.



13. In the given figure, if ABCD is a rectangle in which ∠APB = 100°, Then find the value of x

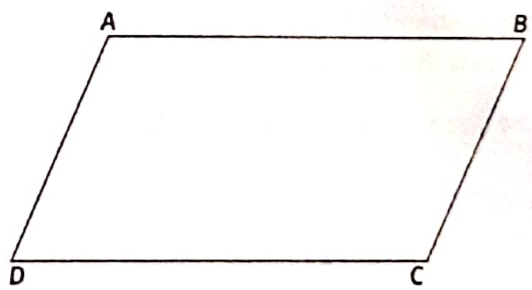


14. If two line intersect at a point and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are__________.

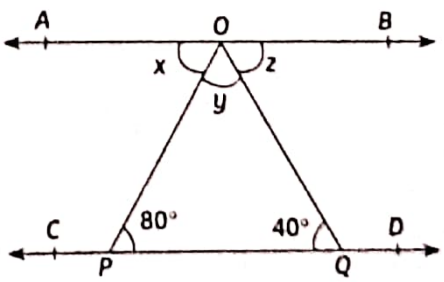
15. An exterior angle of a triangle is 100° and the interior opposite angle are in the ratio 2 : 3, measure of interior opposite angle are:

2 MARKS QUESTIONS

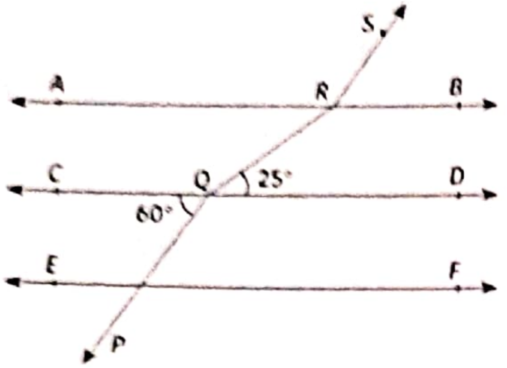
1. In the given figure, AB || DC and AD || BC.
Prove that, ∠DAB = ∠DCB.



2. In the given figure, AB || CD. Determine the values of x, y and z.

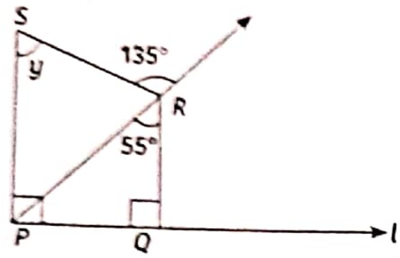


3. In the given figure,

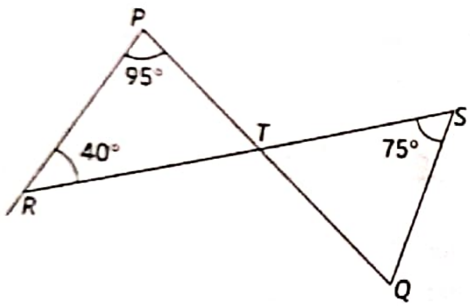


If $AB \parallel CD \parallel EF$, $PQ \parallel RS$, $\angle RQD = 25°$ and $\angle CQP = 60°$, then find $\angle QRS$.

4. In the given figure, $PS \perp l$ and $RQ \perp l$, find the measure of y.



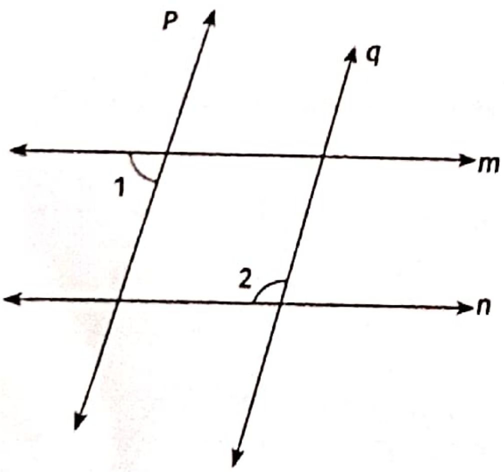
5. In the given figure, line segments PQ and RS intersect each other at a point T, such that $\angle PRT = 40°$ $\angle RPT = 95°$ and $\angle TSQ = 75°$. Find $\angle SQT$.



