CHAPTER 9

TRIGONOMETRIC RATIOS

REVISION OF KEY CONCEPTS AND FORMULAE

1. An angle is considered as the figure obtained by rotating a given ray about its end-point. The revolving ray is called the generating line of the angle. In Fig. 9.1, the initial position OA is called the initial side and the final position OB is called the terminal side of the angle.

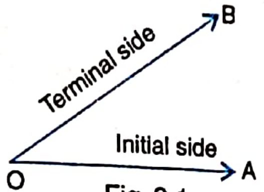


Fig. 9.1

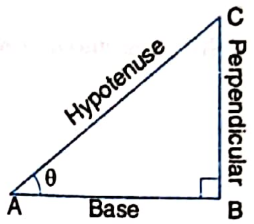


Fig. 9.2

2. The measure of an angle is the amount of rotation from the initial side to the terminal side.
3. If ABC is a right triangle right angled at B and $\angle BAC = \theta$ (see Fig. 9.2), then with reference to angle θ, Base $= AB$, Perpendicular $= BC$ and, Hypotenuse $= AC$. Therefore,

(i) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$ (ii) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

(iii) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB}$ (iv) $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{BC}$

(v) $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB}$ (vi) $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{AB}{BC}$

4. (i) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ (ii) $\sec\theta = \dfrac{1}{\cos\theta}$ (iii) $\cot\theta = \dfrac{1}{\tan\theta}$ (iv) $\sin\theta\operatorname{cosec}\theta = 1$

(v) $\cos\theta\sec\theta = 1$ (vi) $\tan\theta\cot\theta = 1$ (vii) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (viii) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

5. The trigonometric ratios for angles 0°, 30°, 45°, 60° and 90° are given in the following table.

θ / T. ratios	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

6. The values of $\sin\theta$ and $\cos\theta$ never exceed 1, whereas the values of $\sec\theta$ and $\mathrm{cosec}\,\theta$ are always greater than or equal to 1.

SOLVED EXAMPLES

MULTIPLE CHOICE QUESTIONS (MCQs)

EXAMPLE 1 *Which of the following is not defined?*

(a) $\cos 0°$ (b) $\tan 45°$ (c) $\sec 90°$ (d) $\sin 90°$

Ans. (c)

SOLUTION By using definitions of various trigonometric ratios, we find that sec 90° is undefined.

ALITER $\sec 90° = \frac{1}{\cos 90°} = \frac{1}{0}$, which is undefined.

EXAMPLE 2 *For* $0° \le \theta < 90°$, *the maximum value of* $\frac{1}{\sec\theta}$ *is*

(a) 1 (b) 0 (c) undefined (d) $\frac{\sqrt{3}}{2}$

Ans. (a)

SOLUTION We find that $\frac{1}{\sec\theta} = \cos\theta$, which attains all values between 0 and 1 (including 1 but excluding 0) as θ varies between 0° to 90°. Hence, the maximum value of $\frac{1}{\sec\theta}$ is 1.

EXAMPLE 3 *If* $\cos\theta = \frac{1}{2}$, *then* $\cos\theta - \sec\theta$ *is equal to*

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $-\frac{\sqrt{3}}{2}$

Ans. (b)

SOLUTION We have, $\cos\theta = \frac{1}{2}$. Therefore, $\sec\theta = \frac{1}{\cos\theta} = 2$. Hence, $\cos\theta - \sec\theta = \frac{1}{2} - 2 = -\frac{3}{2}$

ALITER $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60° \Rightarrow \sec\theta = \sec 60° = 2$

Hence, $\cos\theta - \sec\theta = \frac{1}{2} - 2 = -\frac{3}{2}$.

EXAMPLE 4 *If* $0 \le A, B \le 90°$ *such that* $\sin A = \frac{1}{2}$ *and* $\cos B = \frac{1}{2}$, *then* $A + B =$

(a) 0° (b) 60° (c) 90° (d) 30°

Ans. (c)

SOLUTION We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{1}{2} \Rightarrow A = 30° \text{ and } B = 60° \Rightarrow A + B = 90°$$

EXAMPLE 5 *In Fig. 9.3,* $\Delta\, ABC$ *is right-angled at B and* $\tan A = \frac{4}{3}$. *If* $AC = 5$ *cm, then the length of BC is*

(a) 4 cm (b) 12 cm (c) 3 cm (d) 9 cm

Ans. (a)

SOLUTION We have,

$$\tan A = \frac{4}{3} \Rightarrow \frac{BC}{AB} = \frac{4}{3} \Rightarrow AB = \frac{3}{4}BC$$

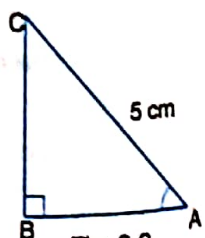


Fig. 9.3

Applying Pythagoras Theorem in ΔABC, we obtain

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \left(\frac{3}{4}BC\right)^2 + BC^2 = 25 \Rightarrow 9BC^2 + 16BC^2 = 400 \Rightarrow 25\,BC^2 = 400 \Rightarrow BC^2 = 16 \Rightarrow BC = 4$$

EXAMPLE 6 *If A is an acute angle in a right triangle ABC, right angled at B, then the value of* $\sin A + \cos A$ *is*

(a) *equal to* 1 (b) *greater than* 1 (c) *less than* 1 (d) 2

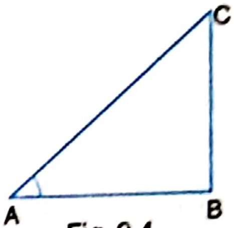


Fig. 9.4

Ans. (b)

SOLUTION In ΔABC, we find that

$$\sin A = \frac{BC}{AC} \text{ and } \cos A = \frac{AB}{AC}$$

$$\Rightarrow \quad \sin A + \cos A = \frac{BC}{AC} + \frac{AB}{AC} = \frac{AB + BC}{AC} \quad \text{...(i)}$$

