CHAPTER 12

CONGRUENT TRIANGLES

REVISION OF KEY CONCEPTS AND FORMULAE

1. Two figures are congruent, if they are of the same shape and of the same size.
2. Two line segments are congruent iff their lengths are equal.
3. Two angles are congruent iff they are of the same measure.
4. Two circles of the same radii are congruent.
5. Two squares of the same size are congruent.
6. If two triangles ABC are DEF are congruent under the correspondence $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, then we write $\Delta ABC \cong \Delta DEF$ or $\Delta ABC \leftrightarrow \Delta DEF$.
7. Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angle of the other triangle (SAS congruence criterion).
8. Two triangles are congruent if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle. (ASA congruence criterion).
9. If any two angles and non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the triangles are congruent (AAS congruence criterion).
10. If three sides of one triangle are equal to three of the other triangle, then the two triangles are congruent (SSS congruence criterion).
11. If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent (RHS congruence criterion).
12. Angles opposite to equal sides of a triangle are equal.
13. Sides opposite to equal angles of a triangle are equal.
14. Each angle of an equilateral triangle is of 60°.
15. If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles.
16. In an isosceles triangle altitude from the vertex bisects the base.
17. If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is isosceles.
18. If the altitudes of a triangles are equal, then it is equilateral.
19. In a triangle, angle opposite to the longer side is larger.
20. In a triangle, side opposite to the larger angle is longer.
21. Sum of any two sides of a triangle is greater than the third side.
22. In a right triangle, the hypotenuse is the longest side.
23. If D is any point on side BC of a ΔABC, then $AB + BC + CA > 2\,AD$.
24. The sum of the altitudes of a triangle is less than the perimeter of the triangle.

25. The difference of any two sides of a triangle is less than the third side of the triangle.
26. The sum of two sides of a triangle is greater than twice the median drawn to the third side.
27. The perimeter of a triangle is greater than the sum of its three medians
28. Of all line segments that can be drawn to a given line, from a point, not lying on it, the perpendicular line segment is the shortest.

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *Which of the following is not a criterion for congruence of triangles?*

(a) SAS (b) ASA (c) SSA (d) SSS

Ans. (c) [NCERT EXEMPLAR]

SOLUTION We have, SSS, SAS, ASA, AAS and RHS as criteria for congruence of triangles. Hence, SSA is not a criterion for congruence of triangles.

EXAMPLE 2 *In two triangles ABC and PQR, if* $AB = QR$, $BC = RP$ *and* $CA = PQ$, *then*

(a) $\Delta ABC \cong \Delta PQR$ (b) $\Delta CBA \cong \Delta PRQ$ (c) $\Delta BAC \cong \Delta RPQ$ (d) $\Delta PQR \cong \Delta BCA$

Ans. (b) [NCERT EXEMPLAR]

SOLUTION We have, $AB = QR, BC = RP$ and $CA = PQ$

$\Rightarrow$ $A \leftrightarrow Q, B \leftrightarrow R$ and $C \leftrightarrow P$

$\Rightarrow$ $\Delta ABC \cong \Delta QRP, \Delta CBA \cong \Delta PRQ, \Delta BAC \cong \Delta RQP$ and $\Delta BCA \cong \Delta RPQ$

Clearly, option (b) is correct.

EXAMPLE 3 *In Fig. 12,1, two triangles ABC and PQR shown. Which congruence criterion can be used to show that the triangles are congruent?*

(a) SAS (b) SSS (c) ASA (d) AAA

Ans. (c)

SOLUTION In triangles ABC and PQR, we find that

$\Rightarrow$ $\angle A = \angle R, AC = RQ$ and $\angle C = \angle Q$

Therefore, by SAS congruence criterion, we obtain $\Delta ABC \cong \Delta RPQ$.

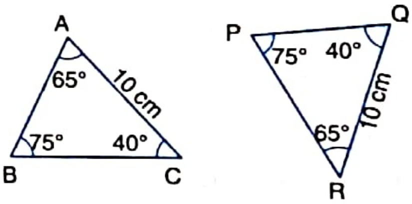
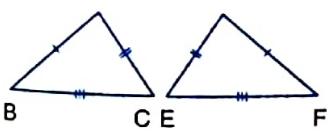
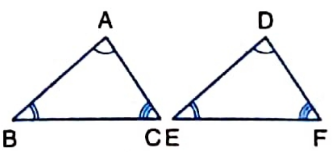
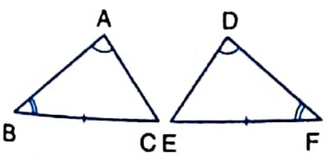


