CHAPTER 14

SURFACE AREAS AND VOLUMES

14.1 INTRODUCTION

What had been learnt in previous classes regarding surface areas and volumes of solids like cuboid, cube, right circular cylinder, right circular cone and sphere has been reviewed in the previous chapter. In this chapter, we shall discuss problems on conversion of one of these solids in another.

In our day-to-day life we come across various solids which are combinations of two or more such solids. For example, a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, also an ice-cream cone is a combination of a cone and a hemisphere. We shall discuss problems on finding surface areas and volumes of such solids. We also come across solids which are a part of a cone. For example, a bucket, a glass tumbler, a friction clutch etc. These solids are known as frustums of a cone. In the end of the chapter, we shall discuss problems on surface area and volume of frustum of a cone.

14.2 SOME USEFUL FORMULAE

CUBOID *Let l, b and h denote respectively the length, breadth and height of a cuboid. Then,*

(i) *Total surface area of the cuboid* $= 2(lb + bh + lh)$ *square units*

(ii) *Volume of the cuboid* = *Area of the base* × *Height* = *Length* × *Breadth* × *Height*

$= lbh$ *cubic units*

(iii) *Diagonal of the cuboid* $= \sqrt{l^2 + b^2 + h^2}$ *units.*

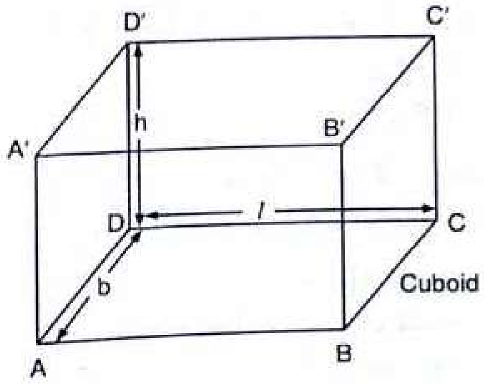

Fig. 14.1

(iv) *Area of four walls of a room* $= lh + lh + bh + bh = 2(l + b)h$ *square units.*

CUBE *If the length of each edge of a cube is 'a' units, then*

(i) *Total surface area of the cube* $= 6a^2$ *square units*

(ii) *Volume of the cube* $= a^3$ *cubic units*

(iii) *Diagonal of the cube* $= \sqrt{3}\,a$ *units*

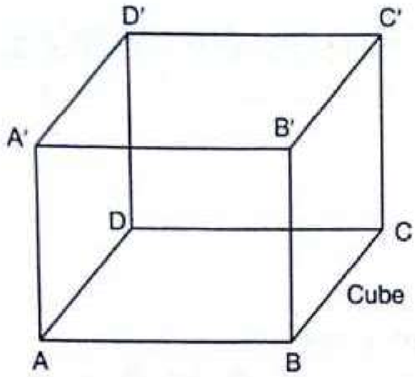

Fig. 14.2

RIGHT CIRCULAR CYLINDER *For a right circular cylinder of base radius r and height (or length) h, we have*

(i) *Area of each end* = *Area of base* $= \pi r^2$

(ii) *Curved surface area* $= 2\pi r h$

$= 2\pi r \times h$

= *Perimeter of the base × Height*

(iii) *Total surface area* = *Curved surface area + Area of circular ends*

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

(iv) *Volume* $= \pi r^2 h$

= *Area of the base × Height*

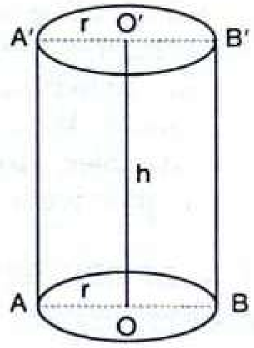

Fig. 14.3

RIGHT CIRCULAR HOLLOW CYLINDER *Let R and r be the external and internal radii of a hollow cylinder of height h. Then,*

(i) *Area of each end* $= \pi(R^2 - r^2)$

(ii) *Curved surface area of hollow cylinder*

= *External surface area + Internal surface area*

$= 2\pi Rh + 2\pi rh$

$= 2\pi h(R + r)$

(iii) *Total surface area* $= 2\pi Rh + 2\pi rh + 2(\pi R^2 - \pi r^2)$

$= 2\pi h(R + r) + 2\pi(R + r)(R - r)$

$= 2\pi(R + r)(R + h - r)$

(iv) *Volume of material* = *External volume – Internal volume*

$= \pi R^2 h - \pi r^2 h$

$= \pi h(R^2 - r^2)$

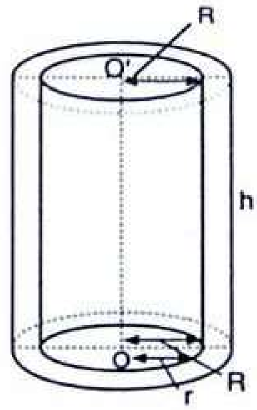

Fig. 14.4

RIGHT CIRCULAR CONE *For a right circular cone of height h, slant height l and radius of base r, we have*

(i) $l^2 = r^2 + h^2$

(ii) *Curved surface area* $= \pi r l$ *sq. units*

(iii) *Total surface area* = *Curved surface area* + *Area of the base*

$= \pi r l + \pi r^2$

$= \pi r (l + r)$ *sq. units*

(iv) *Volume* $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}$ (*Area of the base*) × *Height*

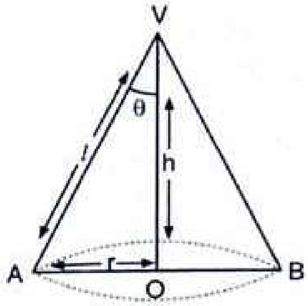

Fig. 14.5

SPHERE *For a sphere of radius r, we have*

(i) *Surface area* $= 4\pi r^2$

(ii) *Volume* $= \frac{4}{3}\pi r^3$

For a hemisphere of radius r, we have

(i) *Surface area* $= 2\pi r^2$

(ii) *Total surface area* $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

(iii) *Volume* $= \frac{2}{3}\pi r^3$

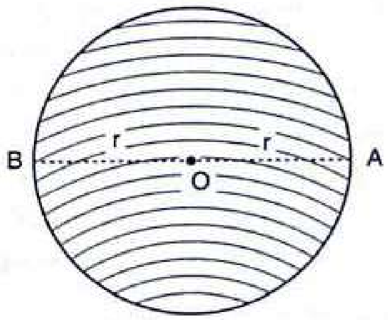

Fig. 14.6

SPHERICAL SHELL *If R and r are respectively the outer and inner radii of a spherical shell, then*