In ΔABC, the sum of any two sides is greater than the third side.

$$\therefore \quad AB + BC > AC \Rightarrow \frac{AB + BC}{AC} > 1 \Rightarrow \sin A + \cos A > 1 \quad \text{[Using (i)]}$$

EXAMPLE 7 *If* $\operatorname{cosec}\theta = 2$ *and* $\cot\theta = \sqrt{3}a$, *then the value of a is*

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $2\sqrt{3}$

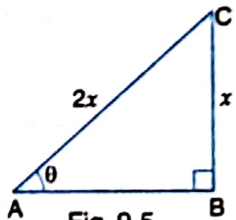


Fig. 9.5

Ans. (a)

SOLUTION We have, $\operatorname{cosec}\theta = 2$ and $\cot\theta = \sqrt{3}a$

$$\Rightarrow \quad \frac{AC}{BC} = 2 \text{ and } \frac{AB}{BC} = \sqrt{3}a$$

$$\Rightarrow \quad BC = \frac{1}{2}AC \text{ and } AB = \sqrt{3}a\,BC$$

$$\Rightarrow \quad BC = \frac{1}{2}AC \text{ and } AB = \frac{\sqrt{3}}{2}a\,AC \quad \text{...(i)}$$

Applying Pythagoras Theorem in ΔABC, we obtain

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow \quad \frac{3}{4}a^2AC^2 + \frac{1}{4}AC^2 = AC^2 \quad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{3}{4}a^2 + \frac{1}{4} = 1 \Rightarrow \frac{3}{4}a^2 = \frac{3}{4} \Rightarrow a^2 = 1 \Rightarrow a = 1$$

ALITER We have, $\operatorname{cosec}\theta = \frac{2}{1}$. So, consider a right triangle ABC with hypotenuse $(= AC) = 2x$ and perpendicular $(= BC) = x$. Applying Pythagoras Theorem in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2 \Rightarrow 4x^2 = AB^2 + x^2 \Rightarrow AB = \sqrt{3}x$$

$$\therefore \quad \cot\theta = \frac{AB}{BC} \Rightarrow \cot\theta = \frac{\sqrt{3}x}{x} \Rightarrow \cot\theta = \sqrt{3} \Rightarrow \sqrt{3}a = \sqrt{3} \Rightarrow x = 1$$

EXAMPLE 8 *In Fig. 9.6,* $\tan A - \cot C$ *is equal to*

(a) 0 (b) $\frac{5}{12}$ (c) $\frac{7}{13}$ (d) $-\frac{7}{13}$

Ans. (a)

SOLUTION In right triangle ABC, we have $AB = 12$ cm, $AC = 13$ cm.

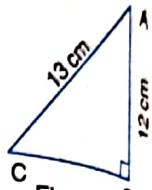


Fig. 9.6

$\therefore \quad AC^2 = AB^2 + BC^2 \Rightarrow BC = \sqrt{AC^2 - AB^2} = \sqrt{169 - 144} = 5$

$\therefore \quad \tan A = \frac{BC}{AB}$ and $\cot C = \frac{BC}{AB}$

$\Rightarrow \quad \tan A = \frac{5}{12}$ and $\cot C = \frac{5}{12} \Rightarrow \tan A - \cot C = 0$

EXAMPLE 9 *If* $\cos\theta = \frac{2}{3}$, *then* $2\sec^2\theta + 2\tan^2\theta - 7$ *is equal to*

(a) 1 (b) 0 (c) 3 (d) 4

Ans. (b)

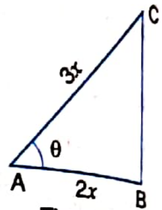


Fig. 9.7

SOLUTION We have

$$\cos\theta = \frac{2}{3} \Rightarrow \frac{\text{Base}}{\text{Hypotenuse}} = \frac{2}{3}$$

So, consider a right triangle ABC with base $AB = 2x$ and hypotenuse $= AC = 3x$. Using Pythagoras Theorem, we obtain

$$AC^2 = AB^2 + BC^2 \Rightarrow (3x)^2 = (2x)^2 + BC^2 \Rightarrow BC = \sqrt{5}x$$

In ΔABC, we obtain

$$\sec\theta = \frac{3x}{2x} = \frac{3}{2}, \quad \tan\theta = \frac{\sqrt{5}x}{2x} = \frac{\sqrt{5}}{2}$$

$$\therefore \quad 2\sec^2\theta + 2\tan^2\theta - 7 = 2 \times \left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7 = \frac{9}{2} + \frac{5}{2} - 7 = 0$$

EXAMPLE 10 *In* ΔABC, *right-angled at C, if* $\tan A = 1$, *then the value of* $2\sin A\cos A$ *is*

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{\sqrt{3}}{2}$

Ans. (a)

SOLUTION In ΔABC, it is given that $\tan A = 1 \Rightarrow \frac{BC}{AC} = 1$

So, let $BC = x$ and $AC = x$. Using Pythagoras Theorem in ΔABC, we obtain

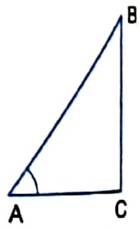


Fig. 9.8

$$AB^2 = AC^2 + BC^2 \Rightarrow AB^2 = x^2 + x^2 \Rightarrow AB = \sqrt{2}x$$

$$\therefore \quad \sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} \text{ and } \cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

Hence, $2\sin A\cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$.

