

# MATHEMATICS

## WORKSHEET\_010225 - CHAPTER 10 CIRCLES (2025-26)

### (ANSWERS)

**SUBJECT: MATHEMATICS**

**MAX. MARKS : 40**

**CLASS : X**

**DURATION : 1½ hrs**

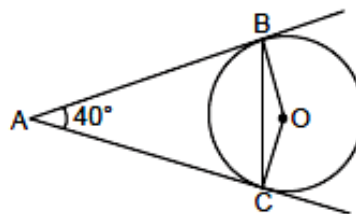
**General Instructions:**

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

### SECTION – A

Questions 1 to 10 carry 1 mark each.

1. In the given figure, AB and AC are tangents to the circle with centre O such that  $\angle BAC = 40^\circ$ , then  $\angle BOC$  is equal to



- (a)  $40^\circ$                       (b)  $50^\circ$                       (c)  $140^\circ$                       (d)  $150^\circ$

Ans. (c)  $140^\circ$

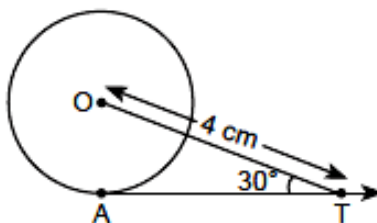
In quadrilateral ABOC

$$\angle ABO + \angle BOC + \angle OCA + \angle BAC = 360^\circ$$

$$\Rightarrow 90^\circ + \angle BOC + 90^\circ + 40^\circ = 360^\circ$$

$$\Rightarrow \angle BOC = 360^\circ - 220^\circ = 140^\circ$$

2. In figure AT is a tangent to the circle with centre O such that  $OT = 4$  cm and  $\angle OTA = 30^\circ$ . Then AT is equal to



- (a) 4 cm                      (b) 2 cm                      (c)  $2\sqrt{3}$  cm                      (d)  $4\sqrt{3}$  cm

Ans.

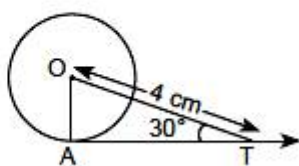
$\angle OAT = 90^\circ$  [ $\because$  Tangent and radius are  $\perp$  to each other at the point of contact]

In right-angled  $\triangle OAT$ ,

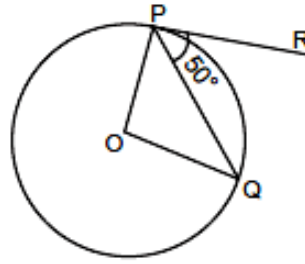
$$\frac{AT}{OT} = \cos 30^\circ$$

$$\Rightarrow \frac{AT}{4} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AT = 2\sqrt{3} \text{ cm.}$$



3. In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of  $50^\circ$  with PQ, then  $\angle POQ$  is equal to



- (a)  $100^\circ$  (b)  $80^\circ$  (c)  $90^\circ$  (d)  $75^\circ$

Ans. (a)  $100^\circ$

$OP \perp PR$  [ $\because$  Tangent and radius are  $\perp$  to each other at the point of contact]

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

$OP = OQ$  [Radii]

$$\therefore \angle OPQ = \angle OQP = 40^\circ$$

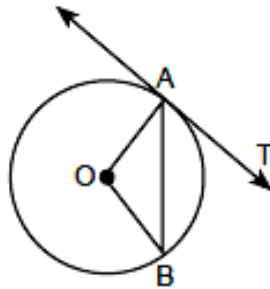
In  $\triangle OPQ$ ,

$$\Rightarrow \angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$\Rightarrow \angle POQ + 40^\circ + 40^\circ = 180^\circ$$

$$\angle POQ = 180^\circ - 80^\circ = 100^\circ.$$

4. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If  $\angle AOB = 100^\circ$ , then  $\angle BAT$  is equal to



- (a)  $100^\circ$  (b)  $40^\circ$  (c)  $50^\circ$  (d)  $90^\circ$

Ans. (c)  $50^\circ$

$$\angle AOB = 100^\circ$$

$\angle OAB = \angle OBA$  ( $\because$  OA and OB are radii)

