CHAPTER 11

TRIANGLE AND ITS ANGLES

REVISION OF KEY CONCEPTS AND FORMULAE

1. A plane figure formed by three lines in a plane is called a triangle.
   A triangle is generally denoted by the symbol Δ. In ΔABC, A, B and C are known as vertices.
   A triangle ABC has six elements, namely, three sides AB, BC and CA and three angles $\angle A, \angle B$ and $\angle C$.
   Equilateral Triangle: A triangle having all sides equal is called an equilateral triangle.
   The measure of each angle of an equilateral triangle is 60°.
   Isosceles Triangle: A triangle having two sides equal is called an isosceles triangle.
   Angles opposite to equal sides of an isosceles triangle are equal. Sides opposite to equal angles of a triangle are equal.
   Scalene triangle: A triangle, no two of whose sides are equal is called a scalene triangles.
   All angles of a scalene triangle are distinct.
   Right angled triangle: A triangle with one angle a right angle is called a right triangle or a right angled triangle.

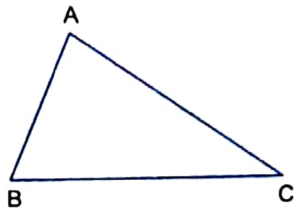

Fig.11.1

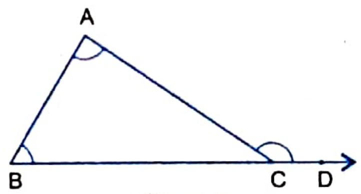

Fig.11.2

2. The sum of the angles of a triangle is 180°.
3. If the side of a triangle is produced then the exterior angle so formed is equal to the sum of the two interior opposite angles. i.e. $\angle ACD = \angle CAB + \angle ABC$.
   The exterior angle of a triangle is greater than each of the interior opposite angles.
4. In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Then,

$$\angle BOC = 90° + \frac{1}{2}\angle A$$

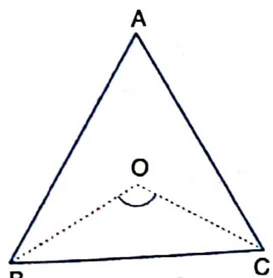

Fig.11.3

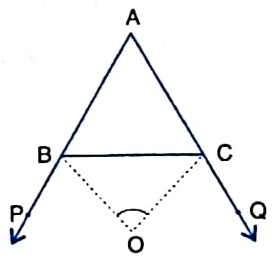

Fig.11.4

5. In a ΔABC, the sides AB and AC are produced to the points P and Q respectively. If the bisectors of $\angle PBC$ and $\angle QCB$ intersect at O, then

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$\angle BOC = 90° - \frac{1}{2}\angle A.$

6. In a ΔABC, if the bisectors $\angle B$ and $\angle C$ intersect at O and the bisectors of ext $\angle B$ and ext $\angle C$ meet at O', then $\angle BOC + \angle BO'C = 180°$ i.e. $\angle BOC$ and $\angle BO'C$ are supplementary.

7. In a ΔABC, the angle between the internal bisector of one base angle and the external bisector of other base angle is equal to one-half of the vertical angle i.e. $\angle E = \frac{1}{2}\angle A$ (see Fig. 11.5)

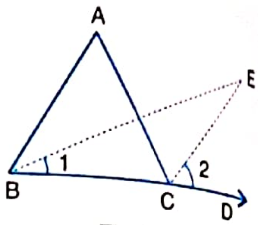

Fig.11.5

SOLVED EXAMPLES

MULTIPLE CHOICE

EXAMPLE 1 *In a ΔABC, if $\angle A = \angle B + \angle C$, then ΔABC is*
(a) *isosceles triangle* (b) *equilateral triangle* (c) *right triangle* (d) *none of these*
Ans. (c)

SOLUTION We have,

$$\angle A = \angle B + \angle C \Rightarrow \angle A + \angle A = \angle A + \angle B + \angle C \Rightarrow 2\angle A = 180° \Rightarrow \angle A = 90°$$

Hence, ΔABC is a right triangle.

