

MATHEMATICS
WORKSHEET_090925
CHAPTER 08 INTRODUCTION TO TRIGONOMETRY
(ANSWERS)

SUBJECT: MATHEMATICS

MAX. MARKS : 40

CLASS : X

DURATION : 1½ hrs

General Instructions:

- (i). All questions are compulsory.
- (ii). This question paper contains 20 questions divided into five Sections A, B, C, D and E.
- (iii). **Section A** comprises of 10 MCQs of 1 mark each. **Section B** comprises of 4 questions of 2 marks each. **Section C** comprises of 3 questions of 3 marks each. **Section D** comprises of 1 question of 5 marks each and **Section E** comprises of 2 Case Study Based Questions of 4 marks each.
- (iv). There is no overall choice.
- (v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The value of $\sin^2 30^\circ - \cos^2 30^\circ$ is

- (a) $-\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

Ans. (a) $-\frac{1}{2}$

$$\begin{aligned}\sin^2 30^\circ - \cos^2 30^\circ &= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} - \frac{3}{4} \\ &= \frac{-2}{4} = \frac{-1}{2}\end{aligned}$$

2. If $\sec A + \tan A = x$ then $\tan A =$

- (a) $\frac{2}{x}$ (b) $\frac{1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{2x}{x^2-1}$

Ans: (c) $\frac{x^2-1}{2x}$

$$\sec A + \tan A = x$$

$$\text{Also } \sec^2 A - \tan^2 A = 1$$

$$\Rightarrow (\sec A - \tan A)(\sec A + \tan A) = 1$$

$$\Rightarrow x(\sec A - \tan A) = 1$$

$$\therefore \sec A - \tan A = \frac{1}{x}$$

Now, subtracting (ii) from (i), we have

$$\tan A = \frac{x^2-1}{2x}$$

3. If $\sin x + \operatorname{cosec} x = 2$, then $\sin^{19} x + \operatorname{cosec}^{20} x =$

- (a) 2^{19} (b) 2^{20} (c) 2 (d) 2^{39}

Ans. (c) 2

$$\sin x + \operatorname{cosec} x = 2$$

$$\Rightarrow \sin x + \frac{1}{\sin x} = 2 \Rightarrow \sin^2 x + 1 = 2 \sin x$$

$$\Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$$

$$\Rightarrow \operatorname{cosec} x = 1$$

$$\therefore \sin^{19} x + \operatorname{cosec}^{20} x = 1 + 1 = 2$$

4. If $\sin \theta = \sqrt{3} \cos \theta$, $0^\circ < \theta < 90^\circ$, then θ is equal to
 (a) 30° (b) 45° (c) 60° (d) 90°

Ans. (c) 60°

$$\sin \theta = \sqrt{3} \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = \sqrt{3}$$

$$\Rightarrow \tan \theta = \sqrt{3} = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

5. If $\tan \theta = \frac{a}{b}$ then the value of $\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$ is

- (a) $\frac{a^2 - b^2}{a^2 + b^2}$ (b) $\frac{a^2 + b^2}{a^2 - b^2}$ (c) $\frac{a}{a^2 + b^2}$ (d) $\frac{b}{a^2 + b^2}$

Ans. (b) $\frac{a^2 + b^2}{a^2 - b^2}$

$$\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta} = \frac{\frac{a \sin \theta}{\cos \theta} + \frac{b \cos \theta}{\cos \theta}}{\frac{a \sin \theta}{\cos \theta} - \frac{b \cos \theta}{\cos \theta}}$$

$$= \frac{a \tan \theta + b}{a \tan \theta - b} = \frac{a \times \frac{a}{b} + b}{a \times \frac{a}{b} - b} = \frac{a^2 + b^2}{a^2 - b^2}$$

6. $\sec A = ?$

- (a) $\frac{1}{\cot A}$ (b) $\frac{1}{\operatorname{cosec} A}$ (c) $\frac{1}{\sqrt{1 + \cot^2 A}}$ (d) $\frac{\sqrt{1 + \cot^2 A}}{\cot A}$

Ans: (d) $\frac{\sqrt{1 + \cot^2 A}}{\cot A}$

$$\sec A = \frac{1}{\cos A} = \frac{\frac{1}{\sin A}}{\frac{\cos A}{\sin A}} = \frac{\operatorname{cosec} A}{\cot A} = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

7. If $\sin \theta + \cos \theta = \sqrt{2}$, then $\tan \theta + \cot \theta =$

- (a) 1 (b) 2 (c) 3 (d) 4

Ans: (b) 2

$$\sin \theta + \cos \theta = \sqrt{2}$$

Squaring on both sides, we get

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = 2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow 2 \sin \theta \cos \theta = 1 \Rightarrow \sin \theta \cos \theta = \frac{1}{2}$$

$$\text{But } \tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = 2$$

8. The value of $\frac{1}{\tan \theta + \cot \theta} =$

- (a) $\cos \theta \sin \theta$ (b) $\sec \theta \sin \theta$ (c) $\tan \theta \cot \theta$ (d) $\sec \theta \operatorname{cosec} \theta$

Ans: (a) $\cos \theta \sin \theta$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The value of each of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

Reason (R): In right $\triangle ABC$, $\angle B = 90^\circ$ and $\angle A = \theta$, $\sin \theta = \frac{BC}{AC} < 1$ and $\cos \theta = \frac{AB}{AC} < 1$ as hypotenuse is the longest side.

Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

10. Assertion (A): In a $\triangle ABC$, right angled at B, if $\sin A = \frac{8}{17}$, then $\cos A = \frac{15}{17}$ and $\tan A = \frac{8}{15}$

Reason (R): For acute angle θ , $\cos \theta = \frac{\text{Hypotenuse}}{\text{Adjacent}}$ and $\tan \theta = \frac{\text{Adjacent}}{\text{Opposite}}$

Ans. (c) Assertion (A) is true but reason (R) is false.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. If $\tan \theta = 3/4$, evaluate $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

Ans: $\tan \theta = \frac{3}{4} \Rightarrow \cot \theta = \frac{4}{3}$

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

12. Find an acute angle θ when $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

Ans: $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

Dividing the numerator and denominator of LHS by $\cos \theta$, we get

$$\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

Which on simplification (or comparison) gives

$$\tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

13. If $\sin(A + B) = 1$ and $\cos(A - B) = \sqrt{3}/2$, $0^\circ < A + B \leq 90^\circ$ and $A > B$, then find the measures of angles A and B.

Ans: $\sin(A + B) = 1 = \sin 90^\circ$, so $A + B = 90^\circ$ (i)

$\cos(A - B) = \sqrt{3}/2 = \cos 30^\circ$, so $A - B = 30^\circ$ (ii)

Solving (i) & (ii) $\angle A = 60^\circ$ and $\angle B = 30^\circ$

14. If $A = 60^\circ$ and $B = 30^\circ$, verify that: $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\text{Ans. LHS} = \sin(A + B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1$$

$$\text{RHS} = \sin A \cos B + \cos A \sin B$$

$$= \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$\therefore \text{LHS} = \text{RHS}$$

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Prove that: $\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$

Ans:

$$\begin{aligned} \text{LHS} &= \frac{\frac{\sin^3 \theta}{\cos^3 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} + \frac{\frac{\cos^3 \theta}{\sin^3 \theta}}{1 + \frac{\cos^2 \theta}{\sin^2 \theta}} = \frac{\frac{\sin^3 \theta}{\cos^3 \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}} + \frac{\frac{\cos^3 \theta}{\sin^3 \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}} \\ &= \frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta} = \frac{\sin^4 \theta + \cos^4 \theta}{\cos \theta \sin \theta} = \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} - \frac{2 \sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta} \\ &= \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta = \text{RHS} \end{aligned}$$

OR

Prove that: $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

$$\begin{aligned} \text{Ans. LHS} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\ &= \frac{1}{\sin \theta - \cos \theta} \left[\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] = \frac{1}{\sin \theta - \cos \theta} \left[\frac{\sin^3 \theta - \cos^3 \theta}{\cos \theta \sin \theta} \right] \\ &= \frac{1}{\sin \theta - \cos \theta} \times \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\cos \theta \sin \theta} \\ &= \frac{1 + \sin \theta \cos \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} + \frac{\sin \theta \cos \theta}{\cos \theta \sin \theta} = 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS} \end{aligned}$$

16. Prove that: $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \frac{(1 - \tan A)^2}{(1 - \cot A)^2}$

$$\text{Ans. LHS} = \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}} = \frac{1 + \tan^2 A}{\frac{1 + \tan^2 A}{\tan^2 A}} = \tan^2 A$$

$$\text{RHS} = \frac{(1 - \tan A)^2}{(1 - \cot A)^2} = \frac{(1 - \tan A)^2}{\left(1 - \frac{1}{\tan A} \right)^2} = \frac{(1 - \tan A)^2}{\left(\frac{\tan A - 1}{\tan A} \right)^2} = \frac{(1 - \tan A)^2}{\frac{(\tan A - 1)^2}{\tan^2 A}} = \tan^2 A$$

$$\therefore \text{LHS} = \text{RHS}$$

17. Prove that: $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}$

Ans. LHS = $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta}$
 $= \frac{\sec^2 \theta - \tan^2 \theta + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{(\sec \theta - \tan \theta)(\sec \theta + \tan \theta + 1)}{1 + \sec \theta + \tan \theta}$
 $= \sec \theta - \tan \theta = \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta} = \text{RHS}$

OR

Prove that $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Ans: LHS = $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} \right)$
 $\left[\because \sec A = \frac{1}{\cos A}, \operatorname{cosec} A = \frac{1}{\sin A}, \frac{\sin \theta}{\cos \theta} = \tan \theta \right]$
 $= \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A} \times \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$
 $= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} \times \frac{1}{\sin A \cos A} \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right] c$
 $= 1 = \text{RHS}$

SECTION – D

Questions 18 carry 5 marks.