EXAMPLE 11 *If for some angle* θ, $\cot 2\theta = \frac{1}{\sqrt{3}}$, *then the value of* $\sin 3\theta$, *where* $3\theta \leq 90°$, *is*

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) 0 (d) $\frac{\sqrt{3}}{2}$

Ans. (b)

SOLUTION We have, $\cot 2\theta = \frac{1}{\sqrt{3}} \Rightarrow 2\theta = 60° \Rightarrow \theta = 30° \Rightarrow 3\theta = 90° \Rightarrow \sin 3\theta = \sin 90° = 1$

EXAMPLE 12 *In Fig. 9.9, if* $AD = 14$ cm, *then the value of* $\tan\theta$ *is*

(a) $\frac{14}{3}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{13}{3}$

Ans. (b)

SOLUTION We have, $AE = BC = 5$ cm

$AD = 14$ cm $\Rightarrow AE + DE = 14$ cm $\Rightarrow DE = (14 - 5)$ cm $= 9$ cm

$\therefore$ In right triangle ABC, we obtain

$AC^2 = AB^2 + BC^2$

$\Rightarrow 13^2 = AB^2 + 5^2 \Rightarrow AB^2 = 169 - 25 = 144 \Rightarrow AB = 12$ cm $\Rightarrow CE = 12$ cm

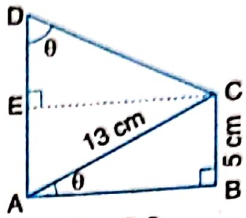


Fig. 9.9

In right triangle CED, we obtain

$$\tan\theta = \frac{CE}{DE} = \frac{12}{9} = \frac{4}{3}$$

EXAMPLE 13 *If* $\tan\theta = \frac{4}{5}$, *then the value of* $\frac{5\sin\theta - 2\cos\theta}{5\sin\theta + 2\cos\theta}$ *is*

(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) 6

Ans. (a)

SOLUTION We have, $\tan\theta = \frac{4}{5}$.

Dividing numerator and denominator by $\cos\theta$, we obtain

$$\frac{5\sin\theta - 2\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{\frac{5\sin\theta}{\cos\theta} - \frac{2\cos\theta}{\cos\theta}}{\frac{5\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}} = \frac{5\tan\theta - 2}{5\tan\theta + 2} = \frac{5\times\frac{4}{5} - 2}{5\times\frac{4}{5} + 2} = \frac{4-2}{4+2} = \frac{1}{3}$$

EXAMPLE 14 *If* $2\tan A = 3$, *then the value of* $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A}$ *is*

(a) $\frac{7}{\sqrt{13}}$ (b) $\frac{1}{\sqrt{13}}$ (c) 3 (d) *does not exist*

Ans. (c) [CBSE 2023]

SOLUTION We have, $2\tan A = 3 \Rightarrow \tan A = \frac{3}{2}$

$$\therefore \quad \frac{4\sin A + 3\cos A}{4\sin A - 3\cos A} = \frac{\frac{4\sin A + 3\cos A}{\cos A}}{\frac{4\sin A - 3\cos A}{\cos A}} \quad \text{[Dividing numerator and denominator by } \cos A\text{]}$$

$$= \frac{4\tan A + 3}{4\tan A - 3} = \frac{4\times\frac{3}{2} + 3}{4\times\frac{3}{2} - 3} = \frac{9}{3} = 3$$

EXAMPLE 15 *In Fig. 9.10, ABCD is an isosceles trapezium, its perimeter is*

(a) $(8 + 4\sqrt{2})$ units (b) $(10 + 2\sqrt{2})$ units (c) $(10 + 4\sqrt{2})$ units (d) $(11 + 4\sqrt{2})$ units

Ans. (c)

SOLUTION In ΔAED, we obtain

$$\tan 45^\circ = \frac{AE}{DE} \Rightarrow 1 = \frac{2}{DE} \Rightarrow ED = 2 \Rightarrow CF = 2$$

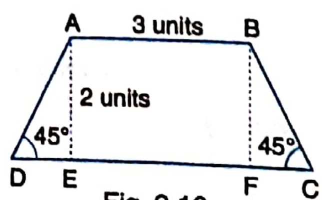


Fig. 9.10

Again in ΔAED, we obtain

$$\sin 45^\circ = \frac{AE}{AD} \Rightarrow \frac{1}{\sqrt{2}} = \frac{2}{AD} \Rightarrow AD = 2\sqrt{2}$$

$CD = CF + EF + ED = 2 + 3 + 2 = 7$

$\therefore$ Perimeter $= AB + BC + CD + AD = 3 + 2\sqrt{2} + 7 + 2\sqrt{2} = (10 + 4\sqrt{2})$ units

EXAMPLE 16 *In Fig. 9.11, AM = MC and* $\angle C$ *is a right angle, then* $\sin^2\alpha - \cos^2\alpha$ *is equal to*

(a) $\dfrac{4b^2 - 3a^2}{5a^2 - 4b^2}$ (b) $\dfrac{5a^2 - 4b^2}{4b^2 - 3a^2}$ (c) $\dfrac{4a^2 - 5b^2}{3b^2 - 4a^2}$ (d) $\dfrac{3b^2 - 4a^2}{4a^2 - 5b^2}$

Ans. (b)

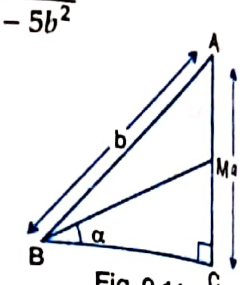


Fig. 9.11

SOLUTION Applying Pythagoras Theorem in right triangle ABC, we obtain

$$AB^2 = AC^2 + BC^2 \Rightarrow b^2 = a^2 + BC^2 \Rightarrow BC = \sqrt{b^2 - a^2}$$

Thus, in right triangle BCM, we obtain: $BC = \sqrt{b^2 - a^2}$ and $CM = a/2$.

