# 2

# POLYNOMIALS

## INTRODUCTION

- In earlier classes, we have studied algebraic expression and their operations (i.e., addition, subtraction, multiplication and division) and some algebraic identities used in factorization (i.e. $(x+y)^2 = x^2 + y^2 + 2xy$, $x^2 - y^2 = (x-y)(x+y)$ etc.)
- But in this chapter, we will study about the polynomial of one variable and their classification.
- Here, we also study about zeroes of polynomials, remainder and factor theorem and their use in factorization of a polynomial, along with some more algebraic identities.

## 1 POLYNOMIALS IN ONE VARIABLE AND THEIR CLASSIFICATIONS

Earlier, we have studied about two types of symbols, viz. variables and constants. Variables usually denoted by the letters $x$, $y$, $z$ etc., can take various numerical values, i.e., the value of a variable can keep changing. On the other hand, constants generally denoted by the letters $a$, $b$, $c$ etc., have a fixed numerical value throughout a particular solution.

e.g., $-4$, $3$, $\pi$ etc., are all constants.

### 1.1 Algebraic Expression

A combination of constants and variable, connected by the four fundamental arithmetical operations +, –, × and ÷ is called an algebraic expression.

e.g.,

(i) $6x^2 - 5y^2 + 2xy$ is an algebraic expression, where $x$ and $y$ are variables.

(ii) $6x^3 - \frac{5}{y} + 3x$ is an algebraic expression, where $x$ and $y$ are variables.

### 1.2 Polynomial

An algebraic expression in which the variables involved, have only non-negative integral power, is called a polynomial.

### 1.3 Polynomial in One Variable

An algebraic expressions involving single variable, which have only whole numbers (or non-negative integers) as the exponent of the variable, are called polynomials in one variable. If the variable in a polynomial is $x$, then we denote the polynomial by $p(x)$ or $q(x)$ or $r(x)$ etc.

A polynomial in variable $x$ is an algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$$

Where $a_0, a_1, a_2, \ldots, a_n$ an are constants and $a_n \neq 0$.

**e.g.,**

(i) $p(x) = 3x^3 + 2x^2 - 7x + 5$ is a polynomial in the variable $x$ (one variable).

(ii) $q(y) = 5y^2 + 3y$ is a polynomial in the variable $y$ (one variable).

**Example 1:** Which of the following are polynomials? Justify your answer.

(i) $x^3 + 3x^2 + 2$ (ii) $\sqrt{x^5} + 4x + 2$ (ii) $\frac{x^4 + x^3 + 3x}{x} + 2$

**Solution:** (i) Given expression is a $x^3 + 3x^2 + 2$
Here, we see that a variable has all positive integer powers.
Hence, it is a polynomial.

(ii) Given expression is $\sqrt{x^5} + 4x + 2$ or $x^{5/2} + 4x + 2$.
Here, we see that a variable $x$ has not all integer powers, i.e., $x^{5/2}$ is not of integral power.
Hence, it is not a polynomial.

(iii) Given expression is $\frac{x^4 + x^3 + 3x}{x} + 2$ or $x^3 + x^2 + 5$.
Here, we see that a variable has all positive integer powers. Hence, it is a polynomial.

**Example 2:** Which of the following expressions are polynomials in one or more variable(s)? State reasons for your answer.

(i) $3x^2 - 5x$ (ii) $5y^2 + 8x$

(iii) $\sqrt{t} + 3y$ (iv) $x^3 + x^2 + \frac{1}{x}$

**Solution** : (i) Given, expression is $3x^2 - 5x$
Here, expression is in one variable and all powers of a variable are integers.
Hence, it is a one variable polynomial.

(ii) Given, expression is $5y^2 + 8x$
Here, expression is in two variable and all powers of variables are non-negative integers.
Hence, it is a two variable polynomial.

(iii) Given, expression is $\sqrt{t} + 3y$ or $t^{1/2} + 3y$ all powers of variables are not non-negative integers. Hence, it is n
polynomial.

(iv) Given, expression is $x^3 + x^2 + \frac{1}{x}$ or $x^3 + x^2 + x^{-1}$
Here, expression is in one variable and all powers of a variable are not non-negative integers.
Hence, it is not a polynomial.

## 1.4 Term and Coefficient of a Polynomial

The part of a polynomial separated by '+'; or '–' sign is called a term of the polynomial. Each term of a polynomial has a coeffic
which is the constant associated with that term.

**e.g.,**

(i) In polynomial $x^2 - 4x + 7$, the expression $x^2, 4x$ and 7 are called terms of the polynomial and here coefficient of $x^2$ i
coefficient of $x$ is – 4 and coefficient of $x^0$, i.e., constant term, is 7.

(ii) The polynomial $4x^3 + 3x^2 - 7x + 5$ has four terms, namely, $4x^3, 3x^2, 7x$ and 5 and the coefficients of are 4, 3, -7 an
respectively.

$x + \frac{1}{x}, \sqrt{x+3}$ and $\sqrt[3]{y} + y^2$ polynomial because the power of polynomial are not whole.

**Example 3:** Find the coefficient of $x^2$ in $(3x + x^3)\left(x + \frac{1}{x}\right)$.

**Solution:** We have $(3x + x^3)\left(x + \frac{1}{x}\right)$.

$$= 3x \times x + 3x \times \frac{1}{x} + x^3 \times x + x^3 \times \frac{1}{x} = 3x^2 + 3 + x^4 + x^2 = x^4 + 4x^2 + 3$$

So, the coefficient of $x^2$ is 4.

## 1.5 Degree of a Polynomial

Highest power of the variable in a polynomial is known as the degree of that polynomial.

(1) **Degree of a polynomial in One Variable:** For a polynomial in one variable, the highest power of the variable is called the degree of a polynomial.

**Examples:** (i) $2x^4 - 6x^3 + 4x + 1$ is a polynomial in $x$ of degree 4. [Since, the highest power of $x$ is 4]

(ii) $\frac{4}{3} + \frac{3}{5}x - 5x^2 + \frac{1}{2}x^3 + x^5$ is a polynomial in $x$ of degree 5. [Since, the highest power of $x$ is 5]

(iii) $6x + \sqrt{3}$ is a polynomial in $x$ of degree 1. [Since, the highest power of $x$ is 1]

(2) **Degree of a Polynomial in Two or More Variables:** For a polynomial in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of a polynomial.

**e.g.,** (i) $7x^3 - 5x^2y^2 + 3xy + 6y + 8$ is a polynomial in $x$ and $y$ of degree 4. [since, the highest sum of powers of $x$ and $y$ is 2+2, i.e., 4]

(ii) $4x^2y^3 - 5xy^2 + 7 - \sqrt{2}x$ is a polynomial in $x$ and $y$ of degree 5. [since, the highest sum of powers of $x$ and $y$ is 2+3, i.e., 5]

- The degree of a non-zero constant polynomial is zero.
- The degree of a zero polynomial is not defined.

**Example 4:** Write the degree of the following polynomials.

(i) $7x^2 + 2x + 5$ (ii) $8xy^2 - 4y + 6$

(iii) $7n^2m + 4m$ (iv) $2z + 5$

(v) 4

The highest power of the variable in a polynomial, is the degree of the polynomial.

**Solution :** (i) We have, $7x^2 + 2x + 5$

Here, the highest power of $x$ is 2, so its degree is 2.

(ii) We have, $8xy^2 - 4y + 6$

Here, sum of the powers of variable in the first term is 1+2, i.e., 3, which is the highest, so its degree is 3.

(iii) We have, $7n^2m + 4m$

Here, polynomial is in terms of $m$ and $n$, so sum of powers of $m$ and $n$ in first term is 2+1, i.e., 3, which is the highest.

Hence, the degree of polynomial is 3.

(iv) We have, $2z + 5$

Here, the highest power of $z$ is 1, so its degree is 1.

(v) We have, 4 i.e, $x^0$

Here, the highest power of $x$ is 0, so its degree is 0.

## 1.6 Classifications of Polynomials

We can categorize polynomial according to their characteristics, which are described below:

(1) **On the Basis of Number of Terms**

On the basic of number of terms, the polynomials can be classified as:

(i) **Monomial:** A polynomial containing one non-zero terms, is called a monomial ('mono' means 'one')

**Example:** $5x, 7, 3x^3, -7x^2$ and $u^4$ are all monomials.

(ii) **Binomial:** A polynomial containing two non-zero terms, is called a binomial ('bi' means 'two').

**Example:** $(5 + 7x)$, $(7x^2y + 3y)$, $y^{30} + 1$ and $z^{23} - z^2$ are binomials.

(iii) **Trinomial:** A polynomial containing three non-zero terms, is called a **trinomial** ('tri' means 'three').