Now, in  $\triangle AOB$ ,  $\angle AOB + \angle OAB + \angle OBA = 180^\circ$  (Angle sum property of  $\triangle$ )

$$\Rightarrow 100^\circ + x + x = 180^\circ$$

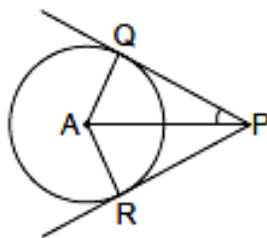
[Let  $\angle OAB = \angle OBA = x$ ]

$$\Rightarrow 2x = 180^\circ - 100^\circ \Rightarrow 2x = 80^\circ \Rightarrow x = 40^\circ$$

Also,  $\angle OAB + \angle BAT = 90^\circ$  [ $\because$  OA is radius and TA is tangent at A]

$$\Rightarrow 40^\circ + \angle BAT = 90^\circ \Rightarrow \angle BAT = 50^\circ$$

5. In figure, PQ and PR are tangents to a circle with centre A. If  $\angle QPA = 27^\circ$ , then  $\angle QAR$  equals to



- (a)  $63^\circ$  (b)  $153^\circ$  (c)  $126^\circ$  (d)  $117^\circ$

Ans. (c)  $126^\circ$

$$\angle QPA = \angle RPA [\because \triangle AQP \cong \triangle ARP \text{ (RHS congruence rule)}]$$

$$\Rightarrow \angle RPA = 27^\circ$$

$$\therefore \angle QPR = \angle QPA + \angle RPA = 27^\circ + 27^\circ = 54^\circ$$

$$\text{Now, } \angle QAR + \angle AQP + \angle ARP + \angle QPR = 360^\circ$$

$$\Rightarrow \angle QAR = 90^\circ + 90^\circ + 54^\circ = 360^\circ$$

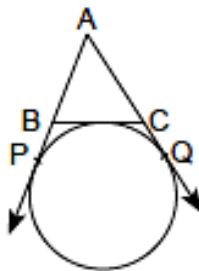
$$\Rightarrow \angle QAR = 360^\circ - 234^\circ = 126^\circ$$

6. At one end A of diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is:

(a) 4 cm (b) 6 cm (c) 8 cm (d) 5 cm

Ans. (c) 8 cm

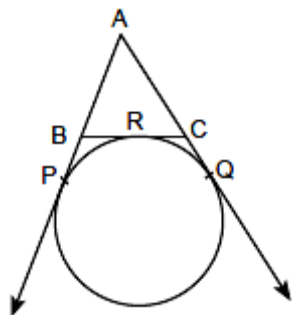
7. In figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is



(a) 7.5 (b) 15 (c) 10 (d) 9

Ans. (a) 7.5

$$AP = AQ$$



$$\Rightarrow AB + BP = AC + CQ \Rightarrow 5 + BP = 6 + CQ$$

$$\Rightarrow BP = 1 + CQ \Rightarrow BP = 1 + CR \quad (\because CQ = CR)$$

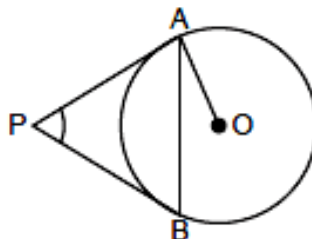
$$\Rightarrow BP = 1 + (BC - BR)$$

$$\Rightarrow BP = 1 + (4 - BP) \quad (\because BR = BP)$$

$$\Rightarrow 2BP = 5 \Rightarrow BP = \frac{5}{2} = 2.5 \text{ cm}$$

$$\text{Now, } AP = AB + BP = 5 + 2.5 = 7.5 \text{ cm}$$

8. In the figure PA and PB are tangents to the circle with centre O. If  $\angle APB = 60^\circ$ , then  $\angle OAB$  is



(a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $15^\circ$

Ans. (a)  $30^\circ$

$$\text{Given } \angle APB = 60^\circ$$

$$\therefore \angle APB + \angle PAB + \angle PBA = 180^\circ$$

$$\Rightarrow \angle APB + x + x = 180^\circ [\because PA = PB \therefore \angle PAB = \angle PBA = x \text{ (say)}]$$

$$\Rightarrow 60^\circ + 2x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 60^\circ \Rightarrow 2x = 120^\circ \Rightarrow x = 60^\circ$$

Also,  $\angle OAP = 90^\circ$

$$\Rightarrow \angle OAB + \angle PAB = 90^\circ$$

$$\Rightarrow \angle OAB + 60^\circ = 90^\circ \Rightarrow \angle OAB = 30^\circ$$

**In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

**9. Assertion (A):** If the angle between two tangents drawn from an external point P to a circle of radius 5 cm and centre O is  $90^\circ$ , then length of each tangent is 10 cm.

