CHAPTER 10

# TRIGONOMETRIC IDENTITIES

## REVISION OF KEY CONCEPTS AND FORMULAE

1. An equation is called an identity if it is true for all values of the variable (s) involved.
2. An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.
3. Following are some trigonometric identities:

(i) $\sin^2\theta + \cos^2\theta = 1$ or, $1-\cos^2\theta = \sin^2\theta$ or, $1-\sin^2\theta = \cos^2\theta$

(ii) $1+\tan^2\theta = \sec^2\theta$ or, $\sec^2\theta - \tan^2\theta = 1$

(iii) $1+\cot^2\theta = \text{cosec}^2\theta$ or, $\text{cosec}^2\theta - \cot^2\theta = 1$.

(iv) $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$ and, $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

i.e. $\sec\theta + \tan\theta$ and $\sec\theta - \tan\theta$ are reciprocal of each other.

(v) $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta) = 1$

$\Rightarrow \text{cosec}\,\theta + \cot\theta = \dfrac{1}{\text{cosec}\,\theta - \cot\theta}$ and, $\text{cosec}\,\theta - \cot\theta = \dfrac{1}{\text{cosec}\,\theta + \cot\theta}$

i.e. $\text{cosec}\,\theta + \cot\theta$ and $\text{cosec}\,\theta - \cot\theta$ are reciprocal of each other.

## SOLVED EXAMPLES

### MULTIPLE CHOICE QUESTIONS (MCQs)

EXAMPLE 1 *The value of* $\cos^2\theta + \dfrac{1}{1+\cot^2\theta}$ *is*

(a) 0 (b) 1 (c) 2 (d) −1

Ans. (b)

SOLUTION $\cos^2\theta + \dfrac{1}{1+\cot^2\theta} = \cos^2\theta + \dfrac{1}{\text{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$

EXAMPLE 2 *The value of* $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)(1+\cos\theta)(1-\cos\theta)(1+\cot^2\theta)$ *is*

(a) 1 (b) −1 (c) 0 (d) −2

Ans. (a)

SOLUTION $(1+\tan^2\theta)(1+\sin\theta)(1-\sin\theta)(1+\cos\theta)(1-\cos\theta)(1+\cot^2\theta)$

$= \sec^2\theta(1-\sin^2\theta)(1-\cos^2\theta)\,\text{cosec}^2\theta = \dfrac{1}{\cos^2\theta}\times\cos^2\theta\times\sin^2\theta\times\dfrac{1}{\sin^2\theta} = 1$

EXAMPLE 3 $(1+\tan A - \sec A)(1+\tan A + \sec A) =$

(a) $2\tan A$ (b) $2\sin A$ (c) $2\sec A$ (d) $2\cot A$

**Ans.** (a)

**SOLUTION** $(1+\tan A-\sec A)(1+\tan A+\sec A)$

$=(1+\tan A)^2-\sec^2 A=1+\tan^2 A+2\tan A-\sec^2 A=\sec^2 A+2\tan A-\sec^2 A=2\tan A$

**EXAMPLE 4** *If* $\cos A=\frac{3}{5}$, *then the value* $9+9\tan^2 A$ *is*

(a) 9 (b) 16 (c) 25 (d) 34

**Ans.** (c)

**SOLUTION** $9+9\tan^2 A=9(1+\tan^2 A)=9\sec^2 A=\frac{9}{\cos^2 A}=9\times\left(\frac{5}{3}\right)^2=25$

**EXAMPLE 5** $\tan^4\theta+\tan^2\theta=$

(a) $\sec^2\theta-2\sec^4\theta$ (b) $2\sec^2\theta-2\sec^4\theta$ (c) $\sec^2\theta-\sec^4\theta$ (d) $\sec^4\theta-\sec^2\theta$

**Ans.** (d)

**SOLUTION** $\tan^4\theta+\tan^2\theta=\tan^2\theta(\tan^2\theta+1)=\tan^2\theta\sec^2\theta=(\sec^2\theta-1)\sec^2\theta=\sec^4\theta-\sec^2\theta.$