**Example:** $(8 + 3x + x^2)$, $(7 + 5xy + 6xy^2)$ and $(\sqrt{2} + x^2 - x)$ are trinomials.

(2) **On the Basis of Degree of Variables**

On the basis of degree of variable, the polynomials can be classified as:

(i) **Constant Polynomial:** A polynomial of degree zero, is called constant polynomial.

or

A polynomial containing only constant term, is called a constant polynomial.

e.g., 3, -7 and $\frac{7}{4}$ are constant polynomials.

(ii) **Linear Polynomial:** A polynomial of degree 1, is called a linear polynomial.

e.g.,

(a) $2x+5$ is a linear polynomial in $x$.

(b) $7y-9$ is a linear polynomial in $y$.

(c) $5z+\sqrt{7}$ is a linear polynomial in $z$.

Thus, we observe that a linear polynomial in $x$ will have atmost two terms. So standard form of a linear polynomial in $x$ will be $ax+b$, where $a$, $b$ are constants and $a \neq 0$ similarly, $ay+b$ is a linear polynomial in $y$.

(iii) **Quadratic Polynomial:** A polynomial of degree 2, is called a quadratic polynomial.

e.g.,

(a) $3x^2+7x+9$ is a quadratic polynomial in $x$.

(b) $5y^2-6y-3$ is a quadratic polynomial in $y$.

Thus, we observe that a quadratic polynomial in $x$ will have atmost 3 terms. So, a quadratic polynomial in $x$ is of the form $ax^2+bx+c$, where $a$, $b$, $c$ are constants and $a \neq 0$.

Similarly, quadratic polynomial in $y$ will be of the form $ay^2+by+c$, provided $a \neq 0$ and $a$, $b$, $c$ are constants.

(iv) **Cubic Polynomial:** A polynomial of degree 3, is called a cubic polynomial.

e.g.,

(a) $7x^3-5x^2+3x-9$ is a cubic polynomial in $x$.

(b) $6y^3+7y^2-5y+1$ is a cubic polynomial in $y$.

Thus, we observe that a cubic polynomial in $x$ will have atmost 4 terms. So, a cubic polynomial in $x$ is of the form $ax^3+bx^2+cx+d$, where $a$, $b$, $c$, $d$ are constants and $a \neq 0$.

Similarly, cubic polynomial in $y$ will be of the form $ay^3+by^2+cy+d$, where $a$, $b$, $c$, $d$ are constants and $a \neq 0$.

(v) **Biquadratic Polynomial:** A polynomial of degree 4, is called a biquadratic polynomial.

**e.g.,** $5x^4-7x^3+8x^2-12x-10$ is a biquadratic polynomial in x.

A biquadratic polynomial in x is of the form $ax^4+bx^3+cx^2+dx+e$, where $a$, $b$, $c$, $d$, $e$ are constants and $a \neq 0$.

(vi) **$n^{th}$ Degree Polynomial:** A polynomial of degree n in x is an expression of the form $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$.

Where, $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are constants and $a_n \neq 0$. Here, $a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, \ldots a_2x^2, a_1x, a_0$ are known as the terms of the polynomial $p(x)$ $a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0$ are known as their coefficients.

(vii) **Zero Polynomial:** If $a_0 = a_1 = a_2 = \ldots = a_n = 0$ (all constants are zero), then we get the zero polynomial, which is denoted by 0. If $p(x)=0$, then it is called the zero polynomial. But the degree of zero polynomial is not defined, as $p(x)$ can be written as

$$p(x) = 0 = 0, x = 0, x^2 = 0, x^3 = 0, \ldots$$

**Example 5:** Identify the following types of polynomials on the basis of terms.

(i) $4y+3y^2$ (ii) $3x$ (iii) $5t^3+6t+2$

**Hint:** The number of terms of polynomial are 1, 2 and 3, then the corresponding polynomials are called monomial, binomial and trinomial.

**Solution:** (i) We have $4y+3y^2$

Here, number of terms of in given polynomial is 2. Hence, it is a binomial.

(ii) We have $3x$. Here, number of terms of in given polynomial is 1. Here, it is a monomial.

(iii) We have $5t^3+6t+2$. Here, number of terms in given polynomial is 3. Here, it is a trinomial.

**Example 6:** Identify the following types of polynomials, on the basis of degree.

(i) $3x^2+5$ (ii) $z^3+4z+1$ (iii) $4t$

If the degree of polynomials are 1, 2 and 3, then the corresponding polynomials are called linear, quadratic and cubic.

**Solution:** (i) We have, $3x^2+5$

Here, degree of polynomial is 2. Hence, it is a quadratic polynomial.

(ii) We have, $z^3+4z+1$

Here, degree of polynomial $z^3+4z+1$ is 3. Hence, it is a cubic polynomial.

(iii) We have, $4t$

Here, degree of polynomial $4t$ is 1. Hence, it is a linear polynomial.

## 2 ZEROES OF A POLYNOMIAL

### 2.1 Value of a Polynomial

The value of a polynomial obtained on putting a particular value of the variable is called the value of a polynomial. The value of a polynomial $p(x)$ at $x=a$ (say) is denoted by $p(a)$.

**e.g.,** Let $p(x)=5x^3-2x^2+3x-2$

At $x=1$, $p(1)=5(1)^3-2(1)^2+3(1)-2=5-2+3-2=8-4=4$

So, 4 is the value of given polynomial $p(x)$ at $x=1$.

**Example 1:** Find the value of each of the following polynomials at the indicated value of variables.

(i) $q(y)=3y^3-4y+\sqrt{11}$ at $y=2$.

(ii) $p(t)=4t^4+5t^3-t^2+6$ at $t=a$.

**Solution** : (i) We have, $q(y)=3y^3-4y+\sqrt{11}$

On Putting $y=2$ in $q(y)$, we get

$q(2)=3(2)^3-4(2)+\sqrt{11}$

$=3\times8-8+\sqrt{11}$

$=24-8+\sqrt{11}=16+\sqrt{11}$

Which is the required value of $q(y)$ at $y=2$.

(ii) We have, $p(t)=4t^4+5t^3-t^2+6$

On putting $t=a$ in $p(t)$, we get

$p(a)=4(a)^4+5(a)^3-(a)^2+6=4a^4+5a^3-a^2+6$

Which is the required value of $p(t)$ at $t=a$.

### 2.2 Zero of a Polynomial

Zero of a polynomial $p(x)$ is a number $\alpha$, Such that $p(\alpha)=0$.

Zero of a polynomial is also called the root of polynomial equation $p(x)=0$.

**e.g.,** Let $p(x)=5x+7$

At $x=\frac{-7}{5}$, $p\left(\frac{-7}{5}\right)=5\left(\frac{-7}{5}\right)+7=-7+7=0$

Hence, $x=-\frac{7}{5}$ is zero (or root) of $p(x)$.

**Example 2:** Find the zero of a polynomial $2x+4$.

**Solution** Given polynomial is $p(x)=2x+4$.

On Putting $p(x)=0$, we get $2x+4=0$

$\Rightarrow 2x=-4 \Rightarrow x=-\frac{4}{2}=-2$

Hence, $x=-2$ is the zero of the polynomial $2x+4$.

(1) **Method to Check Whether the given Value is a Zero of a Polynomial or Not:** If a polynomial in one variable (say $x$) is given to us and a value of variable $x = c$ (say) is also given, then to check that given value of x is a zero of given polynomial or not, we use the following steps:

**Step I :** First, consider the given polynomial say $p(x)$.

**Step II:** Put $x = c$ in given polynomial $p(x)$ and find the value of $p(c)$.

**Step III:** If $p(c) = 0$, then $x = c$ will be a zero of given polynomial and if $p(c) \neq 0$, then $x = c$ will not be a zero of given polynomial.

**Example 3:** Verify that whether –2 and –3 are zeroes of the polynomial $x^2 - x - 6$.

**Solution:** Let given polynomial be $p(x) = x^2 - x - 6$ ...(i)

On Putting $x = -2$ in Eq. (i), we get $p(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$

Again, on putting $x = 3$ in Eq (i), we get $p(3) = (3)^2 - (3) - 6 = 9 - 3 - 6 = 0$

Here, $p(-2) = 0$ and $p(3) = 0$.

So, $x = -2$ and $x = 3$ are zeroes of the given polynomial.

**Example 4:** If $x = \frac{3}{2}$ is a zero of the polynomial $2x^2 + kx - 12$, find the value of $k$.

**Solution:** Let $p(x) = 2x^2 + kx - 12$

Since, $x = \frac{3}{2}$ is a zero of the polynomial.

$\therefore p\left(\frac{3}{2}\right) = 0 \Rightarrow 2\left(\frac{3}{2}\right)^2 + k\left(\frac{3}{2}\right) - 12 = 0 \Rightarrow \frac{9}{2} + \frac{3k}{2} - 12 = 0 \Rightarrow \frac{3k}{2} = \frac{24-9}{2}$

$\Rightarrow 3k = 15 \Rightarrow k = 5$

Hence, the value of $k$ is 5.