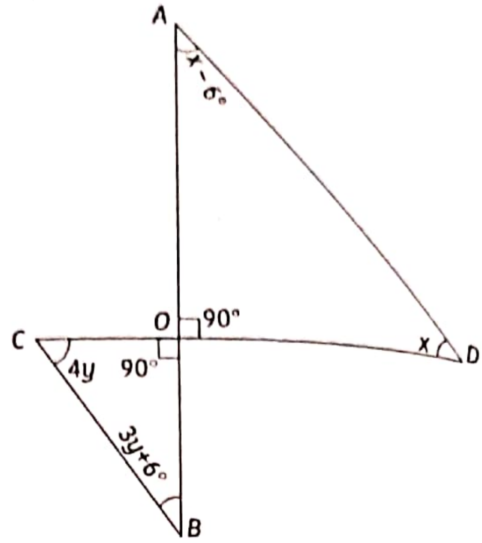
6. If the angles of a triangle are in the ratio 5 : 3 : 7, then show that the triangle is an acute angled triangles.

7. In the given figure, $m \parallel n$ and $p \parallel q$. If $\angle 1 = 75°$, then

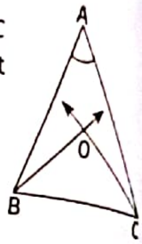
Prove that $\angle 2 = \angle 1 + \frac{1}{3}$ of the right angle.



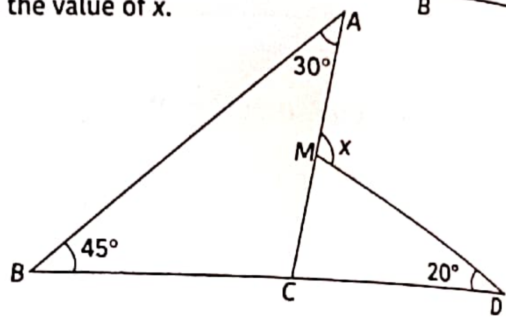
8. In the given figure, find x and y



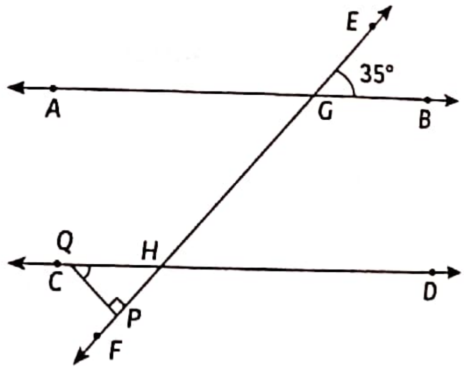
9. In the given figure, the bisectors of $\angle ABC$ and $\angle BCA$, intersect each other at point O. If $\angle BOC = 100°$, then find $\angle A$.



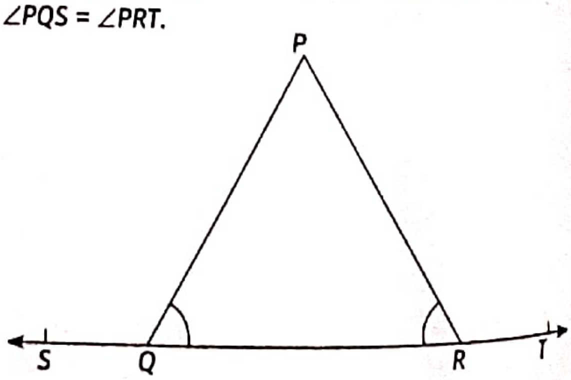
10. Find the value of x.



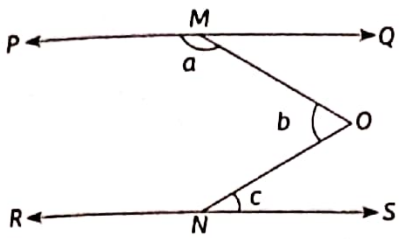
11. In the given figure, $AB \parallel CD$ and EF is a transversal, which intersects them at G and H respectively. If $\angle EGB = 35°$ and $QP \perp EF$, then find $\angle PQH$.



12. In the given figure, if $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

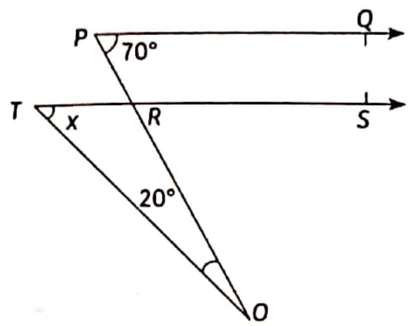


13. In the given figure, if PQ ∥ RS, then find the relationship between a, b and c.

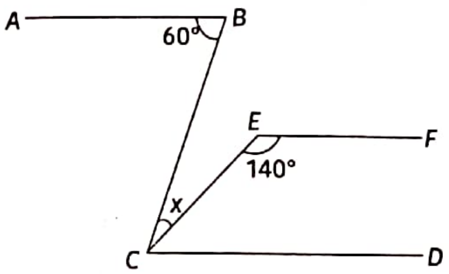


14. In the given figure, PQ ∥ RS.

If ∠QPR = 70° and ∠ROT = 20°, then find the value of x.

