CHAPTER 11

HEIGHTS AND DISTANCES

REVISION OF KEY CONCEPTS AND FORMULAE

1. The line drawn from the eye of an observer to a point in the object where the person is viewing is called the line of sight.
2. The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.
3. The angle formed by the line of sight with the horizontal when the object is below the horizontal level is called the angle of depression.
4. The height of an object or the distance between distant objects can be determined with the help of trigonometric ratios.

SOLVED EXAMPLES

MULTIPLE CHOICE QUESTIONS (MCQs)

EXAMPLE 1 *Figure 11.1, shows the observation of point C from point A. The angle of depression from A is*

(a) 60° (b) 30° (c) 45° (d) 75°

Ans. (b)

SOLUTION Let θ be the angle of depression of point C from A. In right triangle ABC, we obtain

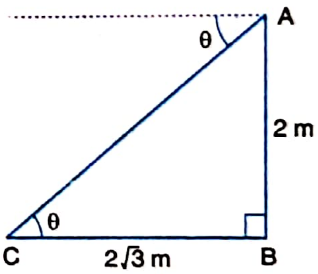

Fig. 11.1

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

Hence, the angle of depression is 30°.

EXAMPLE 2 *The measure of the angle of elevation of the top of the tower $75\sqrt{3}$ m high from a point at a distance of 75 m from the foot of the tower in a horizontal plane is*

(a) 30° (b) 60° (c) 90° (d) 45°

Ans. (b)

SOLUTION Let PQ be a tower of height $75\sqrt{3}$ m and let θ be the angle of elevation of its top Q from a point R at a distance of 75 m from the tower.

In right triangle RPQ, we obtain

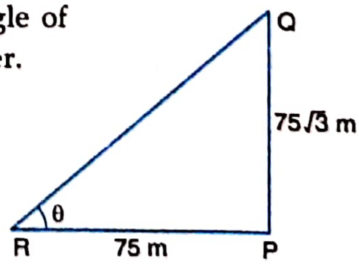

Fig. 11.2

$$\tan\theta = \frac{PQ}{PR} \Rightarrow \tan\theta = \frac{75\sqrt{3}}{75} = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

Hence, the angle of elevation is 60°.

EXAMPLE 3 *If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of Sun's altitude is*

(a) 45° (b) 60° (c) 30° (d) 75°

Ans. (a)

SOLUTION Let AB be a vertical pole of height x m and let AC be the length of its shadow when the angle of elevation of Sun is $\theta°$. It is given that $AC = x$ m. In ΔACB, we obtain

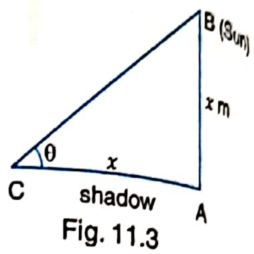

Fig. 11.3

$$\tan\theta = \frac{AB}{AC} = \frac{x}{x} = 1 = \tan 45° \Rightarrow \theta = 45°$$

Hence, Sun's altitude is 45°.

EXAMPLE 4 *If the angle of elevation of the top of a tower from a point on the ground, 100 m away from the foot of the tower is 30°, then the height of the tower is*

(a) 100 m (b) $100\sqrt{3}$ m (c) $\frac{100}{\sqrt{3}}$ m (d) $75\sqrt{3}$ m

Ans. (c)

SOLUTION Let PQ be a tower of height h metre such that the angle of elevation of its top Q at a point R, 100 m away from the foot P of the tower, is 30°.

In right triangle RPQ, we obtain

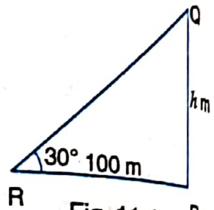

Fig. 11.4

$$\tan 30° = \frac{PQ}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100} \Rightarrow \frac{100}{\sqrt{3}} \text{ m}$$

EXAMPLE 5 *When the Sun's elevation is 30°, the shadow of a tower is 30 m long, if the Sun's elevation is 60°, then the length of the shadow is*

(a) 35 m (b) 20 m (c) 10 m (d) 15 m

Ans. (c)

SOLUTION Let QP be a tower of height h metre and let $QS = 30$ m be the length of the shadow of the tower when Sun's elevation is 30°. Let QR be the shadow when Sun's elevation is 60°.

In triangles PQS and PQR, we obtain

$$\tan 30° = \frac{PQ}{QS} \text{ and } \tan 60° = \frac{PQ}{QR}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30} \text{ and } \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \text{ and } x = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}} \times \frac{30}{\sqrt{3}} = 10 \text{ m}$$

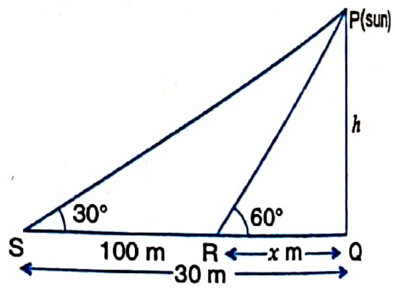

Fig. 11.5

Hence, the length of the shadow is 10 m.