**Example 5:** If $x = 3$ and $x = 0$ are zeroes of the polynomial $2x^3 - 8x^2 + ax + b$, then find the value of $a$ and $b$. Consider the given polynomial as $p(x)$. If $x = 3$ and $x = 0$ are the zeroes of the polynomial $p(x)$. these values will satisfy $p(x)$, i.e., $p(3)=0$ and $p(0) = 0$.

**Solution:** Let $p(x) = 2x^3 - 8x^2 + ax + b$

Since, $x = 3$ is a zero of the polynomial.

$\therefore p(3) = 0$

$\Rightarrow 2(3)^3 - 8(3)^2 + a \times 3 + b = 0 \Rightarrow 2 \times 27 - 8 \times 9 + 3a + b = 0$

$\Rightarrow 54 - 72 + 3a + b = 0 \Rightarrow -18 + 3a + b = 0$

$\Rightarrow 3a + b = 18$ ...(i)

Also, $x = 0$ is a zero of the polynomial.

$\therefore p(0) = 0$

$\Rightarrow 2(0)^3 - 8(0)^2 + a \times 0 + b = 0 \Rightarrow 0 - 0 + 0 + b = 0$

$\Rightarrow b = 0$ ...(ii)

On putting $b = 0$ in Eq. (i), we get

$3a + 0 = 18 \Rightarrow 3a = 18$

$\therefore a = \frac{18}{3} = 6$

Hence, $a = 6$ and $b = 0$.

**Important Points on Zeroes of a polynomial :**

- Zero may be a zero of a polynomial.
- Every linear polynomial has one and only one zero.
- A non-zero constant polynomial has no zero.
- Every real number is a zero of the zero polynomial.
- A polynomial can have more than one zero.
- Maximum number of zeroes of a polynomial is a equal to its degree.

# 3 FACTORISATION OF THE POLYNOMIALS

## 3.1 Factor Theorem

Let $f(x)$ be polynomial of degree n and a be any real number.

(i) If $f(a) = 0$, then $(x - a)$ is a factor $f(x)$.

(ii) If $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

This theorem can be proved in the following ways

**Proof**

(i) Given, $f(a)=0$

Now, suppose that $f(x)$ is divided by $(x - a)$, then quotient is $g(x)$. By remainder theorem, when $f(x)$ is divided by $(x - a)$, then remainder is $f(a)$.

$\therefore f(x) = (x - a)\, g(x) + f(a)$

$\Rightarrow f(x) = (x - a).g(x) \quad [\because f(a) = 0, \text{ given}]$

So, $(x - 1)$ is a factor of $f(x)$.

(ii) Let $(x - a)$ be a factor of $f(x)$.

On dividing $f(x)$ by $(x - a)$, let $g(x)$ be the quotient.

$\therefore f(x) = (x - a).g(x)$

On Putting $x = a$, we get

$f(a) = (a - a).g(a) = 0.g(a) \Rightarrow f(a) = 0$

Thus, if $(x - a)$ is factor of $f(x)$, then $f(a) = 0$.

- If $(x + a)$ is a factor of $f(x)$, then $f(-a) = 0$.
- If $(ax - b)$ is a factor of $f(x)$, then $f\left(\frac{b}{a}\right) = 0$
- If $(x - a)(x - b)$ is a factor of $f(x)$, then $f(a) = 0$ and $f(b) = 0$.

## 3.2 Different Types of Problems Based on Factor Theorem

Sometimes a polynomial having unknown value and one of its factor are given to us and we have to find the value of that unknown. Also, sometimes a linear polynomial is given to us and we have to check (verify) whether it is a factor of given polynomial $f(x)$ of degree more than 1 or not. For solving these types of problems, some types are given below.

(1) **Type I:** Suppose a polynomial of degree more than 1, have an unknown constant and one of its factor is given. Then, firstly consider the given polynomial as $f(x)$ and put the given factor equal to zero, say $x = a$. further, we put the value of $x = a$ in given polynomial such that $f(a) = 0$. And finally we simplify it to get the result.

**Example 1:** Find the value of $p$, if $(2x - 1)$ is a factor of $2x^3 + px^2 + 11x + p + 3$.

**Solution:** Let $q(x) = 2x^3 + px^2 + 11x + p + 3$ be the given polynomial. If $q(x)$ is exactly divisible $2x - 1$, then $(2x - 1)$ is a factor of $q(x)$

$\therefore 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

On putting $x = \frac{1}{2}$ in $q(x)$ we get

$$q\left(\frac{1}{2}\right) = 2\times\left(\frac{1}{2}\right)^3 + p\left(\frac{1}{2}\right)^2 + 11\times\frac{1}{2} + p + 3 = 0$$

$$\Rightarrow 2\times\frac{1}{8} + p\times\frac{1}{4} + \frac{11}{2} + p + 3 = 0$$

$$\Rightarrow \frac{1}{4} + \frac{p}{4} + \frac{11}{2} + p + 3 = 0 \Rightarrow \frac{1 + p + 22 + 4p + 12}{4} = 0$$

$$\Rightarrow 5p + 35 = 0 \Rightarrow 5p = -35 \Rightarrow p = -7$$

Thus, the given polynomial is divisible by $2x - 1$, if $p = -7$.

**Example 2.** If $(x-a)$ is a factor of $4x^2-mx-na$, prove that $a=\frac{m+n}{4}$.

**Solution** Let $p(x)=4x^2-mx-na$

If $(x-a)$ is a factor of $p(x)$, then $p(a)=0$.

$\therefore 4a^2-ma-na=0 \Rightarrow a(4a-m-n)=0$

$\Rightarrow 4a-m-n=0$ $[\because a\neq 0]$

$\Rightarrow 4a-(m+n)=0 \Rightarrow 4a=m+n$

$\therefore a=\frac{m+n}{4}$

Hence proved

**Example 3:** Find $m$ and $n$, if $(x+2)$ and $(x+1)$ are the factor of $x^3+3x^2-2mx+n$.

**Solution** Let $f(x)=x^3+3x^2-2mx+n$

Since, $(x+2)$ and $(x+1)$ are the factors of $f(x)$.

$\therefore f(-2)=0$ and $f(-1)=0 \Rightarrow (-2)^3+3(-2)^2-2m(-2)+n=0$ and $(-1)^3+3(-1)^2-2m(-1)+n=0$

$\Rightarrow -8+12+4m+n=0$ and $-1+3+2m+n=0$

$\Rightarrow 4m+n=-4$ ...(i)

and $2m+n=-2$ ...(ii)

On multiplying Eq. (ii) by 2 and then subtracting Eq. (i) from Eq. (ii), we get $4m+2n-(4m+n)=-4-(-4) \Rightarrow n=0$

On putting $n=0$ in Eq. (i), we get $4m+0=-4 \Rightarrow m=-1$

Hence, $m=-1$ and $n=0$

(2) **Type II:** If polynomial of degree more than one, says $p(x)$ and a linear polynomial, say $q(x)$ are given to us, then to check (verify) that $q(x)=0$ is a factor of $p(x)$ or not, we firstly put $q(x)=0$ and find the value of variable $x$ [i.e. zero of $q(x)$]. Further, on putting this value of variable in $p(x)$, if we get zero, then $q(x)$ is a factor of $p(x)$, otherwise not.

**Example 4:** Using factor theorem, show that $(x+1)$ is a factor of $x^{19}+1$.

**Solution:** Let $p(x)=x^{19}+1$ and $q(x)=x+1$

On putting $q(x)=0$, we get $x+1=0 \Rightarrow x=-1$

On putting $x=-1$ in $p(x)$, we get $p(-1)=(-1)^{19}+1=-1+1=0$

Hence, by factor theorem, $(x+1)$ is a factor of $x^{19}+1$.

## 3.3 Factorisation of a Quadratic Polynomial

Quadratic polynomial of the type $ax^2+bx+c$, where $a$, $b$, $c$ are constants and $a\neq 0$, can be factored by different methods like splitting the middle term and by factor theorem, both of these are discussed ahead.

(1) **By Splitting the Middle Term:** Let factors of the quadratic polynomial $ax^2+bx+c$ be $(px+q)$ and $(rx+s)$.

Then, $ax^2+bx+c=(px+q)(rx+s)=prx^2+(ps+qr)x+qs$

On comparing the coefficients of $x^2$, $x$ and constant terms from both sides, we get $a=pr, b=ps+qr$ and $c=qs$

Here, $b$ is the sum of two numbers $ps$ and $qr$, whose product is $(ps)(qr)=(pr)(qs)=ac$.

Thus, to factors $ax^2+bx-c$, write $b$ as the sum of two numbers, whose product is $ac$.

To factorize $ax^2+bx-c$ and $ax^2-bx-c$, write $b$ as the difference of two numbers whose product is $(-ac)$

**Example 5:** Factorising $2x^2+7x+3$

**Solution:** Given polynomial is $2x^2+7x+3$.

On comparing with $ax^2+bx+c$, we get $a=2, b=7$ and $c=3$.