(i) *Outer surface area* $= 4\pi R^2$

(ii) *Volume of material* $= \frac{4}{3}\pi(R^3 - r^3)$

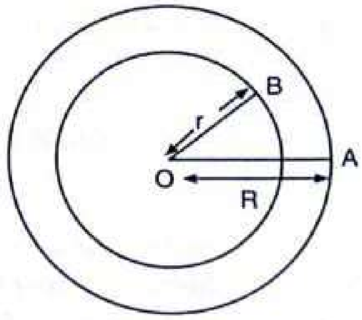

Fig. 14.7

14.3 CONVERSION OF SOLIDS

In this section, we shall discuss problems pertaining to conversion of a solid (discussed in the previous classes) into another solid of different shape. For example, a metallic sphere is melted and recast into a cylindrical wire, the earth taken out by digging a well and spreading it uniformly around the well to form an embankment in the form of a cylindrical shell from its original shape of right circular cylinder, etc. The computation of surface areas and volumes in such cases are illustrated below.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Two cubes each of 10 cm edge are joined end to end. Find the surface area of the resulting cuboid.*

SOLUTION If two cubes are joined end to end, we get a cuboid such that

l = Length of the resulting cuboid = 10 cm + 10 cm = 20 cm

b = Breadth of the resulting cuboid = 10 cm

h = Height of the resulting cuboid = 10 cm

$\therefore$ Surface area of the cuboid $= 2(lb + bh + lh)$

$\Rightarrow$ Surface area of the cuboid $= 2(20 \times 10 + 10 \times 10 + 20 \times 10)\ \text{cm}^2 = 1000\ \text{cm}^2$

EXAMPLE 2 *Three cubes whose edges measure 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge. Also, find the surface area of the new cube.*

SOLUTION Let x cm be the edge of the new cube. Then,

Volume of the new cube = Sum of the volumes of three cubes.

$\Rightarrow$ $x^3 = 3^3 + 4^3 + 5^3 = 27 + 64 + 125$

$\Rightarrow$ $x^3 = 216$

$\Rightarrow$ $x^3 = 6^3 \Rightarrow x = 6$ cm

$\therefore$ Edge of the new cube is 6 cm long.

Surface area of the new cube $= 6x^2 = 6 \times (6)^2\ \text{cm}^2 = 216\ \text{cm}^2$

EXAMPLE 3 *Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.*

SOLUTION The dimensions of the cuboid so formed are as under:

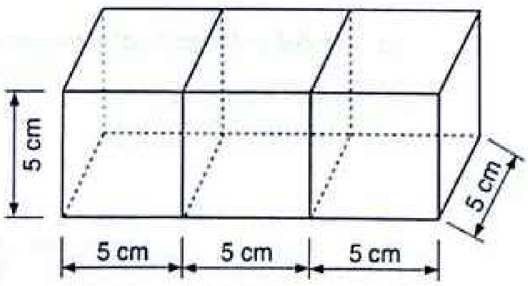

Fig. 14.8

l = Length = 15 cm, b = Breadth = 5 cm, and h = Height = 5 cm

$\therefore$ Surface area of the cuboid $= 2(15 \times 5 + 5 \times 5 + 15 \times 5)\ \text{cm}^2$

$\Rightarrow$ Surface area of the cuboid $= 2(75 + 25 + 75)\ \text{cm}^2 = 350\ \text{cm}^2$

EXAMPLE 4 *Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area and volume of the resulting cuboid.* **[NCERT, NCERT EXEMPLAR]**

SOLUTION Let the length of each edge of the cube of volume 64 cm^3 be x cm. Then,

Volume $= 64\ \text{cm}^3$

$\Rightarrow$ $x^3 = 64$

$\Rightarrow$ $x^3 = 4^3$

$\Rightarrow$ $x = 4$ cm

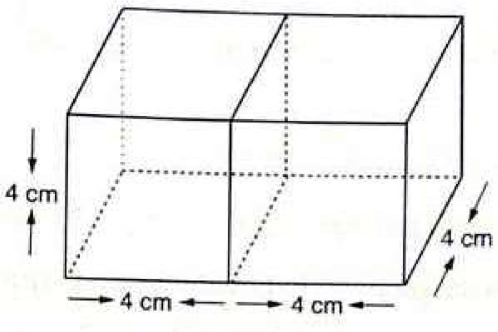

Fig. 14.9

The dimensions of the cuboid so formed are:

L = Length = $(4 + 4)$ cm = 8 cm, b = Breadth = 4 cm and, h = Height = 4 cm

$\therefore$ Surface area of the cuboid $= 2(lb + bh + lh)$

$= 2(8 \times 4 + 4 \times 4 + 8 \times 4)\text{ cm}^2 = 160\text{ cm}^2$

Volume of the cuboid $= lbh = 8 \times 4 \times 4\text{ cm}^3 = 128\text{ cm}^3$

EXAMPLE 5 *The dimensions of a metallic cuboid are: 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the surface area of the cube.*

SOLUTION Let the length of each edge of the recasted cube be a cm.

Volume of the metallic cuboid $= 100 \times 80 \times 64\text{ cm}^3 = 512000\text{ cm}^3$

The metallic cuboid is melted and is recasted into a cube.

$\therefore$ Volume of the cube = Volume of the metallic cuboid

$\Rightarrow$ $a^3 = 512000$

$\Rightarrow$ $a^3 = 8^3 \times 10^3 = (8 \times 10)^3$

$\Rightarrow$ $a = 8 \times 10\text{ cm} = 80\text{ cm}$

$\therefore$ Surface area of the cube $= 6a^2\text{ cm}^2 = 6 \times (80)^2\text{ cm}^2 = 38400\text{ cm}^2$

EXAMPLE 6 *Three metallic solid cubes whose edges are 3 cm, 4 cm and 5 cm, are melted and formed into a single cube. Find the edge of the cube so formed.* **[NCERT EXEMPLAR]**

SOLUTION Let the length of the edge of the new cube formed be x cm. Then,

Volume of the new cube = Sum of the volumes of three metallic cubes

$\Rightarrow$ $x^3 = 3^2 + 4^3 + 5^3$

$\Rightarrow$ $x^3 = 27 + 64 + 125$

$\Rightarrow$ $x^3 = 216$

$\Rightarrow$ $x^3 = 6^3$

$\Rightarrow$ $x = 6$

Hence, the length of an edge of the new cube is 6 cm.

EXAMPLE 7 *A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.* **[NCERT EXEMPLAR]**

SOLUTION Let the length of the pipe be h cm. Then, volume of iron in the pipe is equal to the volume of iron in the block.