Applying Pythagoras Theorem in ΔBCM, we obtain

$$BM^2 = BC^2 + CM^2 \Rightarrow BM^2 = b^2 - a^2 + \frac{a^2}{4} = \frac{4b^2 - 3a^2}{4} \Rightarrow BM = \frac{\sqrt{4b^2 - 3a^2}}{2}$$

In ΔBCM, we obtain

$$\sin\alpha = \frac{CM}{BM} = \frac{a/2}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{a}{\sqrt{4b^2 - 3a^2}} \text{ and } \cos\alpha = \frac{BC}{BM} = \frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}$$

$$\therefore \quad \sin^2\alpha - \cos^2\alpha = \frac{a^2}{4b^2 - 3a^2} - \frac{4(b^2 - a^2)}{4b^2 - 3a^2} = \frac{5a^2 - 4b^2}{4b^2 - 3a^2}$$

EXAMPLE 17 *In Fig. 9.12, the value of DE is*

(a) $5\sqrt{2}$ units (b) 10 units (c) $10\sqrt{2}$ units (d) $15\sqrt{2}$ units

Ans. (c)

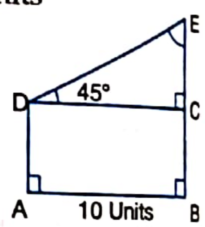


Fig. 9.12

SOLUTION Clearly, $CD = AB = 10$ units. In right triangle DCE, we obtain

$$\tan 45° = \frac{CE}{CD} \Rightarrow 1 = \frac{CE}{10} \Rightarrow CE = 10 \text{ units}$$

Again in ΔDCE, we obtain

$$\sin 45° = \frac{CE}{DE} \Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{DE} \Rightarrow DE = 10\sqrt{2} \text{ units}$$

EXAMPLE 18 *A pendulum of length* $\sqrt{3}$ *m is attached to a point 2.3 m from the ground. It swings through an angle of 30° on each side of the vertical. The height above the ground at ends of its path is*

(a) 0.9 m (b) 0.6 m (c) 0.7 m (d) 0.8 m

Ans. (d)

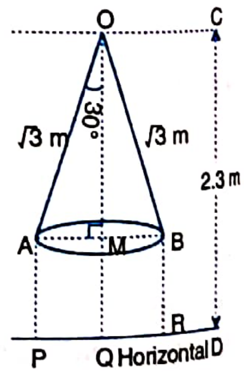


Fig. 9.13

SOLUTION In right triangle AMO, we obtain

$$\cos 30° = \frac{OM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{\sqrt{3}} \Rightarrow OM = \frac{3}{2} \text{ m} = 1.5 \text{ m}$$

$\therefore \quad AP = BR = MQ$

$\Rightarrow \quad AP = BR = OQ - OM = 2.3 \text{ m} - 1.5 \text{ m} = 0.8 \text{ m}$

EXAMPLE 19 *Given that* $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$ *and* $\cos(A + 4B) = 0$, *where A and B are acute angles. The value of A is*

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. (a)

SOLUTION We have, $\sin(A+2B)=\frac{\sqrt{3}}{2}$ and $\cos(A+4B)=0$

$\sin(A+2B)=\sin 60°$ and $\cos(A+4B)=\cos 90°$

$\therefore$ $A+2B=60°$ and $A+4B=90°$

$\Rightarrow$ $2A+4B=120°$ and $A+4B=90° \Rightarrow (2A+4B)-(A+4B)=120°-90° \Rightarrow A=30°$

$\Rightarrow$

EXAMPLE 20 $\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°$ *is equal to*

(a) -1 (b) $\frac{5}{6}$ (c) $-\frac{3}{2}$ (d) $\frac{1}{6}$

[CBSE 2023]

Ans. (a)

SOLUTION $\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°$

$$=\frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2-\left(\sqrt{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}\times\frac{1}{3}-2+\frac{3}{4}=1-2=-1$$

EXAMPLE 21 *If* $\sin\alpha=\frac{\sqrt{3}}{2}$ *and* $\cos\beta=0$, *then the value of* $\tan(\beta-\alpha)$ *is*

(a) 1 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (c)

SOLUTION We have,

$\sin\alpha=\frac{\sqrt{3}}{2}$ and $\cos\beta=0 \Rightarrow \sin\alpha=\sin 60°$ and $\cos\beta=\cos 90° \Rightarrow \alpha=60°$ and $\beta=90°$

$\therefore$ $\tan(\beta-\alpha)=\tan(90°-60°)=\tan 30°=1/\sqrt{3}$

EXAMPLE 22 *If* $\sin\theta+\cos\theta=\sqrt{2}\cos\theta,\ \theta\neq 90°$, *then* $\tan\theta=$

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

Ans. (a)

SOLUTION We have,

$$\sin\theta+\cos\theta=\sqrt{2}\cos\theta \Rightarrow \sin\theta=(\sqrt{2}-1)\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta}=\sqrt{2}-1 \Rightarrow \tan\theta=\sqrt{2}-1$$

EXAMPLE 23 *In a* ΔABC, *right angled at B, the value of* $\sin(A+C)$ *is*

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (b)

SOLUTION In ΔABC, it is given that $\angle B=90°$

$\therefore$ $A+B+C=180° \Rightarrow A+90°+C=180° \Rightarrow A+C=90° \Rightarrow \sin(A+C)=\sin 90°=1$

EXAMPLE 24 *If* $\sin\theta-\cos\theta=0$, *then the value of* $\sin^4\theta+\cos^4\theta$ *is*

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Ans. (b)

SOLUTION We have,

$\therefore$ $\sin\theta-\cos\theta=0 \Rightarrow \sin\theta=\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta}=1 \Rightarrow \tan\theta=1 \Rightarrow \theta=45°$

