MATHEMATICS
WORKSHEET_130625
CHAPTER 12 SURFACE AREAS AND VOLUMES
(ANSWERS)

SUBJECT: MATHEMATICS BASIC **MAX. MARKS : 40**
CLASS : X **DURATION : 1½ hrs**

General Instructions:
(i). **All** questions are compulsory.
(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.
(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.
(iv). There is no overall choice.
(v). Use of Calculators is not permitted

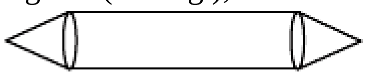
SECTION – A
Questions 1 to 10 carry 1 mark each.

1. If two solid hemispheres of the same base radius *r* are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$
Ans. (a) $4\pi r^2$

2. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is __________ .
(a) 4.2 (b) 2.1 (c) 8.1 (d) 1.05
Ans. (b) 2.1
Edge of the cube = 4.2 cm
Diameter of base of largest cone = 4.2 cm
∴ Radius = 4.2/2 = 2.1 cm

3. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is
(a) 3 : 4 (b) 5 : 3 (c) 27 : 20 (d) 20 : 27
Ans. (d) 20 : 27
Let radii be 2x and 3x and heights be 5y and 3y respectively.
$$\frac{\text{Volume of cylinder I}}{\text{Volume of cylinder II}} = \frac{\pi \times 2x \times 2x \times 5y}{\pi \times 3x \times 3x \times 3y} = 20:27$$

4. The shape of a gilli, in the gilli-danda game (see Fig.), is a combination of



(a) two cylinders (b) a cone and a cylinder
(c) two cones and a cylinder (d) two cylinders and a cone
Ans. (c) two cones and a cylinder

5. Two identical solid cubes of side k units are joined end to end. What is the volume, in cubic units, of the resulting cuboid?
(a) k^3 (b) $2k^3$ (c) $3k^3$ (d) $6k^3$
Ans. (b) $2k^3$
Length of resulting cuboid, l = k + k = 2k
Height of resulting cuboid, h = k
Breadth of resulting cuboid, b = k

Volume of cuboid = l × b × h = 2k × k × k = $2k^3$

6. Volumes of two spheres are in the ratio 27 : 64. The ratio of their surface areas is:
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9
Ans. (c) 9 : 16
Let the radius of two spheres be r_1 and r^2.
Given, the ratio of the volume of two spheres = 27 : 64

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{27}{64} \Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$$

Let the surface areas of the two spheres be S1 and S2.

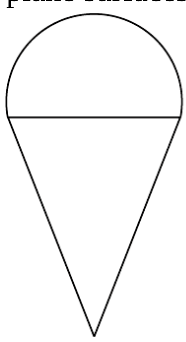
$$\therefore \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

7. The radius of a sphere (in cm) whose volume is 12π cm^3, is
(a) 3 cm (b) $3\sqrt{3}$ cm (c) $3^{2/3}$ cm (d) $3^{1/3}$ cm
Ans. (c) $3^{2/3}$ cm

8. Two cubes each with 6 cm edge are joined end to end. The surface area of the resulting cuboid is:
(a) 180 cm² (b) 360 cm² (c) 300 cm² (d) 260 cm²
Ans. (b) 360 cm²
When two cubes with side length of 6 cm are joined end to end, they form a cuboid. The resulting cuboid has different dimensions.
∴ Surface area of the resulting cuboid
A = 2lb + 2lh + 2bh
= 2 (12) 6 + 2 (12) (6) + 2(6) (6)
= 144 + 144 + 72
= 360 cm²

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.
Reason (R): Top is obtained by joining the plane surfaces of the hemisphere and cone together.



Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

10. **Assertion (A):** If two identical solid cube of side 7 cm are joined end to end. Then the total surface area of the resulting cuboid is 490 cm^2.