Now, $ac=2\times 3=6$

So, all possible pairs of factors of 6 are 1 and 6, 2 and 3.

Clearly, pair 1 and 6 gives $1+6=7=b$

$\therefore 2x^2+7x+3=2x^2+(1+6)x+3$

$=2x^2+x+6x+3=x(2x+1)+3(2x+1)=(2x+1)(x+3)$

(2) **By Using Factor Theorem:** Write the given polynomial $p(x) = ax^2 + bx + c$ in the form

$$p(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\,g(x) \quad ...(i)$$

where, $g(x) = x^2 + \frac{b}{a}x + \frac{c}{a}$

i.e., Firstly make the coefficient of $x^2$ equal to one if it is not one.

Find all the possible factors of constant term $\left(\frac{c}{a}\right)$ of $g(x)$. Using trial method, find the factors at which $g(x) = 0$ say, $x = \alpha$ and $x = \beta$. Further, write $g(x)$ as the product of factors, then $g(x) = (x - \alpha)(x - \beta)$ and put this value of $g(x)$ in Eq. (i) to get required factors of $p(x)$.

**Example 6:** Factorise $x^2 - 5x + 6$ by using factor theorem.

**Solution:** Let given polynomial be $f(x) = x^2 - 5x + 6$.

Here, coefficient of $x^2$ is 1, so we do not need to write it in the form of $g(x)$.

Now, constant term is 6 and all factors of 6 are $\pm 1, \pm 2, \pm 3$ and $\pm 6$.

At $x = 2$, $f(2) = 2^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 10 - 10 = 0$

At $x = 3$, $f(3) = 3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 15 - 15 = 0$

Hence, $(x - 2)$ and $(x - 3)$ are the factors of given quadratic polynomial.

### 3.4 Factorisation of a Cubic Polynomial

To factorise a cubic polynomial, we use the following steps:

**Step I:** Write the given cubic polynomial

$p(x) = ax^3 + bx^2 + cx + d$ in the form

$$p(x) = a\left(x^3 + \frac{b}{a}x^2 + \frac{c}{a} + \frac{d}{a}\right) = ag(x) \quad ...(i)$$

where, $g(x) = x^3 + \frac{b}{a}x^2 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}$

i.e., first make the coefficient of $x^3$ equal to one if it is not one and then find the constant term.

**Step II:** Find all the possible factors of constant term $\left(\frac{d}{a}\right)$ of $g(x)$.

**Step III:** Check at which factor of constant term, $p(x)$ is zero by using trial method and get one factor of $p(x)$, (i.e., $x - \alpha$).

**Step IV:** Write $p(x)$ as the product of this factor and a quadratic polynomial,

i.e., $p(x) = (x - \alpha)(a_1x^2 + b_1x + c_1)$

**Step V:** Apply splitting the middle term method or factor theorem in quadratic polynomial to get other two factors. Thus, we get all the three factors of given cubic polynomial.

**Example 7:** Using factor theorem, factorise $x^3 - 6x^2 + 3x + 10$.

**Solution** Let $p(x) = x^3 - 6x^2 + 3x + 10$

Here, constant term=10 and coefficient of $x^3$ is one.

All possible factors of 10 are $\pm 1, \pm 2, \pm 5$ and $\pm 10$.

At $x = -1$, $p(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$

$= -1 - 6 - 3 + 10 = 0$

So, $(x + 1)$ is a factor of $p(x)$.

On dividing $p(x)$ by $(x + 1)$, we get

Quotient $= x^2 - 7x + 10$

So, $p(x) = (x + 1)(x^2 - 7x + 10)$

By splitting the middle term, we get

$p(x) = (x + 1)\{x^2 - (5 + 2)(x) + 10\}$ [$\because 2 + 5 = 7$ and $2 \times 5 = 10$]

$= (x + 1)\{x^2 - 5x - 2x + 10\}$

$= (x + 1)\{x(x - 5) - 2(x - 5)\}$ $= (x + 1)(x - 2)(x - 5)$

**Example 8:** Factorise $2x^3 - 5x^2 - 19x + 42$.

**Solution:** Let $p(x) = 2x^3 - 5x^2 - 19x + 42$

Here, we see that coefficient of $x^3$ is not one, so firstly we make the coefficient of $x^3$ is one.

i.e., $p(x) = 2\left(x^3 - \frac{5}{2}x^2 - \frac{19}{2}x + \frac{42}{2}\right) = 2g(x)$ ...(i)

where $g(x) = x^3 - \frac{5}{2}x^2 - \frac{19}{2}x + 21$. Here, constant term is 21 and its all possible factors are $\pm1, \pm3, \pm7, \pm21$.

At $x = 1$, $g(1) = (1)^3 - \frac{5}{2}(1)^2 - \frac{19}{2}(1) + 21 = 1 - \frac{5}{2} - \frac{19}{2} + 21 = -12 + 22 = 10 \neq 0$

So, $(x - 1)$ is not a factor of $g(x)$.

At $x = -3$, $g(-3) = (-3)^3 - \frac{5}{2}(-3)^2 - \frac{19}{2}(-3) + 21 = -27 - \frac{5}{2}\times 9 + \frac{57}{2} + 21 = 0$

So, $(x + 3)$ is a factor of $g(x)$. On dividing $g(x)$ by $(x + 3)$, we get Quotient $= \left(x^2 - \frac{11}{2}x + 7\right)$

$\because g(x) = (x+3)\left(x^2 - \frac{11}{2}x + 7\right)$

From Eq. (i), $p(x) = 2(x+3)\left(\frac{2x^2 - 11x + 14}{2}\right)$

$= (x+3)(2x^2 - 11x + 14) = (x+3)(2x^2 - 4x - 7x + 14) = (x+3)[2x(x-2) - 7(x-2)] = (x+3)(x-2)(2x-7)$

**Example 9:** If $p(x) = x^3 - 4x^2 + x + 6$, then show that $p(3) = 0$ and hence factorise $p(x)$.

**Solution:** Given, $p(x) = x^3 - 4x^2 + x + 6$

Put $x = 3$ in Eq. (i), we get $p(3) = (3)^3 - 4(3)^2 + 3 + 6 = 27 - 36 + 9 = 0$

Since, $p(3) = 0$, therefore $x - 3$ is a factor $p(x)$.

$p(x) = (x - 3)(x^2 - x - 2) = (x - 3)(x^2 - 2x + x - 2)$ $[\because -2 + 1 = -1 \text{ and } -2 \times 1 = -2]$

$= (x-3)[x(x-2) + 1(x-2)] = (x-3)(x+1)(x+2)$

**Hence proved.**

## 4 ALGEBRAIC IDENTITIES

An identity is an equality, which is true for all value of its variable in the equality, i.e. an identity is a universal truth. Some useful algebraic identities are given below:

(1) $(x+y)^2 = x^2 + 2xy + y^2$

(2) $(x-y)^2 = x^2 - 2xy + y^2$

(3) $x^2 - y^2 = (x-y)(x+y)$

(4) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(5) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = \sum x^2 + 2\sum xy$

(6) $(x+y)^3 = x^3 + y^3 + 3xy(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$

(7) $(x-y)^3 = x^3 - y^3 - 3xy(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

(8) $x^3 - y^3 = (x-y)(x^2 + xy + y^2) = (x-y)[(x-y)^2 + 3xy]$

(9) $x^3 + y^3 = (x+y)(x^2 - xy + y^2) = (x+y)[(x+y)^2 - 3xy]$

(10) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx) = (x+y+z)[(x+y+z)^2 - 3(xy + yz + zx)]$

if $(x+y+z) = 0$ then $x^3 + y^3 + z^3 = 3xyz$

(11) $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

The right hand side expression is called the expanded form of the left hand side expression.

## 5.1 Problems Based on Algebraic Identities

We can use these identities to solve many problems, such as to factorize the algebraic expression, to find product without multiplying directly and to evaluate the value of number having exponent.

**Example 1:** Using appropriate identity, factorize $4x^2 - \frac{y^2}{9}$.

**Solution:** $4x^2 - \frac{y^2}{9} = (2x)^2 - \left(\frac{y}{3}\right)^2 = \left(2x - \frac{y}{3}\right)\left(2x + \frac{y}{3}\right)$ $[\because a^2 - b^2 = (a-b)(a+b)]$

**Example 2:** Evaluate without multiplying directly $(105 \times 106)$

**Solution:** Firstly, write 105 as 100 + 5 and 106 as 100 + 6.

$105 \times 106 = (100+5) \times (100+6) = (100)^2 + (5+6)100 + (5 \times 6)$ $[\because (x+a)(x+b) = x^2 + (a+b)x + ab]$

$= 10000 + 11 \times 100 + 30 = 10000 + 1100 + 30 = 11130$

**Example 3:** Expand $\left(x - \frac{1}{2}y + \frac{1}{3}z\right)^2$.

**Solution:** $\left(x - \frac{1}{2}y + \frac{1}{3}z\right)^2$

$= (x)^2 + \left(\frac{-1}{2}y\right)^2 + \left(\frac{1}{3}z\right)^2 + 2x\left(-\frac{1}{2}y\right) + 2\left(-\frac{1}{2}y\right)\left(\frac{1}{3}z\right) + 2\left(\frac{1}{3}z\right)(x)$ $[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= x^2 + \frac{1}{4}y^2 + \frac{1}{9}z^2 - xy - \frac{yz}{3} + \frac{2zx}{3}$

**Example 4:** Evaluate $(104)^3$ by using suitable identity.

**Solution:** Given number without power is 104. Since, it is greater than 100, so it can be written as 100 + 4.

$\therefore (104)^3 = (100+4)^3$

On comparing $(100+4)^3$ with $(x+y)^3$, we get $x = 100$ and $y = 4$.

