MATHEMATICS
WORKSHEET_051125
CHAPTER 07 COORDINATE GEOMETRY
(ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : X **DURATION : $1\frac{1}{2}$ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). **Use of Calculators is not permitted**

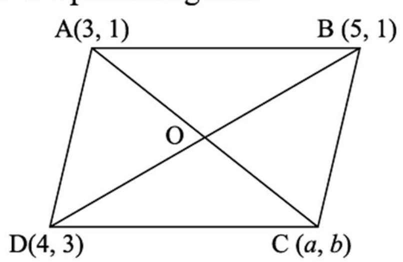
SECTION – A
Questions 1 to 10 carry 1 mark each.

1. Points A(3, 1), B(5, 1), C(a, b) and D(4, 3) are vertices of a parallelogram ABCD. The values of a and b are respectively

(a) a = 6, b = 3 (b) a = 2, b = 1 (c) a = 4, b = 2 (d) None of these

Ans: (a) a = 6, b = 3

ABCD is a parallelogram.



Since, the diagonals of a parallelogram bisect each other.

$$\therefore \left(\frac{3+a}{2}, \frac{1+b}{2}\right) = \left(\frac{4+5}{2}, \frac{3+1}{2}\right)$$

$$\Rightarrow \frac{3+a}{2} = \frac{9}{2} \Rightarrow 3 + a = 9$$

$$\Rightarrow a = 6$$

and $\frac{1+b}{2} = \frac{4}{2} \Rightarrow 1 + b = 4 \Rightarrow b = 3$

Hence, $a = 6$ and $b = 3$.

2. If the distance between the points (x, –1) and (3, 2) is 5, then the value of x is

(a) –7 or –1 (b) –7 or 1 (c) 7 or 1 (d) 7 or –1

Ans: (d) 7 or –1

3. The ratio in which x-axis divides the join of (2, -3) and (5, 6) is:

(a) 1: 2 (b) 3 : 4 (c) 1: 3 (d) 1: 5

Ans: (a) 1 : 2

Let P(x, 0) be the point on x-axis which divides the join of (2, -3) and (5, 6) in the ratio k : 1.

∴ By section formula,

$$P(x, 0) = \left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

$$\Rightarrow y = 0 \Rightarrow \frac{6k-3}{k+1} = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = \frac{1}{2}$$

4. Points A(–1, y) and B(5, 7) lie on a circle with centre O(2, –3y). The values of y are

(a) 1, –7 (b) –1, 7 (c) 2, 7 (d) –2, –7

Ans: (b) –1, 7

5. If C(1, –1) is the mid-point of the line segment AB joining points A(4, x) and B(– 2, 4), then value of x is :

(a) 5 (b) – 5 (c) 6 (d) – 6

Ans. (d) – 6

6. The coordinate of point P on X-axis equidistant from the points A (–1, 0) and B (5, 0) is

(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

Ans: (a) (2, 0)

7. The ratio in which the line segment joining the points P(-3, 10) and Q(6, –8) is divided by O(-1, 6) is:

(a) 1:3 (b) 3:4 (c) 2:7 (d) 2:5

Ans: (c) 2:7

Let k :1 be the ratio in which the line segment joining P(-3, 10) and Q(6, -8) is divided by point O(-1, 6).

By the section formula, we have -1 = (6k – 3)/(k + 1)

⇒ –k – 1 = 6k – 3

⇒ 7k = 2 ⇒ k = 2/7

Hence, the required ratio is 2:7.