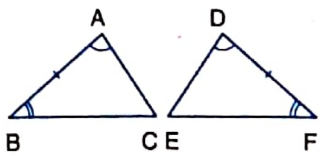
Fig. 12.1

EXAMPLE 4 *Which one of the following pairs of triangles is not a congruent pair?*

(a)


(b)


(c)


(d)


Ans. (b)

SOLUTION In option (a), we find that in triangles ABC and DEF, we have

$AB = DF, AC = DE$ and $BC = EF$

So, by using SSA criterion of congruence, we obtain $\Delta ABC \cong \Delta DFE$

In option (c), we find that in triangles ABC and DEF, we have

$$\angle A = \angle D, \angle B = \angle F \text{ and } BC = FE$$

So, by using AAS criterion of congruence, we obtain $\Delta ABC \cong \Delta DFE$.

In option (d), we observe that $\angle A = \angle D, \angle B = \angle F$ and $AB = DF$. So, by SAS congruence criterion, we obtain $\Delta ABC \cong \Delta DFE$.

In option (b) we observe that $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$. But, ΔABC need not be congruent to ΔDEF as the corresponding sides of two triangles may not be same.

EXAMPLE 5 *If $\Delta PQR \cong \Delta ACB$, then AB is equal to*

(a) QR (b) PR (c) PQ (d) *none of these*

Ans. (b)

SOLUTION We have, $\Delta PQR \cong \Delta ACB$

$\Rightarrow$ $PQ = AC, QR = CB$ and $PR = AB$

EXAMPLE 6 *By which congruence criterion the following triangles are congruent?*

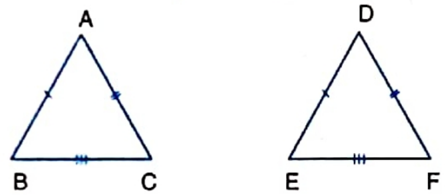


Fig. 12.2

(a) SAS (b) ASS (c) AAS (d) SSS

Ans. (d)

SOLUTION In triangles ABC and DEF, we find that $AB = DE, BC = EF$ and $AC = DF$. Hence by SSS congruence criterion, we obtain $\Delta ABC \cong \Delta DEF$.

EXAMPLE 7 *In triangles ABC and PQR, $AB = PQ$ and $\angle B = \angle Q$. The two triangles are congruent by SAS criterion, if*

(a) $AC = PR$ (b) $BC = PQ$ (c) $AC = QR$ (d) $BC = QR$

Ans. (d)

SOLUTION In triangles ABC and PQR, we have $AB = PQ$ and $\angle B = \angle Q$. Therefore, two triangles will be congruent by SAS criterion, if $BC = QR$.

EXAMPLE 8 *In Fig, 12.3, PQRS is a parallelogram. Can it be concluded that $\Delta RPS \cong \Delta QSP$? Why or why not?*

(a) *Yes, because $PS = PS$, $\angle SPR = \angle PSQ$, $\angle RSP = \angle SPQ$*

(b) *Yes because $PQ = RS, PS = QR, \angle SPR = \angle RPQ, \angle SRP = \angle PRQ, \angle PSR = \angle PQR$ and $PR = PR$*

(c) *No, because angle measures are not equal*

(d) *No, because side lengths are not equal.*

Ans. (a)

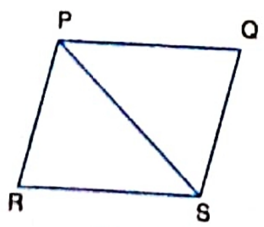


Fig. 12.3

SOLUTION In triangles RPS and QSP, we find that

$PS = PS, \angle SPR = \angle PSQ, \angle RSP = \angle SPQ$ [$\because$ $PQ \parallel RS$ and PS is a transversal]

So, by SAS congruence criterion, we obtain $\Delta RPS \cong \Delta QSP$.

EXAMPLE 9 *In Fig, 12.4, by which criterion triangles OAC and OBD are congruent?*

(a) SAS (b) ASA (c) AAS (d) SSS

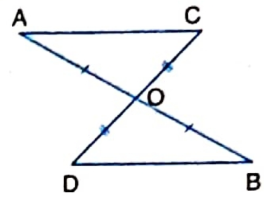


Fig. 12.4

Ans. (a)

SOLUTION In triangles OAC and OBD, we find that

$OA = OB$, $\angle AOC = \angle BOD$ and $OC = OD$

So, by using SAS congruence criterion, we obtain $\Delta OAC \cong \Delta OBD$.

EXAMPLE 10 *In a* ΔABC, *if* $AB = AC$ *and* $\angle B = 50°$, *then* $\angle A =$

(a) 40° (b) 50° (c) 80° (d) 130°

Ans. (c)

SOLUTION We have,

$AB = AC \Rightarrow \angle C = \angle B \Rightarrow \angle C = 50°$ [Angles opposite to equal sides are equal]

Thus, we obtain $\angle B = \angle C = 50°$.