**Reason (R):** Opposite angles of a cyclic quadrilateral are supplementary.

Ans. (d) Assertion (A) is false but reason (R) is true.

**10. Assertion (A):** If the radius of a circle is 5 cm and distance of a point outside the circle from its centre is 13 cm, then the length of the tangent drawn from that external point to the circle is 12 cm.

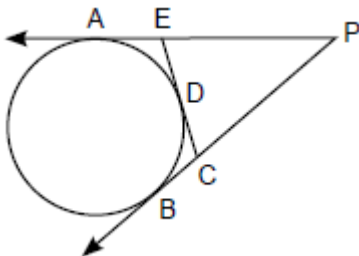
**Reason (R):** In a circle, tangent is always perpendicular to its radius at the point of contact.

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

## SECTION – B

**Questions 11 to 14 carry 2 marks each.**

**11.** In the given figure, PA and PB are tangents to the circle. CE is a tangent to the circle at D. If AP = 15 cm, find the perimeter of the triangle PEC.



Ans.  $PA = PB = 15 \text{ cm}$  (Tangent from P)

Perimeter of  $\triangle PEC = PE + EC + PC$

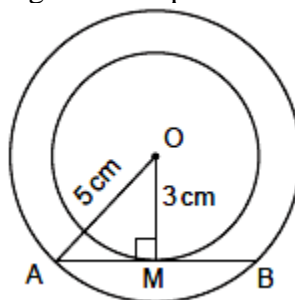
$= PE + ED + DC + PC$

$= PE + EA + CB + PC$  [ED = EA and DC = CB]

$= PA + PB = 15 + 15 = 30 \text{ cm}$

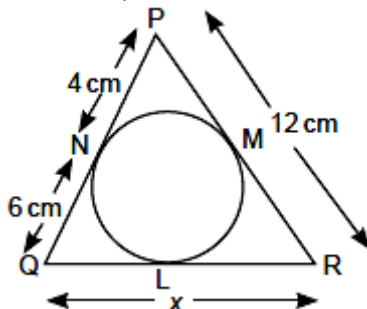
**12.** The radii of two concentric circles are 3 cm and 5 cm. Find the length of the chord of the outer circle which is tangent to the smaller circle.

Ans. Radius is perpendicular to the tangent at the point of contact.



$\therefore (OA)^2 = (AM)^2 + (OM)^2$  [Using Pythagoras Theorem]  
 $\Rightarrow (5)^2 = (AM)^2 + (3)^2$   
 $\Rightarrow 25 = (AM)^2 + 9$   
 $\Rightarrow (AM)^2 = 25 - 9$   
 $\Rightarrow (AM)^2 = 16$   
 $\therefore AM = 4 \text{ cm}$   
 But perpendicular from centre bisects the chord  
 $\therefore AB = 2AM = 2 \times 4 = 8 \text{ cm}$

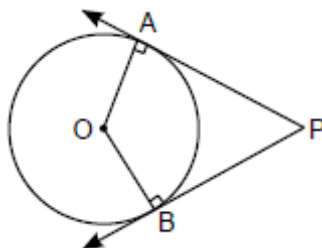
13. In the given figure,  $\Delta PQR$  is circumscribed, find  $x$ .



Ans. Tangents from an external point have equal length.

$\therefore PN = PM = 4 \text{ cm}$   
 $\Rightarrow MR = 12 - 4 = 8 \text{ cm}$   
 $QL = QN = 6 \text{ cm}$   
 and  $RL = RM = 8 \text{ cm}$   
 $\Rightarrow x = QL + RL = 6 \text{ cm} + 8 \text{ cm} = 14 \text{ cm}$

14. In the given figure, O is the centre of the circle, PA and PB are tangent segments. Show that the quadrilateral AOBP is a cyclic quadrilateral.



Ans. Radius is perpendicular to the tangent at the point of contact.

$\therefore \angle A = 90^\circ$  and  $\angle B = 90^\circ$

In quadrilateral APBO,

$\angle A + \angle P + \angle B + \angle AOB = 360^\circ$  [Angle sum property of quadrilateral]

$\Rightarrow 90^\circ + \angle P + 90^\circ + \angle AOB = 360^\circ$

$\angle P + \angle AOB + 180^\circ = 360^\circ$

$\angle P + \angle AOB = 180^\circ$

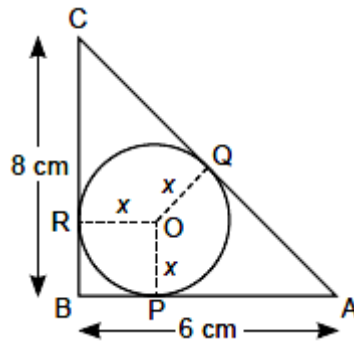
But these are opposite angles of quadrilateral APBO

$\therefore$  AOBP is a cyclic quadrilateral.