$\therefore$ $\sin^4\theta+\cos^4\theta=(\sin 45°)^4+(\cos 45°)^4=\left(\frac{1}{\sqrt{2}}\right)^4+\left(\frac{1}{\sqrt{2}}\right)^4=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

EXAMPLE 25 *$\sec\theta$ when expressed in terms of $\cot\theta$, is equal to*

(a) $\dfrac{1+\cot^2\theta}{\cot\theta}$ (b) $\sqrt{1+\cot^2\theta}$ (c) $\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$ (d) $\dfrac{\sqrt{1-\cot^2\theta}}{\cot\theta}$

[CBSE 2023]

Ans. (c)

SOLUTION $1+\cot^2\theta = \operatorname{cosec}^2\theta \Rightarrow \sqrt{1+\cot^2\theta} = \operatorname{cosec}\theta$

$$\therefore \quad \frac{\sqrt{1+\cot^2\theta}}{\cot\theta} = \frac{\operatorname{cosec}\theta}{\cot\theta} = \frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$

EXAMPLE 26 *If* $\tan\theta = \frac{5}{12}$, *then the value of* $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$ *is*

(a) $-\dfrac{17}{7}$ (b) $\dfrac{17}{7}$ (c) $\dfrac{17}{13}$ (d) $-\dfrac{7}{13}$

[CBSE 2023]

Ans. (a)

SOLUTION $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \dfrac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}$ [Dividing the numerator and denominator by $\cos\theta$]

$$= \frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1} = \frac{\tan\theta+1}{\tan\theta-1} = \frac{\frac{5}{12}+1}{\frac{5}{12}-1} = \frac{17}{-7} = -\frac{17}{7}$$

EXAMPLE 27 *If* $\tan(A+B) = \sqrt{3}$ *and* $\tan(A-B) = \frac{1}{\sqrt{3}}$, *$A > B$, then the value of A is*

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. (b)

SOLUTION We have,

$$\tan(A+B) = \sqrt{3} \text{ and } \tan(A-B) = \frac{1}{\sqrt{3}} \Rightarrow \tan(A+B) = \tan 60^\circ \text{ and } \tan(A-B) = \tan 30^\circ$$

$$\Rightarrow \quad A+B = 60^\circ \text{ and } A-B = 30^\circ \Rightarrow (A+B)+(A-B) = 60^\circ+30^\circ \Rightarrow 2A = 90^\circ \Rightarrow A = 45^\circ$$

EXAMPLE 28 $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ *is equal to*

(a) $\sin 60^\circ$ (b) $\cos 60^\circ$ (c) $\tan 60^\circ$ (d) $\sin 30^\circ$

[CBSE 2023]

Ans. (a)

SOLUTION $$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

EXAMPLE 29 *In Fig. 9.14, lengths of sides BC and AB are respectively*

(a) 12 cm, $3\sqrt{3}$ cm (b) 3 cm, $3\sqrt{3}$ cm (c) 12 cm, $6\sqrt{3}$ cm (d) 18 cm, $9\sqrt{3}$ cm

Ans. (b)

SOLUTION In ΔABC, we have

$$\sin 30^\circ = \frac{BC}{AC} \text{ and } \cos 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{BC}{6} \text{ and } \frac{\sqrt{3}}{2} = \frac{AB}{6} \Rightarrow BC = 3 \text{ cm}, AB = 3\sqrt{3} \text{ cm}$$

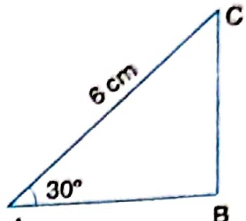


Fig. 9.14

EXAMPLE 30 *In an acute angled triangle ABC, if* $\sin(A + B - C) = \frac{1}{2}$ *and* $\cos(B + C - A) = \frac{1}{\sqrt{2}}$. *Then measure of angle B is*

(a) $37\frac{1}{2}^\circ$ (b) 45° (c) 75° (d) 62.5°

Ans. (a)

SOLUTION We have, $\sin(A + B - C) = \frac{1}{2}$ and, $\cos(B + C - A) = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin(A + B - C) = \sin 30^\circ$ and $\cos(B + C - A) = \cos 45^\circ$

$\Rightarrow \quad A + B - C = 30^\circ$ and $B + C - A = 45^\circ \Rightarrow (A + B - C) + (B + C - A) = 30^\circ + 45^\circ$

$\Rightarrow \quad 2B = 75^\circ \Rightarrow B = 37\frac{1}{2}^\circ$

EXAMPLE 31 *In a* ΔABC, *if* $\angle B = 90^\circ$, $BC = 5$ *cm,* $AC - AB = 1$ *cm. Then the value of* $\frac{1 + \sin C}{1 + \cos C}$ *is*

(a) $\frac{18}{25}$ (b) $\frac{36}{31}$ (c) $\frac{25}{18}$ (d) $\frac{31}{36}$

Ans. (c)

SOLUTION Let $AB = x$ cm. Then, $AC - AB = 1$ cm gives $AC = (x + 1)$ cm.

Applying Pythagoras Theorem in ΔABC, we obtain

$$AC^2 = AB^2 + BC^2 \Rightarrow (x+1)^2 = x^2 + 25 \Rightarrow 2x + 1 = 25 \Rightarrow x = 12$$

$\therefore \quad AB = 12$ cm and $AC = 13$ cm

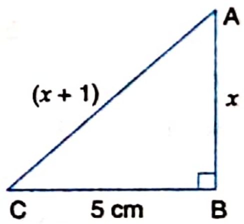


Fig. 9.15

Thus, $\sin C = \frac{AB}{AC} = \frac{12}{13}$ and $\cos C = \frac{BC}{AC} = \frac{5}{13}$

Hence, $$\frac{1 + \sin C}{1 + \cos C} = \frac{1 + \frac{12}{13}}{1 + \frac{5}{13}} = \frac{25}{18}$$

ASSERTION-REASON BASED MCQs

Each of the following examples contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 and Statement-2 are True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 and Statement-2 are True; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is True, Statement-2 is False.
(d) Statement-1 is False, Statement-2 is True.