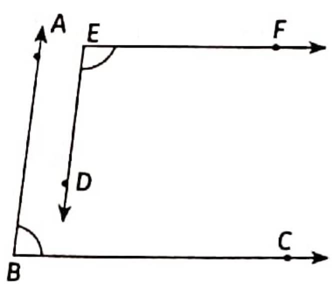


15. In the given figure, find the value of x, if AB ∥ CD ∥ EF.



3 MARKS QUESTIONS

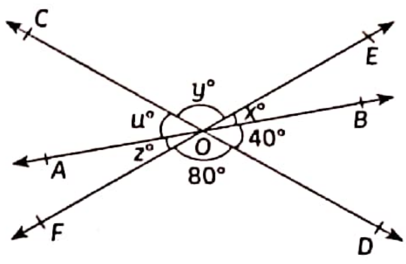
1. In the given figure, BA ∥ ED and BC ∥ EF. Show that ∠DEF + ∠ABC = 180°



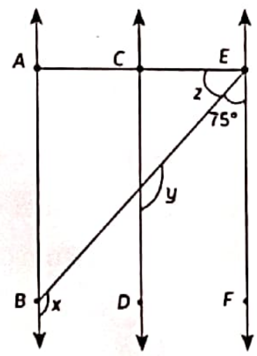
2. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m. Show that AP ∥ BQ.

3. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 3 : 2, then find the greater of the two angles.

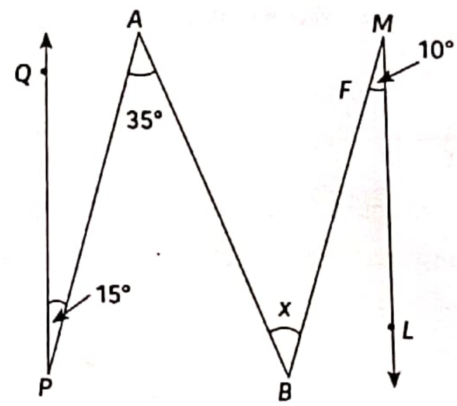
4. Three lines AB, CD and EF meet at a point O, forming angles as shown in the figure. Find the values of x, y, z and u.



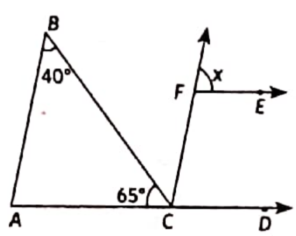
5. In the given figure, AB ∥ CD, CD ∥ EF, and EA ⊥ AB. If ∠BEF = 75°, then find the values of x, y and z.



6. In the given figure, if QP ∥ ML, then find the value of x.



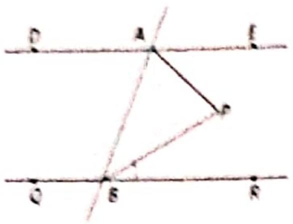
7. In the given figure, if AB ∥ CF and CD ∥ EF, then find the value of x.



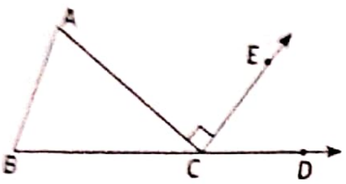
8. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

9. Prove that through a given point, we can draw only one perpendicular to a given line.

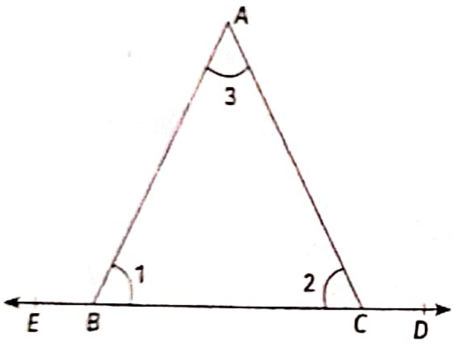
10. In the given figure, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$, respectively. Then, find $\angle APB$.



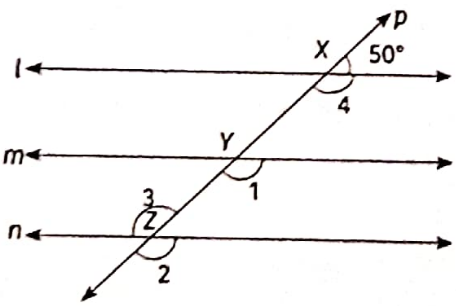
11. In the given figure, $AC \perp CE$ and in ΔABC, $\angle A : \angle B : \angle C = 5 : 3 : 2$. Find the value of $\angle ECD$.