## SECTION – C

Questions 15 to 17 carry 3 marks each.

15. ABC is a right-angled triangle in which  $\angle B = 90^\circ$  with  $AB = 6 \text{ cm}$  and  $BC = 8 \text{ cm}$ . A circle with centre O has been inscribed inside the triangle. Find the value of  $x$ .



Ans. **Method I:**

Using Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$$

$$\Rightarrow AC = 10 \text{ cm}$$

Join OA, OB and OC.

$$\text{Then, } ar(\triangle AOB) + ar(\triangle BOC) + ar(\triangle AOC) = ar(\triangle ABC)$$

$$\Rightarrow \frac{1}{2} \times 6x + \frac{1}{2} \times 8x + \frac{1}{2} \times 10x = \frac{1}{2} (6 \times 8) \Rightarrow x = 2 \text{ cm}$$

(As OR, OP and OQ are respectively perpendicular to BC, AB and AC.  $\therefore$  OR, OP and OQ are altitudes)

**Method II:**

Each angle of quadrilateral BROP =  $90^\circ$  and OR = OP

$\Rightarrow$  BROP is a square.

$$\therefore BR = x \Rightarrow CR = (8 - x) \text{ cm}$$

$$\text{and } BP = x \Rightarrow AP = (6 - x) \text{ cm}$$

$$\text{Also, } CQ = CR \quad (\text{Tangents from external point are equal})$$

$$\text{and } AP = AQ \quad (\text{Tangents from external point are equal})$$

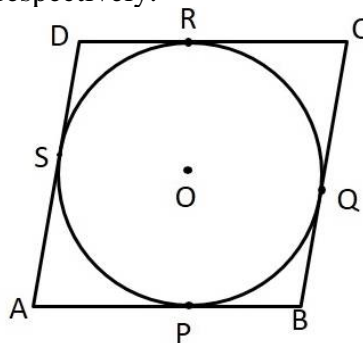
$$AQ + CQ = 10 \text{ cm}$$

$$\Rightarrow 6 - x + 8 - x = 10 \text{ cm}$$

$$\Rightarrow x = 2 \text{ cm}$$

**16. Prove that the parallelogram circumscribing a circle is a rhombus.**

Ans. Consider a parallelogram ABCD circumscribing a circle such that it touches the sides AB, BC, CD and DA at P, Q, R and S respectively.



Now, we know lengths of tangents drawn from an external point to a circle are equal.

$$\therefore AP = AS, BP = BQ, CR = CQ \text{ and } DR = DS$$

Adding the above equations, we get

$$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + CB$$

But AB = CD and AD = CB [Since, opposite sides of parallelogram are equal]

$$\Rightarrow AB + AB = AD + AD$$

$$\Rightarrow 2AB = 2AD \Rightarrow AB = AD$$

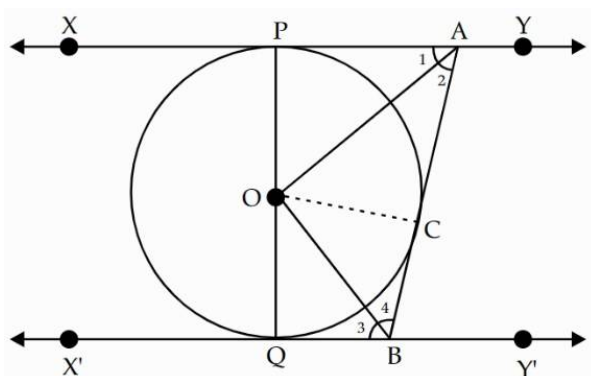
$$\Rightarrow AB = BC = CD = AD$$

Hence, ABCD is a rhombus.