**EXAMPLE 6** $\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=$

(a) 1 (b) $2\tan^2\theta$ (c) $2\cot^2\theta$ (d) $2\sec^2\theta$

**Ans.** (a)

**SOLUTION** $\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=\frac{\tan^2\theta}{\sec^2\theta}+\frac{\cot^2\theta}{\text{cosec}^2\theta}=\tan^2\theta\times\frac{1}{\sec^2\theta}+\cot^2\theta\times\frac{1}{\text{cosec}^2\theta}$

$=\frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta+\frac{\cos^2\theta}{\sin^2\theta}\times\sin^2\theta=\sin^2\theta+\cos^2\theta=1$

**EXAMPLE 7** *If* $\sin\theta+\cos\theta=\sqrt{2}$, then $\tan\theta+\cot\theta=$

(a) 1 (b) 2 (c) 3 (d) 4

**Ans.** (b)

**SOLUTION** We have, $\sin\theta+\cos\theta=\sqrt{2}$

$\Rightarrow\quad(\sin\theta+\cos\theta)^2=2$

$\Rightarrow\quad\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=2\Rightarrow 1+2\sin\theta\cos\theta=2\Rightarrow 2\sin\theta\cos\theta=1\Rightarrow\sin\theta\cos\theta=\frac{1}{2}$

$\therefore\quad\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}=2$

**EXAMPLE 8** $\frac{\cos^2\theta}{\sin^2\theta}-\frac{1}{\sin^2\theta}$, *in simplified form, is*

(a) $\tan^2\theta$ (b) $\sec^2\theta$ (c) 1 (d) $-1$

**Ans.** (d) [CBSE 2023]

**SOLUTION** $\frac{\cos^2\theta}{\sin^2\theta}-\frac{1}{\sin^2\theta}=\left(\frac{\cos\theta}{\sin\theta}\right)^2-\left(\frac{1}{\sin\theta}\right)^2=\cot^2\theta-\text{cosec}^2\theta=-(\text{cosec}^2\theta-\cot^2\theta)=-1$

**EXAMPLE 9** *If* $15\tan^2\theta+4\sec^2\theta=23$, *then the value of* $(\sec\theta+\text{cosec}\,\theta)^2-\sin^2\theta$ *is*

(a) 7 (b) $\frac{15}{2}$ (c) $\frac{9}{2}$ (d) $\frac{11}{2}$

**Ans.** (b)

**SOLUTION** We have,

$15\tan^2\theta+4\sec^2\theta=23$

$\Rightarrow\quad 15\tan^2\theta+4(1+\tan^2\theta)=23\Rightarrow 19\tan^2\theta=19\Rightarrow\tan^2\theta=1\Rightarrow\tan\theta=1\Rightarrow\theta=45°$

$\therefore\quad (\sec\theta+\operatorname{cosec}\theta)^2-\sin^2\theta=(\sec 45^\circ+\operatorname{cosec}45^\circ)^2-\sin^2 45^\circ=(\sqrt{2}+\sqrt{2})^2-\left(\frac{1}{\sqrt{2}}\right)^2=8-\frac{1}{2}=\frac{15}{2}$

EXAMPLE 10 *If* $\sqrt{3}\cot^2\theta-4\cot\theta+\sqrt{3}=0$, *then the value of* $3(\cot^2\theta+\tan^2\theta)$ *is*

(a) 15 (b) 7 (c) 10 (d) 4

Ans. (c)

SOLUTION We have,

$$\sqrt{3}\cot^2\theta-4\cot\theta+\sqrt{3}=0$$

$$\Rightarrow\quad \sqrt{3}\cot^2\theta-3\cot\theta-\cot\theta+\sqrt{3}=0$$

$$\Rightarrow\quad \sqrt{3}\cot\theta(\cot\theta-\sqrt{3})-(\cot\theta-\sqrt{3})=0$$

$$\Rightarrow\quad (\sqrt{3}\cot\theta-1)(\cot\theta-\sqrt{3})=0$$

$$\Rightarrow\quad \sqrt{3}\cot\theta-1=0 \text{ or, } \cot\theta-\sqrt{3}=0 \Rightarrow \cot\theta=\frac{1}{\sqrt{3}} \text{ or, } \cot\theta=\sqrt{3} \Rightarrow \theta=60^\circ \text{ or } \theta=30^\circ$$