Volume of the block = $(4.4 \times 2.6 \times 1)\ m^3 = (440 \times 260 \times 100)\ m^3$

We have,

r = Internal radius of the pipe = 30 cm

R = External radius of the pipe = (30 + 5) cm = 35 cm

∴ Volume of iron in the pipe = (External Volume) – (Internal Volume)

$$= \pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$$

$$= \pi(R + r)(R - r)h$$

$$= \pi \times (35 + 30) \times (35 - 30) \times h \text{ cm}^3$$

$$= \pi \times 65 \times 5 \times h \text{ cm}^3$$

Now, Volume of iron in the pipe = Volume of iron in the block

$$\Rightarrow \quad \pi \times 65 \times 5 \times h = 440 \times 260 \times 100$$

$$\Rightarrow \quad \frac{22}{7} \times 65 \times 5 \times h = 440 \times 260 \times 100$$

$$\Rightarrow \quad h = \left(440 \times 260 \times 100 \times \frac{7}{22} \times \frac{1}{65} \times \frac{1}{5}\right) \text{cm} = 11200 \text{ cm} = 112 \text{ m}.$$

Hence, the length of the pipe is 112 m.

EXAMPLE 8 *The radii of the bases of two right circular solid cones of same height are r_1 and r_2 respectively. The cones are melted and recast into a solid sphere of radius R. Show that the height of each cone is given by* $h = \dfrac{4R^3}{r_1^2 + r_2^2}$

SOLUTION Let h be the height of each cone. Then,

Sum of the volumes of two cones = Volume of the sphere

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad (r_1^2 + r_2^2)h = 4R^3$$

$$\Rightarrow \quad h = \frac{4R^3}{r_1^2 + r_2^2}$$

EXAMPLE 9 *Two solid right circular cones have the same height. The radii of their bases are r_1 and r_2. They are melted and recast into a cylinder of same height. Show that the radius of the base of the cylinder is* $\sqrt{\dfrac{r_1^2 + r_2^2}{3}}$

SOLUTION Let h be the height of two given cones of base radii r_1 and r_2 respectively. Further, let R be the radius of the cylinder. It is given that the cylinder is also of height h.

∴ Volume of the cylinder = Sum of the Volumes of two cones

$$\Rightarrow \quad \pi R^2 h = \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$$

$$\Rightarrow \quad \pi R^2 h = \frac{1}{3}\pi h (r_1^2 + r_2^2)$$

$$\Rightarrow \quad R^2 = \frac{1}{3}\left(r_1^2 + r_2^2\right)$$

$$\Rightarrow \quad R = \sqrt{\frac{r_1^2 + r_2^2}{3}}$$

EXAMPLE 10 *The diameter of a metallic sphere is 6 cm. It is melted and drawn into a wire having diameter of the cross-section as 0.2 cm. Find the length of the wire.*

SOLUTION We have,

Diameter of metallic sphere = 6 cm

∴ Radius of metallic sphere = 3 cm

Also, we have

Diameter of cross-section of cylindrical wire = 0.2 cm

∴ Radius of cross-section of cylindrical wire = 0.1 cm

Let the length of the wire be h cm. Since metallic sphere is converted into a cylindrical shaped wire of length h cm.

∴ Volume of the metal used in wire = Volume of the sphere

$$\Rightarrow \quad \pi \times (0.1)^2 \times h = \frac{4}{3} \times \pi \times 3^3$$

$$\Rightarrow \quad \pi \times \left(\frac{1}{10}\right)^2 \times h = \frac{4}{3} \times \pi \times 27$$

$$\Rightarrow \quad \pi \times \frac{1}{100} \times h = 36\pi$$

$$\Rightarrow \quad h = \frac{36\pi \times 100}{\pi} \text{ cm} = 3600 \text{ cm} = 36 \text{ metres}$$

EXAMPLE 11 *The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform cross-section. If the length of the wire is 36 m, find its radius.* **[CBSE 2013]**

SOLUTION We have,

Diameter of the sphere = 6 cm

∴ Radius of the sphere $= \frac{6}{2}$ cm = 3 cm

$\Rightarrow$ Volume of the sphere $= \frac{4}{3} \times \pi \times 3^3 \text{ cm}^3 = 36\pi \text{ cm}^3$ $\left[\text{Using } V = \frac{4}{3}\pi r^3\right]$

Let the radius of cross-section of wire be r cm. It is given that the length of the cylindrical shaped wire is 36 m.

$\therefore$ Volume of the wire $= (\pi r^2 \times 3600)\text{ cm}^3$ $\quad$ [Using $V = \pi r^2 h$]

Since metallic sphere is converted into cylindrical shaped wire. Therefore,

Volume of the wire = Volume of the sphere

$$\Rightarrow \quad \pi r^2 \times 3600 = 36\pi$$

$$\Rightarrow \quad r^2 = \frac{36\pi}{3600\pi} = \frac{1}{100}$$

$$\Rightarrow \quad r = \frac{1}{10}\text{ cm} = 1\text{ mm}$$

EXAMPLE 12 *How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm?* **[NCERT EXEMPLAR]**

SOLUTION Volume of the lead in cubical solid $= (9 \times 11 \times 12)\text{ cm}^3 = 1188\text{ cm}^3$

Suppose x shots can be made from the cubical solid. Then

Volume of lead in x spherical shots = Volume of the solid

$$\Rightarrow \quad \left\{\frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3\right\} x = 1188$$

$$\Rightarrow \quad \left(\frac{4}{3} \times \frac{22}{7} \times \frac{27}{8}\right) x = 1188$$

$$\Rightarrow \quad x = \frac{1188 \times 3 \times 7 \times 8}{4 \times 22 \times 27} = 84$$

Hence, 84 shots can be made from the cubical solid.

EXAMPLE 13 *A right circular cone of radius 3 cm had a curved surface area of 47.1 cm^2. Find the volume of the cone. (Use $\pi = 3.14$)* **[CBSE 2016]**

SOLUTION Let the height and the slant height of the cone be h cm and l cm respectively. It is given that the radius of the base is $r = 3$ cm. It is also given that the curved surface area of the cone is 47.1 cm^2

$$\therefore \quad \pi r l = 47.1$$

$$\Rightarrow \quad 3.14 \times 3 \times l = 47.1$$

$$\Rightarrow \quad l = \frac{47.1}{9.42}\text{ cm} = 5\text{ cm}$$

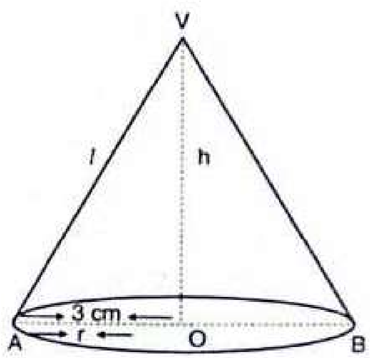

Fig. 14.10

Thus, we obtain $l = 5$ cm and $r = 3$ cm

$$\therefore \quad l^2 = r^2 + h^2$$

$$\Rightarrow \quad 25 = 9 + h^2$$

$$\Rightarrow \quad h^2 = 16$$

$$\Rightarrow \quad h = 4\text{ cm}$$

Let V be the volume of the cone. Then,

$$V = \frac{1}{3}\pi r^2 h$$

$\Rightarrow \quad V = \frac{1}{3} \times 3.14 \times 3^2 \times 4 \text{ cm}^3 = 37.68 \text{ cm}^3$

Hence, the volume of the cone is 37.68 cm^3.