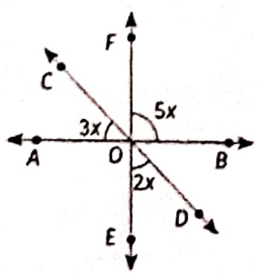
12. In the given figure, side BC of ΔABC is produced in both the directions. Prove that the sum of the two exterior angles, so formed is greater than 180°.



13. In the given figure, l, m and n are parallel lines intersected by a transversal p at X, Y and Z, respectively. Then, find $\angle 1$, $\angle 2$ and $\angle 3$.



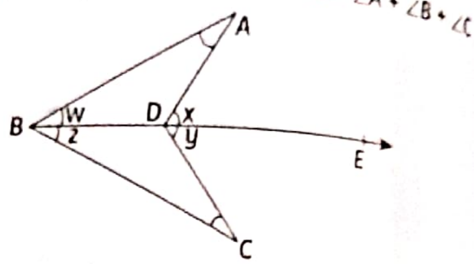
14. In the figure, find the value of x.



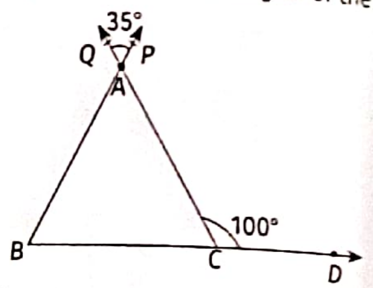
5 MARKS QUESTIONS

1. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

2. In the given figure, prove that $\angle ADC = \angle A + \angle B + \angle C$

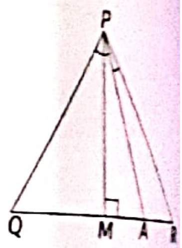


3. Sides BC, CA and BA of a ΔABC are produced to D, Q and P respectively as shown in the given figure. If $\angle ACD = 100°$, $\angle QAP = 35°$, then find all the angles of the triangles.



4. In the given figure $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and $PM \perp QR$. Prove that

$$\angle APM = \frac{1}{2}(\angle Q - \angle R).$$

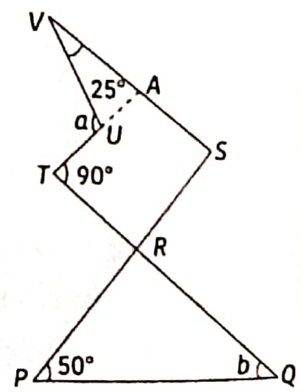


5. If the arms of one angle are respectively parallel to the arms of another angle, then show that the two angles are either equal or supplementary.

6. Bisectors of interior $\angle B$ and exterior $\angle ACD$ of a ΔABC intersect at the point T.

Prove that $\angle BTC = \frac{1}{2}\angle BAC$.

7. In the given figure, if $TU \parallel SR$ and $TR \parallel SV$ then find $\angle a$ and $\angle b$.



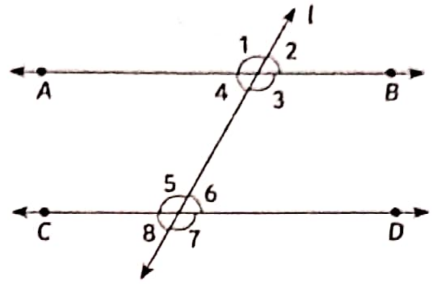
8. If the bisectors of the base angles of a triangle enclose an angle of 135°, then prove that the triangle is a right angled triangle.

9. In $\triangle ABC$, the sides AB and AC of $\triangle ABC$ are produced to points E and D, respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$, respectively meet at point O, then prove that $\angle BOC = 90° - \frac{1}{2}\angle A$.

10. Prove the bisectors of a pair of vertically opposite angles are in the same straight line.

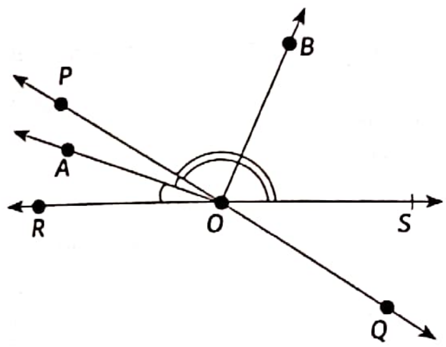
11. If two straight lines intersect each other, prove that the ray opposite to the bisector of one of the angles this formed bisects the vertically opposite angle.