EXAMPLE 2 *In Fig. 11.6, $BC \parallel PQ$, BP and CQ intersect at O. If $x + y = 80°$ and $x - y = 55°$, then $z =$*
(a) 80° (b) 55° (c) 90° (d) 100°
Ans. (d)

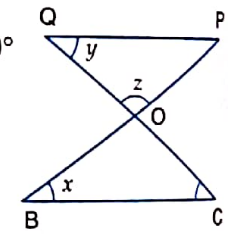

Fig.11.6

SOLUTION It is given that $BC \parallel PQ$ and transversal BP cuts them at B and P respectively.

$\therefore \quad \angle CBP = \angle BPQ$ [Alternate angles]

$\Rightarrow \quad \angle BPQ = x$

Using angle sum property in ΔOPQ, we obtain

$\angle P + \angle Q + \angle POQ = 180° \Rightarrow x + y + z = 180° \Rightarrow 80° + z = 180° \Rightarrow z = 100°$ [$\because x + y = 80°$ (given)]

EXAMPLE 3 *In Fig. 11.7, $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F =$*
(a) 180° (b) 360° (c) 540° (d) 90°
Ans. (b)

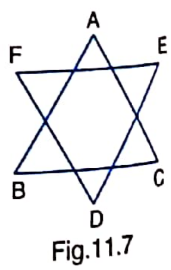

Fig.11.7

SOLUTION Using angle sum property in Δ's ABC and DEF, we obtain

$\angle A + \angle B + \angle C = 180°$ and $\angle D + \angle E + \angle F = 180°$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 180° + 180° = 360°$

EXAMPLE 4 *In Fig. 11.8, $\angle ACD = 120°$ and $\angle ABC = 40°$, then $\angle BAC =$*
(a) 80° (b) 60° (c) 50° (d) 40°
Ans. (a)

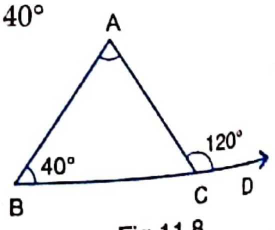

Fig.11.8

SOLUTION In ΔABC, side BC is produced to D. Using exterior angle property, we obtain

$\angle ACD = \angle ABC + \angle BAC$

$\Rightarrow \quad 120° = 40° + \angle BAC \Rightarrow \angle BAC = 80°$

EXAMPLE 5 *In Fig. 11.9, sides CB and BA of ΔABC are produced to D and E respectively. If $\angle ABD = 105°$ and $\angle CAE = 130°$, then $\angle ACB =$*
(a) 50° (b) 55° (c) 75° (d) 130°

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Ans. (b)

SOLUTION We have, $\angle CAE = 130°$

$\therefore$ $\angle BAC = 180° - 130° = 50°$ $[\because \angle BAC + \angle CAE = 180°]$

Using exterior angle property in ΔABC, we obtain

$\angle ABD = \angle BAC + \angle ACB$

$\Rightarrow$ $105° = 50° + \angle ACB \Rightarrow \angle ACB = 105° - 50° = 55°$

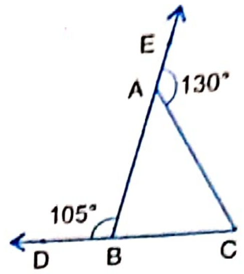

Fig.11.9

EXAMPLE 6 *In a ΔABC, it is given that $\angle A : \angle B : C = 3:2:1$ and $\angle ACD = 90°$. If BC is produced to E, then $\angle ECD =$*

(a) 60° (b) 30° (c) 50° (d) 40°

Ans. (a)

SOLUTION We have, $\angle A : \angle B : \angle C = 1:2:3$

So, let $\angle A = 3x$, $\angle B = 2x$ and $\angle C = x$.