If $\theta=60^\circ$, then $3(\cot^2\theta+\tan^2\theta)=3(\cot^2 60^\circ+\tan^2 60^\circ)=3\left(\frac{1}{3}+3\right)=10$

If $\theta=30^\circ$, then $3(\cot^2\theta+\tan^2\theta)=3(\cot^2 30^\circ+\tan^2 30^\circ)=3\left(\frac{1}{3}+3\right)=10$

EXAMPLE 11 *If* $1+\sin^2\theta=3\sin\theta\cos\theta$, *then* $\tan\theta$ *can take values*

(a) $1,\frac{1}{2}$ (b) $1, 2$ (c) $\frac{1}{2}, 2$ (d) *none of these*

Ans. (a)

SOLUTION We have, $1+\sin^2\theta=3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get

$$\frac{1}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{3\sin\theta\cos\theta}{\cos^2\theta}$$

$$\Rightarrow \sec^2\theta+\tan^2\theta=3\tan\theta$$

$$\Rightarrow 1+\tan^2\theta+\tan^2\theta=3\tan\theta \Rightarrow 2\tan^2\theta-3\tan\theta+1=0 \Rightarrow (2\tan\theta-1)(\tan\theta-1)=0 \Rightarrow \tan\theta=1,\frac{1}{2}$$

EXAMPLE 12 $(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$ *is equal to*

(a) $2\sqrt{2}$ (b) 0 (c) 1 (d) $\sqrt{2}$

Ans. (c)

SOLUTION $(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$

$$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=\frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\times\frac{1}{\sin\theta\cos\theta}=1$$

EXAMPLE 13 *If* $k+1=\sec^2\theta(1+\sin\theta)(1-\sin\theta)$, *then* $k=$

(a) 1 (b) 2 (c) 0 (d) $-1$

Ans. (c)

SOLUTION We have,

$$k+1=\sec^2\theta(1+\sin\theta)(1-\sin\theta)$$

$$\Rightarrow\quad k+1=\frac{1}{\cos^2\theta}\times(1-\sin^2\theta)\Rightarrow k+1=\frac{\cos^2\theta}{\cos^2\theta}\Rightarrow k+1=1\Rightarrow k=0$$

EXAMPLE 14 *If* $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=k+\tan^2\theta+\cot^2\theta$, *then* $k=$

(a) 1 (b) 2 (c) 5 (d) 7

Ans. (d)

SOLUTION We find that

$$(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2$$

$$= \sin^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \text{cosec}^2\theta + \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$$
$$= \sin^2\theta + 2 + 1 + \cot^2\theta + \cos^2\theta + 2 + 1 + \tan^2\theta$$
$$= (\sin^2\theta + \cos^2\theta) + 6 + \cot^2\theta + \tan^2\theta = 7 + \tan^2\theta + \cot^2\theta$$

Hence, $k = 7$.

**EXAMPLE 15** *If* $\sin\theta - \cos\theta = 0$, *then the value of* $\sin^6\theta + \cos^6\theta$ *is*

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

**Ans.** (d)

SOLUTION We have,

$$\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$$

$$\therefore \quad \sin^6\theta + \cos^6\theta = (\sin\theta)^6 + (\cos\theta)^6 = (\sin 45°)^6 + (\cos 45°)^6 = \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

**EXAMPLE 16** *If* $\theta$ *is an acute angle of a right angled triangle, then which of the following equation is not true?*

(a) $\sin\theta\cot\theta = \cos\theta$ (b) $\cos\theta\tan\theta = \sin\theta$ (c) $\text{cosec}^2\theta - \cot^2\theta = 1$ (d) $\tan^2\theta - \sec^2\theta = 1$

**Ans.** (d) [CBSE 2023]

SOLUTION $\sin\theta\cot\theta = \sin\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta$. So, option (a) is true.