EXAMPLE 6 *If the angles of elevation of the top of a tower from two points at a distance of 4 m and 16 m from the base of a tower and in the same line are complementary, then the height of the tower is*

(a) 20 m (b) 12 m (c) 8 m (d) 16 m

Ans. (c)

SOLUTION Let PQ be a tower of height h metre such that the angles of elevation of its top at two points R and S are complementary. Let θ be the angle of elevation at R. Then, the angle of elevation at S is $90° - \theta$. In right triangles PQR and PQS, we obtain

$$\tan\theta = \frac{PQ}{QR} \text{ and } \tan(90° - \theta) = \frac{PQ}{QS}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{4} \text{ and } \cot\theta = \frac{h}{16}$$

$$\Rightarrow \quad \tan\theta \times \cot\theta = \frac{h}{4} \times \frac{h}{16} \Rightarrow 1 = \frac{h^2}{64} \Rightarrow h = 8 \text{m}.$$

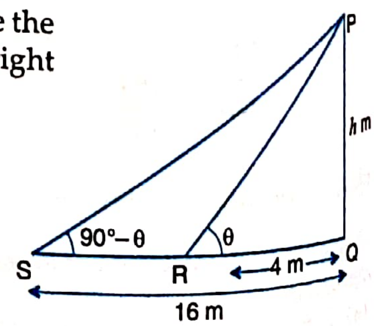

Fig. 11.6

EXAMPLE 7 *A man on the top of a cliff 'x' metres high observes that the angles of elevation of the top of a tower is equal to the angle of depression of the foot of the tower. The height of the tower in metres is*

(a) $2\sqrt{2}x$ (b) $2x$ (c) $\sqrt{2}x$ (d) $\frac{x}{2}$

Ans. (b)

SOLUTION Let AB be a Cliff of height x metres and PQ be a tower of height h metres. Let θ be the angle of elevation of the top Q of the tower observed from the top of the cliff and θ be the angle of depression of the foot of the tower. In right triangles BRQ and PAB, we obtain

$$\tan\theta = \frac{QR}{BR} \text{ and } \tan\theta = \frac{AB}{PA}$$

$$\Rightarrow \tan = \frac{h-x}{AP} \text{ and } \tan\theta = \frac{x}{AP}$$

$$\Rightarrow \frac{h-x}{AP} = \frac{x}{AP} \Rightarrow h-x = x \Rightarrow h = 2x$$

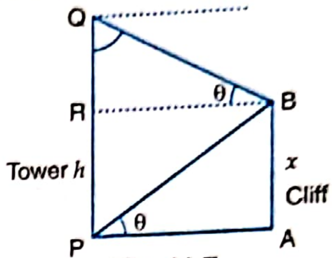

Fig. 11.7

EXAMPLE 8 *If the height of a flagstaff is twice the height of the tower on which it is fixed and the angle of elevation of the top of the tower as seen from a point on the ground is 30°, then the angle of elevation of the top of the flag staff as seen from the same point is*

(a) 45° (b) 30° (c) 60° (d) 90°

Ans. (c)

SOLUTION Let AB be a tower of height h metres and BC be a flagstaff of a height $2h$ metres. The angle of elevation of the top B of the tower as seen from a point P on the ground is 30°. Let the angle of elevation of the top of the flagstaff as seen from point P be θ. In right triangles PAB and PAC, we obtain

$$\tan 30° = \frac{AB}{AP} \text{ and } \tan\theta = \frac{AC}{AP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AP} \text{ and } \tan\theta = \frac{3h}{AP}$$

$$\Rightarrow AP = \sqrt{3}h \text{ and } AP = \frac{3h}{\tan\theta}$$

$$\Rightarrow \sqrt{3}h = \frac{3h}{\tan\theta} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60°$$

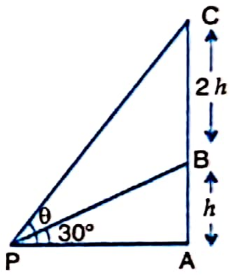

Fig. 11.8

EXAMPLE 9 *Two friends Rohit and Mohit are standing on the opposite sides of a tower of height 60 metres. If their angles of depression seen from the top of the tower are 30° and 45° respectively, then the distance between the two friends is*

(a) $60(\sqrt{3}-1)$ m (b) $60(\sqrt{3}+1)$ m (c) $30(\sqrt{3}-1)$ m (d) $30(\sqrt{3}+1)$ m

Ans. (b)

SOLUTION Let AB be a tower of height 60 metres. In right triangles PAB and QAB, we obtain

$$\tan 30° = \frac{AB}{AP} \text{ and, } \tan 45° = \frac{AB}{AQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60}{AP} \text{ and } 1 = \frac{60}{AQ}$$

$$\Rightarrow AP = 60\sqrt{3} \text{ and } AQ = 60$$

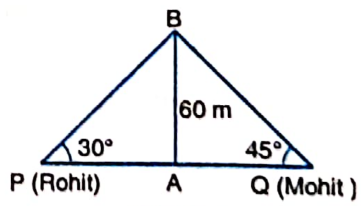

Fig. 11.9

Hence, $PQ = PA + AQ = 60\sqrt{3} + 60 = 60(\sqrt{3}+1)$ m.

EXAMPLE 10 *A man is climbing a ladder which is inclined to the wall at an angle of 30°. If he ascends at the rate of 2 m/sec, then he approaches the wall at the rate of*

(a) 2 m/sec (b) 2.5 m/sec (c) 1 m/sec (d) 1.5 m/sec

Ans. (c)

SOLUTION Let PQ be the wall and QR be the ladder. It is given that the ladder makes an angle of 30° with the wall. Let the man take t seconds to reach to the point Q with the speed of 2m/sec. Then, $PQ = 2t$. In ΔQPR, we obtain

$$\cos 60° = \frac{PR}{QR} \Rightarrow PR = t$$

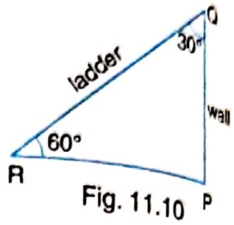

Fig. 11.10

Thus, the man covers distance $PR = t$ metres in t seconds. Hence, the rate at which he approaches the wall is $\frac{t}{t} = 1$ m/sec.