18. (a) Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$ (3)

Ans: LHS = $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$
 $= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A(1 + \sin A)} = \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{\cos A(1 + \sin A)}$
 $= \frac{2 + 2 \sin A}{\cos A(1 + \sin A)} = \frac{2}{\cos A} = 2 \sec A = \text{RHS}$

(b) Prove that: $\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A$ (2)

Ans:

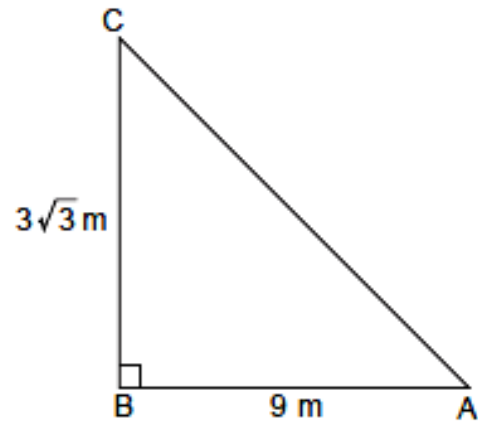
L.H.S. = $\frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A}$
 $= \frac{\sin A(1 - 2 \sin^2 A)}{\cos A(2 \cos^2 A - 1)} = \frac{\sin A[1 - 2 \sin^2 A]}{\cos A[2(1 - \sin^2 A) - 1]}$
 $= \frac{\sin A(1 - 2 \sin^2 A)}{\cos A(1 - 2 \sin^2 A)} = \tan A = \text{R.H.S}$

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Three friends – Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A,

B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9$ m, $BC = 3\sqrt{3}$ m and $\angle B = 90^\circ$



Now based on the above answer the following:

- (i) Find the measure of $\angle A$.
- (ii) Find the measure of $\angle C$.
- (iii) Find the length of AC.

OR

Find the $\cos 2A$.

Ans. (i) With the help of trigonometric ratios and angles; $A = 30^\circ$

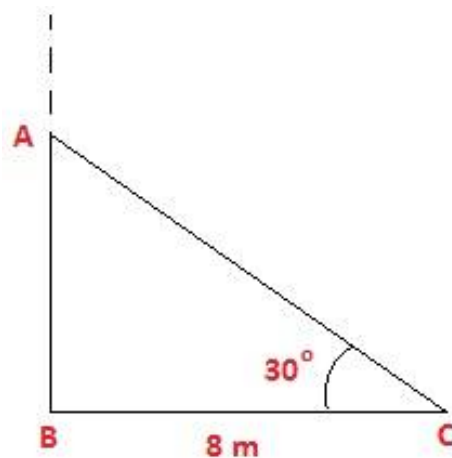
(ii) $C = 60^\circ$

(iii) $AC = 6\sqrt{3}$ m

OR

$$\cos 2A = \cos 60^\circ = \frac{1}{2}$$

- 20.** During a sudden, violent storm, a tall old tree in a neighborhood park is struck by strong winds. It breaks off near its base, and the top portion of the tree topples over, resting on the ground. A group of children playing nearby are curious to know the original height of the tree. They grab a measuring tape and measure the distance from the foot of the tree to the point where the top touched the ground. The broken part bends so that the top of the tree touches the ground, making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 8 meters.



- (a) What is the height of the point where the tree broke from the ground? (1 mark)
- (b) What is the length of the broken part of the tree? (1 mark)
- (c) What was the total height of the tree before it broke? (2 marks)

Ans. (a) In right $\triangle ABC$, $\tan 30^\circ = AB/BC$

$$\Rightarrow 1/\sqrt{3} = AB/8$$

$$\Rightarrow AB = 8/\sqrt{3}$$

(b) In right $\triangle ABC$,

$$\cos 30^\circ = BC/AC$$

We know that, $\cos 30^\circ = \sqrt{3}/2$

$$\Rightarrow \sqrt{3}/2 = 8/AC$$

$$\Rightarrow AC = 16/\sqrt{3}$$

(c) Total height of the tree = $AB + AC = 16/\sqrt{3} + 8/\sqrt{3} = 24/\sqrt{3} = 8\sqrt{3}$ m.

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