EXAMPLE 14 *A right circular cone is of height 8.4 cm and the radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.*

SOLUTION We have,

r = Radius of the base of the cone = 2.1 cm, h = Height of the cone = 8.4 cm

$\therefore \quad \text{Volume of the cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (2.1)^2 \times 8.4 \text{ cm}^3$

Let R cm be the radius of the sphere obtained by recasting the melted cone. Then,

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

Since the volume of the material in the form of cone and sphere remains the same.

$\therefore \quad \frac{4}{3}\pi R^3 = \frac{1}{3} \times \pi \times (2.1)^2 \times (8.4)$

$\Rightarrow \quad R^3 = \frac{(2.1)^2 \times 8.4}{4} = (2.1)^3$

$\Rightarrow \quad R = 2.1$

Hence, the radius of the sphere is 2.1 cm.

EXAMPLE 15 *A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.* **[NCERT]**

SOLUTION Let the height of the cylinder be h cm. Then,

Volume of the cylinder = Volume of the sphere

$\Rightarrow \quad \pi \times 6^2 \times h = \frac{4}{3} \times \pi \times (4.2)^3$

$\Rightarrow \quad h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$

$\Rightarrow \quad h = 4 \times 0.7 \times 0.7 \times 1.4 \text{ cm}$

EXAMPLE 16 *Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.* **[NCERT]**

SOLUTION Let the radius of the resulting sphere be r cm. Then,

Volume of the resulting sphere = Sum of the volumes of three spheres of radii 6 cm, 8 cm and 10 cm

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6^3 + \frac{4}{3}\pi \times 8^3 + \frac{4}{3}\pi \times 10^3$

$\Rightarrow \quad r^3 = 216 + 512 + 1000$

$\Rightarrow \quad r^3 = 1728$

$\Rightarrow \quad r^3 = 12^3$

$\Rightarrow \quad r = 12 \text{ cm}.$

EXAMPLE 17 *A solid sphere of radius 3 cm is melted and then cast into small spherical balls each of diameter 0.6 cm. Find the number of balls thus obtained.*

SOLUTION Let the total number of balls be x.

$$\text{Volume of the solid sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 3^3 \text{ cm}^3 = 36\pi \text{ cm}^3$$

$$\text{Radius of spherical ball} = \frac{0.6}{2}\text{ cm} = 0.3 \text{ cm}$$

$$\text{Volume of a spherical ball} = \frac{4}{3}\pi \times (0.3)^3 \text{ cm}^3 = \frac{4}{3}\pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\text{ cm}^3 = \frac{36\pi}{1000}\text{ cm}^3$$

$\therefore$ $$\text{Volume of } x \text{ spherical balls} = \frac{36\pi}{1000}x \text{ cm}^3$$

Clearly, Volume of the solid sphere = Volume of x spherical balls.

$\Rightarrow$ $$36\pi = \frac{36\pi}{1000}x \Rightarrow x = 1000$$

Hence, 1000 spherical balls are obtained by melting the given solid sphere.

EXAMPLE 18 *Find the number of coins, 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.*

SOLUTION Let the total number of coins be x. Clearly, each coin is a cylinder of radius $r = 0.75$ cm and height $h = 0.2$ cm.

$$\text{Volume of a coin} = [\pi \times (0.75)^2 \times 0.2] \text{ cm}^3 \qquad [\text{Using: } V = \pi r^2 h]$$

$\therefore$ $$\text{Volume of } x \text{ coins} = [\pi \times (0.75)^2 \times 0.2]\, x \text{ cm}^3$$

$$\text{Volume of the cylinder} = [\pi \times (2.25)^2 \times 10] \text{cm}^3 \quad \left[\because \text{Radius} = \frac{4.5}{2}\text{ cm} = 2.25 \text{ cm}\right]$$

Clearly, Volume of metal in x coins = Volume of the cylinder

$\Rightarrow$ $$\{\pi \times (0.75)^2 \times 0.2\}\, x = [\pi \times (2.25)^2 \times 10] \text{ cm}^3$$

$\Rightarrow$ $$x = \frac{\pi(2.25 \times 2.25 \times 10)}{\pi(0.75 \times 0.75 \times 0.2)} = 3 \times 3 \times 50 = 450$$

EXAMPLE 19 *How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.*

SOLUTION Let the total number of bullets be x.

$$\text{Radius of a spherical bullet} = \frac{4}{2}\text{ cm} = 2 \text{ cm}$$

Now, $$\text{Volume of a spherical bullet} = \frac{4}{3}\pi \times (2)^3 \text{ cm}^3 = \left(\frac{4}{3} \times \frac{22}{7} \times 8\right)\text{cm}^3$$

$\therefore$ $$\text{Volume of } x \text{ spherical bullets} = \left(\frac{4}{3} \times \frac{22}{7} \times 8 \times x\right)\text{cm}^3$$

$$\text{Volume of the solid cube} = (44)^3 \text{ cm}^3$$

Clearly, Volume of x spherical bullets = Volume of cube

$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7} \times 8 \times x = (44)^3$

$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7} \times 8 \times x = 44 \times 44 \times 44$

$\Rightarrow \quad x = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 8} = 2541$

Hence, total number of spherical bullets = 2541

EXAMPLE 20 *How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm, 21 cm. (Use $\pi = 22/7$).*

SOLUTION Let the number of lead shots be x

Volume of lead in the rectangular solid $= (66 \times 42 \times 21)\ \text{cm}^3$

Radius of a lead shot $= \frac{4.2}{2}\ \text{cm} = 2.1\ \text{cm}$

Volume of a spherical lead shot $= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3\ \text{cm}^3$

$\therefore$ Volume of x spherical lead shots $= \left\{\frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \times x\right\}\text{cm}^3$

$\because$ Volume of x spherical lead shots = Volume of lead in rectangular solid

$\therefore \quad \left\{\frac{4}{3} \times \frac{22}{7} \times (2.1)^3 \times x\right\} = 66 \times 42 \times 21$

$\Rightarrow \quad x = \frac{66 \times 42 \times 21 \times 3 \times 7}{4 \times 22 \times (2.1)^3} = \frac{66 \times 42 \times 21 \times 21 \times 1000}{4 \times 22 \times 21 \times 21 \times 21} = 1500$

Hence, the number of spherical lead shots is 1500.