$\therefore$ $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $3x + 2x + 3x = 180° \Rightarrow 6x = 180° \Rightarrow x = 30°$

$\therefore$ $\angle A = 90°$, $\angle B = 60°$ and $\angle C = 30°$

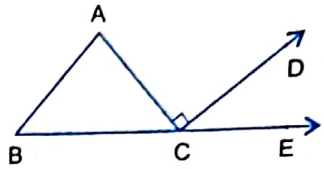

Fig.11.10

Using exterior angle property in ΔACE, we obtain

$\angle ACE = \angle A + \angle B \Rightarrow \angle ACD + \angle ECD = 90° + 60° \Rightarrow 90° + \angle ECD = 150° \Rightarrow \angle ECD = 60°$

EXAMPLE 7 *In Fig. 11.11, if the sides BC, CA and AB of ΔABC have been produced to D, E and F respectively, then $\angle BAE + \angle CBF + \angle ACD =$*

(a) 180° (b) 360° (c) 240° (d) 300°

Ans. (b)

SOLUTION Using exterior angle property in ΔABC, we obtain

$\angle ACD = \angle A + \angle B, \angle CBF = \angle A + \angle C$ and, $\angle BAE = \angle B + \angle C$

$\therefore$ $\angle BAE + \angle CBF + \angle ACD = (\angle B + \angle C) + (\angle A + \angle C) + (\angle A + \angle B)$

$= 2(\angle A + \angle B + \angle C) = 2 \times 180° = 360°$

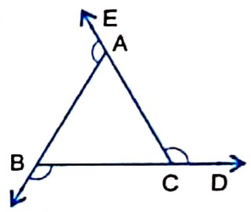

Fig.11.11

EXAMPLE 8 *In Fig. 11.12, if PT is the bisector of $\angle QPR$ in ΔPQR, $\angle PQR = 50°, \angle PRQ = 30°$ and $PS \perp QR$, then $x =$*

(a) 40° (b) 20° (c) 30° (d) 10°

Ans. (d)

SOLUTION Using angle sum property in ΔPQR, we obtain

$\angle PQR + \angle PRQ + \angle RPQ = 180°$

$\Rightarrow$ $50° + 30° + \angle RPQ = 180° \Rightarrow \angle RPQ = 100°$

$\therefore$ $\angle QPT = \frac{1}{2} \angle RPQ = 50°$ $[\because$ PT is bisector of $\angle QPR]$

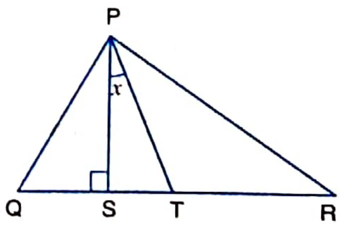

Fig.11.12

Using exterior angle property in ΔPQS, we obtain

$\angle PST = \angle PQS + \angle QPS \Rightarrow 90° = 50° + \angle QPS \Rightarrow \angle QPS = 40°$

$\therefore$ $x = \angle QPT - \angle QPS = 50° - 40° = 10°$

EXAMPLE 9 *In Fig.11.13, $a + b =$*

(a) 117° (b) 130° (c) 127° (d) 158°

Ans. (c)

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SOLUTION Since ARB is a straight line.

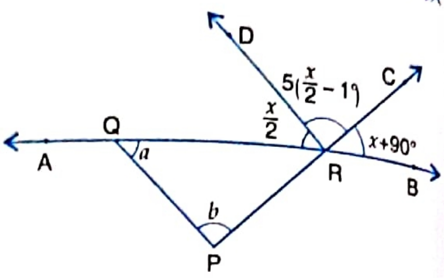

Fig.11.13

$$\therefore \quad \frac{x}{2}+5\left(\frac{x}{2}-1°\right)+x+9°=180°$$

$$\Rightarrow \quad 4x+4°=180° \Rightarrow 4x=176° \Rightarrow x=44°$$

Using exterior angle property in PQR, we obtain

$$\angle QRC = a+b$$

$$\Rightarrow \quad \frac{x}{2}+5\left(\frac{x}{2}-1°\right)=a+b \Rightarrow a+b=3x-5° \Rightarrow a+b=3\times 44°-5=127°$$