By using the algebraic identity

$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$, we get

$(104)^3 = (100+4)^3 = (100)^3 + (4)^3 + 3(100)(4)(100+4)$

$= 1000000 + 64 + 1200(104) = 1000000 + 64 + 124800 = 1124864$

While finding the squares, cubes, if given number is greater than 10 or 100 or 1000 or 10000, then we write it as 10+a or 100 + $a$ or 1000 + $a$ or 10000 + $a$ and if given number is less than 10 or 100 or 1000 or 10000, then we write it as 10 − $a$ or 100 − $a$ or 1000 − $a$ or 10000 − $a$ to make calculation easy, where a is any number.

**Example 5:** Factorize $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30zx$ by using suitable identity.

**Solution:** We have $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30zx$

We can rewrite the given expression as

$(5x)^2 + (2y)^2 + (3z)^2 - 2 \times 5x \times 2y - 2 \times 2y \times 3z + 2 \times 3z \times 5x$

$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$

$= (5x - 2y + 3z)^2$ $[\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2]$

**Example 6:** Factorize $\frac{x^3}{8} - 64 - 3x^2 + 24x$ by using suitable identity.

**Solution:** We have, $\frac{x^3}{8} - 64 - 3x^2 + 24x$

We can rewrite the given expression as $\left(\frac{x}{2}\right)^3 - (4)^3 - 3x(x-8)$

$= \left(\frac{x}{2}\right)^3 - (4)^3 - 3 \times \frac{x}{2} \times 4\left(\frac{x}{2} - 4\right) = \left(\frac{x}{2} - 4\right)^3$ $[\because a^3 - b^3 - 3ab(a-b) = (a-b)^3]$

$= \left(\frac{x}{2} - 4\right)\left(\frac{x}{2} - 4\right)\left(\frac{x}{2} - 4\right)$

**Example 7:** Simplify $(x+y)^3-(x-y)^3-6y(x+y)(x-y)$.

**Solution:** $(x+y)^3-(x-y)^3-6y(x+y)(x-y)$

$=(x+y)^3-(x-y)^3-3(x+y)(x-y)[(x+y)-(x-y)]$

$=[x+y-(x-y)]^3$ $[\because a^3-b^3-3ab(a-b)=(a-b)^3]$

$=(2y)^3=8y^3$

**Example 8:** If $x^2+\frac{1}{x^2}=7$, find the value of $x^3+\frac{1}{x^3}$, using only positive value of $x+\frac{1}{x}$.

**Solution:** Given, $x^2+\frac{1}{x^2}=7\Rightarrow x^2+\frac{1}{x^2}+2=7+2$ [Adding both side by 2]

$\Rightarrow\left(x+\frac{1}{x}\right)^2=(3)^2$

Taking posiive square root, we get $x+\frac{1}{x}=3$ ...(i)

Now, cubing both sides, we get $x^3+\left(\frac{1}{x}\right)^3+3x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=(3)^3$

$\Rightarrow x^3+\frac{1}{x^3}+3(3)=27\Rightarrow x^3+\frac{1}{x^3}=21$ [from Eq. (i)]

**Example 9:** If $a+b+c=9$ and $ab+bc+ca=40$, then find the value of $a^2+b^2+c^2$

**Solution:** Given that, $a+b+c=9$

On squaring both sides, we get $(a+b+c)^2=(9)^2$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca=81$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=81$

On putting $ab+bc+ca=40$, we get

$a^2+b^2+c^2+2\times 40=81$

$\therefore a^2+b^2+c^2=81-80=1$

**Example 10:** Find the following product by using suitable identity.

$(3x-5y-4)(9x^2+25y^2+15xy+12x-20y+16)$

**Solution** $(3x-5y-4)(9x^2+25y^2+15xy+12x-20y+16)$

$=[3x+(-5y)+(-4)][(3x)^2+(-5y)^2+(-4)^2-(3x)(-5y)-(-5y)(-4)-(-4)(3x)]$

$=(3x)^3+(-5y)^3+(-4)^3-3\times 3x\times(-5y)\times(-4)$ $[\because a^3+b^3+c^3-3abc=(a+b+c)\ (a^2+b^2+c^2-ab-bc-ca)]$

$=27x^3-125y^3-64\ -180$

## 5.2 Problems Based on Geometrical Figure

Sometimes the problem is given in the form of an area and volume of some geometrical figure (square, cube etc.,) in polynomial form.

For this, firstly we determine all the factors of the given polynomial and further consider any of the factor as any of the dimension.

**Example 11:** Give possible expression for the length and breadth of the rectangle whose area is given by $4a^2+4a-3$.

**Solution** Given, area of rectangle $=4a^2+4a-3$

$=4a^2+6a-2a-3$ [by slitting the middle term]

$=2a(2a+3)-1(2a+3)=(2a-1)(2a+3)$

Hence, possible expression for length/breadth = $(2a-1)$ and possible expression for breadth / length= $(2a+3)$.

**Example 12:** What are the possible expressions for the dimensions a cuboid, whose volume is $36kx^2y-21kxy^2+13ky^3$?

**Solution** $\because$ Volume $=36kx^2y-21kxy^2+3ky^3$

$=3ky[12x^2-7xy+y^2]=3ky[12x^2-4xy-3xy+y^2]$

$=3ky[4x(3x-y)-y(3x-y)]=3ky(4x-y)(3x-y)$

Hence, possible expression for length / breadth / height $=3ky$, possible expression for breadth / height / length $=4x-y$ and possible expression for height / length / breadth $=3x-y$.

# Chapter at Glance

(1) A combination of constants and variables, connected four fundamental arithmetical operations '+', '–', and '÷' is called an algebraic expression. **Example:** $6x^2 - 5y^2 + 2xy$

(2) An algebraic expression which have only whole numbers as the exponent of one variable is called polynomial in one variable. **Example:** $3x^3 + 2x^2 - 7x + 5$ etc.

(3) The part of a polynomial separated from each other by '+' or '–' sign is called a term and each term of a polynomial has a coefficient.

(4) Highest power of the variable in a polynomial, is known as degree of that polynomial.

(5) On the Basis of Number of Terms

(i) A polynomial containing one non-zero term, is called a monomial.

(ii) A polynomial containing two non-zero terms, is called a binomial.

(iii) A polynomial containing three non-zero terms, is called a trinomial.

(6) On the Basis of Degree of Variables

(i) A polynomial of degree 0, is called a constant polynomial.

(ii) A polynomial of degree 1, is called a linear polynomial.

(iii) A polynomial of degree 2, is called a quadratic polynomial.

(iv) A polynomial of degree 3, is called a cubic polynomial.

(v) A polynomial of degree 4, is called a biquadratic polynomial.

(7) The value obtained on putting a particular value of the variable in polynomial is called value of the polynomial at that value of variable.

(8) Zero of a polynomial $p(x)$ is a number $a$, such that $p(a) = 0$. It is also called root of polynomial equation $p(x) = 0$.

(9) Let $f(x)$ be any polynomial of degree $n$, $(n \geq 1)$ and $a$ be any real number. If $f(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $f(a)$.

(10) Let $f(x)$ be a polynomial of degree $n$, $(n \geq 1)$ and a be any real number.