EXAMPLE 21 *The radii of the internal and external surfaces of a metallic spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid right circular cylinder of height $10\frac{2}{3}$ cm. Find the diameter of the base of the cylinder.*

SOLUTION Let the radius of the base of the cylinder be r cm. Then,

Volume of the metallic solid cylinder of height $10\frac{2}{3}$ cm

= Volume of the metal in the spherical shell

$\Rightarrow \quad \pi \times r^2 \times \frac{32}{3} = \frac{4}{3}\pi(5^3 - 3^3)$

$\Rightarrow \quad \frac{32}{3} r^2 = \frac{4}{3}(125 - 27)$

$\Rightarrow \quad r^2 = \frac{3}{32} \times \frac{4}{3} \times 98$

$\Rightarrow \quad r^2 = \frac{49}{4}$

$\Rightarrow \quad r = \frac{7}{2}$ cm

Hence, diameter of the base of the cylinder = 7 cm

EXAMPLE 22 *A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?*

SOLUTION We have,

Radius of hemispherical bowl = 18 cm

$\therefore$ Volume of hemispherical bowl $= \frac{2}{3}\pi \times (18)^3$ cm^3 $\quad \left[\because V = \frac{2}{3}\pi r^3\right]$

and, Radius of a cylindrical bottle = 3 cm
Height of a cylindrical bottle = 6 cm

$\therefore$ Volume of a cylindrical bottle $= (\pi \times 3^2 \times 6)$ cm^3 $\quad \left[\because V = \pi r^2 h\right]$

Suppose x bottles are required to empty the bowl.

Volume of x cylindrical bottles $= (\pi \times 9 \times 6 \times x)$ cm^3

Clearly, Volume of liquid in x bottles = Volume of bowl

$$\Rightarrow \quad \pi \times 9 \times 6 \times x = \frac{2\pi}{3} \times (18)^3$$

$$\Rightarrow \quad x = \frac{2\pi \times 18^3}{3 \times \pi \times 9 \times 6} = 72$$

Hence, 72 bottles are required to empty the bowl.

EXAMPLE 23 *A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Find its height.*

SOLUTION We have,

	First cone	*Second cone*
Radii	$r_1 = 1.6$ cm	$r_2 = 1.2$ cm
Heights	$h_1 = 3.6$ cm	$h_2 = ?$
Volumes	V_1	V_2

Clearly, two cones have the same volume.

$$\therefore \quad V_1 = V_2$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

$$\Rightarrow \quad r_1^2 h_1 = r_2^2 h_2$$

$$\Rightarrow \quad h_2 = \frac{r_1^2 h_1}{r_2^2}$$

$$\Rightarrow \quad h_2 = \frac{1.6 \times 1.6 \times 3.6}{1.2 \times 1.2} \text{ cm} = \frac{16 \times 16 \times 36}{12 \times 12 \times 10} = 6.4 \text{ cm}$$

Hence, the height of new cone is 6.4 cm.

EXAMPLE 24 *Solid cylinder of brass 8 m high and 4 m diameter is melted and recast into a cone of diameter 3 m. Find the height of the cone.*

SOLUTION We have,

	Cylinder	Cone
Radii	$r_1 = 2\text{m}$	$r_2 = 1.5\text{ m}$
Heights	$h_1 = 8\text{ m}$	$h_2 = ?$
Volumes	V_1	V_2

Clearly, Volume of the cone = Volume of the cylinder

i.e., $V_1 = V_2$

$$\Rightarrow \quad \frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1$$

$$\Rightarrow \quad r_2^2 h_2 = 3 r_1^2 h_1$$

$$\Rightarrow \quad h_2 = \frac{3r_1^2 h_1}{r_2^2} \Rightarrow h_2 = \frac{3 \times 2^2 \times 8}{(1.5)^2}\text{ m} \Rightarrow h_2 = \frac{96}{2.25}\text{ m} = 42.66\text{ m}$$

Hence, the height of the cone is 42.66 m.

EXAMPLE 25 *A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?*

SOLUTION We have, Radius of the sphere = 3 cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi \times (3)^3\text{ cm}^3 = 36\pi\text{cm}^3$ $\left[\because V = \frac{4}{3}\pi r^3\right]$

Radius of the cylindrical vessel = 6 cm

Suppose water level rises by h cm in the cylindrical vessel. Then,

Volume of the cylinder of height h cm and radius 6 cm

$= (\pi \times 6^2 \times h)\text{ cm}^3 = 36\pi h\text{ cm}^3$ $\left[\because V = \pi r^2 h\right]$

This is the volume of water displaced by the sphere. Clearly, volume of water displaced by the sphere is equal to the volume of the sphere.

$\therefore$ $36\pi h = 36\pi \Rightarrow h = 1\text{ cm}$

Hence, water level rises by 1 cm.

EXAMPLE 26 *A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cms. Find the height to which the water rises.*

SOLUTION We have,

r_1 = radius of the conical vessel = 5 cm, h_1 = height of the conical vessel = 24 cm

and, r_2 = radius of the cylindrical vessel = 10 cm

Suppose water rises upto the height of h_2 cm in the cylindrical vessel.

Clearly, Volume of water in conical vessel = Volume of water in cylindrical vessel

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow \quad r_1^2 h_1 = 3 r_2^2 h_2$$

$\Rightarrow$ $$5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$$

$\Rightarrow$ $$h_2 = \frac{5 \times 5 \times 24}{3 \times 10 \times 10} = 2 \text{ cm}$$

Hence, the height of water in the cylindrical vessel is 2 cm.

EXAMPLE 27 *A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. (Take $\pi = 3.142$)*

SOLUTION Suppose the water rises by h cm. Clearly, water in the cylinder forms a cylinder of height h cm and radius 10 cm.

$\therefore$ Volume of the water displaced = Volume of the cube of edge 8 cm

$\Rightarrow$ $$\pi r^2 h = 8^3$$

$\Rightarrow$ $$3.142 \times 10^2 \times h = 8 \times 8 \times 8 \qquad [\because r = 10 \text{ cm}]$$

$\Rightarrow$ $$h = \frac{8 \times 8 \times 8}{3.142 \times 10 \times 10} \text{ cm} = 1.6 \text{ cm}$$

EXAMPLE 28 *500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is $0.04\ m^3$?*

[NCERT EXEMPLAR]

SOLUTION Suppose level of water rises by h metres in the pond. Then, clearly, water risen in the pond forms a cuboidal of dimensions 80 m $\times$ 80 m $\times$ h m.