12. In figure, AB, CD and $\angle 1$ and $\angle 2$ are in the ratio 3 : 2. Determine all angles from 1 to 8.



HOTS QUESTIONS

1. In the given figure, lines PQ and RS intersect each other at point O, ray OA and ray OB bisect $\angle POR$ and $\angle POS$, respectively. If $\angle POA : \angle POB = 2 : 7$, then $\angle SOQ + \angle BOQ$ is equal to



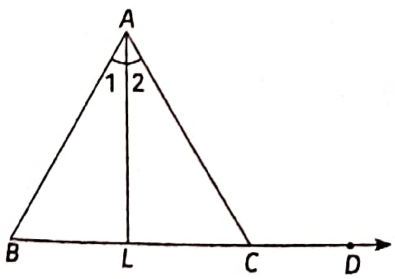
(a) 110°

(b) 70°

(c) 150°

(d) 135°

2. The side BC of a $\triangle ABC$ is produced such that D is on ray BC. If the bisector of $\angle A$ meets BC at L as shown in the figure, then $\angle ABC + \angle ACD$ is equal to



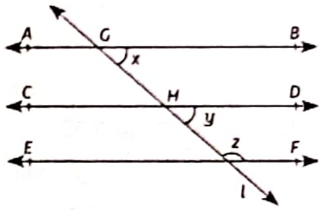
(a) $\angle ALC$

(b) $2\angle ALC$

(c) $3\angle ALC$

(d) None of these

3. In the given figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 2 : 3$, then x is equal to



(a) 50° (b) 82°

(c) 70° (d) 72°

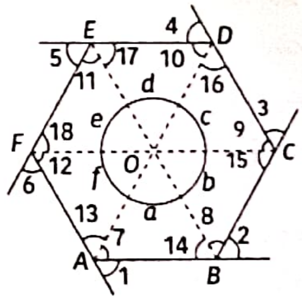
4. $ABCDE$ is a regular pentagon and bisector of $\angle BAE$ meets CD at M. If bisector of $\angle BCD$ meets AM at P, then $\angle CPM$ is equal to

(a) 72° (b) 36°

(c) 108° (d) 48°

5. In the given figure, sides of a regular hexagon are produced, then value of the

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6$ is



(a) 270° (b) 180°

(c) 360° (d) 720°

6. Student are asked to establish a relation between vertically opposite angles. They draw various figures, measure the angle and observe that vertically opposite angles are equal.

In this case. Student according to Van Hiele though are at

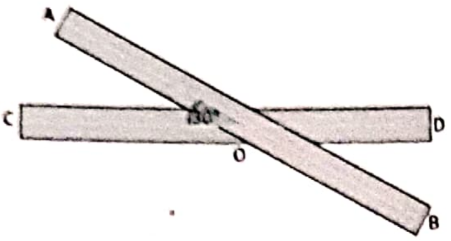
(a) Visualization level (b) Analytic level

(c) Informal deduction level (d) Deduction level

CASE BASED QUESTIONS

Read the following and answer any four questions from 1 (I-V)
Case - 1:

Harry was going on a road trip with his father. They were travelling on a straight road. After riding for some distance, they reach a crossroad where one straight road cuts the other at 30°. Now using the given information, answer the following questions.



I. Find the measure of angle *AOD*.
(a) 130° (b) 150°
(c) 120° (d) 50°

II. Find the measure of angle *BOD*.
(a) 30° (b) 150°
(c) 120° (d) 50°

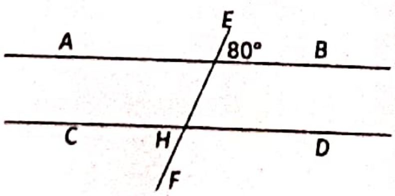
III. Find the measure of angle *BOC*.
(a) 30° (b) 150°
(c) 120° (d) 50°

IV. Which of the following is incorrect?
(a) Sum of a linear pair of angles is 180°
(b) Linear pair of angles are supplementary to each other
(c) Both angles in a linear pair are acute
(d) Angles in a linear pair can be equal

V. Which of the following is correct?
(a) Vertically opposite angles are always supplementary
(b) Vertically opposite angles are always complementary
(c) Vertically opposite angles are made using straight lines
(d) Vertically opposite angles have common arms

Read the following and answer any four questions from 2 (I-V)
Case - 2:

An electric post was tilted due to heavy winds by an angle of 80°. Now despite the tilt, the electric wire lines remained parallel to each other and the ground. Now, using the given information, answer the following questions.