EXAMPLE 32 *Statement-1 (A): In Fig. 9.16, the trigonometric ratios of angle* θ *depend only on the value of* θ *and are independent of the position of the point P on the terminal side AY of angle* θ.

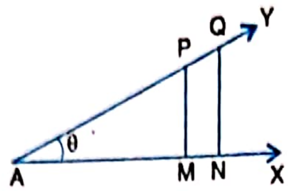


Fig. 9.16

Statement-2 (R): In a right triangle ABC right angled at B, if $\angle BAC = \theta$, *then* $\sin\theta = \frac{BC}{AC} < 1$ *and* $\cos\theta = \frac{AB}{AC} < 1$ *because the hypotenuse AC is the longest side.*

Ans. (b)

SOLUTION Statement-1 is true (see Theorem on page 462 of the main book). Statement-2 is also true but it is not a correct explanation for statement-1. Hence, option (b) is correct.

EXAMPLE 33 *Statement-1 (A): For any acute angle* θ, *the value of* $\sin\theta$ *cannot be greater than 1.*

Statement-2 (R): Hypotenuse is the longest side in any right angled triangle.

Ans. (a)

SOLUTION Both statements are true and statement-2 is the correct explanation for statement-1, because $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} < 1$.

EXAMPLE 34 *Statement-1 (A): For* $0 \leq \theta < 90°$, $\sec x + \cos x \geq 2$.

Statement-2 (R): For any $x > 0, x + \frac{1}{x} \geq 2$.

Ans. (a)

SOLUTION For any $x > 0$, we find that

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \geq 0 \Rightarrow x + \frac{1}{x} - 2 \geq 0 \Rightarrow x + \frac{1}{2} \geq 2$$

So, statement-2 is true. Since, $\sec x = \frac{1}{\cos x}$. Therefore,

$$\sec x + \cos x = \cos x + \frac{1}{\cos x} \geq 2$$

So, statement-1 is also true and statement-2 is the correct explanation for statement-1. Hence, option (a) is correct.

PRACTICE EXERCISES

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. If $\sin\theta = x$ and $\sec\theta = y$, then $\tan\theta$ is equal to

   (a) xy (b) x/y (c) y/x (d) $1/xy$

2. Given that $\sin\theta = \frac{a}{b}$, then $\tan\theta$ is equal to

   (a) $\frac{b}{\sqrt{a^2+b^2}}$ (b) $\frac{b}{\sqrt{b^2-a^2}}$ (c) $\frac{a}{\sqrt{a^2-b^2}}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

   [CBSE Sample Paper 2023]

3. If $4\tan\beta = 3$, then $\dfrac{4\sin\beta - 3\cos\beta}{4\sin\beta + 3\cos\beta} =$

(a) 0 (b) 1/3 (c) 2/3 (d) 7/25

4. If ΔABC right angled at B. If $\tan A = \sqrt{3}$, then $\cos A\cos C - \sin A\sin C = 0$

(a) −1 (b) 0 (c) 1 (d) $\sqrt{3}/2$

5. If the angle of ΔABC are in the ratio 1 : 1 : 2 respectively (the largest angle being angle C), then the value of $\dfrac{\sec A}{\operatorname{cosec} B} - \dfrac{\tan A}{\cot B}$ is

(a) 0 (b) 1/2 (c) 1 (d) $\sqrt{3}/2$

6. If θ is an acute angle such that $\cos\theta = \dfrac{3}{5}$, then $\dfrac{\sin\theta\tan\theta - 1}{2\tan^2\theta} =$

(a) $\dfrac{16}{625}$ (b) $\dfrac{1}{36}$ (c) $\dfrac{3}{160}$ (d) $\dfrac{160}{3}$

7. If $\tan\theta = \dfrac{a}{b}$, then $\dfrac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is equal to

(a) $\dfrac{a^2+b^2}{a^2-b^2}$ (b) $\dfrac{a^2-b^2}{a^2+b^2}$ (c) $\dfrac{a+b}{a-b}$ (d) $\dfrac{a-b}{a+b}$

8. If $5\tan\theta - 4 = 0$, then the value of $\dfrac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is

(a) $\dfrac{5}{3}$ (b) $\dfrac{5}{6}$ (c) 0 (d) $\dfrac{1}{6}$

9. If $16\cot x = 12$, then $\dfrac{\sin x - \cos x}{\sin x + \cos x}$ equals

(a) $\dfrac{1}{7}$ (b) $\dfrac{3}{7}$ (c) $\dfrac{2}{7}$ (d) 0

10. If $8\tan x = 15$, then $\sin x - \cos x$ is equal to

(a) $\dfrac{8}{17}$ (b) $\dfrac{17}{7}$ (c) $\dfrac{1}{17}$ (d) $\dfrac{7}{17}$

11. If $\tan\theta = \dfrac{1}{\sqrt{7}}$, then $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} =$

(a) $\dfrac{5}{7}$ (b) $\dfrac{3}{7}$ (c) $\dfrac{1}{12}$ (d) $\dfrac{3}{4}$

12. If $\tan\theta = \dfrac{3}{4}$, then $\cos^2\theta - \sin^2\theta =$

(a) $\dfrac{7}{25}$ (b) 1 (c) $\dfrac{-7}{25}$ (d) $\dfrac{4}{25}$

13. If θ is an acute angle such that $\tan^2\theta = \dfrac{8}{7}$, then the value of $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is