$\therefore$ $\angle A + \angle B + \angle C = 180° \Rightarrow \angle A + 50° + 50° = 180° \Rightarrow \angle A = 80°$

EXAMPLE 11 *In a* ΔABC, $BC = AB$ *and* $\angle B = 80°$, *then* $\angle A =$

(a) 80° (b) 50° (c) 40° (d) 100°

Ans. (b)

SOLUTION We have, $BC = AB$ and $\angle B = 80°$

$\therefore$ $\angle A = \angle C$ and $\angle B = 80°$ [Angles opposite to equal sides are equal]

Now $\angle A + \angle B + \angle C = 180° \Rightarrow \angle A + 80° + \angle A = 180° \Rightarrow 2\angle A = 100 \Rightarrow \angle A = 50°$

EXAMPLE 12 *In a* ΔPQR, *if* $\angle P = \angle R$, $PR = 5$ cm *and* $QR = 4$ cm, *then* $PQ =$

(a) 4 cm (b) 5 cm (c) 2 cm (d) 2.5 cm

Ans. (a)

SOLUTION In ΔPQR, it is given that

$\angle R = \angle P \Rightarrow PQ = QR \Rightarrow PQ = 4$ cm [Sides opposite to equal angles]

EXAMPLE 13 *In triangles ABC and PQR, if* $AB = AC$, $\angle C = \angle P$ *and* $\angle B = \angle Q$. *Then, the two triangles are*

(a) isosceles but not congruent (b) isosceles and congruent

(c) congruent but not isosceles (d) neither congruent nor isosceles

Ans. (a)

SOLUTION In ΔABC, we have

$AB = AC \Rightarrow \angle C = \angle B \Rightarrow \angle P = \angle Q \Rightarrow QR = PR$

Thus, ΔPQR is isosceles.

$\Delta ABC \cong \Delta RQP$ only if $BC = QP$, otherwise not.

Hence, two triangles are isosceles but not congruent.

EXAMPLE 14 *In Fig. 12.5, if $AB = FC, EF = BD$ and $\angle AFE = \angle CBD$. Then, the rule by which $\Delta AFE \cong \Delta CBD$ is*

(a) SAS (b) ASA (c) SSS (d) AAS

Ans. (a)

SOLUTION We have,

$$AB = FC \Rightarrow AB + BF = BF + FC \Rightarrow AF = BC$$

Thus, in triangles AFE and CBD, we obtain

$$AF = BC$$
$$\angle AFE = \angle CBD$$

and, $FE = BD$

So, by SAS congruence criterion, we obtain $\Delta AFE \cong \Delta CBD$.

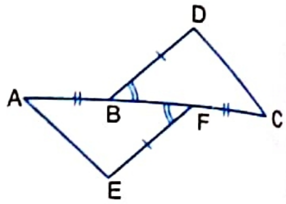


Fig. 12.5

EXAMPLE 15 *In Fig. 12.6, if $AB = AC$ and $BD = CD$, then $\angle ABD : \angle ACD =$*

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

Ans. (a)

SOLUTION In triangles ABC and DBC, we have

$AB = AC$ and $DB = DC$

$\Rightarrow$ $\angle C = \angle B$ and $\angle DCB = \angle DBC$

$\Rightarrow$ $\angle C - \angle DCB = \angle B - \angle DBC$

$\Rightarrow$ $\angle ABD = \angle ACD \Rightarrow \angle ABD : \angle ACD = 1:1$

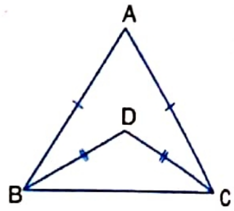


Fig. 12.6

EXAMPLE 16 *In Fig. 12.7, if $\Delta ABC \cong \Delta ADC, \angle BAC = 30°$ and $\angle ABC = 100°$, then $\angle ACD =$*

(a) 30° (b) 80° (c) 50° (d) 70°

Ans. (c)

SOLUTION In ΔABC, we have

$\angle BAC = 30°$ and $\angle ABC = 100°$

$\therefore$ $\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow$ $30° + 100° + \angle ACB = 180° \Rightarrow \angle ACB = 50°$

Now, $\Delta ABC \cong \Delta ADC \Rightarrow \angle ACD = \angle ACB \Rightarrow \angle ACD = 50°$

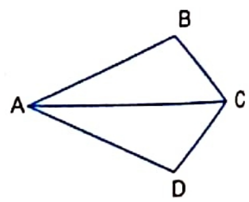


Fig. 12.7

EXAMPLE 17 *In Fig. 12.8, ABC is an isosceles triangle with $AB = AC$ and LM is parallel to BC. If $\angle A = 50°$, then $\angle LMC =$*

(a) 65° (b) 115° (c) 130° (d) 50°

Ans. (b)

SOLUTION In ΔABC, we have

$AB = AC \Rightarrow \angle C = \angle B$

$\therefore$ $\angle A + \angle B + \angle C = 180° \Rightarrow 50° + \angle B + \angle B = 180° \Rightarrow 2\angle B = 130° \Rightarrow \angle B = 65°$

Thus, we have $\angle B = \angle C = 65°$.