ASSERTION-REASON

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

EXAMPLE 10 *Statement-1 (Assertion): In Fig. 11.14, side BC of ΔABC is produced to D. If $\angle ACD = 110°$, then $x = 40°$.*

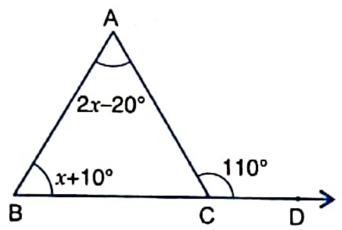

Fig.11.14

Statement-2 (Reason): If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Ans. (a)

SOLUTION Statement-2 is the Exterior angle theorem. So, it is true.
Using statement-2, we obtain

$$2x-20°+x+10°=110° \Rightarrow 3x=120° \Rightarrow x=40°$$

So, statement-1 is also true. Also, statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

EXAMPLE 11 *Statement-1 (Assertion): In Fig. 11.15, if the bisectors of angles $\angle B$ and $\angle C$ of ΔABC meet at O, then $\angle BOC = 140°$*

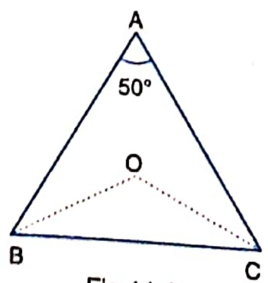

Fig.11.15

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Statement-2 (Reason): *If bisectors of angles B and C of a ΔABC meet at O, then $\angle BOC = 90° + \frac{\angle A}{2}$.*

Ans. (d)

SOLUTION Statement-2 is true (See S.No. 4 on Page 96).

Using statement-2, we obtain

$$\angle BOC = 90° + \frac{1}{2} \times 50° = 115°$$

So, statement-1 is not true. Hence, option (d) is correct.

EXAMPLE 12 *Statement-1 (Assertion): In a ΔABC, the bisectors of $\angle B$ and $\angle C$ meet a point O and the bisectors of ext $\angle B$ and ext $\angle C$ meet a point O'. If $\angle BOC = 135°$, then $\angle BO'C = 45°$*

Statement-2 (Reason): In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ meet at a point O and the bisectors of ext $\angle B$ and ext $\angle C$ meet at a point O'. Then, $\angle BOC$ and $\angle BO'C$ are supplementary.

Ans. (a)

SOLUTION Statement-2 is true see S.No. 6 on page 96). Using statement-2, we have,

$$\angle BOC + \angle BO'C = 180° \Rightarrow 135° + \angle BO'C = 180° \Rightarrow \angle BO'C = 45°$$

So, statement-1 is true and statement-2 is a correct explanation for statement-1.
Hence, option (a) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE

Mark the correct alternative in each of the following:

1. If all the three angles of a triangle are equal, then each one of them is equal to
   (a) 90° (b) 45° (c) 60° (d) 30°
2. If two acute angles of a right triangle are equal, then each acute is equal to
   (a) 30° (b) 45° (c) 60° (d) 90°
3. An exterior angle of a triangle is equal to 100° and two interior opposite angles are equal. Each of these angles is equal to
   (a) 75° (b) 80° (c) 40° (d) 50°
4. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
   (a) an isosceles triangle (b) an obtuse triangle
   (c) an equilateral triangle (d) a right triangle
5. Side BC of a triangle ABC has been produced to a point D such that $\angle ACD = 120°$. If $\angle B = \frac{1}{2}\angle A$, then $\angle A$ is equal to
   (a) 80° (b) 75° (c) 60° (d) 90°
6. In ΔABC, $\angle B = \angle C$ and ray AX bisects the exterior angle $\angle DAC$. If $\angle DAX = 70°$, then $\angle ACB =$
   (a) 35° (b) 90° (c) 70° (d) 55°
7. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angle is 55°, then the measure of the other interior angle is
   (a) 55° (b) 85° (c) 40° (d) 9.0°