17. Prove that the intercepts of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Ans: In the below figure, Join OC. Since, the tangents drawn to a circle from an external point are equal.

$$\therefore AP = AC$$



In  $\triangle PAO$  and  $\triangle AOC$ , we have:

$$AO = AO \text{ [Common]}$$

$$OP = OC \text{ [Radii of the same circle]}$$

$$AP = AC$$

$$\Rightarrow \triangle PAO \cong \triangle AOC \text{ [SSS Congruency]}$$

$$\therefore \angle PAO = \angle CAO = \angle 1$$

$$\angle PAC = 2 \angle 1 \quad \dots(1)$$

$$\text{Similarly } \angle CBQ = 2 \angle 2 \quad \dots(2)$$

Again, we know that sum of internal angles on the same side of a transversal is  $180^\circ$ .

$$\therefore \angle PAC + \angle CBQ = 180^\circ$$

$$\Rightarrow 2 \angle 1 + 2 \angle 2 = 180^\circ \text{ [From (1) and (2)]}$$

$$\Rightarrow \angle 1 + \angle 2 = 180^\circ / 2 = 90^\circ \quad \dots(3)$$

$$\text{Also } \angle 1 + \angle 2 + \angle AOB = 180^\circ \text{ [Sum of angles of a triangle]}$$

$$\Rightarrow 90^\circ + \angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ \Rightarrow \angle AOB = 90^\circ.$$

## SECTION – D

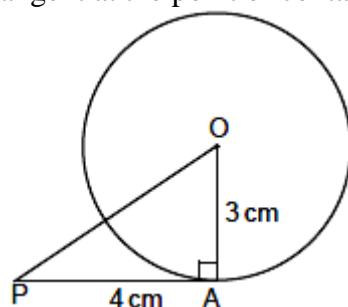
Questions 18 carry 5 marks.

18. (a) Prove that the tangent to a circle is perpendicular to the radius through the point of contact. (3)  
 (b) The length of the tangent to a circle of radius 3 cm is 4 cm from a point P. Find the distance of P from the centre of the circle. (2)

Ans. (a) Given, To prove, Construction and figure of  $1\frac{1}{2}$  marks

Proof of  $1\frac{1}{2}$  marks

- (b) Radius is perpendicular to the tangent at the point of contact.



$$\therefore (OP)^2 = (OA)^2 + (PA)^2$$

$$= (3)^2 + (4)^2 = 9 + 16$$

$$(OP)^2 = 25$$

$$\therefore OP = 5 \text{ cm}$$

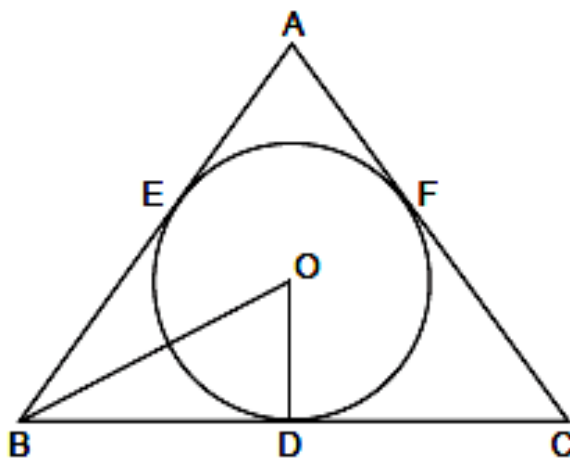
[Using Pythagoras Theorem]

## SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

### 19. Case Study -1:

An ambitious surveyor, tasked with designing a new park, discovered a triangular plot of land. To maximize the usable space, they planned a perfect circular garden at its center. This circular garden, with its center at point O, touches each of the three straight boundary fences, with the line from O to D being perfectly perpendicular to the base fence BC.



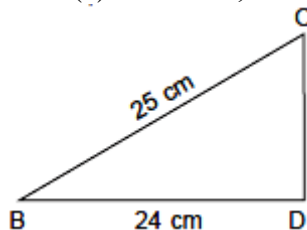
Answer the questions based on above

(a) What will be the radius of the circle, if  $BD = 24$  cm and  $OB = 25$  cm? (1)

(b) Determine CD, if  $OC = 26$  cm. (1)

(c) As AB and AC act as tangents to the circle at E and F and  $AE = 8$  cm, then what is the perimeter of  $\triangle ABC$ . (2)