It is given that $LM \parallel BC$.

$\therefore$ $\angle ALM = \angle B$ and $\angle AML = C \Rightarrow \angle ALM = 65° = \angle AML$

$\therefore$ $\angle LMC = 180° - \angle AML = 180° - 65 = 115°$

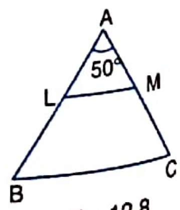


Fig. 12.8

EXAMPLE 18 *In Fig. 12.9, ΔABC is an isosceles triangle with $AB = AC$. If AD is the bisector of $\angle A$, which one of the following options has words that correctly complete the following statement? By congruence criterion, $\Delta ABD \cong \Delta ACD$ and using c.p.c.t, we obtain $\angle ABC = ...$*

(a) SAS; $\angle ACD$ (b) SAS; $\angle ADC$ (c) ASA; $\angle ACD$ (d) ASA; $\angle ADC$

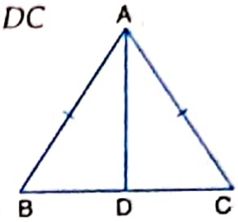


Fig. 12.9

Ans. (a)

SOLUTION In Δ's ABD and ACD, we have,

$$AB = AC, \angle BAD = \angle CAD \text{ and } AD = AD$$

So, by SAS congruence criterion, we obtain

$$\angle ABD \cong \Delta ACD \Rightarrow \angle ABD = \angle ACD \Rightarrow \angle ABC = \angle ACD.$$

Hence, option (a) is correct.

EXAMPLE 19 *In Fig. 12.9, if ΔABC is an isosceles triangle with $AB = AC$ and $BD = CD$, then the congruence criterion by which $\Delta ADB \cong \Delta ADC$ is*

(a) AAS (b) RHS (c) ASA (d) SSS

Ans. (b)

SOLUTION In ΔABC, we have, $AB = AC$ and D is the mid-point of BC. Therefore, $AD \perp BC$.

Thus, in ΔABD and ΔACD, we have

$$AB = AC, BD = CD \text{ and } \angle ADB = \angle ADC = 90°$$

So, by RHS congruence criterion, we obtain $\Delta ADB \cong \Delta ADC$.

EXAMPLE 20 *In Fig. 12.10, ABCD is a quadrilateral in which BN and DM are perpendiculars drawn to AC such that BN = DM. If OB = 4 cm, then BD =*

(a) 6 cm (b) 8 cm (c) 10 cm (d) 12 cm

Ans. (b)

SOLUTION In triangles ONB and OMD, we have

$\angle ONB = \angle OMD$ [Each equal to 90°]

$\angle BON = \angle DOM$ [Vertically opposite angles]

and, $BN = DM$

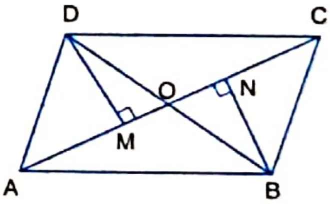


Fig. 12.10

So, by using AAS congruence criterion, we obtain

$$\Delta ONB \cong \Delta OMD \Rightarrow OB = OD \Rightarrow OD = OB = 4 \text{ cm}$$

$\therefore$ $BD = OB + OD = 4 \text{ cm} + 4 \text{ cm} = 8 \text{ cm}$

EXAMPLE 21 *In Fig, 12.11, ABCD is a quadrilateral in which AD = CB and AB = CD, then $\angle ACB$ is equal to*

(a) $\angle ACD$ (b) $\angle BAC$ (c) $\angle CAD$ (d) $\angle BAD$

Ans. (c)

SOLUTION In triangles ACD and CAB, we have

$AD = CB$ [Given]

$AB = CD$ [Given]

and, $AC = AC$ [Common]

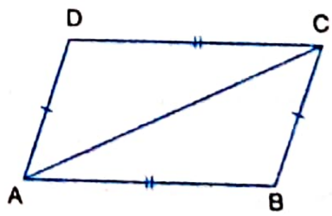


Fig. 12.11

So, by SSS congruence criterion, we obtain

$\Delta ACD \cong \Delta CAB \Rightarrow \angle ACB = \angle CAD$ [c.p.c.t.]