$\cos\theta\tan\theta = \cos\theta \times \frac{\sin\theta}{\cos\theta} = \sin\theta$. So, option (b) is true.

$1 + \cot^2\theta = \text{cosec}^2\theta$ and $1 + \tan^2\theta = \sec^2\theta \Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1$ and $\sec^2\theta - \tan^2\theta = 1$

So, option (c) is true but option (d) is not true.

## ASSERTION-REASON BASED MCQs

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

**EXAMPLE 17** *Statement-1 (A): For* $0 \le \theta < 90°$, $\sec^2\theta + \cos^2\theta \ge 2$.

*Statement-2 (R): For* $x > 0$, $x + \frac{1}{x} \ge 2$

**Ans.** (a)

SOLUTION For any $x > 0$, we know that

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \ge 0 \Rightarrow x + \frac{1}{x} - 2 \ge 0 \Rightarrow x + \frac{1}{x} \ge 2.$$

So, statement-2 is true.

For $0 \le \theta < 90°$, we find that $\sec^2\theta$ and $\cos^2\theta$ are positive real numbers such that $\sec^2\theta = \frac{1}{\cos^2\theta}$. Using statement-2, we obtain

$$\cos^2\theta + \frac{1}{\cos^2\theta} \ge 2 \Rightarrow \sec^2\theta + \cos^2\theta \ge 2$$

Thus, statement-1 is true and statement-2 is a correct explanation for statement-2. Hence, option (a) is correct.

EXAMPLE 18 *Statement-1 (A): The value of the product* $P_1 = \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ \cos 180^\circ$ *is zero.*

*Statement-2 (R): The value of the product* $P_2 = \tan 1^\circ \tan 2^\circ \ldots \tan 89^\circ$ *is 1.*

Ans. (b)

SOLUTION We find that in the product $P_1 = \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ \cos 180^\circ$ one of the terms is $\cos 90^\circ$ which is equal to zero. Hence, product $P_1 = 0$. So, statement-1 is true.

The product $P_2$ can be written as

$$P_2 = (\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)\ldots(\tan 44^\circ \tan 46^\circ)\ \tan 45^\circ$$

$$\Rightarrow \quad P_2 = (\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ)\ldots(\tan 44^\circ \cot 44^\circ)\ \tan 45^\circ = 1 \quad \left[\begin{array}{l}\text{Using} : \tan(90^\circ - \theta) = \cot\theta \\ \text{for } \theta = 1^\circ, 2^\circ, \ldots, 44^\circ\end{array}\right]$$

So, statement $P_2$ is also true. But, it is not a correct explanation for $P_1$.

EXAMPLE 19 *Statement-1 (A): For* $0 < \theta < 90^\circ$, $\sec\theta + \tan\theta$ *and* $\sec\theta - \tan\theta$ *are reciprocal of each other.*

*Statement-2 (R):* $\tan^2\theta - \sec^2\theta = 1$

Ans. (c)

SOLUTION Clearly, statement-2 is not true. We know that

$$\sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1 \Rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} \text{ and, } \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

i.e. $\sec\theta + \tan\theta$ and $\sec\theta - \tan\theta$ are reciprocal of each other. Thus, statement-1 is true.

EXAMPLE 20 *Statement-1 (A): Let a, b be non-zero real numbers. Then,* $\sec^2\theta = \dfrac{4ab}{(a+b)^2}$ *is true if and only if* $a = b$.

*Statement-2 (R):* $\sec^2\theta \geq 1$ *for* $0 \leq \theta < 90^\circ$.

Ans. (a)

SOLUTION In a right triangle if $\theta$ is one of the acute angles, then

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} \geq 1 \Rightarrow \sec^2\theta \geq 1 \qquad [\because \text{Hypotenuse} \geq \text{Base}]$$

So, statement-2 is true.