I. Find the measure of angle *DHF*.
(a) 80° (b) 100°
(c) 180° (d) 50°

II. Find the measure of angle *GHD*.
(a) 80° (b) 100°
(c) 180° (d) 50°

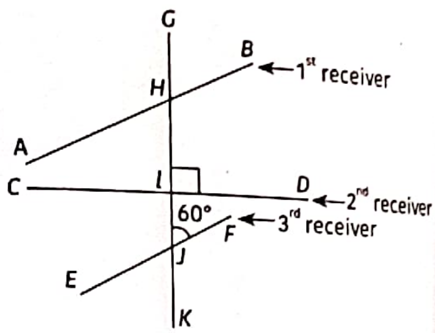
III. Find the measure of angle *AGE*.
(a) 80° (b) 100°
(c) 180° (d) 50°

IV. Find the measure of angle *CHD*.
(a) 80° (b) 100°
(c) 180° (d) 50°

V. Find the measure of angle *AGH*.
(a) 80° (b) 100°
(c) 180° (d) 50°

Read the following and answer any four questions from 3 (I-V)
Case - 3:

An old TV antenna had three parallel receivers mounted on a pole, but due to heavy rainfall, the receivers became disoriented. If the first and the third receivers are still parallel, then answer the following questions.



I. Find the measure of angle *BHJ*.
(a) 60° (b) 120°
(c) 80° (d) 100°

II. Find the measure of angle *HIC*.
(a) 60° (b) 120°
(c) 90° (d) 80°

III. Find the measure of angle *EJK*.
(a) 60° (b) 120°
(c) 90° (d) 80°

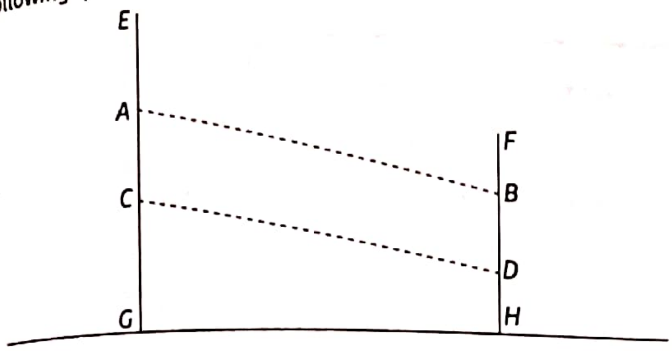
IV. Find the measure of angle *AHJ*.
(a) 60° (b) 120°
(c) 90° (d) 80°

V. Find the measure of angle *BHG*.
(a) 60° (b) 120°
(c) 90° (d) 80°

Read the following and answer any four questions from 4 (I-V)

Case - 4:

Two poles *EG* and *FH* are erected perpendicularly on the ground and are tied by ropes *AB* and *CD* as shown in the figure. Now if the two ropes are parallel to each other, then answer the following questions.



I. Find $m\angle BDC$, if $m\angle ABD = 120°$.

(a) 50° (b) 60°
(c) 70° (d) 80°

II. Find $m\angle FBA$, if $m\angle ABD = 120°$.

(a) 50° (b) 60°
(c) 70° (d) 80°

III. Find $m\angle CDH$, if $m\angle ABD = 120°$.

(a) 120° (b) 60°
(c) 50° (d) 80°

IV. Find $m\angle EAB$, if $m\angle ABD = 120°$.

(a) 120° (b) 60°
(c) 50° (d) 80°

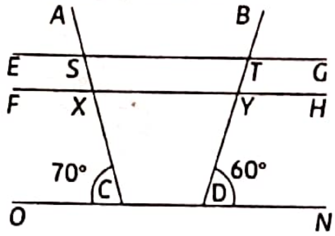
V. Find $m\angle GCD$, if $m\angle ABD = 120°$.

(a) 120° (b) 60°
(c) 50° (d) 80°

Read the following and answer any four questions from 5 (I-V)

Case - 5:

Two parallel beams (*EG* and *FH*) are supported using two poles (*AC* and *BD*). If the beams are also parallel to the ground (*ON*), then answer the following questions.