EXAMPLE 22 *In Fig. 12.12, if $AC = BD$ and $\angle CAB = \angle DBA$, then $\angle ACB =$*

(a) $\angle BAD$ (b) $\angle ABC$ (c) $\angle ABD$ (d) $\angle BDA$

Ans. (d)

SOLUTION In triangles ABC and ABD, we have

$AB = AB$ [Common]

$\angle CAB = \angle DBA$ [Given]

and, $AC = BD$ [Given]

So, by SAS congruence criterion, we obtain

$\Delta ABC \cong \Delta BAD \Rightarrow \angle ACB = \angle BDA$

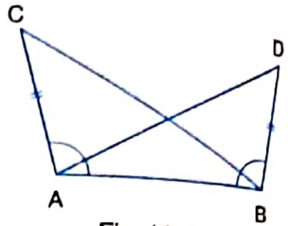


Fig. 12.12

CASE STUDY BASED

EXAMPLE 23 *Engineers often use the familiar triangular shape for strength in bridge design. Triangles are effective tools for architecture and are used in the design of bridges, buildings and other structures as they provide strength and stability. The triangle is common in all sorts of building supports and trusses. Following are some questions on triangles:*

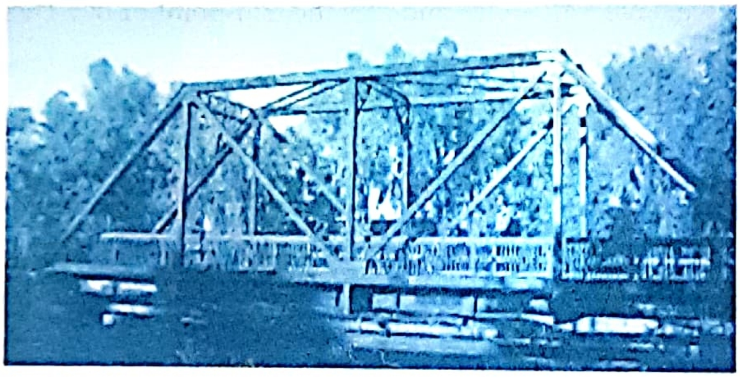


Fig. 12.13

(i) *In triangles ABC and DEF, if $AB = DE$, $AC = EF$ and $\angle A = \angle E$. Then,*

(a) $\Delta ABC \cong \Delta DEF$ by SAS criterion (b) $\Delta ABC \cong \Delta EFD$ by SSS criterion

(c) $\Delta ABC \cong \Delta EDF$ by SAS criterion (d) $\Delta ABC \cong \Delta EDF$ by ASA criterion

(ii) *If $\Delta PRQ \cong \Delta DEF$, then $DE =$*

(a) PR (b) RQ (c) PQ (d) DF

(iii) *Is it possible to construct a triangle with lengths of sides as 5 cm, 4 cm and 8 cm?*

(iv) *In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. Then the two triangles will be congruent by SAS axiom, if*

(a) $BC = EF$ (b) $AC = DE$ (c) $AC = EF$ (d) $BC = DE$

(v) *In ΔPQR, if $\angle R > \angle Q$, then*

(a) $QR > PR$ (b) $PQ > PR$ (c) $PQ < PR$ (d) $CR < PR$

SOLUTION (i) Ans. (c): In Δ's ABC and DEF, we have

$AB = DE, \angle A = \angle F$ and $AC = EF$

So, by SAS congruence criterion $\Delta ABC \cong \Delta EDF$.

(ii) Ans. (a): $\Delta PRQ \cong \Delta DEF \Rightarrow PR = DE$ [c.p.c.t.]

(iii) Ans. No, sum of any two sides must be greater than the third side.

(iv) Ans. (b): $\Delta ABC \cong \Delta FDE$ by SAS criterion

$\therefore$ $AC = DE$

(v) Ans. (b): Since side opposite to greater angle is greater.