Now, $\quad \sec^2\theta = \dfrac{4ab}{(a+b)^2}$

$$\Rightarrow \quad \frac{4ab}{(a+b)^2} \geq 1 \qquad [\because \sec^2\theta \geq 1]$$

$$\Rightarrow \quad 1 - \frac{4ab}{(a+b)^2} \geq 0$$

$$\Rightarrow \quad \frac{(a+b)^2 - 4ab}{(a+b)^2} \leq 0$$

$$\Rightarrow \quad \frac{(a-b)^2}{(a+b)^2} \leq 0 \Rightarrow \left(\frac{a-b}{a+b}\right)^2 \leq 0 \Rightarrow \left(\frac{a-b}{a+b}\right)^2 = 0 \qquad \left[\because \left(\frac{a-b}{a+b}\right)^2 \text{ cannot be negative}\right]$$

$$\Rightarrow \quad \frac{a-b}{a+b} = 0 \Rightarrow a - b = 0 \Rightarrow a = b.$$

So, statement-1 is true and statement-2 is a correct explanation for statement-1.

**EXAMPLE 21** *Statement-1 (A): If* $\sin\theta + \sin^2\theta = 1$, *then* $\cos^2\theta + \cos^4\theta = 1$

*Statement-2 (R):* $1 - \sin^2\theta = \cos^2\theta$.

**Ans.** (a)

**SOLUTION** Clearly, statement-2 is true as $\sin^2\theta + \cos^2\theta = 1$.

Now, $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$ [Using statement-2]

$\Rightarrow \sin^2\theta = \cos^4\theta$ [Squaring both sides]

$\Rightarrow 1 - \cos^2\theta = \cos^4\theta$ [Using statement-2]

$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

So, statement-1 is also correct and statement-2 is a correct explanation for statement-1. Hence, option (a) is correct.

**EXAMPLE 22** *Statement-1 (A): If* $\tan\theta + \cot\theta = 2$, *then* $\tan^2\theta + \cot^2\theta = 4$.

*Statement-2 (R): If* $\operatorname{cosec} A = \sqrt{2}$, *then* $\dfrac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - 2\cos^2 A} = \dfrac{4}{3}$.

**Ans.** (d)

**SOLUTION** We have, $\tan\theta + \cot\theta = 2$

$\Rightarrow (\tan\theta + \cot\theta)^2 = 2^2$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4 \Rightarrow \tan^2\theta + \cot^2\theta + 2 = 4 \Rightarrow \tan^2\theta + \cot^2\theta = 2$

So, statement-1 is not true.

We have, $\operatorname{cosec} A = \sqrt{2} \Rightarrow \sin A = \dfrac{1}{\sqrt{2}} \Rightarrow \sin A = \sin 45° \Rightarrow A = 45°$

$$\therefore \quad \frac{2\sin^2 A + 3\cot^2 A}{4\tan^2 A - 2\cos^2 A} = \frac{2\sin^2 45 + 3\cot^2 45°}{4\tan^2 45° - 2\cos^2 45°} = \frac{2\left(\frac{1}{2}\right) + 3(1)}{4 \times 1 - 2\left(\frac{1}{2}\right)} = \frac{4}{3}$$

So, statement-2 is true.

## PRACTICE EXERCISES

### MULTIPLE CHOICE QUESTIONS (MCQs)

*Mark the correct alternative in each of the following:*

1. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then $x + y$ is equal to

   (a) 3 (b) 2 (c) 1 (d) 1/2

2. If $\tan\alpha + \cot\alpha = 2$, then $\tan^{2020}\alpha + \cot^{2020}\alpha =$

   (a) 0 (b) 2 (c) 2020 (d) $2^{2020}$

3. $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}}$ is equal to

   (a) $\sec\theta + \tan\theta$ (b) $\sec\theta - \tan\theta$ (c) $\sec^2\theta + \tan^2\theta$ (d) $\sec^2\theta - \tan^2\theta$

4. The value of $\sqrt{\dfrac{1 + \cos\theta}{1 - \cos\theta}}$ is

   (a) $\cot\theta - \operatorname{cosec}\theta$ (b) $\operatorname{cosec}\theta + \cot\theta$ (c) $\operatorname{cosec}^2\theta + \cot^2\theta$ (d) $(\cot\theta + \operatorname{cosec}\theta)^2$