$\therefore$ $80 \times 50 \times h$ = Volume of water displaced by 500 persons

$\Rightarrow$ $80 \times 50 \times h = 500 \times 0.04$

$\Rightarrow$ $4000h = 20$

$\Rightarrow$ $h = \frac{1}{200} \text{ m} = 0.5 \text{ cm}$

EXAMPLE 29 *The barrel of a fountain-pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre ?*

SOLUTION We have,

$$\text{Volume of a barrel} = \left(\frac{22}{7} \times 0.25 \times 0.25 \times 7\right) \text{cm}^3 = 1.375 \text{ cm}^3$$

$$\text{Volume of ink in the bottle} = \frac{1}{5} \text{ litre} = \frac{1000}{5} \text{ cm}^3 = 200 \text{ cm}^3$$

$\therefore$ Total number of barrels that can be filled from the given volume of ink $= \frac{200}{1.375}$

So, required number of words $= \frac{200}{1.375} \times 330 = 48000$

EXAMPLE 30 *The cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise per sq. dm is ₹237.60. The height of the tank is 6 times the radius of the base of the tank. Find its volume correct to two decimal places.*

SOLUTION Let r dm be the radius of the base and h dm be the height of the cylindrical tank. Then, $h = 6r$ (given)

Total surface area $= 2\pi r(r+h) = 2\pi r(r+6r) = 14\pi r^2$

$\Rightarrow$ Cost of painting $= ₹\,(14\pi r^2) \times \frac{60}{100} = ₹\,\frac{42}{5}\pi r^2$

It is given that the cost of painting is ₹ 237.60

$\therefore \quad \frac{42}{5}\pi r^2 = 237.60$

$\Rightarrow \quad \frac{42}{5} \times \frac{22}{7} \times r^2 = 237.60$

$\Rightarrow \quad r^2 = 237.60 \times \frac{5}{42} \times \frac{7}{22} = 9 \Rightarrow r = 3 \text{ dm}$

$\therefore \quad h = 6r = 18 \text{ dm}$

Hence, Volume of the cylinder $= \pi r^2 h = (\pi \times 3 \times 3 \times 18)\,\text{dm}^3 = \left(\frac{22}{7} \times 9 \times 18\right)\text{dm}^3 = 509.14\,\text{dm}^3$

EXAMPLE 31 *A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it is spread all a round to a width of 5 m to form an embankment. Find the height of embankment.*

SOLUTION We have,

Volume of the earth dugout $= (\pi r^2 h)\text{ m}^3$

$\Rightarrow$ Volume of the earth dugout $= \frac{22}{7} \times 5 \times 5 \times 14 \text{ m}^3 = 1100 \text{ m}^3$

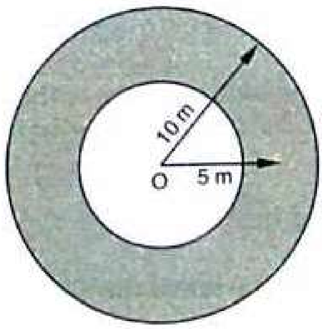

Fig. 14.11

Area of the embankment (shaded region) $= \pi(R^2 - r^2) = \pi(10^2 - 5^2)\,\text{m}^2 = \frac{22}{7} \times 75\,\text{m}^2$

$\therefore$ Height of the embankment $= \frac{\text{Volume of the earth dugout}}{\text{Area of the embankment}} = \frac{1100}{\frac{22}{7} \times 75} = \frac{7 \times 1100}{22 \times 75} = 4.66\text{ m.}$

EXAMPLE 32 *A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.* **[NCERT, CBSE 2015]**

SOLUTION We have,

Volume of the earth taken out of the well

= Volume of a cylinder of radius $\frac{7}{2}$ m and height 20 m

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20\ m^3 = 770\ m^3$

Let the height raised of 22 m × 14 m platform be equal to h metres. Then,

Volume of the earth in platform = Volume of the earth taken out of the well

$\Rightarrow \quad 22 \times 14 \times h = 770$

$\Rightarrow \quad h = \frac{770}{22 \times 14}\ m \Rightarrow h = \frac{5}{2}\ m = 2.5\ m.$

EXAMPLE 33 *An agriculture field is in the form of a rectangle of length 20 m width 14 m. A 10 m deep well of diameter 7m is dug in a corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in its level.*

SOLUTION We have,

Radius of the well $= \frac{7}{2}$, Depth of the well = 10 m

$\therefore$ Volume of the earth dug $= \pi\left(\frac{7}{2}\right)^2 \times 10\ m^3 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10\ m^3 = 358\ m^3$

Also, we have

Length of the field = 14 m, Breadth of the field = 14 m

$\therefore$ Area of the field $= 20 \times 14\ m^2 = 280\ m^2$

Area of the base of the well $= \pi \times \left(\frac{7}{2}\right)^2 m^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ m^2 = \frac{77}{2}\ m^2$

$\therefore$ Area of the remaining part of the field = Area of the field – Area of the base of the field

$= \left(280 - \frac{77}{2}\right) m^2 = \left(\frac{560 - 77}{2}\right) m^2 = \frac{483}{2}\ m^3$

Let the rise in the level of the field be h metres.

$\therefore$ Volume of the raised field = Area of the base × Height $= \left(\frac{483}{2} \times h\right) m^3$

But, Volume of the raised field = Volume of the earth dugout

$\therefore \quad \frac{483}{2} \times h = 385$

$\Rightarrow \quad h = \frac{2 \times 385}{483} = \frac{770}{483} = 1.594\ m$

Hence, rise in the level of the field = 1.594 m.

EXAMPLE 34 *A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow in kilometres per hour.* **[CBSE 2013]**

SOLUTION We have,

Volume of water that flows per hour = (192.50 × 60) litres

$= (192.50 \times 60 \times 1000)\ cm^3 \quad \ldots(i)$

Inner diameter of the pipe = 7 cm

$\Rightarrow$ Inner radius of the pipe $\frac{7}{2}$ cm = 3.5 cm

Let h cm be the length of the column of water that flows in one hour.

Clearly, water column forms a cylinder of radius 3.5 cm and length h cm.

$\therefore$ Volume of water that flows in one hour = Volume of the cylinder of radius 3.5 cm and length h cm

$$= \left(\frac{22}{7} \times (3.5)^2 \times h\right) \text{cm}^3 \quad \ldots \text{(ii)}$$

From (i) and (ii), we have

$$\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.50 \times 60 \times 1000$$

$$\Rightarrow \quad h = \frac{192.50 \times 60 \times 1000 \times 7}{22 \times 3.5 \times 3.5} \text{ cm} = 300000 \text{ cm} = 3 \text{ km}$$

Hence, the rate of flow of water is 3 km per hour.