$\therefore$ $\angle R > \angle Q \Rightarrow PQ > PR.$

EXAMPLE 24 *A ladder manufacturing company manufactures foldable step ladders of aluminum as shown in Fig. 12.14. The lengths of two legs AB and AC are both equal to 110 cm and the angle between the two legs is 30°. On the basis of the above information answer the following questions:*

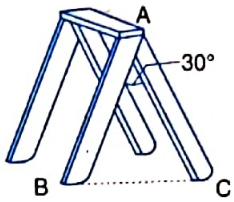


Fig. 12.14

(i) $\angle ABC$ *is equal to*

(a) 70° (b) 75° (c) 85° (d) 60°

(ii) *If* $\angle BAC = 60°$, *then* $BC =$

(a) 120 cm (b) 55 cm (c) 110 cm (d) 100 cm

(iii) $\Delta\, ABC$ *is*

(a) isosceles acute angled (b) right angled isosceles

(c) isosceles obtuse angled (d) equilateral

(iv) *In two triangles ABC and DEF, if* $\angle A = \angle D$, $AB = DE$ *and* $AC = DF$, *then the criterion by which two triangles are congruent is*

(a) SSS (b) ASA (c) AAS (d) SAS

(v) *In a* $\Delta\, ABC$, *if* $AB = AC$ *and* $\angle B = 85°$, *then* $\angle C$ *is equal to*

(a) 25° (b) 75° (c) 85° (d) 35°

SOLUTION (i) Ans. (b): We have, $AB = AC$. So, $\Delta\, ABC$ is isosceles.

$\therefore$ $\angle ABC = \angle ACB$

Now, $\angle BAC + \angle ABC + \angle ACB = 180° \Rightarrow 30° + 2\angle ABC = 180° \Rightarrow \angle ABC = 75°$

(ii) Ans. (c): If $\angle BAC = 60°$, then

$AB = AC \Rightarrow \angle ACB = \angle ABC$

$\therefore$ $\angle ABC + \angle ACB + \angle BAC = 180° \Rightarrow \angle ABC + \angle ABC + 60° = 180° \Rightarrow 2\angle ABC = 120°$

$\Rightarrow$ $\angle ABC = 60°$

Thus, we have, $\angle ABC = \angle ACB = \angle BAC = 60°$. So, $\Delta\, ABC$ is equilateral.

Hence, $AB = BC = AC \Rightarrow BC = 110$ cm

(iii) Ans. (a): Since $AB = AC$, Therefore $\Delta\, ABC$ is isosceles acute angled triangle.

(iv) Ans. (d): We have, $\angle A = \angle D$, $AB = DE$ and $AC = DF$. So, by SAS criterion of congruence, $\Delta\, ABC \cong \Delta\, DEF$.

(v) Ans. (c): Given that $\Delta\, ABC$ is isosceles with $AB = AC$.

$\therefore$ $\angle B = \angle C \Rightarrow \angle C = 85°$ $[\because \angle B = 85°]$

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

EXAMPLE 25 *Statement-1 (Assertion): If M is a point on side BC of a triangle ABC such that AM is the bisector of $\angle BAC$, then perimeter of ΔABC is greater than 2 AM.*

Statement-2 (Reason): The sum of any two sides of a triangle is greater than the third side.

Ans. (a)

SOLUTION Statement-2 is true. Using statement-2, in the triangles ABM and ACM we obtain

$$AB + BM > AM \text{ and } AC + MC > AM$$

$\Rightarrow$ $AB + BM + AC + MC > AM + AM$

$\Rightarrow$ $AB + AC + (BM + MC) > 2\,AM$

$\Rightarrow$ $AB + AC + BC > 2\,AM$

$\Rightarrow$ Perimeter of $\Delta ABC > 2\,AM$

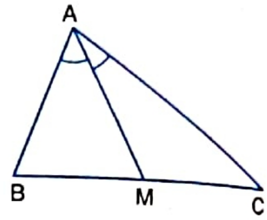


Fig. 12.15

Thus, statement-1 is true. Also, statement-2 is a correct explanation for statement-1.

Hence, option (a) is correct.

EXAMPLE 26 *Statement-1 (Assertion): It is not possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm.*

Statement-2 (Reason): The difference of any two sides of a triangles is less than the third side.

Ans. (b)

SOLUTION Statement-2 is true (See Ex. 16 on page 295 of main book).

If possible let ABC be a triangle such that $AB = 9$ cm, $BC = 7$ cm and $AC = 17$ cm. We find that $AB + BC \not> AC$. So, it is not possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm. Hence, statement-1 is true. Thus, both the statements are true but statement-2 is not a correct explanation for statement-1. Hence, option (b) is correct.

EXAMPLE 27 *Statement-1 (Assertion): It is possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm.*

Statement-2 (Reason): The sum of any two sides of a triangle is greater than the third side.

Ans. (a)

SOLUTION Statement-2 is true. If possible, let ABC be a triangle such that $AB = 8$ cm, $BC = 7$ cm and $AC = 4$ cm. We find that these lengths satisfy $AB + BC > AC$, $BC + CA > AB$ and $AC + AB > BC$. Hence, it is possible to construct a triangle with lengths of its sides as 8 cm, 7 cm, and 4 cm. So, statement-1 is true. Also, statement-2 is a correct explanation for statement-1.