(a) $\dfrac{7}{8}$ (b) $\dfrac{8}{7}$ (c) $\dfrac{7}{4}$ (d) $\dfrac{64}{49}$

14. If $3\cos\theta = 5\sin\theta$, then the value of $\dfrac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is

(a) $\dfrac{271}{979}$ (b) $\dfrac{316}{2937}$ (c) $\dfrac{542}{2937}$ (d) none of these

15. If $\tan^2 45° - \cos^2 30° = x\sin 45°\cos 45°$, then $x =$

(a) 2 (b) −2 (c) $-\dfrac{1}{2}$ (d) $\dfrac{1}{2}$

16. If $\dfrac{x\,\text{cosec}^2 30^\circ \sec^2 45^\circ}{8\cos^2 45^\circ \sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$, then $x =$

(a) 1 (b) -1 (c) 2 (d) 0

17. If $x \tan 45^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$, then x is equal to

(a) 1 (b) $\sqrt{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

18. If angles A, B, C of a ΔABC form an increasing AP, then $\sin B =$

(a) $\dfrac{1}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) 1 (d) $\dfrac{1}{\sqrt{2}}$

19. If θ is an acute angle such that $\sec^2\theta = 3$, then the value of $\dfrac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta}$ is

(a) $\dfrac{4}{7}$ (b) $\dfrac{3}{7}$ (c) $\dfrac{2}{7}$ (d) $\dfrac{1}{7}$

20. $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ is equal to

(a) $\sin 60^\circ$ (b) $\cos 60^\circ$ (c) $\tan 60^\circ$ (d) $\sin 30^\circ$

[NCERT, CBSE 2023]

21. $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ is equal to

(a) $\tan 90^\circ$ (b) 1 (c) $\sin 45^\circ$ (d) $\sin 0^\circ$

[NCERT]

22. $\sin 2A = 2\sin A$ is true when $A =$

(a) 0° (b) 30° (c) 45° (d) 60°

[NCERT]

23. $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ is equal to

(a) $\cos 60^\circ$ (b) $\sin 60^\circ$ (c) $\tan 60^\circ$ (d) $\sin 30^\circ$

[NCERT]

24. If $\cos\theta = \dfrac{2}{3}$, then $2\sec^2\theta + 2\tan^2\theta - 7$ is equal to

(a) 1 (b) 0 (c) 3 (d) 4

25. In Fig. 9.17, the value of $\cos\phi$ is

(a) $\dfrac{5}{4}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{5}$

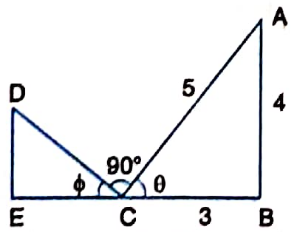


Fig. 9.17

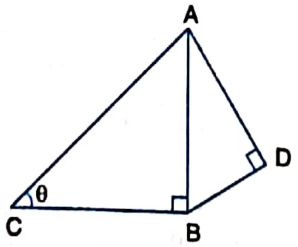


Fig. 9.18

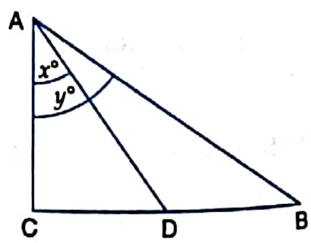


Fig. 9.19

26. In Fig. 9.18, if $AD = 4$ cm $BD = 3$ cm and $CB = 12$ cm, then $\cot\theta =$

(a) $\dfrac{12}{5}$ (b) $\dfrac{5}{12}$ (c) $\dfrac{13}{12}$ (d) $\dfrac{12}{13}$

[CBSE 2008]

27. In Fig. 9.19, if D is the mid-point of BC, then the value of $\frac{\cot y°}{\cot x°}$ is

(a) 2 (b) 1/2 (c) 1/3 (d) 3/4

CASE STUDY BASED MCQs

28. In structural design a structure is composed of triangles that are interconnecting. A truss is one of the major types of engineering structures and is especially used in the design of bridges and buildings. Trusses are designed to support loads, such as the weight of people. A truss is exclusively made of long, straight members connected by joints at the end of each member.

Fig. 9.20

This is a single repeating triangle in a truss system.

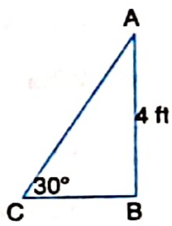


Fig. 9.21

(i) In above triangle, what is the length of AC?

(a) 5 ft (b) 6 ft (c) 8 ft (d) $\frac{8}{\sqrt{3}}$ ft

(ii) What is the length of BC?

(a) $\frac{4}{\sqrt{3}}$ ft (b) $4\sqrt{3}$ ft (c) 8 ft (d) $8\sqrt{3}$ ft

(iii) If $\sin A = \sin C$, what will be the length of BC?

(a) 2 ft (b) 4 ft (c) 8 ft (d) $4\sqrt{2}$ ft

(iv) Which of the following relation will be true in the triangle?

(a) $\sin\left(\frac{A+C}{2}\right) = \cos\left(\frac{B}{2}\right)$ (b) $\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{C}{2}\right)$

(c) $\cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{C}{2}\right)$ (d) $\cos\left(\frac{A-B}{2}\right) = \cos\left(\frac{C}{2}\right)$

(v) If the length of AB doubles what will happen to the length of AC?

(a) remains same (b) doubles the original length

(c) become three times the original length (d) become half of the original length

29. A trolley carries passengers from the ground level located at point A to up to the top of mountain chateau located at P as shown in Fig. 9.22. The point A is at a distance of 2000 m from point C at the base of mountain. Here $\alpha = 30°, \beta = 60°$.