CASE STUDY BASED EXAMPLES

EXAMPLE 11 *A group of students of class X visited India Gate on an educational trip. The teacher as well as students had interest in history behind India Gate. The teacher narrated that India Gate's, official name is Delhi Memorial, Originally called All India War Memorial, a monumental sand stone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Kartavya Path (formerly called the Rajpath), is about 138 feet (42 metres) in height.*

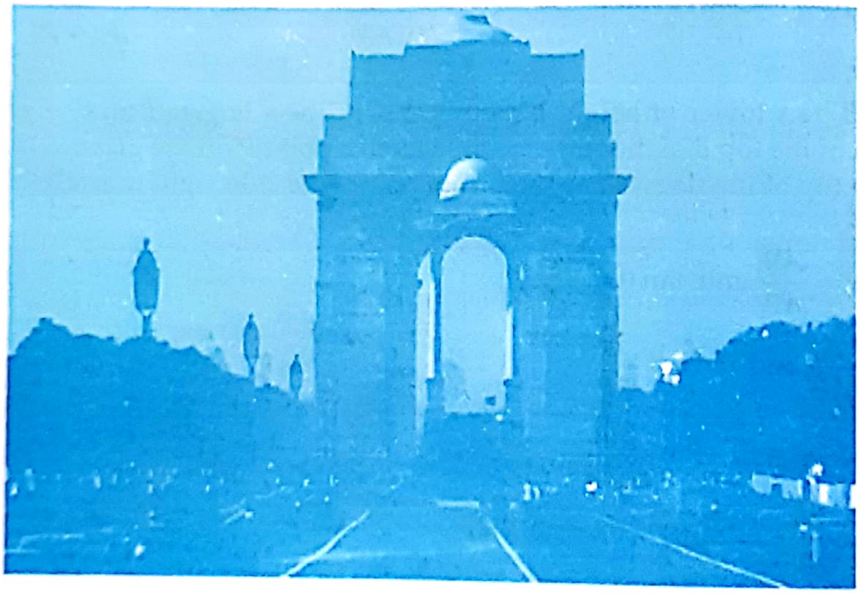

Fig. 11.11

(i) *What is the angle of elevation if they are standing at a distance of 42 metres away from the monument?*
(a) 30° (b) 45° (c) 60° (d) 0°

(ii) *If they want to see the top at an angle of 60°, then the distance where they should stand is*
(a) 24.24 m (b) 20.12 m (c) 24.64 m (d) 25.24 m

(iii) *If the altitude of the Sun is at 60°, then the height of the vertical tower that will cast a shadow of length 20 m, is*
(a) $20\sqrt{3}$ m (b) $\frac{20}{\sqrt{3}}$ m (c) $\frac{15}{\sqrt{3}}$ m (d) $15\sqrt{3}$ m

(iv) *If the ratio of the length of a rod and its shadow is 1 : 1, the angle of elevation of the Sun is*
(a) 30° (b) 45° (c) 60° (d) 90°

(v) *The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal line is*
(a) *corresponding angle* (b) *angle of elevation*
(c) *angle of depression* (d) *complete angle*

SOLUTION (i) **Ans.** (b): Let θ be the angle of elevation of the top of the India Gate. In ΔPQR, we have $PQ = 42$ m and $QR = 42$ m.

$\therefore \quad \tan\theta = \frac{QR}{PQ} = \frac{42}{42} = 1 \Rightarrow \theta = 45°$

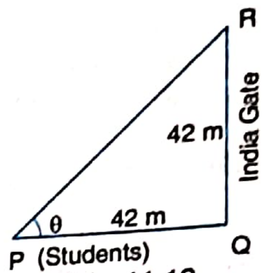

Fig. 11.12

(ii) **Ans.** (a): In this case, $\theta = 60°$, $QR = 42$ m and we have to find PQ. In ΔPQR, we obtain

$$\tan\theta = \frac{QR}{PQ} \Rightarrow \tan 60° = \frac{42}{PQ} \Rightarrow PQ = \frac{42}{\sqrt{3}} = 14\sqrt{3} = 14 \times 1.732 = 24.24 \text{ m}$$

(iii) **Ans.** (a): Let PQ be a tower that casts a shadow of length 20 m when altitude of sun is 60°. In ΔQPR, we obtain

$$\tan 60 = \frac{PQ}{20} \Rightarrow PQ = 20\sqrt{3} \text{ m}$$

(iv) **Ans.** (b): Let PQ be a rod and PR be the length of its shadow. It is given that $PQ : PR : 1 : 1$.

In ΔQPR, we obtain

$$\tan\theta = \frac{PQ}{PR} \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$$

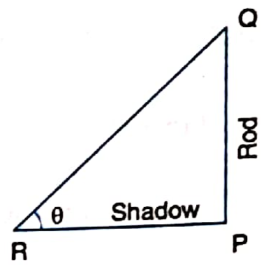

Fig. 11.13

(v) **Ans.** (c): Trivial.