Hence, option (a) is correct.

EXAMPLE 28 *Statement-1 (Assertion): In a ΔPQR, if $\angle P = 70°$ and $\angle R = 30°$, then PR is the longest side of ΔPQR.*

Statement-2 (Reason): In a triangle, side opposite to the greater angle is greater.

Ans. (a)

SOLUTION Clearly, statement-2 is true (see Theorem 2 on page 290 of the main book). In ΔPQR, we have $\angle P = 70°$ and $\angle R = 30°$.

But, $\angle P + \angle Q + \angle R = 180° \Rightarrow 70 + \angle Q + 30° = 180° \Rightarrow \angle Q = 180° - 100° = 80°$

Thus, we have,

$\angle P = 70°, \angle Q = 80°$ and $\angle R = 30°$

$\Rightarrow \quad \angle Q > \angle P > \angle R$

$\Rightarrow \quad PR > QR > PQ$ [$\because$ In a triangle, side opposite to the greater angle is greater]

$\Rightarrow \quad PR$ is the longest side of ΔPQR

Thus, statement-1 is true and statement-2 is a correct explanation for statement-1.

Hence, option (a) is correct.

EXAMPLE 29 *Statement-1 (Assertion): In a ΔABC, if $\angle A = 65°$ and $\angle C = 30°$, then AC is the longest side of ΔABC.*

Statement-2 (Reason): Sum of the angles of a triangle is 180°.

Ans. (b)

SOLUTION We have, $\angle A = 65°$ and $\angle C = 30°$.

$\therefore \quad \angle A + \angle B + \angle C = 180° \Rightarrow 65° + \angle B + \angle 30° = 180° \Rightarrow \angle B = 85°$

Thus, $\angle B > \angle A > \angle C$

$\Rightarrow \quad AC > BC > AB$ [In a triangle, side opposite to greater angle is greater]

$\Rightarrow \quad AC$ is the longest side of ΔABC

Thus, statement-1 is true.

Statement-2 is also true, but is not a correct explanation for statement-1. Hence, option (b) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. If $\Delta ABC \cong \Delta LKM$, then side of ΔLKM equal to side AC of ΔABC is
   (a) LK (b) KM (c) LM (d) None of these
2. If $\Delta ABC \cong \Delta ACB$, then ΔABC is isosceles with
   (a) $AB = AC$ (b) $AB = BC$ (c) $AC = BC$ (d) None of these
3. If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ, then which of the following is not true:
   (a) $BC = PQ$ (b) $AC = PR$ (c) $AB = PQ$ (d) $QR = BC$
4. In triangles ABC and PQR three equality relations between some parts are as follows:
   $$AB = QP, \angle B = \angle P \text{ and } BC = PR$$
   State which of the congruence conditions applies:
   (a) SAS (b) ASA (c) SSS (d) RHS
5. In triangles ABC and PQR, if $\angle A = \angle R$, $\angle B = \angle P$ and $AB = RP$, then which one of the following congruence conditions applies:
   (a) SAS (b) ASA (c) SSS (d) RHS

6. If $\Delta PQR \cong \Delta EFD$, then $ED =$

(a) PQ (b) QR (c) PR (d) None of these

7. If $\Delta PQR \cong \Delta EFD$, then $\angle E =$

(a) $\angle P$ (b) $\angle Q$ (c) $\angle R$ (d) None of these

8. In a ΔABC, if $AB = AC$ and BC is produced to D such that $\angle ACD = 100°$ then $\angle A =$

(a) 20° (b) 40° (c) 60° (d) 80°

9. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is

(a) 100° (b) 120° (c) 110° (d) 130°

10. Which of the following is not a criterion for congruence of triangles? [NCERT EXEMPLAR]

(a) SAS (b) SSA (c) ASA (d) SSS

11. If ABC and DEF are two triangles such that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$. Then, which of the following is true? [NCERT EXEMPLAR]