Ans. (a) In  $\triangle OBD$ ,  $OD \perp BD$



(Radius is perpendicular to tangent)

$$OB^2 = BD^2 + OD^2$$

$$\Rightarrow (25)^2 = (24)^2 + (OD)^2$$

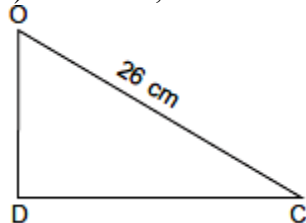
$$\Rightarrow 525 = 576 + OD^2$$

$$\Rightarrow OD^2 = 625 - 576 = 49 \Rightarrow OD = 7 \text{ cm}$$

Radius of circle = 7 cm

(b) In  $\triangle OCD$ ,  $OD \perp CD$

(Radius is perpendicular to tangent)



$$\therefore OC^2 = OD^2 + CD^2$$

$$\Rightarrow (26)^2 = (7)^2 + CD^2$$

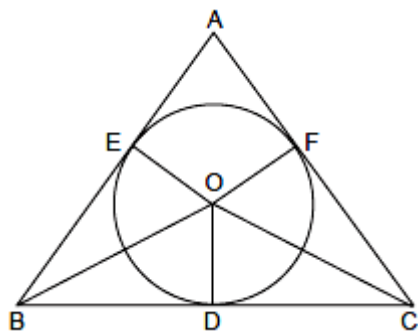
$$\Rightarrow 676 = 49 + CD^2$$

$$\Rightarrow CD^2 = 676 - 49 = 627 \text{ cm}$$

$$\Rightarrow CD = 25.04 \text{ cm}$$

(c)





As  $BD = BE$

$CD = CF \Rightarrow BF = 24 \text{ cm}$

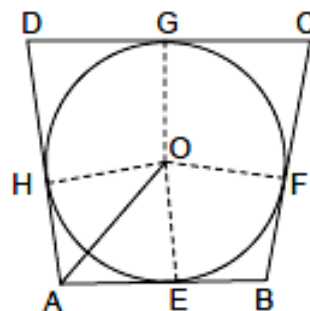
$CF = 25.0 \text{ cm}$

$AE = AF = 8 \text{ cm}$

Perimeter of  $\triangle ABC = AB + BC + AC$   
 $= (AE + BE) + (BD + CD) + (AF + FC)$   
 $= (8 + 24) + (24 + 25.04) + (8 + 25.04)$   
 $= 32 + 49.04 + 33.04 = 114.08 \text{ cm}$

## 20. Case Study – 2:

A community of birds, seeking a safe place to drink and bathe, banded together to build a circular watering hole. They constructed it within the confines of a quadrilateral-shaped area of land, with each of the four sides of their property acting as a perfect boundary for the new tank. The sides AB, BC, CD, and DA measured 5m, 7m, 6m, and an unknown length respectively, each serving as a tangent to their communal bird bath.



Answer the questions based on above.

(a) Find AD. (1)

(b) If O is centre of tank and AH & AE inclined to each other at angle  $100^\circ$ , then find  $\angle HOE$ . (1)

(c) If  $\angle GOF = (3x - 8)^\circ$  and  $\angle GCF = (2x + 3)^\circ$  then find the value of x. (1)

(d) Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm and the radius of the circle is 7 cm. (1)

Ans. (a)  $AB + CD = AD + BC$

$\Rightarrow 5 + 6 = AD + 7 \Rightarrow AD + 7 = 11 \Rightarrow AD = 4 \text{ cm}$

(b)  $\angle HOE + \angle HAE = 180^\circ$

$\Rightarrow \angle HOE + 100^\circ = 180^\circ$

$\Rightarrow \angle HOE = 80^\circ$

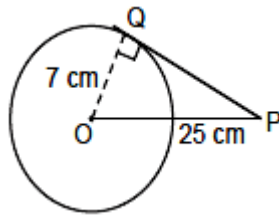
(c)  $\angle GOF + \angle GCF = 180^\circ \Rightarrow 3x - 8 + 2x + 3 = 180^\circ$

$\Rightarrow 5x - 5 = 180^\circ$

$\Rightarrow 5x = 180^\circ + 5^\circ = 185^\circ$

$\Rightarrow x = 37^\circ$

(d) Let O is the centre of the circle and P is a point such that  $OP = 25 \text{ cm}$  and PQ is the tangent to the circle.



$OQ = \text{radius} = 7 \text{ cm}$

In  $\triangle OQP$ , we have  $\angle Q = 90^\circ$

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow (25)^2 = 7^2 + PQ^2$$

$$\Rightarrow PQ^2 = 625 - 49 = 576 \Rightarrow PQ = 24 \text{ cm}$$

Hence, the length of the tangent = 24 cm

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