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8. If the sides of a triangle are produced in order, then the sum of the three exterior angles so formed is
(a) 90° (b) 180° (c) 270° (d) 360°

9. In ΔABC, if $\angle A = 100°$, AD bisects $\angle A$ and $AD \perp BC$. Then, $\angle B =$
(a) 50° (b) 90° (c) 40° (d) 100°

10. An exterior angle of a triangle is 108° and its interior opposite angles are in the ratio 4 : 5. The angles of the triangle are
(a) 48°, 60°, 72° (b) 50°, 60°, 70° (c) 52°, 56°, 72° (d) 42°, 60°, 76°

11. In a ΔABC, if $\angle A = 60°$, $\angle B = 80°$ and the bisectors of $\angle B$ and $\angle C$ meet at O, then $\angle BOC =$
(a) 60° (b) 120° (c) 150° (d) 30°

12. Line segments AB and CD intersect at O such that $AC \parallel DB$. If $\angle CAB = 45°$ and $\angle CDB = 55°$, then $\angle BOD =$
(a) 100° (b) 80° (c) 90° (d) 135°

13. In Fig. 11.16, if $EC \parallel AB$, $\angle ECD = 70°$ and $\angle BDO = 20°$, then $\angle OBD$ is
(a) 20° (b) 50° (c) 60° (d) 70°

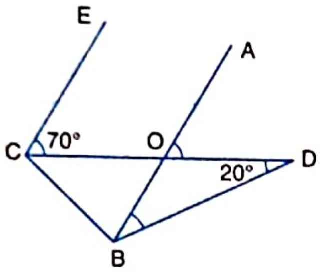

Fig. 11.16

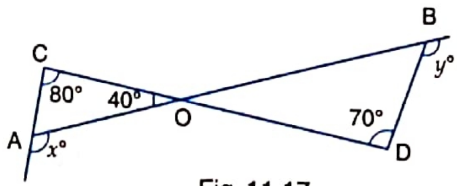

Fig. 11.17

14. In Fig. 11.17, $x + y =$
(a) 270 (b) 230 (c) 210 (d) 190°

15. If the measures of angles of a triangle are in the ratio of 3 : 4 : 5, what is the measure of the smallest angle of the triangle?
(a) 25° (b) 30° (c) 45° (d) 60°

16. In Fig. 11.18, for which value of x is $l_1 \parallel l_2$?
(a) 37 (b) 43 (c) 45 (d) 47

17. In Fig. 11.19, the value of x is
(a) 65° (b) 80° (c) 95° (d) 120°

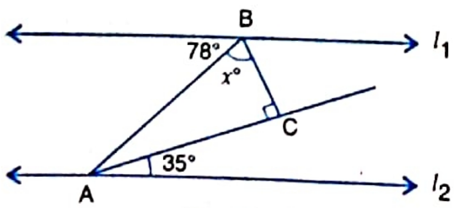

Fig. 11.18

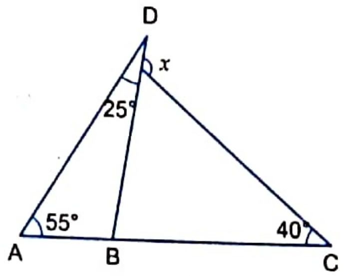

Fig. 11.19

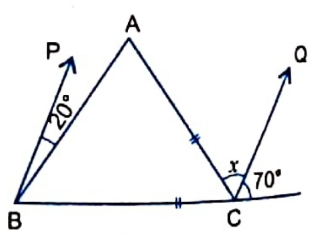

Fig. 11.20

18. In Fig. 11.20, if $BP \parallel CQ$ and $AC = BC$, then the measure of x is
(a) 20° (b) 25° (c) 30° (d) 35°

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19. The base BC of triangle ABC is produced both ways and the measure of exterior angles formed are 94° and 126°. Then, $\angle BAC =$