Then,

(i) If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

(ii) If $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

(11) **Algebraic Identities**

(i) $(x+y)^2 = x^2 + y^2 + 2xy$

(ii) $(x-y)^2 = x^2 + y^2 - 2xy$

(iii) $x^2 - y^2 = (x-y)(x+y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(viii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(ix) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

(xi) $x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

# CHAPTER PRACTICE

## MULTIPLE CHOICE QUESTIONS

1. $\sqrt{2}$ is a polynomial of degree

(a) 2 (b) 0
(c) 1 (d) $\frac{1}{2}$

2. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(a) 0 (b) 1
(c) $4\sqrt{2}$ (d) $8\sqrt{2} + 1$

3. $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of $k$ is

(a) –3 (b) 4
(c) 2 (d) –2

4. The value of $249^2 - 248^2$ is

(a) $1^2$ (b) 477
(c) 487 (d) 497

5. If $x^2 + \frac{1}{x^2} = 7$, then the value of $x^3 + \frac{1}{x^3}$ is

(a) 27 (b) 9
(c) 18 (d) 36

6. If $a + b + c = 9$ and $ab + bc + ca = 40$ then the value of $a^2 + b^2 + c^2$ is

(a) 1 (b) 2
(c) 3 (d) 4

7. Which one of the following algebraic expressions is a polynomials in variable $x$?

(a) $x^2 + \frac{2}{x^2}$ (b) $\sqrt{x} + \frac{1}{\sqrt{x}}$
(c) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (d) None of these

8. One of the dimensions of the cuboid whose volume is $36Kx^2y - 21Kxy^2 + 3Ky^3$, is

(a) $3Ky$ (b) $4x - y$
(c) $3x - y$ (d) All of these

9. $(x + 1)$ is a factor of the Polynomial

(a) $x^3 + x^2 - x + 1$
(b) $x^3 + x^2 + x + 1$
(c) $x^4 + x^3 + x^2 + 1$
(d) $x^4 + 3x^3 + 3x^2 + x + 1$

10. If $49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then value of $b$ is

(a) 0 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

## 1 MARK QUESTIONS

1. Determine the degree of each of the following polynomial.

(i) $3x - 2$ (ii) $-20$
(iii) $x^3(2 - x^3)$

2. Find the factors of the expression $ab + bc + ax + cx$.

3. Check whether $p(x)$ is a multiple of $g(x)$ or not.

$p(x) = 2x^3 - 11x^2 - 4x + 5, g(x) = 2x + 1$.

4. What are the factors of $(x + y)^3 - (x^3 + y^3)$?

5. What is the best way to evaluate $(996)^2$?

6. If the volume of a cuboid is $2x^2 - 16$, then find its possible dimensions.

7. Factorise $(25x^2 - 1) + (1 + 5x)^2$.

8. Find the value of $305 \times 308$ by using suitable identity.

9. If $x^2 + \frac{1}{x^2} = 51$, then find $x + \frac{1}{x}$.

10. If $\frac{x}{y} + \frac{y}{x} = -1$, $x \neq 0, y \neq 0$, then find the value of $x^3 - y^3$.

11. Without actually calculating the cubes, find the value of $-(0.4)^3 - (0.2)^3 + (0.6)^3$.

12. Solve for $x : \sqrt{(3x - 5)} = 7$.

## 2 MARK QUESTIONS

1. Write whether the following statement are true or false.

(i) A binomial can have atmost two terms.
(ii) Every polynomial is a binomial.
(iii) A binomial may have degree 6.
(iv) Zero of a polynomial is a always 1.
(v) A polynomial cannot have more than two zeroes.

2. Find the zeroes of the polynomial $p(y) = (x-2)^2 - (x+2)^2$.

3. Factorise the following expressions.

(i) $25x^2 + 9y^2 + 9z^2 - 30xy - 18yz + 30xz$

(ii) $9x^2 + 16y^2 + 4z^2 - 24xy + 16yz - 12xz$

4. Factorise

(i) $2x^2 - 7x - 15$ (i) $84 - 2r - 2r^2$

5. Factorise

(i) $2x^2 - \frac{5}{6}x + \frac{1}{12}$ (ii) $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

6. Factorise the following $x^3 - x^2 + ax + x - a - 1$

7. Simplify $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$.

8. Factorise $5(3x+y)^2 + 6(3x+y) - 8$.

9. Factorise $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$.

10. Factorise $24\sqrt{3}x^3 - 125y^3$.

11. Factorise $(x^2 + 4) - 2a - a^2 = 5$.

12. Find the product $\left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)\left(a^2 + \frac{1}{a^2}\right)\left(a^4 + \frac{1}{a^4}\right)$ using a suitable identity.

13. If $a^2 + \frac{9}{a^2} = 31$, what is the positive value of $a - \frac{3}{a}$?

## 3 MARK QUESTIONS

1. If $(x+4)$ is a factor of the polynomial $x^3 - x^2 - 14x + 24$, find the its other factors.

2. Factorise $2x^3 - 3x^2 - 17x + 30$.

3. Factorise the following.

(i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

4. Factorise the following.

(i) $1 - 64a^3 - 12a + 48a^2$

(ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

5. Evaluate $(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$.

6. Simplify the following expressions.

(i) $(x+y+z)^2 + (x+y-z)^2$

(ii) $(2x+p-c)^2 - (2x-p+c)^2$

7. Simplify $27x^3 - (3x-y)^3$.

8. Factorise the following

(i) $3a^3b - 243ab^3$ (ii) $x^4 - 625$

(iii) $a^3 + b^3 + a + b$ (iv) $x(x-y)^3 + 3x^2y(x-y)$

9. Factorise $x^4 - 3x^2 + 2$.

10. Factorise $a^7 - ab^6$.

11. If $\sqrt{u} + \sqrt{v} - \sqrt{w} = 0$, find the value of $(u + v - w)$.

12. Find $y^2 + \frac{1}{y^2}$ and $y^4 + \frac{1}{y^4}$, if $y - \frac{1}{y} = 9$.

13. Factorise $a^3(b-c)^3 + b^3(c-a)^3 + c^3(a-b)^3$

14. If $x - y = 5$ and $xy = 84$, find the value of $x^3 - y^3$.

15. If $x^2 + \frac{1}{x^2} = 14$, find $x^3 + \frac{1}{x^3}$.

## 5 MARK QUESTIONS

1. If $a$, $b$ and $c$ are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

2. Find the value of

$9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$, when $x = 1$, $y = 2$ and $z = -1$.

3. Prove that $(a+b+c)^3 - a^3 - b^3 - c^3 = 3(a+b)(b+c)(c+a)$.

4. Let $A$ and $B$ be the remainders, when the polynomials $y^3 + 2y^2 - 5ay - 7$ and $y^3 + ay^2 - 12y + 6$ are divided by $(y+1)$ and $(y-2)$respectively. If $2A + B = 6$, find the value of $a$.

5. Without actual division, prove that $(2x^4 - 6x^3 - 3x^2 + 3x - 2)$ is exactly divisible by $(x^2 - 3x - 2)$

6. Factorise $a^{12}y^4 - a^4y^{12}$.

7. Factorise $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$.

8. If $x + y + z = 1$, $xy + yz + zx = -1$ and $xyz = -1$. find the value of $x^3 + y^3 + z^3$.

9. Find the square root of $(x^2 + 4x + 4)(x^2 + 6x + 9)$.

10. Simplify

$$\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}.$$

11. If $x = (2+\sqrt{5})^{\frac{1}{2}} + (2-\sqrt{5})^{\frac{1}{2}}$ and $y = (2+\sqrt{5})^{\frac{1}{2}} - (2-\sqrt{5})^{\frac{1}{2}}$, evaluate $x^2 + y^2$.

## HOTS QUESTIONS

1. If $a = -5$, $b = -6$ and $c = 10$, then the value of

$$\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2} \text{ is}$$

(a) 0 (b) −1
(c) 1 (d) Non of these

2. If $x + y = 2$, then the value of $x^3 + 6xy + y^3 - 8$ is
(a) 1 (b) 3
(c) 0 (d) 5

3. The factor of

$$\left(3a - \frac{1}{b}\right)^2 - 8\left(3a - \frac{1}{b}\right) + 16 + \left(c + \frac{1}{b} - 2a\right)\left(3a - \frac{1}{b} - 4\right) \text{ is}$$

(a) $\left(3a - \frac{1}{b} - 4\right)(a + c - 4)$

(b) $\left(3a - \frac{1}{b} - 4\right)(3a + c - 4)$

(c) $\left(3a - - - 4\right)(2a + 3c - 4)$

(d) $\left(3a - \frac{1}{b} - 4\right)(a + 3c - 4)$

4. If $x^4 + \frac{1}{x^4} - 8 = 314$, then the value of $x - \frac{1}{x}$ will be.
(a) 4 (b) 7
(c) 8 (d) 6

5. If $ab - b + 1 = 0$ and $bc - c + 1 = 0$, then the value $a - ac$ is
(a) 1 (b) 0
(c) 2 (d) 5

6. If $p(x)$ is a common multiple of degree 6 of the polynomials, $f(x) = x^3 + x^2 - x - 1$ and $g(x) = x^3 - x^2 + x -$ then $p(x)$ is
(a) $x^6 - 1$ (b) $x^6 + x^5 + x^3 + 1$
(c) $x^6 + 1$ (d) $(x - 1)^2 (x + 1)^2 (x^2 + 1)$

7. Let $p$, $q$ and $r$ positive real numbers such that $p + q + r =$
Then $\frac{p}{p^2 + q^3 + r^3} + \frac{q}{q^2 + r^3 + p^3} + \frac{r}{r^2 + p^3 + q^3}$.
(a) $\le \frac{1}{5pqr}$ (b) $\ge \frac{}{pqr}$
(c) $= \frac{1}{5pqr}$ (d) $> \frac{1}{5pqr}$

## CASE BASED QUESTIONS

**Read the following and answer any four questions from 1(I-V)**

**Case 1:** Ankur and Ranjan start a new business together. The amount invested by both partners together is given by the polynomial $p(x) = 4x^2 + 12x + 5$, which is the product of their individual shares.