EXAMPLE 12 *A satellite flying at a height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi (height 7,816 m) and Mullayanagri (height 1930 m). The angle of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are 30° and 60° respectively. If the distance between the peaks of two mountains is 1937 km, and the satellite is vertically above the mid-point of the distance between the two mountains. Based on the above information answer the following questions.*

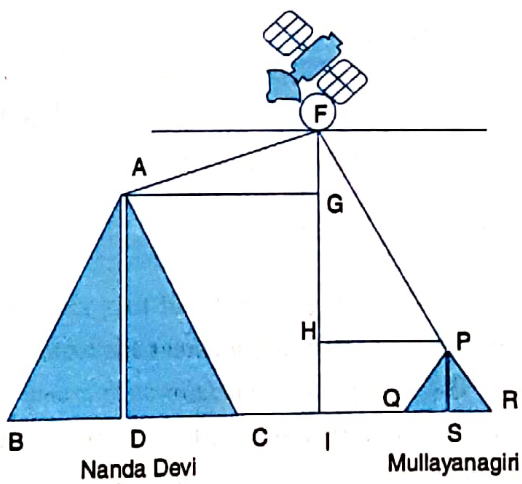

Fig. 11.14

(i) *The distance of the satellite from the top of Nanda Devi is*

(a) 1118.36 km (b) $577\sqrt{2}$ km (c) 1937 km (d) 1025.36 km

(ii) *The distance of the satellite from the top of Mullayanagiri is*

(a) 1139.4 km (b) 577.52 km (c) 1937 km (d) 1025.36 km

(iii) *The distance of the satellite from the ground is*

(a) 1139.4 km (b) 567 km (c) 1937 km (d) 1025.36 km

(iv) *What is the angle of elevation if a man is standing at a distance of 7816 m from Nanda Devi?*

(a) 30° (b) 45° (c) 60° (d) 0°

(v) *If a mile stone very far from, makes 45° to the top of Mullayanagiri mountain. The distance of this mile stone from the mountain is*

(a) 1118.327 m (b) 566.976 m (c) 1930 m (d) 1025.36 m

SOLUTION (i) **Ans.** (a): In ΔAGF, we have: $AG = \frac{1}{2} \times 1937$ km and $\angle GAF = 30°$

$$\therefore \quad \cos 30° = \frac{AG}{AF} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AG}{AF} \Rightarrow AF = \frac{2}{\sqrt{3}} AG = \frac{1937}{\sqrt{3}} \text{ km} = 1118.36 \text{ km}$$

(ii) **Ans.** (c): In ΔPHF, we have: $PH = \frac{1}{2} \times 1937$ km and $\angle HPF = 60°$

$$\therefore \quad \cos 60° = \frac{PH}{PF} \Rightarrow \frac{1}{2} = \frac{PH}{PF} \Rightarrow PF = 2PH = 1937 \text{ km}$$

(iii) **Ans.** (b): In ΔAGF, we have: $AG = \frac{1}{2} \times 1937$ km and $\angle GAF = 30°$

$$\therefore \quad \tan 30° = \frac{GF}{AG} \Rightarrow \frac{1}{\sqrt{3}} = \frac{GF}{AG} \Rightarrow GF = \frac{1}{\sqrt{3}} AG = \frac{1}{\sqrt{3}} \times \frac{1}{2} \times 1937 \text{ km} = 559.18 \text{ km}$$

$$\therefore \quad IF = IG + GF = 7.816 + 559.18 = 566.99 \cong 567 \text{ km}$$

(iv) **Ans.** (b): In ΔADM, we have $AD = 7816$ m and $DM = 7816$ m

$$\therefore \quad \tan\theta = \frac{AD}{DM} = 1 \Rightarrow \theta = 45°$$

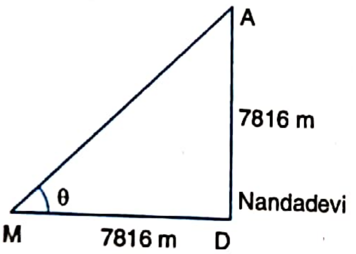

Fig. 11.15

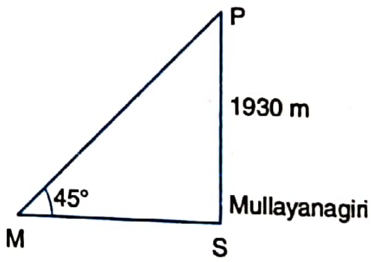

Fig. 11.16

(v) **Ans.** (c): In ΔPSM, we have $PS = 1930$ m and $\angle PMS = 45°$

$$\therefore \quad \tan 45° = \frac{PS}{MS} \Rightarrow MS = PS = 1930 \text{ m}$$

EXAMPLE 13 *Skysails is that genre engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres to 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively. In Fig. 11.17, skysails technology is being used to move a vessel. By observing this figure answer the following questions:*

(i) *The length of the rope of the kite sail in order to pull the vessel at angle of 30° and be at a vertical height of 200 metre, is*

(a) 300 m (b) 400 m (c) 500 m (d) 600 m

(ii) *In (i), the length of BC is*

(a) $400\sqrt{3}$ m (b) $300\sqrt{3}$ m

(c) $200\sqrt{3}$ m (d) $100\sqrt{3}$ m

(iii) *If* $BC = 15$ *metre and* $\theta = 30°$, *then* $AB =$

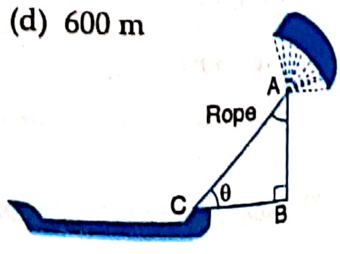

Fig. 11.17

(a) $2\sqrt{3}\,m$ (b) 15 m (c) 24 m (d) $5\sqrt{3}$ m

(iv) *If $AB = BC$, then $\theta =$*

(a) 0° (b) 30° (c) 45° (d) 60°

(v) *If $BC = 6$ m and $\theta = 45°$, then AC is equal to*

(a) $4\sqrt{2}$ m (b) $7\sqrt{3}$ m (c) $9\sqrt{3}$ m (d) $6\sqrt{2}$ m

SOLUTION (i) **Ans.** (b): From Fig. 11.17, we find that $AB = 200$ m and $\theta = 30°$. In right triangle ABC, we obtain

$$\sin\theta = \frac{AB}{AC} \Rightarrow \sin 30° = \frac{200}{AC} \Rightarrow \frac{1}{2} = \frac{200}{AC} \Rightarrow AC = 400 \text{ m}$$