8. The vertices of a parallelogram in order are A(1, 2), B(4, y), C(x, 6) and D(3, 5). Then (x, y) is:

(a) (6, 3) (b) (3, 6) (c) (5, 6) (d) (1, 4)

Ans: (a) (6, 3)

∴ Mid-point of diagonal AC = Mid-point of BD

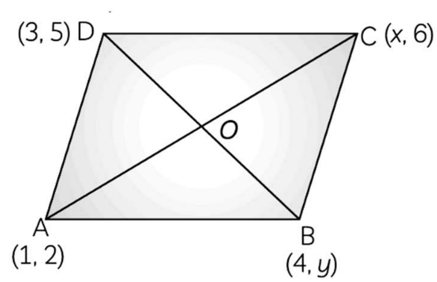
$$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\Rightarrow \frac{1+x}{2} = \frac{4+3}{2}\ ;\ \frac{2+6}{2} = \frac{y+5}{2}$$

$$\Rightarrow x + 1 = 7\ ;\ 8 = y + 5$$

$$\Rightarrow x = 6\ ;\ y = 3$$

$$\therefore (x, y) = (6, 3)$$



In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** The value of y is 3, if the distance between the points P(2, -3) and Q(10, y) is 10.

Reason (R): Distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

For y = 3

Distance PQ $= \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{8^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$ units

10. **Assertion (A):** The point (–1, 6) divides the line segment joining the points (–3, 10) and (6, –8) in the ratio 2 : 7 internally.

Reason (R): Given three points, i.e. A, B, C form an equilateral triangle, then AB = BC = AC.

Ans: (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the ratio in which the line segment joining the points (6, 4) and (1, –7) is divided by x-axis.
Ans. Let x-axis divides the join of (6, 4) and (1, – 7) in the ratio k : 1 at the point (a, 0).

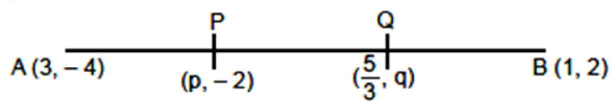
Therefore, $a = \dfrac{k \times (1) + 1 \times 6}{k+1}$

and $0 = \dfrac{k \times (-7) + 1 \times 4}{k+1}$

$\Rightarrow 0 = -7k + 4 \Rightarrow 7k = 4 \Rightarrow k = \dfrac{4}{7}$

Thus, x-axis divides the join of the given points in the ratio 4 : 7.

12. The line segment AB joining the points A(3, –4) and B(1, 2) is trisected at the points P(p, –2) and Q(5/3, q). Find the values of p and q.
Ans: Now AP : PB = 1 : 2



$\therefore\ p = \dfrac{1 \times 1 + 2 \times 3}{1+2} \Rightarrow p = \dfrac{7}{3}$

Also AQ : QB = 2 : 1 $\Rightarrow q = \dfrac{2 \times 2 + 1 \times (-4)}{1+2} = 0$

13. What point on the x-axis is equidistant from (7, 6) and (– 3, 4)?
Ans. Let A(7, 6), B(– 3, 4) be the given points and P(x, 0) be the required point.
Since P is equidistant from A and B, therefore,
$AP = BP \Rightarrow AP^2 = BP^2$
$\Rightarrow (x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2 \Rightarrow x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$
$\Rightarrow -14x - 6x = 25 - 85 \Rightarrow -20x = -60$
$\Rightarrow x = 3$.
$\therefore$ Required point on x-axis is (3, 0).

14. Use distance formula to show that the points A(– 2, 3), B(1, 2) and C(7, 0) are collinear.
Ans. AB = $\sqrt{(1+2)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(7-1)^2 + (0-2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

AC = $\sqrt{(7+2)^2 + (0-3)^2} = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

Since AB + BC = $\sqrt{10} + 2\sqrt{10} = 3\sqrt{10}$ = AC
Hence, the points A, B and C are collinear.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Show that the points A(1, 2), B(5, 4), C(3, 8) and D (–1, 6) are the vertices of a square.
Ans. A(1, 2), B (5, 4),C(3, 8) and D (–1, 6)

AB = $\sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20}$; BC = $\sqrt{(-2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20}$

CD = $\sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$; DA = $\sqrt{(-2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20}$

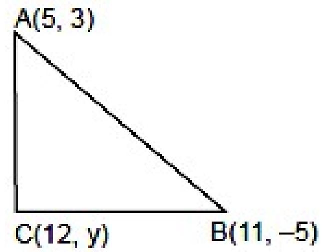
Here AB = BC = CA = DA

AC = $\sqrt{2^2 + 6^2} = \sqrt{40}$ and BD = $\sqrt{(-6)^2 + (2)^2} = \sqrt{36+4} = \sqrt{40}$

All sides of quadrilateral are equal and diagonals are equal.
$\therefore$ ABCD is square.