(a) $DF = 5$ cm, $\angle F = 60°$ (b) $DE = 5$ cm, $\angle E = 60°$

(c) $DF = 5$ cm, $\angle E = 60°$ (d) $DE = 5$ cm, $\angle D = 40°$

12. In Fig. 12.16, the measure of $\angle B'A'C'$ is

(a) 50° (b) 60° (c) 70° (d) 80°

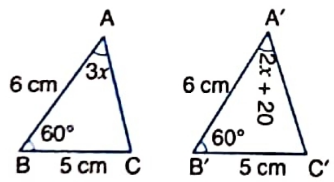


Fig. 12.16

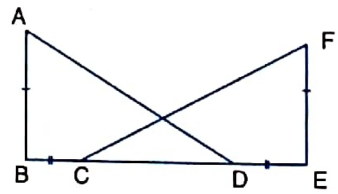


Fig. 12.17

13. In Fig. 12.17, $AB \perp BE$ and $FE \perp BE$. If $BC = DE$ and $AB = EF$, then ΔABD is congruent to

(a) ΔEFC (b) ΔECF (c) ΔCEF (d) ΔFEC

14. In Fig. 12.18, if $AE \parallel DC$ and $AB = AC$, the value of $\angle ABD$ is

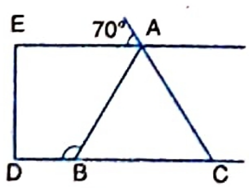


Fig. 12.18

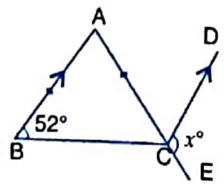


Fig. 12.19

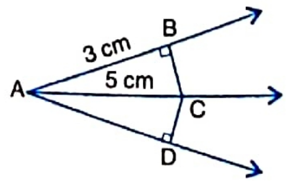


Fig. 12.20

(a) 70° (b) 110° (c) 120° (d) 130°

15. In Fig. 12.19, ABC is an isosceles triangle whose side AC is produced to E. Through C, CD is drawn parallel to BA. The value of x, is

(a) 52° (b) 76° (c) 156° (d) 104°

16. In Fig. 12.20, if AC is bisector of $\angle BAD$ such that $AB = 3$ cm and $AC = 5$ cm, then $CD =$

(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

17. D, E, F are the mid-point of the sides BC, CA and AB respectively of ΔABC. Then ΔDEF is congruent to triangle

(a) ABC (b) AEF (c) BFD, CDE (d) AFE, BFD, CDE

18. In fig. 12.21, ABC is an isosceles triangle such that $AB = AC$ and AD is the median to base BC. Then, $\angle BAD =$

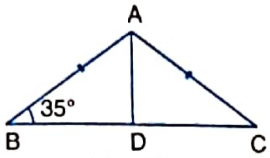


Fig. 12.21

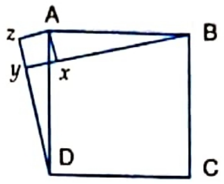


Fig. 12.22

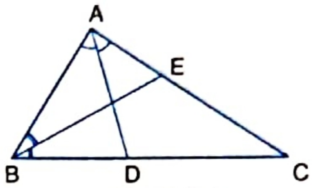


Fig. 12.23

(a) 55° (b) 70° (c) 35° (d) 110°

19. In Fig. 12.22, X is a point in the interior of square $ABCD$. $AXYZ$ is also a square. If $DY = 3$ cm and $AZ = 2$ cm, then $BY =$

(a) 5 cm (b) 6 cm (c) 7 cm (d) 8 cm

20. In Fig. 12.23, ABC is a triangle in which $\angle B = 2\angle C$. D is a point on side BC such that AD bisects $\angle BAC$ and $AB = CD$. BE is the bisector of $\angle B$. The measure of $\angle BAC$ is

(a) 72° (b) 73° (c) 74° (d) 95°

[Hint: $\Delta ABE \cong \Delta DCE$ **]**

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

21. Statement-1 (Assertion): If AD is a median of ΔABC, then perimeter of ΔABC is greater than $2AD$.

Statement-2 (Reason): The sum of any two sides of a triangle is greater than the third side.

22. Statement-1 (Assertion): In an equilateral triangle ABC, if AD is the median, then $AB + AC > 2AD$.

Statement-2 (Reason): In a right triangle, hypotenuse is the longest side.

23. Statement-1 (Assertion): The sum of any two sides of a triangle is greater than the third sides.

Statement-2 (Reason): It is possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm.

24. Statement-1 (Assertion): If $\Delta ABC \cong \Delta RPQ$, then $BC = QR$

Statement-2 (Reason): Corresponding parts of two congruent triangle are equal.

25. Statement-1 (Assertion): If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of the another triangle, then the two triangles are concurrent.

Statement-2 (Reason): If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.

26. Statement-1 (Assertion): If two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle, then the two triangles are concurrent

Statement-2 (Reason): If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles are congruent.

ANSWERS

1. (c)	2. (a)	3. (a)	4. (a)	5. (b)	6. (c)	7. (a)
8. (a)	9. (b)	10. (b)	11. (c)	12. (b)	13. (d)	14. (b)
15. (d)	16. (c)	17. (d)	18. (a)	19. (c)	20. (a)	21. (a)
22. (a)	23. (b)	24. (d)	25. (c)	26. (c)		