(a) 94° (b) 54° (c) 40° (d) 44°

20. In Fig. 11.21 if $AB \perp BC$, then $x =$

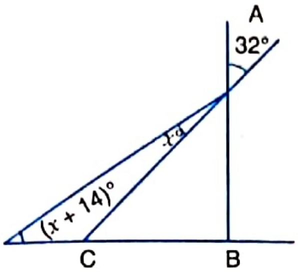

Fig. 11.21

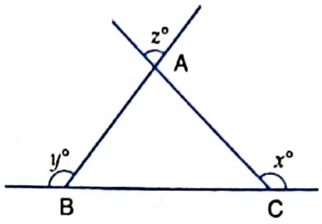

Fig. 11.22

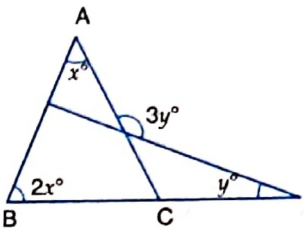

Fig. 11.23

(a) 18 (b) 22 (c) 25 (d) 32

21. In Fig. 11.22, what is z in terms of x and y?

(a) $x + y + 180$ (b) $x + y - 180$ (c) $180° - (x + y)$ (d) $x + y + 360°$

22. In Fig. 11.23, what is y in terms of x?

(a) $\frac{3}{2}x$ (b) $\frac{4}{3}x$ (c) x (d) $\frac{3}{4}x$

23. In Fig. 11.24, what is the value of x?

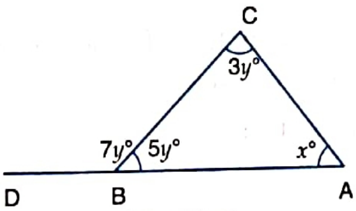

Fig. 11.24

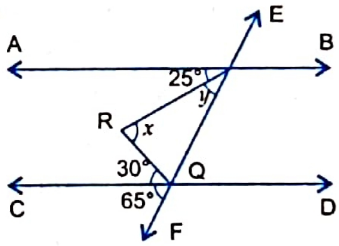

Fig. 11.25

(a) 35 (b) 45 (c) 50 (d) 60

24. In Fig. 11.25, AB and CD are parallel lines and transversal EF intersects them at P and Q respectively. If $\angle APR = 25°$, $\angle RQC = 30°$ and $\angle CQF = 65°$, then

(a) $x = 55°, y = 40°$ (b) $x = 50°, y = 45°$ (c) $x = 60°, y = 35°$ (d) $x = 35°, y = 60°$

25. If the bisectors of the acute angles of a right triangle meet at O, then the angle at O between the two bisectors is

(a) 45° (b) 95° (c) 135° (d) 90°

26. The bisects of exterior angles at B and C of ΔABC meet at O. If $\angle A = x°$, then $\angle BOC =$

(a) $90° + \frac{x°}{2}$ (b) $90° - \frac{x°}{2}$ (c) $180° + \frac{x°}{2}$ (d) $180° - \frac{x°}{2}$

27. In a ΔABC, $\angle A = 50°$ and BC is produced to a point D. If the bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then $\angle E =$

(a) 25° (b) 50° (c) 100° (d) 75°

28. The side BC of ΔABC is produced to a point D. The bisector of $\angle A$ meets side BC in L. If $\angle ABC = 30°$ and $\angle ACD = 115°$, then $\angle ALC =$

(a) 85° (b) $72\frac{1}{2}°$ (c) 145° (d) none of these

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29. In Fig. 11.26, if $l_1 \parallel l_2$, the value of x is