I. Coefficient of $x^2$ in the given polynomial is
(a) 2 (b) 3
(c) 4 (d) 12

II. Total amount invested by both, if $x = 1000$ is
(a) ₹3015065 (b) ₹ 3705615
(c) ₹4012005 (d) ₹ 4906215

III. The shares of Ankur and Ranjan invested individually are
(a) ₹$(2x + 1)$, ₹$(2x + 5)$,
(b) ₹$(2x + 3)$, ₹$(x + 1)$
(c) ₹$(x + 1)$, ₹$(x + 3)$
(d) None of these

IV. Name the polynomial of amounts invested by each partner.
(a) Cubic (b) Quadratic
(c) Linear (d) None of these

V. Find the value of $x$, if the total amount invested is equal to 0.
(a) $\frac{-1}{2}$ (b) $\frac{-5}{2}$
(c) Both (a) and (b) (d) None of these

**Read the following and answer any four questions from 2(I-V)**

**Case 2:** A class teacher decided to organise an educational trip for his class. He asked the students for their preferences where they want to go. $\frac{1}{12}$th times the square of total number of students want to go to old age home, $\frac{7}{12}$ times the total number of students plan to visit historical monuments, while 15 students decide to teach children of orphanage home.

I. Which of the following polynomial represents the above situation, if $x$ is the total number of students?
(a) $\frac{7}{12}x^2 + \frac{1}{12}x + 15$ (b) $\frac{1}{12}x^2 + \frac{7}{12}x + 15$
(c) $7x^2 + 12x + 15$ (d) None of this

II. The coefficient of $x^2$ in the above polynomial is

(a) $\frac{7}{12}$ (b) $-\frac{1}{12}$

(c) $-\frac{7}{12}$ (d) $\frac{1}{12}$

III. Write the coefficient of x in the polynomial.

(a) $-\frac{1}{12}$ (b) $\frac{1}{12}$

(c) $\frac{7}{12}$ (d) $-\frac{7}{12}$

IV. Value of the polynomial at $x = 1$, is

(a) 172 (b) 150

(c) $\frac{176}{12}$ (d) $\frac{47}{3}$

V. Value of the polynomial at $x = 2$ is

(a) $\frac{170}{12}$ (b) $\frac{182}{12}$

(c) 190 (d) $\frac{33}{2}$

**Read the following and answer any four questions from 3(I-V)**

**Case 3:** On one day, principal of a particular school visited the classroom. Class teacher was teaching the concept of polynomial to students. He was very much impressed by her way of teaching. To check, whether the students also understand the concept taught by her or not, he asked various questions to students. Some of them are given below. Answer them.

I. Which one of the following is not a polynomial?

(a) $4x^2 + 2x - 1$ (b) $y + \frac{3}{y}$

(c) $x^3 - 1$ (d) $y^2 + 5y + 1$

II. The polynomial of the type $ax^2 + bx + c$, $a = 0$ is called

(a) Linear polynomial (b) Quadratic polynomial

(c) Cubic polynomial (d) Biquadratic polynomial

III. The value of k, if $(x - 1)$ is a factor of $4x^3 + 3x^2 - 4x + k$, is

(a) 1 (b) -2

(c) -3 (d) 3

IV. If $x + 2$ is the factor of $x^3 - 2ax^2 + 16$, then value of $a$ is

(a) -7 (b) 1

(c) -1 (d) 7

V. The number of zeroes of the polynomial $x^2 + 4x + 2$ is

(a) 1 (b) 2

(c) 3 (d) 4

❑❑❑

# EXAM PRACTICE

## NCERT QUESTIONS

1. Which of the following expressions are polynomials in one variable and which are not? State reason for your answer.
   (i) $4x^2 - 3x + 7$  (ii) $y^2 - \sqrt{2}$
   (iii) $3\sqrt{t} + t\sqrt{2}$  (iv) $y + \frac{2}{y}$
   (v) $x^{10} - y^3 + t^{50}$

2. Write the coefficients of $x^2$ in each of the following polynomials.
   (i) $2 + x^2 + x$  (ii) $2 - x^2 - x^3$
   (iii) $\frac{\pi}{2}x^2 + x$  (iv) $\sqrt{2}x - 1$

3. Give one example each of a binomial of degree 35 and a monomial of degree 100.

4. Write the degree of each of the following polynomials.
   (i) $5x^3 + 4x^2 + 7x$  (ii) $4 - y^2$
   (iii) $5t - \sqrt{7}$  (iv) $3$

5. Classify the following as linear, quadratic and cubic polynomials.
   (i) $x^2 + x$  (ii) $x - x^3$
   (iii) $y + y^2 + 4$  (iv) $1 + x$
   (v) $3t$  (vi) $r^2$
   (vii) $7x^3$

6. Find the value of polynomial $5x - 4x^2 + 3$ at
   (i) $x = 0$  (ii) $x = -1$
   (iii) $x = 2$

7. Find the $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials.
   (i) $p(y) = y^2 - y + 1$  (ii) $p(t) = 2 + t^2 - t^3$
   (iii) $p(x) = x^3$  (iv) $p(x) = (x - 1)(x + 1)$

8. Verify whether the following are zeroes of the polynomial, indicated against them.
   (i) $p(x) = 3x + 1, x = -\frac{1}{3}$
   (ii) $p(x) = 5x - \pi, x = \frac{4}{5}$
   (iii) $p(x) = x^2 - 1, x = 1, -1$
   (iv) $p(x) = (x + 1)(x - 2), x = -1, 2$
   (v) $p(x) = x^2, x = 0$
   (vi) $p(x) = lx + m, x = -$ —
   (vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$
   (viii) $p(x) = 2x + 1, x = \frac{1}{2}$

9. Find the zero of the polynomial in each of the following cases.
   (i) $p(x) = x + 5$  (ii) $p(x) = x - 5$
   (iii) $p(x) = 2x + 5$  (iv) $p(x) = 3x - 2$
   (v) $p(x) = 3x$  (vi) $p(x) = ax, a \neq 0$
   (viii) $p(x) = cx + d, c \neq 0$; $c$ and $d$ are real numbers

10. Determine which of the following polynomial has $(x + 1)$ as a factor?
    (i) $x^3 + x^2 + x + 1$  (ii) $x^4 + x^3 + x^2 + 1$
    (iii) $x^4 + 3x^3 + 3x^2 + x + 1$  (iv) $x^3 - x^2 - (2 + \sqrt{2})x + 2$

11. Use the factor theorem to determine, whether $g(x)$ is a factor of $p(x)$ in each of the following cases.
    (i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$
    (ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$
    (iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

12. Find the value of $k$, if $(x - 1)$ is a factor of $p(x)$ in each of the following cases.
    (i) $p(x) = x^2 + x + k$  (ii) $p(x) = 2x^2 + kx + \sqrt{2}$
    (iii) $p(x) = kx^2 - \sqrt{2}x + 1$  (iv) $p(x) = kx^2 - 3x + k$

13. Factorise the following.
    (i) $12x^2 - 7x + 1$  (ii) $2x^2 + 7x + 3$
    (iii) $6x^2 + 5x - 6$  (iv) $3x^2 - x - 4$

14. Use suitable identities to find the following products.
    (i) $(x + 4)(x + 10)$  (ii) $(x + 8)(x - 10)$
    (iii) $(3x + 4)(3x - 5)$  (iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$
    (v) $(3 - 2x)(3 + 2x)$

15. Evaluate the following products without multiplying directly.
    (i) $103 \times 107$  (ii) $95 \times 96$
    (iii) $104 \times 96$

16. Factorise the following using appropriate identities.
    (i) $9x^2 + 6xy + y^2$  (ii) $4y^2 - 4y + 1$
    (iii) $x^2 - \frac{y^2}{100}$

17. Expand each of the following using suitable identities.
    (i) $(x + 2y + 4z)^2$  (ii) $(2x - y + z)^2$
    (iii) $(-2x + 3y + 2z)^2$  (iv) $(3a - 7b - c)^2$
    (v) $(-2x + 5y - 3z)^2$  (vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

18. Factorise the following.

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

19. Write the following cubes in expanded form.

(i) $(2x + 1)^3$ (ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$ (iv) $\left(x - \frac{2}{3}y\right)^3$

20. Evaluate the following using suitable identities.

(i) $(99)^3$ (ii) $(102)^3$

(iii) $(998)^3$

21. Factorise each of the following.

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

22. Verify that

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

23. Factorise each of the following.

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343n^3$

24. Factorise $27x^3 + y^3 + z^3 - 9xyz$.

25. If $(x + y + z) = 0$, Show that $x^3 + y^3 + z^3 = 3xyz$.

26. Without actually calculating the cubes, find the value of each of the following.

(i) $(-12)^3 + (7)^3 + (5)^3$ (ii) $(28)^3 + (-15)^3 + (-13)^3$

27. Give possible expressions for the length and the breadth of each of the following rectangles, in which their areas are given.