16. If A(5, 3), B(11, –5) and C(12, y) are vertices of a right triangle right angled at C, then find the value of y.
Ans.

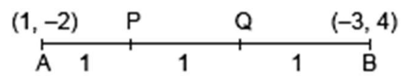
$$AB = \sqrt{(11-5)^2+(-5-3)^2} = \sqrt{100} = 10$$
$$AC = \sqrt{(12-5)^2+(y-3)^2} = \sqrt{49+(y-3)^2}$$
$$BC = \sqrt{(12-11)^2+(y+5)^2} = \sqrt{1+(y+5)^2}$$



Using Pythagoras' theorem, we get
$AB^2 = AC^2 + BC^2$
$(10)^2 = 49 + (y-3)^2 + 1 + (y+5)^2$
$100 = 50 + y^2 + 9 - 6y + y^2 + 25 + 10y$
$\Rightarrow 2y^2 + 4y - 16 = 0 \Rightarrow y^2 + 2y - 8 = 0$
$\Rightarrow (y+4)(y-2) = 0 \Rightarrow y = -4, y = 2$

OR

Find the coordinates of the point of trisection of the line segment joining (1, –2) and (–3, 4).
Ans: Let the points P and Q trisect AB.
$\Rightarrow$ AP : PB = 1 : 2 and AQ : QB = 2 : 1



Using section formula coordinates of P are

$$x = \frac{1\times(-3)+2\times1}{1+2} = \frac{-3+2}{3} = \frac{-1}{3} \text{ and } y = \frac{1\times4+2\times(-2)}{1+2} = \frac{4+(-4)}{3} = \frac{0}{3} = 0$$

Thus, P is $\left(\frac{-1}{3}, 0\right)$,

Coordinates of Q are $x = \frac{2\times(-3)+1\times1}{1+2} = \frac{-6+2}{3} = \frac{-5}{3}$

$$y = \frac{2\times4+1\times(-2)}{1+2} = \frac{8+(-2)}{3} = \frac{6}{3} = 2$$

Thus, Q is $\left(\frac{-5}{3}, 2\right)$

17. In what ratio does the line $x - y - 2 = 0$ divide the line segment joining (3, –1) and (8, 9)?
Ans: Let the line $x - y - 2 = 0$, divides the line segment joining (3, – 1) and (8, 9) in the ratio k : 1 and let the coordinates of the required point be (x_1, y_1).

Then $x_1 = \frac{8k+3}{k+1}$

and $y_1 = \frac{9\times k+1\times(-1)}{k+1} = \frac{9k-1}{k+1}$

This point (x_1, y_1) lies on the line whose equation is $x - y - 2 = 0$.
$\therefore$ It must satisfy the equation of the given line

$$\Rightarrow \frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$

$\Rightarrow 8k + 3 - (9k - 1) - 2(k + 1) = 0$
$\Rightarrow 8k + 3 - 9k + 1 - 2k - 2 = 0$

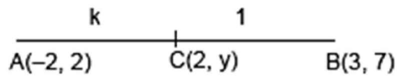
$\Rightarrow -3k + 2 = 0 \Rightarrow k = \frac{2}{3}$

Therefore, the required ratio is $k : 1 = \frac{2}{3} : 1$ or $2 : 3$.

OR

Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y.