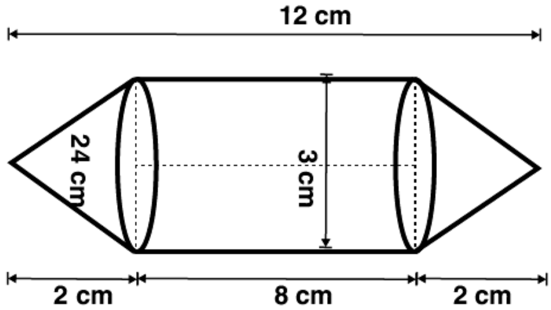
Reason (R): Total surface area of cuboid = lb + bh + hl
Ans. (c) Assertion (A) is true but reason (R) is false.
When cubes are joined end to end, it forms a cuboid.
Here, $l = 2 \times 7 = 14$ cm, b = 7 cm and h = 7 cm
Total surface area of cuboid = 2(lb + bh + hl) = 2 (14 × 7 + 7 × 7 + 7 × 14) = 490 cm^2

SECTION – B

Questions 11 to 14 carry 2 marks each.

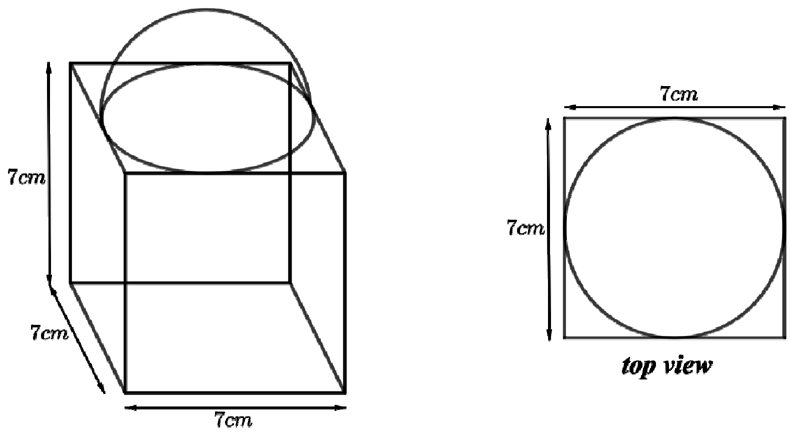
11. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
Ans. Volume of cube = a^3
According to the problem, $a^3 = 64\ cm^3 \Rightarrow$ Side, a = 4 cm
Therefore, the dimensions of the cuboid,
Length, l = 4 + 4 = 8 cm, Breadth, b = 4 cm and Height, h = 4 cm
Surface area of resulting cuboid = 2(lb + bh + hl)
= 2[(8 × 4) + (4 × 4) + (4 × 8)] = 2 (32 + 16 + 32) = 2 × 80 = 160 cm^2

12. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm, and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model are nearly the same.)
Ans. Height of cylinder = 12–4 = 8 cm, Radius = 1.5 cm
Height of cone = 2 cm



Now, the total volume of the air contained will be = Volume of cylinder + 2×(Volume of the cone)
∴ Total volume = $\pi r^2 h + [2\times(\frac{1}{3}\pi r^2 h)]$
= $18\pi + 2(1.5\pi) = 21\pi = 21\times 22/7 = 66\ cm^3$.

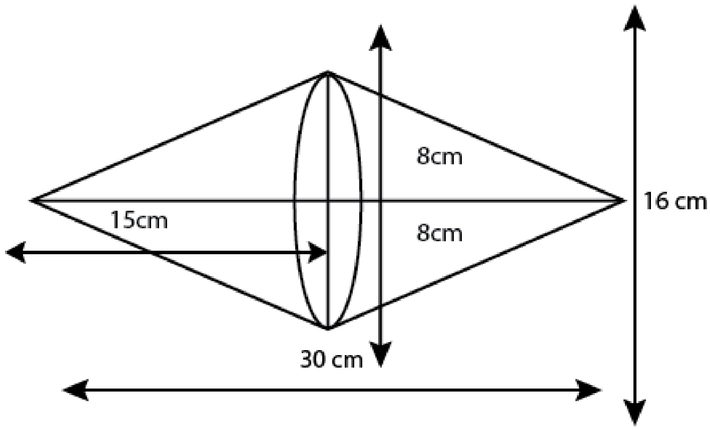
13. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.
Ans. Given that each side of the cube is 7 cm. So, the radius will be 7/2 cm.