(i) Area $= 25a^2 - 35a + 12$ (ii) Area $= 35y^2 + 13y - 12$

28. What are the possible expressions for the dimension of the cuboids whose volume are given below?

(i) Volume $= 3x^2 - 12x$

(ii) Volume $= 12ky^2 + 8ky - 20k$

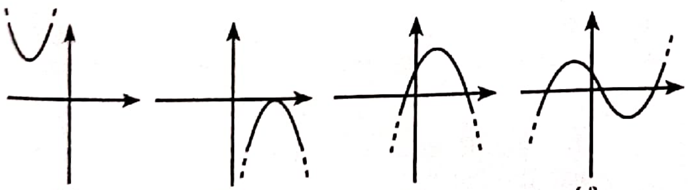
## COMPETENCY QUESTIONS

### (FOR FOUNDATION, NTSE, OLYMPIAD QUESTIONS)

## SECTION A MULTIPLE CHOICE QUESTIONS

**DIRECTIONS:** This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct.

1. Which of the following is not the graph of a quadratic polynomial?



(a) (b) (c) (d)

2. Factors of $(42 - x - x^2)$ are

(a) $(x - 7)(x - 6)$ (b) $(x + 7)(x - 6)$

(c) $(x + 7)(6 - x)$ (d) $(x + 7)(x + 6)$

3. If $4x^4 - 3x^3 - 3x^2 + x - 7$ is divided by $1 - 2x$ then remainder will be

(a) $\frac{57}{8}$ (b) $-\frac{59}{8}$

(c) $\frac{55}{8}$ (d) $-\frac{55}{8}$

4. Factors of $a^2 - b + ab - a$ are

(a) $(a - b)(a + 1)$

(b) $(a + b)(a - 1)$

(c) $(a - b)(a - 1)$

(d) $(a + b)(a + 1)$

5. If $x^2 - x - 42 = (x + k)(x + 6)$ then the value of $k$ is

(a) 6 (b) $-6$

(c) 7 (d) $-7$

6. If one factor of $5 + 8x - 4x^2$ is $(2x + a)$ then the second factor is

(a) $(5 + 2x)$ (b) $(2x - 5)$

(c) $(5 - 2x)$ (d) $-(5 + 2x)$

7. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves remainders $R_1$ and $R_2$, respectively then, value of $a$ if $2R_1 - R_2 = 0$, is

(a) $-\frac{18}{127}$ (b) $\frac{18}{127}$

(c) $\frac{17}{127}$ (d) $-\frac{17}{127}$

8. If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$, then the values of $a$ and $b$ are

(a) 11 and 5 (b) 1 and $-5$

(c) $-1$ and $-5$ (d) $-11$ and 5

9. If $f\left(\frac{-3}{4}\right) = 0$; then for $f(x)$, which of the following is a factor?

(a) $3x - 4$ (b) $4x + 3$

(c) $-3x + 4$ (d) $4x - 3$

10. If $(x - 1)$, $(x + 1)$ and $(x - 2)$ are factors of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of $p$ is

(a) 1 (b) 2

(c) 3 (d) 4

11. If $a$, $b$ are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $a + 3$ and $b + 3$ is

(a) $2x^2 - 11x + 30 = 0$ (b) $-x^2 + 11x = 0$

(c) $x^2 - 11x + 30 = 0$ (d) $x^2 - 22x + 60 = 0$

12. Zeroes of polynomial $p(x) = x^2 - 3x + 2$ are

(a) 3 (b) 1

(c) 4 (d) –1

## SECTION B MATCHING BASED MCQ

**DIRECTIONS (Qs.15 to 17):** Match Column-I with Column-II and select the correct answer using the codes given below the columns.

13.

| Column-I (Polynomials) | | Column-II (Zeroes) | |
|---|---|---|---|
| (A) | $4 - x^2$ | (p) | 7 |
| (B) | $x^3 - 2x^2$ | (q) | –2 |
| (C) | $6x^2 - 3 - 7x$ | (r) | 2 |
| (D) | $-x + 7$ | (s) | 3/2 |

(a) (A) - (p, r); (B) - (r); (C) - (p); (D) - (s)

(b) (A) - (q, r); (B) - (r); (C) - (s); (D) - (p)

(c) (A) - (r, s); (B) - (r); (C) - (q); (D) - (s)

(d) (A) - (p, q); (B) - (r); (C) - (s); (D) - (q)

14.

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | The zeroes of the polynomial $x^2 + x - 2$ are | (p) | $\frac{1}{3}, -4$ |
| (B) | The zeroes of the polynomia $2x^2 - 3x - 2$ are | (q) | $\frac{1}{2}, \frac{1}{2}$ |
| (C) | The zeroes of the polynomial $3x^2 + 11x - 4$ are | (r) | $-\frac{1}{2}, 2$ |
| (D) | The zeroes of the polynomial $4x^2 - 4x + 1$ are | (s) | $1, -2$ |

(a) (A) - (s); (B) - (r); (C) - (p); (D) - (q)

(b) (A) - (q); (B) - (p); (C) - (r); (D) - (s)

(c) (A) - (p); (B) - (r); (C) - (p); (D) - (s)

(d) (A) - (r); (B) - (s); (C) - (p); (D) - (q)

15.

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | $a^3 + b^3$ | (p) | $a^2 + b^2 + 2ab$ |
| (B) | $a^3 - b^3$ | (q) | $a^3 - b^3 - 3ab\,(a - b)$ |
| (C) | $(a + b)^3$ | (r) | $(a + b)\,(a^2 - ab + b^2)$ |
| (D) | $(a - b)^3$ | (s) | $a^3 + b^3 + 3ab\,(a + b)$ |
| (E) | $(a + b)^2$ | (t) | $(a - b)\,(a^2 + ab + b^2)$ |
| (F) | $(a + b)^2$ | (u) | $a^2 + b^2 - 2ab$ |

(a) (A) - (t); (B) - (u); (C) - (s); (D) - (q); (E) - (p); (F) - (r)

(b) (A) - (u); (B) - (q); (C) - (s); (D) - (r); (E) - (t); (F) - (p)

(c) (A) - (r); (B) - (t); (C) - (s); (D) - (q); (E) - (p); (F) - (u)

(d) (A) - (p);(B) - (t); (C) - (s); (D) - (q); (E) - (r); (F) - (u)

## SECTION C STATEMENT BASED MCQ

16. Consider the following statements :

(i) $x - 2$ is a factor of $x^3 - 3x^2 + 4x - 4$

(ii) $x + 1$ is a factor of $2x^3 + 4x + 6$

(iii) $x - 1$ is a factor of $x^5 + x^4 - x^3 + x^2 - x + 1$

Which of these statements given above are correct?

(a) Both (i) and (ii) (b) Both(ii)and (iii)

(c) Both (i) and (iii) (d) Neither (i) nor (ii)

## SECTION D PASSAGE BASED MCQ

**DIRECTIONS (Qs. 17 to 19):** Read the passage(s) given below and answer the questions that follow.

**Passage - 1**

**Let the polynomial be $f(x) = 2x^3 - 9x^2 + x + 12$**

17. The degree of the given polynomial is

(a) 2 (b) 3

(c) 0 (d) 1

18. Zeroes of the given polynomial is

(a) (1, 3/2) (b) (–1, –3/2)

(c) (–1, 3/2) (d) (1, 3/2)

19. If $f(x)$ is divided by $\left(x - \frac{3}{2}\right)$, then the remainder is

(a) 1 (b) $\frac{3}{2}$

(c) 0 (d) none of these

## SECTION E ASSERTION REASON BASED MCQ

**DIRECTIONS (Qs. 20 to 22):** Following questions consist of two statements, one labelled as the 'Assertion' and the other as 'Reason'. You are to examine these two statements carefully and select the answer to these items using the code given below.

**Codes:**

(a) Both A and R are individually true and R is the correct explanation of A:

(b) Both A and R are individually true but R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true.

20. **Assertion:** Zeroes of $f(x) = x^2 - 4x - 5$ are 5, –1.

**Reason:** The polynomial whose zeroes are $2 + \sqrt{3}, 2 - \sqrt{3}$ is $x^2 - 4x + 7$.

21. **Assertion:** The polynomial $x^4 + 4x^2 + 5$ has four zeroes.

**Reason:** If $p(x)$ is divided by $(x - k)$, then the remainder $= p(k)$.

22. **Assertion:** Degree of a zero polynomial is not defined.

**Reason:** Degree of a non-zero constant polynomial is '0'.

□□