EXAMPLE 35 *Water is being pumped out through a circular pipe whose internal diameter is 7 cm. If the flow of water is 72 cm per second, how many litres of water are being pumped out in one hour?*

SOLUTION We have, Radius of the circular pipe $= \frac{7}{2}$ cm

Clearly, water column forms a cylinder of radius $\frac{7}{2}$ cm. It is given that the water flows out at the rate of 72 cm/sec.

$\therefore$ Length of the water column flowing out in one second = 72 cm.

Volume of the water flowing out per second

= Volume of the cylinder of radius $\frac{7}{2}$ cm and length 72 cm.

$$= \pi \times \left(\frac{7}{2}\right)^2 \times 72 \text{ cm}^3 = \pi \times \frac{7}{2} \times \frac{7}{2} \times 72 \text{ cm}^3 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 72 \text{ cm}^3 = 2772 \text{ cm}^3$$

$\therefore$ Volume of the water flowing out in one hour $= (2772 \times 3600) \text{ cm}^3$ [$\because$ 1 hr = 3600 sec.]

$$= 9979200 \text{ cm}^3$$

$$= \frac{9979200}{1000} \text{ litres} = 9979.2 \text{ litres}$$

Hence, 9979.2 litres of water flows out per hour.

EXAMPLE 36 *Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how much time will the cistern be filled?* **[CBSE 2008]**

SOLUTION Suppose the cistern is filled in x hours. Since water is flowing at the rate of 3 km/hr. Therefore,

Length of the water column in x hours = $3x$ km = $3000x$ metres.

Clearly, the water column forms a cylinder of radius

$$r = \frac{20}{2} \text{ cm} = 10 \text{ cm} = \frac{1}{10} \text{ m and } h = \text{height (length)} = 3000x \text{ metres}$$

$\therefore$ Volume of the water that flows in the cistern in x hours

$$= \pi r^2 h = \left(\frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times 3000x\right) m^3$$

Also, Volume of the cistern $= \left(\frac{22}{7} \times 5 \times 5 \times 2\right) m^3$ $[\because r = 5 \text{ m}, h = 2 \text{ m}]$

Since the cistern is filled in x hours.

$\therefore$ Volume of the water that flows in the cistern in x hours = Volume of the cistern.

$$\Rightarrow \quad \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times 3000x = \frac{22}{7} \times 5 \times 5 \times 2$$

$$\Rightarrow \quad x = \left(\frac{5 \times 5 \times 2 \times 10 \times 10}{3000}\right) \text{hrs} = \frac{5}{3} \text{ hours} = 1 \text{ hour } 40 \text{ minutes.}$$

EXAMPLE 37 *Water is flowing at the rate of 7 metres per second through a circular pipe whose internal diameter is 2 cm into a cylindrical tank the radius of whose base is 40 cm. Determine the increase in the water level in 1/2 hour.* **[CBSE 2006C, 2013]**

SOLUTION We have,

Rate of flow of water = 7 m/sec = 700 cm/sec.

Length of the water column in $\frac{1}{2}$ hours $= (700 \times 30 \times 60)$ cm

Internal radius of circular pipe = 1 cm.

Clearly, water column forms a cylinder of radius 1 cm and length $= (700 \times 30 \times 60)$ cm.

$\therefore$ Volume of the water that flows in the tank in $\frac{1}{2}$ hr

$$= \left(\frac{22}{7} \times 1 \times 1 \times 700 \times 30 \times 60\right) cm^3 \quad [\text{Using: } V = \pi r^2 h] \quad \ldots(i)$$

Let h cm be the rise in the level of water in the tank. Then,

$$\text{Volume of the water in the tank} = \frac{22}{7} \times 40 \times 40 \times h \text{ cm}^3 \quad \ldots(ii)$$

From (i) and (ii), we obtain

$$\frac{22}{7} \times 40 \times 40 \times h = \frac{22}{7} \times 1 \times 1 \times 700 \times 30 \times 60$$

$$\Rightarrow \quad h = \frac{700 \times 30 \times 60}{40 \times 40} \text{ cm} = 787.5 \text{ cm}$$

Hence, the rise in the level of water in the tank in $\frac{1}{2}$ hr is 787.5 cm.

EXAMPLE 38 *Water is flowing at the rate of 5 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Determine the time in which the level of the water in the tank will rise by 7 cm.* **[CBSE 2017]**

SOLUTION Suppose the level of the water in the tank will rise by 7 cm in x hours.

Since the water is flowing at the rate of 5 km/hr. Therefore,

Length of the water column in x hours $= 5x$ km $= 5000\,x$ metres

Clearly, the water column forms a cylinder whose radius $r = \frac{14}{2}$ cm $= \frac{7}{100}$ m

and, Length $= h = 5000\,x$ metres

∴ Volume of the water flowing through the cylindrical pipe in x hours

$$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000x \text{ m}^3 = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000x \text{ m}^3 = 77x \text{ m}^3$$

Also,

Volume of the water that falls into the tank in x hours $= 50 \times 44 \times \frac{7}{100} \text{ m}^3 = 154 \text{ m}^3$

But, Volume of the water flowing through the cylindrical pipe in x hours

= Volume of the water that falls in the tank in x hours

$\Rightarrow \quad 77x = 154$

$\Rightarrow \quad x = \frac{154}{77} = 2$

Hence, the level of the water in the tank will rise by 7 cm in 2 hours.

EXAMPLE 39 *The rain water from a roof of 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, find the rain fall in cm.*

[CBSE 2010]

SOLUTION We have,

$r =$ Radius of cylindrical vessel = 1 m, $h =$ Height of cylindrical vessel = 3.5 m

∴ Volume of cylindrical vessel $= \pi r^2 h = \frac{22}{7} \times 1^2 \times 3.5 \text{ m}^3 = 11 \text{ m}^3$

Let the rain fall be x m. Then,

Volume of the water = Volume of a cuboid of base 22 m × 20 m and height x metres

$= (22 \times 20 \times x) \text{ m}^3$

Since the vessel is just full of the water that drains out of the roof into the vessel.

∴ Volume of the water = Volume of the cylindrical vessel

$\Rightarrow \quad 22 \times 20 \times x = 11$

$\Rightarrow \quad x = \frac{11}{22 \times 20} = \frac{1}{40} \text{ m} = \frac{100}{40} \text{ cm} = 2.5 \text{ cm}$

EXAMPLE 40 *Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km/hr. How much area will it irrigate in 30 minutes, if 8 cm of standing water is required for irrigation?*

[CBSE 2014]

SOLUTION We have,

Width of the canal = 30 dm = 300 cm = 3 m

Depth of the canal = 12 dm = 120 cm = 1.2 m.