We know, total surface area of solid (TSA) = surface area of the cubical block + CSA of the hemisphere – Area of the base of the hemisphere
∴ TSA of solid = $6\times(\text{side})^2 + 2\pi r^2 - \pi r^2$

$= 6\times(\text{side})^2 + \pi r^2$
$= 6\times(7)^2 + (22/7)\times(7/2)\times(7/2)$
$= (6\times49) + (77/2) = 294+38.5 = 332.5 \text{ cm}^2$
So, the surface area of the solid is 332.5 cm^2

14. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.
Ans. Given in the question that, Radius, r = 8 cm Height, h = 15 cm
Since, both cones are identical,
∴ Total surface area of shape formed = Curved area of first cone + Curved surface area of second cone = 2(Surface area of cone) = $2\pi rl$

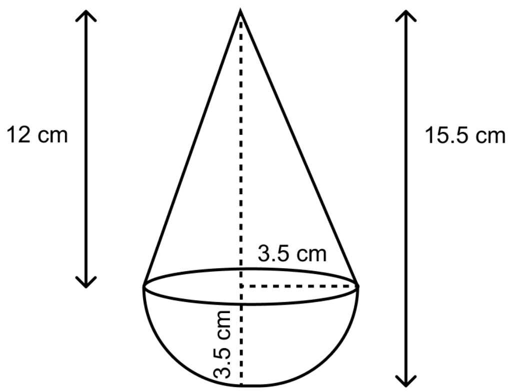


$= 2 \times \pi \times r \times \sqrt{(r^2 + h^2)}$
$= 2 \times (22/7) \times 8 \times \sqrt{(8^2 + 15^2)}$
$= 50.28 \times \sqrt{289} = 854.85 \text{ cm}^2 = 855 \text{ cm}^2$ (approx)
Hence, the surface area of shape so formed is 855cm^2.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
Ans. Radius of the cone (r) = 3.5 cm
Radius of the hemisphere (r) = 3.5 cm
Total height of the toy = 15.5 cm

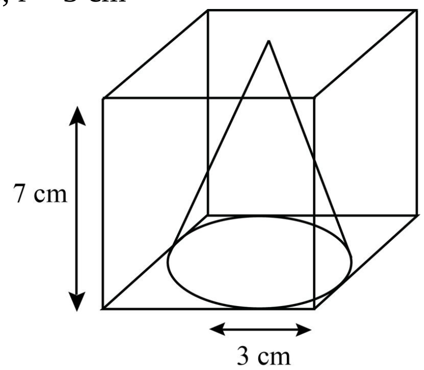


Height of the cone (h) = 15.5 – 3.5 = 12 cm
Slant height of the cone (l) = $\sqrt{r^2 + h^2}$
$= \sqrt{3.5^2 + 12^2} = \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5$ cm
Total Surface Area (TSA) of the toy = CSA of hemisphere + CSA of cone
$= 2\pi r^2 + \pi r l = \pi r\,(2r + l) = \frac{22}{7}\times3.5\left(2\times3.5+12.5\right) = \frac{22}{7}\times3.5\times19.5$
$= 214.5 \text{ cm}^2$

16. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid.

Ans. For cube: side a = 7 cm

For cone: height, h = 7 cm, radius, r = 3 cm



Since, conical cavity is hollowed out from cube,

Volume of remaining solid = Volume of cube – Volume of cone.

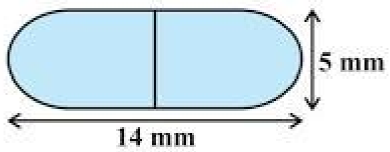
Volume of cube = $a^3 = (7)^3 = 343$

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 7$ = 66 cm^3

∴ Volume of remaining solid = 343 – 66 = 277 cm^3

Hence, the required volume of the solid is 277 cm3.

17. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 5 mm. Find its surface area.