(ii) **Ans.** (c): We have, $AB = 200$ m and $\theta = 30°$. In right triangle ABC, we obtain

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{200}{BC} \Rightarrow BC = 200\sqrt{3} \text{ m}$$

(iii) **Ans.** (d): If $BC = 15$ m and $\theta = 30°$, then

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 30° = \frac{AB}{15} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{15} \Rightarrow AB = 5\sqrt{3} \text{ m}$$

(iv) **Ans.** (c): In ΔABC, we obtain

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = 1 \Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$$

(v) **Ans.** (d): In ΔABC, we obtain

$$\cos\theta = \frac{BC}{AC} \Rightarrow \cos 45° = \frac{6}{AC} \Rightarrow \frac{1}{\sqrt{2}} = \frac{6}{AC} \Rightarrow AC = 6\sqrt{2} \text{ m}$$

EXAMPLE 14 *Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported buy wires from a point O.*

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.

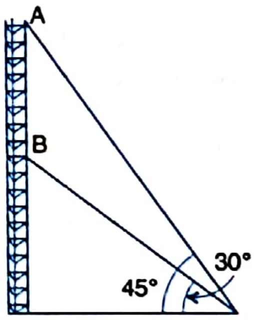

Fig. 11.18

Based on the above information, answer the following questions:

(i) *Find the length of the wire from the point O to the top of Section B.*

(ii) *Find the distance AB.*

(iii) *Find the area of ΔOPB.*

(iv) *Find the height of the Section A from the base of the tower.*

[CBSE 2023]

SOLUTION (i) In ΔOPB, we obtain

$$\cos 30° = \frac{OP}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{OB} \Rightarrow OB = 24\sqrt{3} \text{ cm}$$

(ii) In right triangles OPB and OPA, we obtain

$$\tan 30° = \frac{PB}{OP} \text{ and } \tan 45° = \frac{PA}{OP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{PB}{36} \text{ and } 1 = \frac{PA}{36} \Rightarrow PB = 12\sqrt{3} \text{ cm and } PA = 36 \text{ cm}$$

$\therefore$ $AB = PA - PB = (36 - 12\sqrt{3})$ cm $= 12\sqrt{3}(\sqrt{3} - 1)$ cm

(iii) Area of $\Delta OPB = \frac{1}{2}(OP \times PB) = \frac{1}{2}(36 \times 12\sqrt{3})\text{ cm}^2 = 216\sqrt{3}\text{ cm}^2$

(iv) Height of section A from the base of tower $= PA = 36$ cm.

EXAMPLE 15 *One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80 m. He observed a bird on the tree at an angle of elevation of 45°. When a sixer was hit, a ball flew through the three frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.*

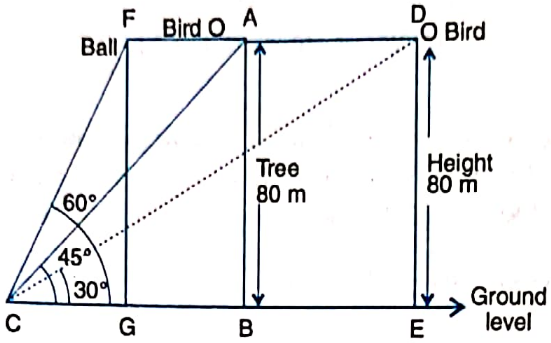

Fig. 11.19

(i) *At what distance from the foot of the tree was he observing the bird sitting on the tree?*

(ii) *How far did the bird fly in the mentioned time?*

(iii) *After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?*

(iv) *What is the speed of the bird in m/min if it had flown $20(\sqrt{3}+1)$ m?*

[CBSE Sample Paper 2024]

SOLUTION (i) In right triangles *CBA* and *CED*, we obtain

$$\tan 45° = \frac{AB}{BC} \text{ and } \tan 30° = \frac{DE}{CE} \Rightarrow 1 = \frac{80}{BC} \text{ and } \frac{1}{\sqrt{3}} = \frac{80}{CE} \Rightarrow BC = 80 \text{ m and } CE = 80\sqrt{3} \text{ m}$$

$\therefore \quad BE = CE - CB = (80\sqrt{3} - 80) \text{ m} = 80(\sqrt{3} - 1) \text{ m}$

Hence, Kaushik was observing the bird at a distance of $80(\sqrt{3}-1)$ m from the foot of the tree.

(ii) The bird flew distance $AD = BE = 80(\sqrt{3}-1)$ m.