Ans: Let C divides AB in the ratio k : 1



$\therefore$ x coordinate of C = $\frac{k \times 3 + 1 \times (-2)}{k+1}$

$\Rightarrow 2 = \frac{3k-2}{k+1} \Rightarrow 2k + 2 = 3k - 2 \Rightarrow k = 4$

$\therefore$ C divides AB in the ratio 4 : 1

Now y coordinate of C = $\frac{4 \times 7 + 1 \times 2}{4+1}$ [$\because k = 4$]

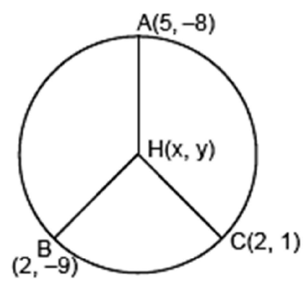
$\Rightarrow y = \frac{28+2}{5} = \frac{30}{5} = 6$

SECTION – D

Questions 18 carry 5 marks.

18. Find the centre of a circle passing through (5, – 8), (2, – 9) and (2, 1).

Ans: Let H(x, y) is centre of circle passing through A, B and C. Since AH, BH and CH are radius of circle.



$\therefore$ AH = BH and BH = CH

Also $AH^2 = BH^2$ and $BH^2 = CH^2$

$AH^2 = (x-5)^2 + (y+8)^2 = x^2 + 25 - 10x + y^2 + 64 + 16y$

$BH^2 = (x-2)^2 + (y+9)^2 = x^2 + 4 - 4x + y^2 + 81 + 18y$

$CH^2 = (x-2)^2 + (y-1)^2 = x^2 + 4 - 4x + y^2 + 1 - 2y$

$\because$ $AH^2 = BH^2$ [Radii of a circle]

$\therefore$ $x^2 + 25 - 10x + y^2 + 64 + 16y = x^2 + 4 - 4x + y^2 + 81 + 18y$

$\Rightarrow -10x + 4x + 16y - 18y = -4$

$\Rightarrow -6x - 2y = -4 \Rightarrow 3x + y = 2$ ...(i)

Also $BH^2 = CH^2$

$\therefore$ $x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 4 - 4x + y^2 + 1 - 2y$

$\Rightarrow 18y + 2y = 1 - 81$

$\Rightarrow 20y = -80 \Rightarrow y = -4$

Putting value of y in (i), we get

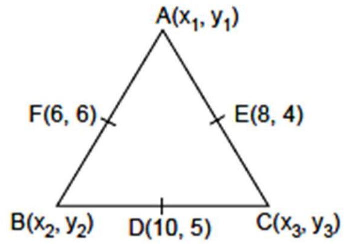
$3x + (-4) = 2 \Rightarrow 3x = 2 + 4 \Rightarrow 3x = 6 \Rightarrow x = 2$

$\therefore$ Coordinates of centre are (2, –4).

OR

If the points (10, 5), (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

Ans: Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of a triangle D(10, 5), E(8, 4) and F(6,6) are mid-points of sides BC, CA and AB respectively.



Therefore, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (6, 6)$

$\Rightarrow x_1 + x_2 = 12$ ...(i)

and $y_1 + y_2 = 12$ ...(ii)

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) = (10, 5)$

$x_2 + x_3 = 20$ ...(iii)

and $y_2 + y_3 = 10$ ...(iv)

and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (8, 4)$

$\Rightarrow x_1 + x_3 = 16$ ...(v)

and $y_1 + y_3 = 8$ ...(vi)

Adding (i), (iii) and (v), we get $2(x_1 + x_2 + x_3) = 48$ ...(vii)

$\Rightarrow x_1 + x_2 + x_3 = 24$

From (i), (iii), (v) and (vii), we get $x_1 = 4$, $x_2 = 8$, $x_3 = 12$ ...(viii)

Adding (ii), (iv) and (vi), we get $2(y_1 + y_2 + y_3) = 30$

$y_1 + y_2 + y_3 = 15$ ...(ix)

From (ii), (iv), (vi) and (ix), we get $y_1 = 5$, $y_2 = 7$, $y_3 = 3$ ...(x)

From (viii) and (x), we get

Coordinates of vertices are A (4, 5), B (8, 7) and C (12, 3).