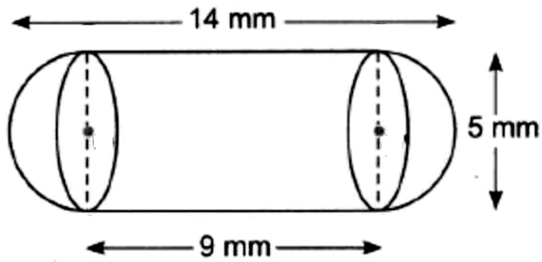


Ans. Here, the diameter of the capsule = 5 mm

∴ Radius = 5/2 = 2.5 mm

Now, the length of the capsule = 14 mm

So, the length of the cylinder = 14 – (2.5 + 2.5) = 9 mm



∴ The surface area of a hemisphere = $2\pi r^2$

Now, the surface area of the cylinder = $2\pi rh$

Thus, the required surface area of the medicine capsule will be

= 2×surface area of hemisphere + surface area of the cylinder

= $(2\times 2\pi r^2) \times 2\pi rh = 2\pi r\,(2r + h) = 2\times\frac{22}{7}\times\frac{5}{2}\left(2\times\frac{5}{2}+9\right) = 2\times\frac{22}{7}\times\frac{5}{2}\times 14$

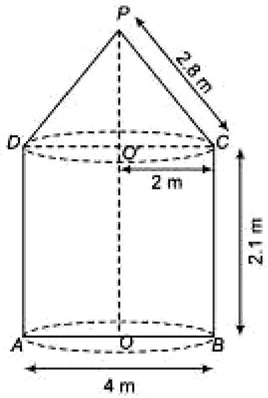
= 44 × 5 = 220 mm^2

SECTION – D

Questions 18 carry 5 marks.

18. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m, respectively, and the slant height of the top is 2.8 m, find the area

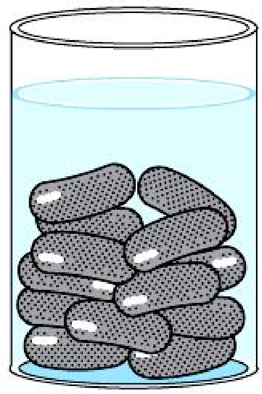
of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)
Ans. We know that a tent is a combination of a cylinder and a cone.



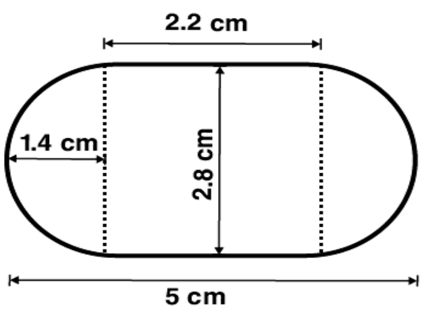
From the question, we know that
Diameter = 4 m
The slant height of the cone (l) = 2.8 m
Radius of the cone (r) = Radius of cylinder = 4/2 = 2 m
Height of the cylinder (h) = 2.1 m
So, the required surface area of the tent = surface area of the cone + surface area of the cylinder
$= \pi r l + 2\pi r h = \pi r(l + 2h)$
$= (22/7)\times 2(2.8 + 2\times 2.1)$
$= (44/7)(2.8 + 4.2)$
$= (44/7)\times 7 = 44\ m^2$
∴ The cost of the canvas of the tent at the rate of ₹500 per m^2 will be
= Surface area × cost per m^2
44×500 = ₹22000
So, Rs. 22000 will be the total cost of the canvas.

OR

A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and a diameter of 2.8 cm (see figure).



Ans. Given that the gulab jamuns are cylindrical shaped with two hemispherical ends.
So, the total height of a gulab jamun = 5 cm.
Diameter = 2.8 cm
So, radius = 1.4 cm