I. Find $m\angle HYD$.

(a) 120° (b) 60°
(c) 180° (d) 70°

II. Find $m\angle BTG$.

(a) 120° (b) 60°
(c) 180° (d) 70°

III. Find $m\angle YXC$.

(a) 120° (b) 60°
(c) 180° (d) 70°

IV. Find $m\angle SXF$.

(a) 120° (b) 60°
(c) 180° (d) 70°

V. Find $m\angle ASE$.

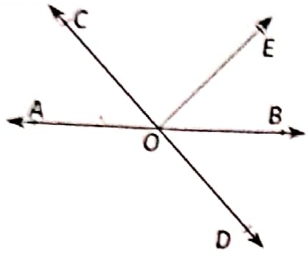
(a) 120° (b) 60°
(c) 180° (d) 70°

❑❑❑

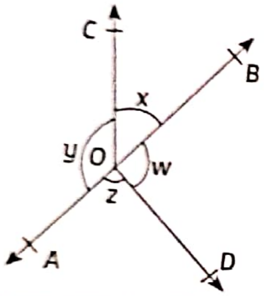
EXAM PRACTICE

NCERT QUESTIONS

1. In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, then find $\angle BOE$ and reflex $\angle COE$.

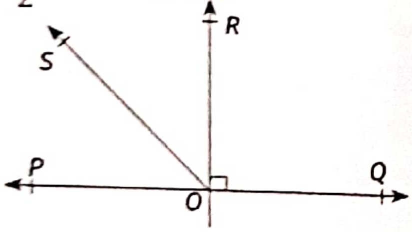


2. In the given figure if $x + y = w + z$, then prove that AOB is line.



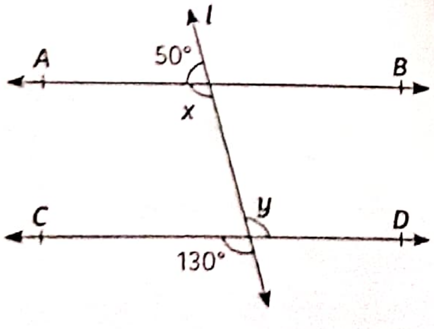
3. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ, OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$

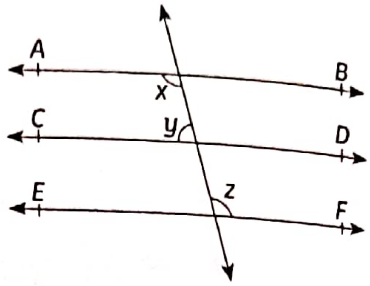


4. It is given that, $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$ then find $\angle XYQ$ and reflex $\angle QYP$.

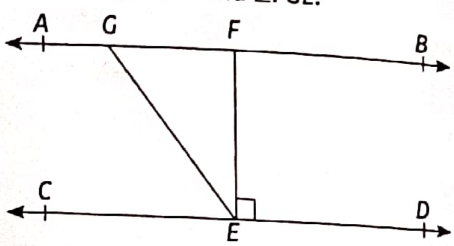
5. In the given figure, find the values of x and y and then show that $AB \parallel CD$.



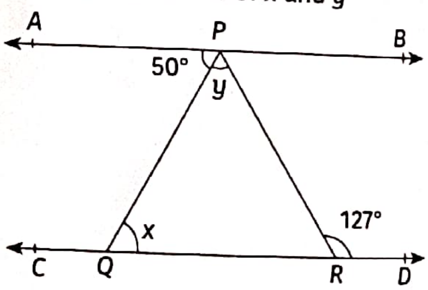
6. In the given figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, then find the value of x.



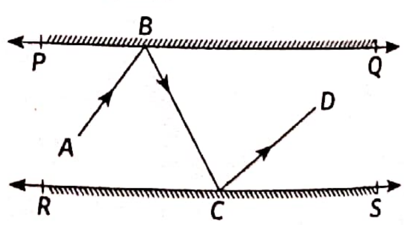
7. In the given figure, if $AB \parallel CD$, $FE \perp CD$ and $\angle GED = 126°$, then find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



8. In the given figure, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, then find the value of x and y



9. In the given figure, PQ and RS are two mirrors placed parallel to each other.



An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflect back along CD. Prove that $AB \parallel CD$.

COMPETENCY QUESTIONS

(FOR FOUNDATION, NTSE, OLYMPIAD QUESTIONS)

SECTION A MULTIPLE CHOICE QUESTIONS

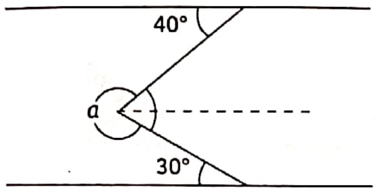
DIRECTIONS: This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct.