(iii) In right triangle *CGF*, we obtain

$$\tan 60° = \frac{GF}{CG} \Rightarrow \sqrt{3} = \frac{80}{CG} \Rightarrow CG = \frac{80}{\sqrt{3}}$$

Distance travelled by the ball after hitting the tree = $FA = GB = CB - CG$

$$= \left(80 - \frac{80}{\sqrt{3}}\right) \text{m} = 80\left(1 - \frac{1}{\sqrt{3}}\right) \text{m}$$

(iv) Speed of the bird $= \dfrac{\text{Distance flown by it}}{\text{Time taken}} = \dfrac{20(\sqrt{3}+1)}{2}$ m/sec

$$= \frac{20(\sqrt{3}+1)}{2} \times 60 \text{ m/min} = 600(\sqrt{3}+1) \text{ m/min}$$

ASSERTION - REASON BASED MCQs

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 16 Statement-1 (A): *If the angles of elevation of the top of a tower from the points at distances of 9 m and 16 m from the base of a tower in the same line are complementary, then the height of the tower is 12 m.*

Statement-2 (R): *If the angle of elevation of a tower from two points at distances of a and b from its foot and in the same straight line with it are complementary, then the height of the tower is $\sqrt{ab}$.*

Ans. (a)

SOLUTION Let PQ be a tower of height h metres such that the angles of elevation of its top observed from points A and B at distances a and b from the base of the tower are complementary. In right triangles APQ and BPQ, we obtain

$$\tan\theta = \frac{h}{a} \text{ and } \tan(90^\circ - \theta) = \frac{h}{b}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{a} \text{ and } \cot\theta = \frac{h}{b}$$

$$\Rightarrow \quad \tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b} \Rightarrow 1 = \frac{h^2}{ab} \Rightarrow h = \sqrt{ab}$$

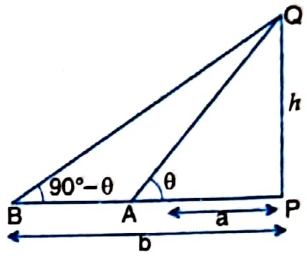

Fig. 11.20

Thus, statement-2 is true.

Using statement-2, we find that the height h of the tower in statement-1 is given by $h = \sqrt{9 \times 16}$ m $= 12$ m. So, statement-1 is also true and statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 17 Statement-1 (A): *If the shadow of a vertical pole is $\frac{1}{\sqrt{3}}$ of its height, then the altitude of the sun is 60°*

Statement-2 (R): *If the sun's altitude is 45°, then the shadow of a vertical pole is same as its height.*

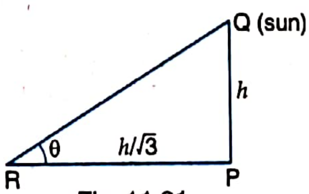

Fig. 11.21

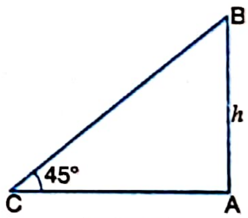

Fig. 11.22

Ans. (b)

SOLUTION Let PQ be a vertical pole of height h such that its shadow is of length $\frac{h}{\sqrt{3}}$. Let the sun's altitude be θ. Then,

$$\tan\theta = \frac{h}{h/\sqrt{3}} = \sqrt{3} \Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

Thus, statement-1 is true.

Let AB be a vertical pole of height h metre and AC be the length of its shadow when sun's altitude is 45°. (see Fig. 11.22).

In ΔBAC, we obtain

$$\tan 45^\circ = \frac{AB}{AC} \Rightarrow 1 = \frac{h}{AC} \Rightarrow AC = h$$

Hence, the shadow $AC$of pole AB is of the same height as that of the pole. So, statement-2 is true.

PRACTICE EXERCISES

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is
   (a) 45° (b) 30° (c) 60° (d) 90°
   [CBSE 2012, 2014]
2. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is
   (a) $25\sqrt{3}$ (b) $50\sqrt{3}$ (c) $75\sqrt{3}$ (d) 150
   [CBSE 2013]
3. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is
   (a) $15\sqrt{3}$ m (b) $\frac{15\sqrt{3}}{2}$ m (c) $\frac{15}{2}$ m (d) 15 m
   [CBSE 2013]
4. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is
   (a) $50\sqrt{3}$ (b) $150\sqrt{3}$ (c) $150\sqrt{2}$ (d) 75
   [CBSE 2014]
5. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then sun's elevation is
   (a) 60° (b) 45° (c) 30° (d) 90°
   [CBSE 2023]
6. The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the tower is 45°. Then the height of the tower (in metres) is
   (a) $50\sqrt{3}$ (b) 50 (c) $\frac{50}{\sqrt{2}}$ (d) $\frac{50}{\sqrt{3}}$
   [CBSE 2014]
7. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is
   (a) $\frac{4}{\sqrt{3}}$ (b) $4\sqrt{3}$ (c) $2\sqrt{2}$ (d) 4
   [CBSE 2014]
8. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sun is
   (a) 30° (b) 45° (c) 60° (d) 90°
9. If the angle of elevation of a tower from a distance of 100 metres from its foot is 60°, then the height of the tower is
   (a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m (c) $50\sqrt{3}$ m (d) $\frac{200}{\sqrt{3}}$ m
10. If the altitude of the sum is at 60°, then the height of the vertical tower that will cast a shadow of length 30 m is
    (a) $30\sqrt{3}$ m (b) 15 m (c) $\frac{30}{\sqrt{3}}$ m (d) $15\sqrt{2}$ m
11. If the angles of elevation of a tower from two points distant a and b $(a > b)$ from its foot and in the same straight line from it are 30° and 60°, then the height of the tower is
    (a) $\sqrt{a+b}$ (b) $\sqrt{ab}$ (c) $\sqrt{a-b}$ (d) $\sqrt{\frac{a}{b}}$

12. If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary, then the height of the tower is

(a) ab (b) $\sqrt{ab}$ (c) $\frac{a}{b}$ (d) $\sqrt{\frac{a}{b}}$

13. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30° and 45°. If the height of the light house is h metres, the distance between the ships is

(a) $(\sqrt{3}+1)\,h$ m (b) $(\sqrt{3}-1)\,h$ m (c) $\sqrt{3}\,h$ m (d) $1+\left(1+\frac{1}{\sqrt{3}}\right)h$ m

14. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. The height of the tower is

(a) $\frac{d}{\cot\alpha+\cot\beta}$ (b) $\frac{d}{\cot\alpha-\cot\beta}$ (c) $\frac{d}{\tan\beta-\tan\alpha}$ (d) $\frac{d}{\tan\beta+\tan\alpha}$

15. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with horizontal, then the length of the wire is

(a) 12 m (b) 10 m (c) 8 m (d) 6 m

16. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is

(a) 25 m (b) 50 m (c) 75 m (d) 100 m

17. The angles of depression of two ships from the top of a light house are 45° and 30° towards east. If the ships are 100 m apart, the height of the light house is

(a) $\frac{50}{\sqrt{3}+1}$ m (b) $\frac{50}{\sqrt{3}-1}$ m (c) $50(\sqrt{3}-1)$ m (d) $50(\sqrt{3}+1)$ m

18. If the angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°, then the height of the cloud above the lake, is

(a) 200 m (b) 500 m (c) 30 m (d) 400 m

19. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is

(a) 100 m (b) $100\sqrt{3}$ m (c) $100(\sqrt{3}-1)$m (d) $\frac{100}{\sqrt{3}}$ m

20. Two persons are a metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then the height of the shorter post is

(a) $\frac{a}{4}$ (b) $\frac{a}{\sqrt{2}}$ (c) $a\sqrt{2}$ (d) $\frac{a}{2\sqrt{2}}$

21. The angle of elevation of a cloud from a point h metre above a lake is θ. The angle of depression of its reflection in the lake is 45°. The height of the cloud is

(a) $h\tan(45°+\theta)$ (b) $h\cot(45°-\theta)$ (c) $h\tan(45°-\theta)$ (d) $h\cot(45°+\theta)$

22. A tower subtends an angle of 30° at a point on the same level as its foot. At a second point h metres above the first, the depression of the foot of the tower is 60°. The height of the tower is

(a) $\frac{h}{2}$ m (b) $\sqrt{3}h$ m (c) $\frac{h}{3}$ m (d) $\frac{h}{\sqrt{3}}$ m

23. It is found that on walking x meters towards a chimney in a horizontal line through its base, the elevation of its top changes from 30° to 60°. The height of the chimney is

(a) $3\sqrt{2}x$ (b) $2\sqrt{3}x$ (c) $\frac{\sqrt{3}}{2}x$ (d) $\frac{2}{\sqrt{3}}x$

24. The length of the shadow of a tower standing on level ground is found to be $2x$ metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower in metres is

(a) $(\sqrt{3}+1)x$ (b) $(\sqrt{3}-1)x$ (c) $2\sqrt{3}x$ (d) $3\sqrt{2}x$

25. Two poles are 'a' metres apart and the height of one is double of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the smaller is

(a) $\sqrt{2}a$ metres (b) $\frac{a}{2\sqrt{2}}$ metres (c) $\frac{a}{\sqrt{2}}$ metres (d) $2a$ metres

26. The tops of two poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then l =

(a) 26 (b) 16 (c) 12 (d) 10

27. If a 1.5 m tall girl stands at a distance of 3 m from a lamp-post and casts a shadow of length 4.5 m on the ground, then the height of the lamp-post is

(a) 1.5 m (b) 2 m (c) 2.5 m (d) 2.8 m

CASE STUDY BASED MCQs

28. We know that during vacation period many people love to go out of the city and gain some experience about historical and scientific values. Keeping in views, Mr. Ramlal decided to go somewhere out of the country and chosen the country USA. In the series of sight seeing, he has first chosen the place sky tower of Mexico City. Then he decided to stand on a building and wanted to see the sky tower. Mr. Ramlal whose height is 2.3 m stood on the top of a building and started to look at the top of sky tower. The horizontal distance between sky tower and the building is 120 mt. as shown in Fig. 11.23. The angle of elevation of the top and angle of depression of the bottom of the sky tower are 60° and 30° respectively. Looking into the above circumstances to give the answer of the following questions:

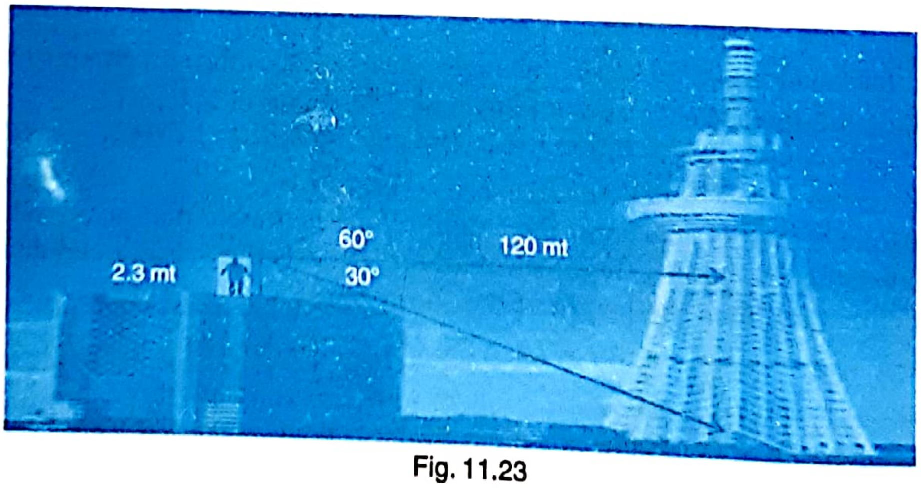

Fig. 11.23

(i) What is the height of the building excluding the height of Mr. Ramlal standing on it?