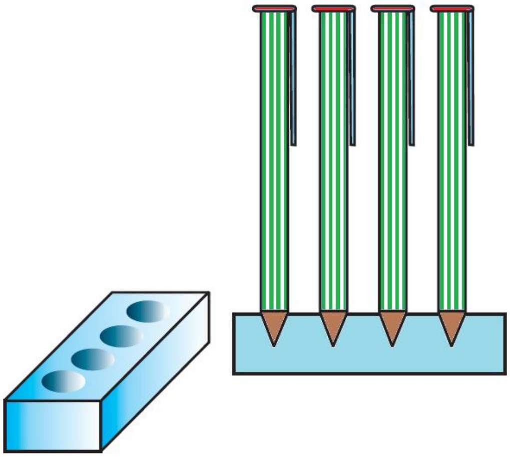


∴ The height of the cylindrical part = 5 cm–(1.4+1.4) cm =2.2 cm
Now, the total volume of one gulab jamun = Volume of cylinder + Volume of two hemispheres
$= \pi r^2 h + (4/3)\pi r^3$
$= 4.312\pi + (10.976/3)\,\pi$
$= 25.05\ cm^3$
We know that the volume of sugar syrup = 30% of the total volume
So, the volume of sugar syrup in 45 gulab jamuns = 45×30%(25.05 cm^3)
= 45×7.515 = 338.184 cm^3

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



Based on the above information, answer the following questions.
(i) Find the volume of four conical depressions in the entire stand [2]
(ii) Find the volume of wood in the entire stand [2]

OR

(ii) Three cubes each of side 15 cm are joined end to end. Find the total surface area of the resulting cuboid. [2]
Ans: (i) Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.
∴ Volume of the cuboid = 15 × 10 × 35/10 cm^3
= 15 × 35 cm^3
= 525 cm^3
Since each depression is conical with base radius (r) = 0.5 cm and depth (h) = 1.4 cm,
∴ Volume of each depression (cone)

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10}\ \text{cm}^3$$

Since there are 4 depressions,
∴ Total volume of 4 depressions

$$= 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{14}{10}\ \text{cm}^3 = \frac{4}{3} \times \frac{11}{10}\ \text{cm}^3 = \frac{44}{30}\ \text{cm}^3$$

(ii) Volume of the wood in entire stand
= [Volume of the wooden cuboid] − [Volume of 4 depressions]

$$= 525\ \text{cm}^3 - \frac{44}{30}\ \text{cm}^3 = \frac{15750 - 44}{30}\ \text{cm}^3 = \frac{15706}{30}\ \text{cm}^3 = \mathbf{523.53\ cm^3}.$$

OR

(ii) New length (l) = 15+15+15 = 45cm,
New breadth (b) = 15cm,
New height (h) = 15cm,
Total surface of the cuboid = 2(lb + bh + hl)
= 2(45 x 15 + 15 x 15 + 15 x 45)
= 2 × 1575 = 3,150 cm^2

20. The word 'circus' has the same root as 'circle'. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical upto a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.
Based on the above, answer the following questions:
(i) Find slant height of the conical part. (1)
(ii) Determine the floor area of the tent. (1)
(iii) (a) Find area of the cloth used for making tent. (2)

OR

(b) Find total volume of air inside an empty tent.
Ans. Given, Cylindrical height = 8 m, Diameter of base = 28 m
Total height of tent = 18.5 m
(i) Radius = 14 m
$(\text{Slant height})^2 = (\text{Height})^2 + (\text{Radius})^2$
$\Rightarrow l^2 = (10.5)^2 + (14)^2 = 110.25 + 196$
$\Rightarrow l^2 = 306.25$
$\Rightarrow l = 17.5$ m
(ii) Floor Area of Tent is = πr^2
= 22/7 ×14 ×14
= 22 × 7 × 14
⇒ Area = 616 m^2
(iii) (a) Area of cloth used for making tent = $2\pi rh \times \pi r l$
= $2\pi r[h + l]$ = 2 × 22/7 ×14[8 + 17.5]
= 2 × 22 × 2[25.5]
= 88 × 25.5
= 2244 m^2

OR

(b) Total volume Inside the Test
$= \pi r^2 h + \frac{1}{3}\pi r^2 h' = \pi r^2\left(h + \frac{1}{3}h'\right)$
$= \frac{22}{7} \times 14 \times 14\left(8 + \frac{1}{3} \times 10.5\right)$
= 22 × 2 × 14(8 + 3.5)
= 616 × 11.5 = 7084 cm^3