SECTION – E (Case Study Based Questions)

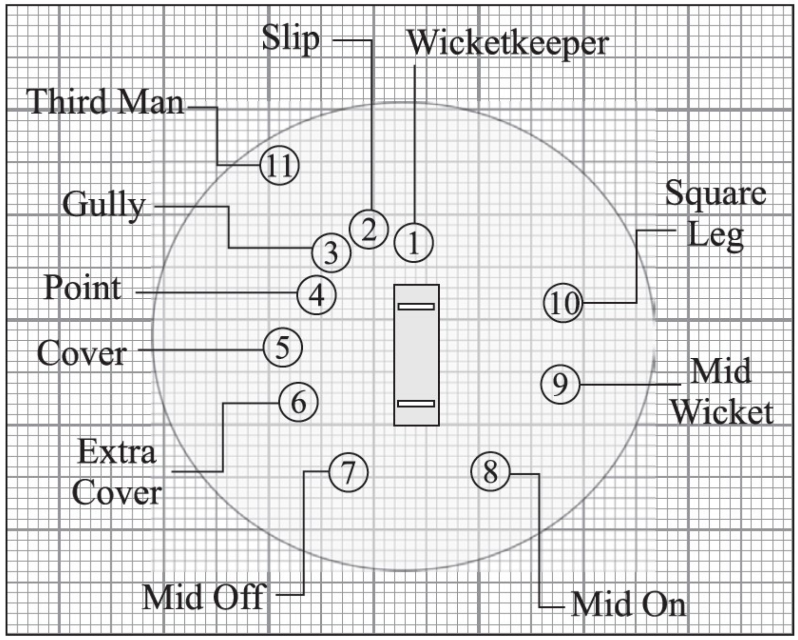
Questions 19 to 20 carry 4 marks each.

19. In the sport of cricket the Captain sets the field according to a plan. He instructs the players to take a position at a particular place. There are two reasons to set a cricket field—to take wickets and to stop runs being scored.

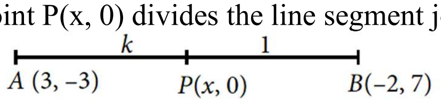
The following graph shows the position of players during a cricket match.

(i) Find the coordinate of the point on y-axis which are equidistant from the points representing the players at Cover P(2, –5) and Mid-wicket Q(–2, 9)

(ii) Find the ratio in which x-axis divides the line segment joining the points Extra Cover S(3, –3) and Fine Leg (–2, 7).