(a) 65.98 m (b) 66.98 m (c) 67.98 m (d) 68.98 m

(ii) Find the height of the building including height of Mr. Ramlal?

(a) $30\sqrt{3}$ m (b) $35\sqrt{3}$ m (c) $40\sqrt{3}$ m (d) $45\sqrt{3}$ m

(iii) What is the length of line of sight of Mr. Ramlal to the base of the Sky tower?

(a) $40\sqrt{3}$ m (b) $80\sqrt{3}$ m (c) $100\sqrt{3}$ m (d) $120\sqrt{3}$ m

(iv) Find the distance from the eye of Mr. Ramlal to top of the sky tower along the line of sight?

(a) 210 m (b) 220 m (c) 230 m (d) 240 m

(v) Find the height of the sky tower?

(a) $120\sqrt{3}$ m (b) $140\sqrt{3}$ m (c) $150\sqrt{3}$ m (d) $160\sqrt{3}$ m

29. A helicopter lifts up 1000 feet over an island and spots a swimmer that need to be rescued. Using a distant land mark, the helicopter pilot determines the angle of depression.

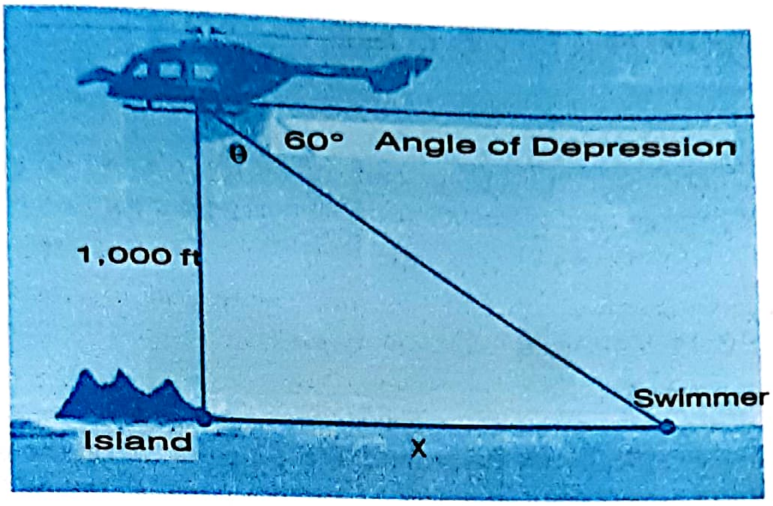

Fig. 11.24

(i) As the angle of depression increases what will be the effect?

(a) The helicopter gets further from the island.

(b) The helicopter gets closer to the island.

(c) The swimmer gets closer to the island.

(d) The swimmer gets further from the island.

(ii) How does the swimmer's distance from island changes as the angle of depression is halved from 60° to 30°?

(a) The swimmer's distance decreases to less than a quarter of his starting distance.

(b) The swimmer's distance from the island doubles

(c) The swimmer's distance from the island increases three times.

(d) The swimmer's distance from the island is halved.

(iii) For which angle of depression both the helicopter and swimmer's will be at same distance?

(a) 30° (b) 45° (c) 60° (d) 90°

(iv) Let the swimmer start out 1019 ft. from the island. If he swims half of the distance, what is angle of depression?)

(a) nearly 30° (b) nearly 45° (c) nearly 60° (d) nearly 90°

(v) How would the angle of depression be affected if the helicopter left its initial position and moved vertically upward?

(a) angle of depression doesn't change

(b) angle of depression increases

(c) angle of depression decreases

(d) none of the above

30. The following TV Tower was built in 1988 and is located in Pitampura, Delhi. It has an observation deck. Observe the picture given below:

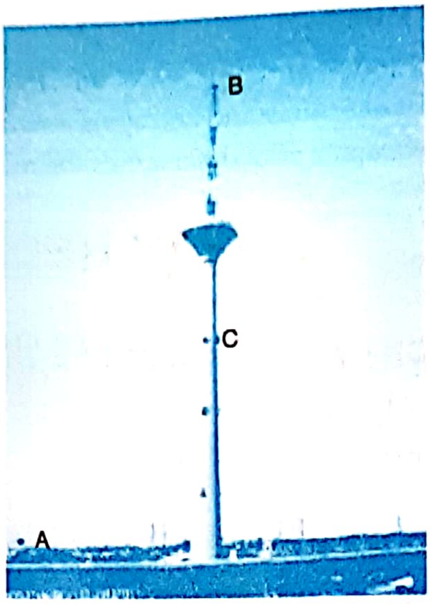

Fig. 11.25

The TV Tower stands vertically on the ground. From a point 'A' on the ground, the angle of elevation of top of the tower (point 'B') is 60°. There is a point 'C' on the tower which is 78 m (approx.) above the ground. The angle of elevation of the point C from point A is found to be 30°.

(i) Draw a well-labelled figure, based on the information given above.

(ii) Find the height of the tower and the distance of the tower from point *A*. [CBSE 2022]

ANSWERS

1. (b) 2. (c) 3. (c) 4. (b) 5. (c) 6. (b) 7. (d) 8. (a) 9. (a) 10. (a) 11. (b)
12. (b) 13. (a) 14. (b) 15. (a) 16. (b) 17. (d) 18. (d) 19. (c) 20. (d) 21. (a) 22. (c)
23. (c) 24. (a) 25. (b) 26. (c) 27. (c)

28. (i) (b) (ii) (c) (iii) (b) (iv) (d) (v) (d)

29. (i) (a) (ii) (c) (iii) (b) (iv) (a) (v) (b)

30. (ii) 234 m, $78\sqrt{3}$ m