5. $\frac{\sin\theta}{1+\cos\theta}$ is equal to

(a) $\frac{1+\cos\theta}{\sin\theta}$ (b) $\frac{1-\cos\theta}{\cos\theta}$ (c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1-\sin\theta}{\cos\theta}$

6. $\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}$ is equal to

(a) 0 (b) 1 (c) $\sin\theta+\cos\theta$ (d) $\sin\theta-\cos\theta$

7. $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ is equal to

(a) $2\tan\theta$ (b) $2\sec\theta$ (c) $2\operatorname{cosec}\theta$ (d) $2\tan\theta\sec\theta$

8. If $x=a\cos\theta$ and $y=b\sin\theta$, then $b^2x^2+a^2y^2=$

(a) $a^2b^2$ (b) $ab$ (c) $a^4b^4$ (d) $a^2+b^2$

9. If $x=a\sec\theta$ and $y=b\tan\theta$, then $b^2x^2-a^2y^2=$

(a) $ab$ (b) $a^2-b^2$ (c) $a^2+b^2$ (d) $a^2b^2$

10. The value of $(\sec A+\tan A)(1-\sin A)$ is

(a) $\sec A$ (b) $\sin A$ (c) $\operatorname{cosec} A$ (d) $\cos A$

[CBSE Sample Paper 2024]

11. If $x=a\sec\theta\cos\phi$, $y=b\sec\theta\sin\phi$ and $z=c\tan\theta$, then $\frac{x^2}{a^2}+\frac{y^2}{b^2}=$

(a) $\frac{z^2}{c^2}$ (b) $1-\frac{z^2}{c^2}$ (c) $\frac{z^2}{c^2}-1$ (d) $1+\frac{z^2}{c^2}$

12. $9\sec^2 A-9\tan^2 A$ is equal to

(a) 1 (b) 9 (c) 8 (d) 0

[NCERT]

13. $(\sec A+\tan A)(1-\sin A)=$

(a) $\sec A$ (b) $\sin A$ (c) $\operatorname{cosec} A$ (d) $\cos A$

[NCERT]

14. $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to

(a) $\sec^2 A$ (b) $-1$ (c) $\cot^2 A$ (d) $\tan^2 A$

[NCERT]

15. If $2\sin^2\beta-\cos^2\beta=2$, then $\beta$ is equal to

(a) 0° (b) 90° (c) 45° (d) 30°

16. If $\Delta ABC$ is right angled at $C$, then the value of $\cos(A+B)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

17. If $\sec\theta+\tan\theta=x$, then $\sec\theta=$

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2+1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{x^2-1}{x}$

18. If $\sec\theta+\tan\theta=x$, then $\tan\theta=$

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2-1}{x}$ (c) $\frac{x^2+1}{2x}$ (d) $\frac{x^2-1}{2x}$

19. $\sec^4 A - \sec^2 A$ is equal to
(a) $\tan^2 A - \tan^4 A$ (b) $\tan^4 A - \tan^2 A$ (c) $\tan^4 A + \tan^2 A$ (d) $\tan^2 A + \tan^4 A$

20. $\cos^4 A - \sin^4 A$ is equal to
(a) $2\cos^2 A + 1$ (b) $2\cos^2 A - 1$ (c) $2\sin^2 A - 1$ (d) $2\sin^2 A + 1$

21. The value of $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$ is
(a) 1 (b) 2 (c) 4 (d) 0

22. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$ is equal
(a) 0 (b) 1 (c) $-1$ (d) none of these

23. If $A$ and $B$ are acute angles such that $\sin(A - B) = 0$ and $2\cos(A + B) - 1 = 0$, then $A =$
(a) 60° (b) 30° (c) 45° (d) 15°

24. If $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ is
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. If $a\cos\theta - b\sin\theta = c$, then $a\sin\theta + b\cos\theta =$
(a) $\pm\sqrt{a^2 + b^2 + c^2}$ (b) $\pm\sqrt{a^2 + b^2 - c^2}$ (c) $\pm\sqrt{c^2 - a^2 - b^2}$ (d) none of these

26. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to
(a) $\cos\beta$ (b) $\cos 2\beta$ (c) $\sin\alpha$ (d) $\sin 2\alpha$

27. If $1 + \sin^2\alpha = 3\sin\alpha\cos\alpha$, then the values of $\cot\alpha$ are
(a) $-1, 1$ (b) $0, 1$ (c) $1, 2$ (d) $-1, -1$

28. $\dfrac{\cot\theta}{\cot\theta - \cot 3\theta} + \dfrac{\tan\theta}{\tan\theta - \tan 3\theta}$ is equal to
(a) 0 (b) 1 (c) $-1$ (d) 2

29. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$ is equal to
(a) 0 (b) 1 (c) $-1$ (d) none of these

30. If $a\cos\theta + b\sin\theta = 4$ and $a\sin\theta - b\cos\theta = 3$, then $a^2 + b^2 =$
(a) 7 (b) 12 (c) 25 (d) none of these

31. If $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$, then $p^2 - q^2 =$
(a) $a^2 - b^2$ (b) $b^2 - a^2$ (c) $a^2 + b^2$ (d) $b - a$

32. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then
(a) $x^2 + y^2 + z^2 = r^2$ (b) $x^2 + y^2 - z^2 = r^2$
(c) $x^2 - y^2 + z^2 = r^2$ (d) $z^2 + y^2 - x^2 = r^2$

33. If $\sin\theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta =$
(a) $-1$ (b) 1 (c) 0 (d) none of these

34. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then $a^2 + b^2 =$
(a) $m^2 - n^2$ (b) $m^2 n^2$ (c) $n^2 - m^2$ (d) $m^2 + n^2$

35. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A =$
(a) $-1$ (b) 0 (c) 1 (d) none of these

## ASSERTION-REASON BASED MCQs

Each of the following questions contains STATEMENT-1 (A) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

36. Statement-1 (A): The value of the product $P = \tan 1^\circ \tan 2^\circ \tan 3^\circ, \ldots\ldots, \tan 89^\circ$ is 1.
Statement-2 (R): For $0 < \theta \leq 90^\circ$, $\tan(90^\circ - \theta) = \cot\theta$ and $\tan 45^\circ = 1$.

37. Statement-1 (A): The value of the product of $P = \cos 1^\circ \cos 2^\circ \ldots\ldots \cos 179^\circ$ as $180^\circ$ is zero.
Statement-2 (R): The value of $\cos 90^\circ$ is zero.

38. Statement-1 (A): For $0 < \theta \leq 90^\circ$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.
Statement-2 (R): $\cot^2\theta - \operatorname{cosec}^2\theta = 1$. [CBSE 2023]

39. Statement-1 (A): For $0 \leq \theta < 90^\circ$, $\sec\theta + \tan\theta$ and $\sec\theta - \tan\theta$ are reciprocal of each other.
Statement-2 (R): $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

40. Statement-1 (A): If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 = a^2b^2$.
Statement-2 (R): $\cos^2\theta + \sin^2\theta = 1$.

41. Statement-1 (A): For $0 < \theta \leq 90^\circ$, $\operatorname{cosec}^2\theta + \sin^2\theta \geq 2$.
Statement-2 (R): For any $x > 0$, $x + \frac{1}{x} \geq 2$.

## ANSWERS

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. (a) | 2. (b) | 3. (a) | 4. (b) | 5. (c) | 6. (c) | 7. (c) |
| 8. (a) | 9. (d) | 10. (d) | 11. (d) | 12. (b) | 13. (d) | 14. (d) |
| 15. (b) | 16. (a) | 17. (b) | 18. (d) | 19. (c) | 20. (b) | 21. (b) |
| 22. (b) | 23. (b) | 24. (c) | 25. (b) | 26. (b) | 27. (c) | 28. (b) |
| 29. (c) | 30. (c) | 31. (b) | 32. (a) | 33. (b) | 34. (d) | 35. (c) |
| 36. (a) | 37. (a) | 38. (c) | 39. (c) | 40. (a) | 41. (a) | |