1. If two lines intersected by a transversal, then each pair of corresponding angles so formed are –
   (a) Equal (b) Complementary
   (c) Supplementary (d) None of these

2. An angle is 14° more than its complementary angle then angle is –
   (a) 38° (b) 52°
   (c) 50° (d) None of these

3. If the supplement of an angle is three times its complement, then angle is –
   (a) 40° (b) 35°
   (c) 50° (d) 45°

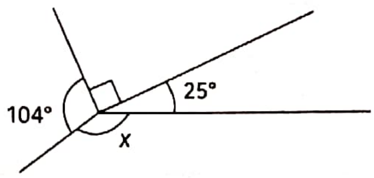
4. In the Fig., angle a is



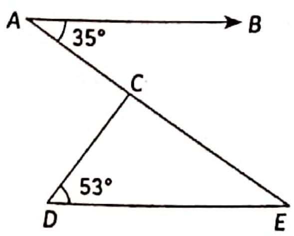
   (a) 290° (b) 70°
   (c) 105° (d) 45°

5. Value of x =
   (a) 270° (b) 70°
   (c) 15° (d) 45°

6. Value of x =
   (a) 141°
   (b) 70°
   (c) 105°
   (d) 45°

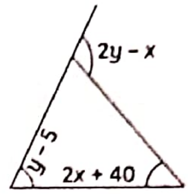


7. In Fig., if $AB \parallel DE$, $\angle BAC = 35°$ and $\angle CDE = 53°$, $\angle DCE = ?$



   (a) 102° (b) 92°
   (c) 80° (d) 72°

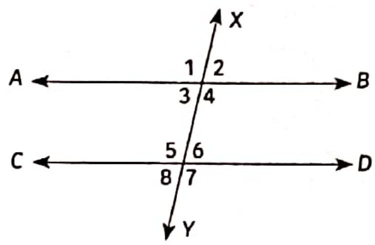
8. In Fig. value y, if $x = 5°$ is –
   (a) 60°
   (b) 50°
   (c) 65°
   (d) 45°



SECTION B MATCHING BASED MCQ

DIRECTIONS (Qs.10): Match Column-I with Column-II and select the correct answer using the codes given below the columns..

9. For the Fig. shown, Match **Column-I** and **Column-II** correctly



	Column-I		Column-II
(A)	corresponding angles	(p)	1 and 5
(B)	alternate interior angles	(q)	4 and 6
(C)	alternate exterior angles	(r)	1 and 7
(D)	interior angles on same side of the transversal	(s)	4 and 5

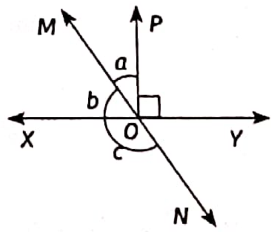
   (a) (A) - (q); (B) - (s); (C) - (s); (D) - (r);
   (b) (A) - (p); (B) - (s); (C) - (r); (D) - (q)
   (c) (A) - (s); (B) - (r); (C) - (q); (D) - (s);
   (d) (A) - (q); (B) - (s); (C) - (r); (D) - (s);

SECTION D PASSAGE BASED MCQ

DIRECTIONS (Qs. 10 to 12): Read the passage(s) given below and answer the questions that follow.

Passage - 1

In Fig., lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$, then



10. Measure of angle 'a' is
   (a) 54° (b) 36°
   (c) 126° (d) none of these

11. Measure of angle 'b' is

(a) 126° (b) 36°

(c) 54° (d) none of these

12. Measure of angle 'c' is

(a) 36° (b) 126°

(c) 54° (d) none of these

SECTION E ASSERTION REASON BASED MCQ

DIRECTIONS (Qs. 13 to 15): Following questions consist of two statements, one labelled as the 'Assertion' and the other as 'Reason'. You are to examine these two statements carefully and select the answer to these items using the code given below.

Codes:

(a) Both Assertion and Reason are individually true and Reason is the correct explanation of Assertion:

(b) Both Assertion and Reason are individually true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false

(d) Assertion is false but Reason is true.

13. **Assertion:** Sum of the pair of angles (like 120°, 60°) is supplementary.

Reason: Two angles, the sum of whose measures is 180°, are called supplementary angles.

14. **Assertion:** If an angle formed by two intersecting lines is 60°, then its vertically opposite angle is 60°

Reason: If two lines intersect each other, then the vertically opposite angles are equal.

15. **Assertion:** The angles 'a' and 'b' form a linear pair of angles and $a = 40°$, then $b = 150°$.

Reason: Sum of linear pair of angles is always 180°

LINES AND ANGLES

Multiple Choice Question

1	c	2	a	3	d	4	d	5	b	6	c	7	b	8	a

HOTS Question

1	c	2	b	3	d	4	b	5	d	6	c

Cased Based Question

Case 1

I	b	II	a	III	b	IV	c	V	c

Case 2

I	b	II	a	III	b	IV	c	V	a

Case 3

I	b	II	c	III	a	IV	a	V	a

Case 4

I	b	II	b	III	a	IV	a	V	b

Case 5

I	a	II	b	III	d	IV	d	V	d

Competency Questions

1	a	2	b	3	d	4	a	5	c	6	a	7	b	8	b	9	b	10	b
11	c	12	c	13	a	14	a	15	d										