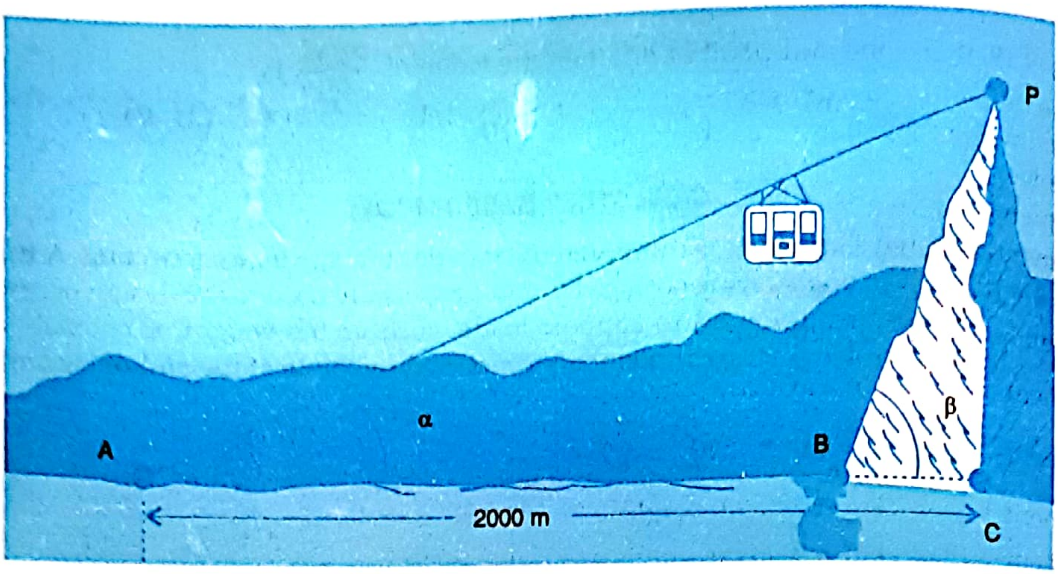


Fig. 9.22

(i) Assuming the cable is held tight what will be the length of cable?

(a) 2000 m (b) $2000\sqrt{3}$ m (c) $4000\sqrt{3}$ m (d) $\frac{4000}{\sqrt{3}}$ m

(ii) What will be height of the mountain?

(a) 1000 m (b) $\frac{2000}{\sqrt{3}}$ m (c) 2000 m (d) $2000\sqrt{3}$ m

(iii) What will be the slant height of the mountain?

(a) 4000 m (b) $\frac{4000}{3}$ m (c) $4000\sqrt{3}$ m (d) $\frac{4000}{\sqrt{3}}$ m

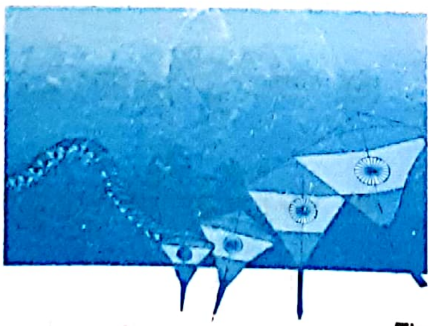
(iv) What will be the length of BC?

(a) 1000 m (b) $\frac{2000}{3}$ m (c) $1000\sqrt{3}$ m (d) $\frac{1000}{\sqrt{3}}$ m

(v) What will be the distance of point A to the foot of the mountain located at B?

(a) $4000\sqrt{3}$ m (b) $4000\sqrt{6}$ m (c) $\frac{4000}{\sqrt{3}}$ m (d) $\frac{4000}{3}$ m

30. Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites. The picture given below, shows kites flying together.



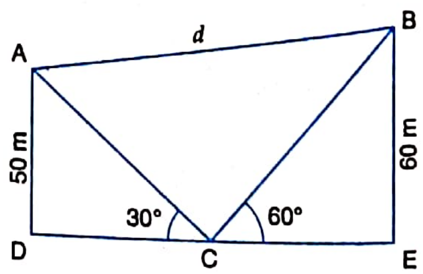


Fig. 9.23

In Fig. 9.23, the angles of elevation of two kites (Points A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking $AD = 50$ m and $BE = 60$ m, find

(i) the lengths of strings used (take them straight) for kites A and B as shown in the figure.

(ii) the distance 'd' between these two kites.

[CBSE 2022]

ASSERTION-REASON BASED MCQs

Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

31. Statement-1 (A): For any acute angle θ, values of $\tan\theta$ never exceeds $\sqrt{3}$.

Statement-2 (R): For $0 \le \theta < 90^\circ$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

32. Statement-1 (A): For any acute angle $\theta\,(0 \le \theta < 90^\circ)$, $\sec\theta \ge 1$

Statement-2 (R): For any acute angle $\theta\,(0 < \theta \le 90^\circ)$, $\operatorname{cosec}\theta \ge 1$

33. Statement-1 (A): For $0 < \theta \le 90^\circ$, $\sin\theta + \operatorname{cosec}\theta \ge 2$.

Statement-2 (R): $x + \dfrac{1}{x} \ge 2$ for all $x > 0$.

ANSWERS

1. (a)	2. (d)	3. (a)	4. (b)	5. (a)	6. (c)	7. (a)
8. (c)	9. (a)	10. (d)	11. (d)	12. (a)	13. (a)	14. (a)
15. (d)	16. (a)	17. (a)	18. (b)	19. (d)	20. (a)	21. (d)
22. (a)	23. (c)	24. (b)	25. (d)	26. (a)	27. (b)	
28. (i) (c)	(ii) (b)	(iii) (b)	(iv) (a)	(v) (b)		
29. (i) (d)	(ii) (b)	(iii) (b)	(iv) (b)	(v) (d)		

30. (i) 100 m, $40\sqrt{3}$ m (ii) 121.65 m

31. (d)	32. (b)	33. (a)