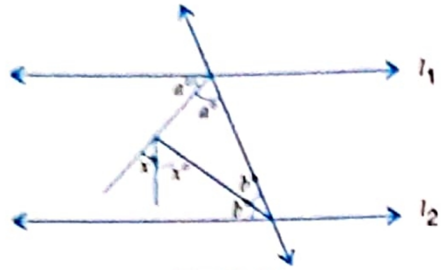

Fig. 11.26

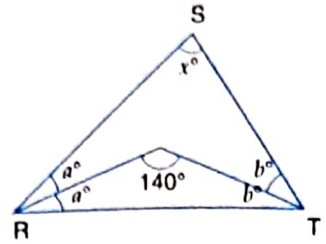

Fig. 11.27

(a) $22\frac{1}{2}$ (b) 30 (c) 45 (d) 60

30. In ΔRST (See Fig. 11.27), what is the value of x?
(a) 40 (b) 90° (c) 80° (d) 100

ASSERTION-REASON

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

31. Statement-1 (Assertion): In a ΔABC, if the bisectors of angles $\angle B$ and $\angle C$ meet at a point O, then $\angle BOC$ is always an obtuse angle.

Statement-2 (Reason): In a ΔABC, if the bisectors of angles $\angle B$ and $\angle C$ meet at a point O, then $\angle BOC = 90° + \frac{\angle A}{2}$.

32. Statement-1 (Assertion): In a ΔABC, sides AB and AC are produced to P and Q respectively. If the bisectors of $\angle PBC$ and $\angle QCB$ intersect at O, then $\angle BOC$ is an acute angle.

Statement-2 (Reason): In a ΔABC, sides AB and AC are produced to P and Q respectively. If the bisectors of $\angle PBC$ and $\angle QCB$ intersect at O, then $\angle BOC = 90° - \frac{A}{2}$.

33. Statement-1 (Assertion): In Fig. 11.28, side BC of ΔABC is produced to a point D such that the bisectors of $\angle ABC$ and $\angle ACD$ meet at a point E. If $\angle BAC = 80°$, then $\angle BEC = 50°$.

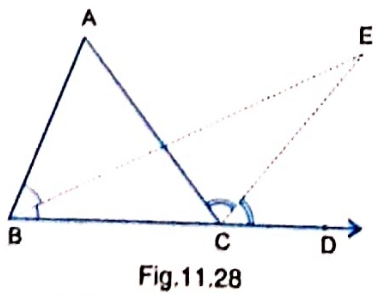

Fig.11.28

Statement-2 (Reason): The angle between the internal bisector of one base angle and the external bisector of the other angle of a triangle is equal to one-half of the vertical angle.

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34. Statement-1 (Assertion): In fig. 11.29, if $CP \parallel BQ$, then $\angle ACP = 140°$.

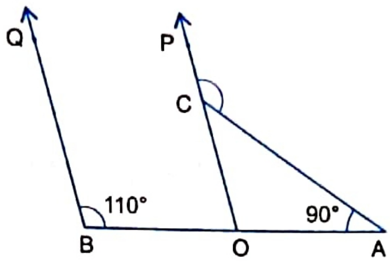

Fig.11.29

Statement-2 (Reason): If two parallel lines are intersected by a transversal, then the corresponding angles are equal.

ANSWERS

1. (a)	2. (b)	3. (d)	4. (d)	5. (a)	6. (c)	7. (c)
8. (d)	9. (c)	10. (a)	11. (b)	12. (b)	13. (b)	14. (b)
15. (c)	16. (d)	17. (d)	18. (c)	19. (c)	20. (a)	21. (b)
22. (a)	23. (d)	24. (a)	25. (c)	26. (b)	27. (a)	28. (b)
29. (c)	30. (d)	31. (a)	32. (a)	33. (d)	34. (b)	

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