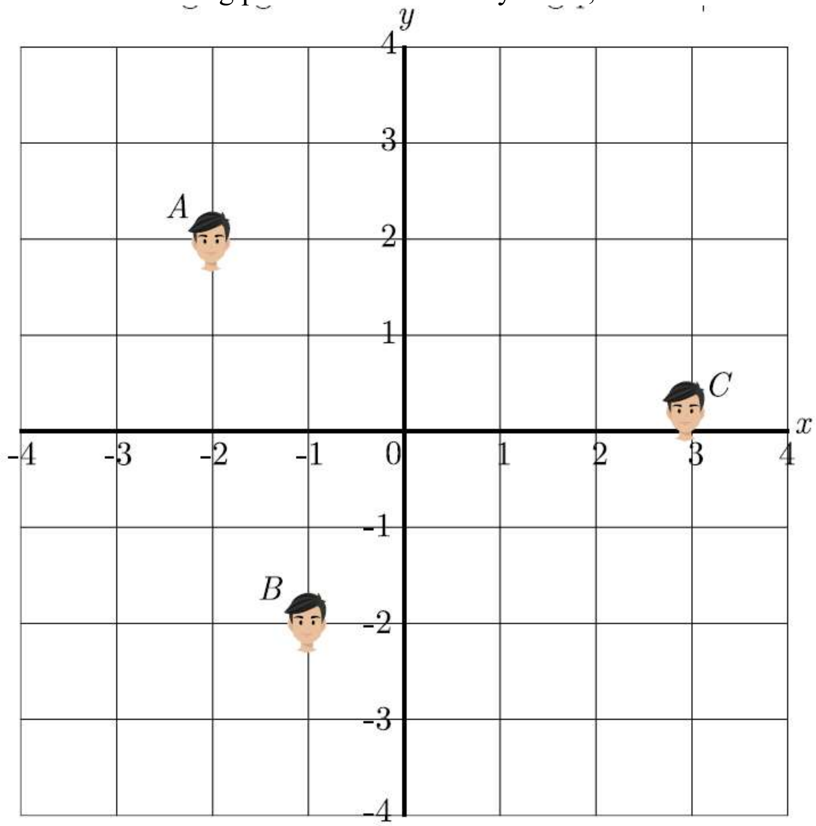


Ans: (i) Let A (0, y) be any point on the y-axis.
Since A (0, y) is equidistant from P (2, –5) and Q (–2, 9)
So AP = AQ ⇒ $AP^2 = AQ^2$
⇒ $(2)^2 + (y + 5)^2 = (2)^2 + (y - 9)^2 \Rightarrow y^2 + 10y + 25 = y^2 - 18y + 81$
⇒ 28y = 81 – 25 ⇒ 28y = 56
⇒ y = 28/56 = 2
So, the point is (0, 2)
(ii) Let point P(x, 0) divides the line segment joining the points A and B in the ratio k : 1



Using section formula,
Coordinates of P are $\left(\dfrac{-2k+3}{k+1}, \dfrac{7k-3}{k+1}\right)$
y-coordinate of $P = \dfrac{7k-3}{k+1} = 0$
$\Rightarrow \quad 7k = 3 \quad \Rightarrow \quad k = \dfrac{3}{7}$
Hence, the point P divides the line segment in the ratio 3 : 7.

20. Aditya, Ritesh and Damodar are fast friend since childhood. They always want to sit in a row in the classroom . But teacher doesn't allow them and rotate the seats row-wise everyday. Ritesh is very good in maths and he does distance calculation everyday. He consider the centre of class as origin and marks their position on a paper in a co-ordinate system. One day Ritesh make the following diagram of their seating position marked Aditya as A, Ritesh as B and Damodar as C.



(i) What is the distance between A and B ? [1]
(ii) What is the distance between B and C ? [1]
(iii) A point D lies on the line segment between points A and B such that AD :DB = 4 : 3 . What are the the coordinates of point D ? [2]

OR

(iii) If the point P(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then find the value of k [2]

Ans:

(i) It may be seen easily from figure that coordinates of point A are (- 2, 2).

$AB^2 = (-2 + 1)^2 + (2 + 2)^2 = 1 + 4^2 = 17$

$\Rightarrow AB = \sqrt{17}$

(ii) It may be seen easily from figure that coordinates of point C are (3, 0).

$BC^2 = (-1 - 3)^2 + (-2 - 0)^2 = 4^2 + 4 = 20$

$\Rightarrow BC = 2\sqrt{5}$

(iii) We have A(- 2, 2) and B(- 1,- 2) and $\frac{m_1}{m_2} = \frac{4}{3}$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{-1(4) + 3(-2)}{4+3} = \frac{-10}{7}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{-2(4) + 3(2)}{4+3} = \frac{-2}{7}$$

Coordinates of D is $\left(\frac{-10}{7}, \frac{-2}{7}\right)$

OR

Using Section Formula, $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ *and* $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$, we get

$$k = \left[\frac{2 \times 2 - 1 \times 7}{1+2}\right] = \frac{4-7}{3} = \frac{-3}{3} = -1$$