It is given that the water is flowing with velocity 10 km/hr. Therefore,

Length of the water column formed in $\frac{1}{2}$ hour = 5 km = 5000 m

∴ Volume of the water flowing in $\frac{1}{2}$ hour = Volume of the cuboid of length 5000 m, width 3 m and depth 1.2 m

$\Rightarrow$ Volume of the water following in ½ hour $= 5000 \times 3 \times 1.2 \text{ m}^3 = 18000 \text{ m}^3$

Suppose x m² area is irrigated in $\frac{1}{2}$ hour. Then,

$$x \times \frac{8}{100} = 18000 \Rightarrow x = \frac{1800000}{8} \text{ m}^2 \Rightarrow x = 225000 \text{ m}^2$$

Hence, the canal irrigates 225000 m^2 area in $\frac{1}{2}$ hour.

EXAMPLE 41 *Water flows at the rate of 10 metre per minute through a cylindrical pipe having its diameter as 5 mm. How much time will it take to fill a conical vessel whose diameter of base is 40 cm and depth 24 cm?*

SOLUTION We have,

Volume of the water that flows out in one minute

= Volume of the cylinder of diameter 5 mm and length 10 metre

= Volume of the cylinder of radius $\frac{5}{2}$ mm $\left(=\frac{1}{4}\right)$ cm and length 1000 cm

$= \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 1000 \text{ cm}^3$

Volume of a conical vessel of base radius 20 cm and depth 24 cm $= \frac{1}{3} \times \frac{22}{7} \times (20)^2 \times 24 \text{ cm}^3$

Suppose the conical vessel is filled in x minutes.

$\therefore$ Volume of the water that flows out in x minutes = Volume of the conical vessel

$\Rightarrow \quad \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 1000 \times x = \frac{1}{3} \times \frac{22}{7} \times 20^2 \times 24$

$\Rightarrow \quad x = \frac{1}{3} \times \frac{400 \times 24 \times 4 \times 4}{1000} = \frac{512}{10}$ minutes

$\Rightarrow \quad x = 51$ minutes 12 seconds.

EXAMPLE 42 *A hemispherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second. How much time will it take to empty the tank completely?*

SOLUTION Suppose the pipe takes x seconds to empty the tank. Then,

Volume of the water that flows out of the tank in x seconds

= Volume of the hemispherical tank.

$\Rightarrow$ Volume of the water that flows out of the tank x in seconds

= Volume of the hemispherical shell of radius 175 cm

$\Rightarrow \quad 7000\,x = \frac{2}{3} \times \frac{22}{7} \times 175 \times 175 \times 175$

$\Rightarrow \quad x = \frac{2}{3} \times \frac{22}{7} \times \frac{175 \times 175 \times 175}{7000} = 1604.16$ seconds

$\Rightarrow \quad x = \frac{1604.16}{60}$ minutes = 26.73 minutes

EXAMPLE 43 *A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to make the tank half-empty, if the tank is 3 m in diameter?*

[CBSE 2016]

SOLUTION We have, Radius of hemispherical tank $= \frac{3}{2}$ m

$\therefore$ Volume of the tank $= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3 \text{ m}^3 = \frac{99}{14} \text{m}^3$

Volume of the water to be emptied $= \frac{1}{2} \times \frac{99}{14} \text{ m}^3 = \frac{99}{28} \text{ m}^3 = \frac{99}{28} \times 1000$ litres $= \frac{99000}{28}$ litres

Since $\frac{25}{7}$ litres of water is emptied in one second. Therefore,

Total time taken to empty half the tank i.e $\frac{99000}{28}$ litres of water $= \frac{99000}{28} \div \frac{25}{7}$ seconds

$= \frac{99000}{28} \times \frac{7}{25}$ seconds

$= \frac{99000}{28} \times \frac{7}{25} \times \frac{1}{60}$ minutes

$= 16.5$ minutes

EXAMPLE 44 *The largest sphere is carved out of a cube of a side 7 cm. Find the volume of the sphere.*

SOLUTION The diameter of the largest sphere which can be carved out of a cube of side 7 cm is 7 cm.

$\therefore$ Radius of the sphere $= r = \frac{7}{2}$ cm

Hence,

Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 \text{ cm}^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{343}{8} \text{ cm}^3 = 179.66 \text{ cm}^3$$

EXAMPLE 45 *Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.*

SOLUTION Let the radius of the sphere which fits exactly into a cube be r units. Then, Length of each edge of the cube $= 2r$ units

Let V_1 and V_2 be the volumes of the cube and sphere respectively. Then,

$$V_1 = (2r)^3 \text{ and } V_2 = \frac{4}{3}\pi r^3$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{8r^3}{\frac{4}{3}\pi r^3} = \frac{6}{\pi} \Rightarrow V_1 : V_2 = 6 : \pi$$

EXAMPLE 46 *Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r.*

SOLUTION Clearly,

Radius of the base of cone = Radius of the hemisphere = r

and, Height of the cone = Radius of the hemisphere = r

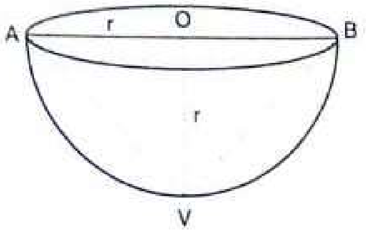

Fig. 14.12

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 \times r = \frac{1}{3}\pi r^3$ cubic units

LEVEL-2

EXAMPLE 47 *The radius of a solid iron sphere is 8 cm. Eight rings of iron plate of external radius $6\frac{2}{3}$ cm and thickness 3 cm are made by melting this sphere. Find the internal diameter of each ring.*

SOLUTION We have,

Volume of solid iron sphere $= \frac{4}{3}\pi \times 8^3 \text{ cm}^3 = \frac{2048}{3}\pi \text{ cm}^3$

External radius of each iron ring $= 6\frac{2}{3}$ cm $= \frac{20}{3}$ cm

Let the internal radius of each ring be r cm. Since each ring forms a hollow cylindrical shell of external and internal radii $\frac{20}{3}$ cm and r cm respectively and height 3 cm.

$\therefore$ Volume of each ring $= \pi\left\{\left(\frac{20}{3}\right)^2 - r^2\right\} \times 3 \text{ cm}^3$

Volume of 8 such rings $= 8\pi\left(\frac{400}{9} - r^2\right) \times 3 \text{ cm}^3 = 24\pi\left(\frac{400}{9} - r^2\right) \text{cm}^3$

Clearly, Volume of 8 rings = Volume of the sphere

$\Rightarrow \quad 24\pi\left(\frac{400}{9} - r^2\right) = \frac{2048}{3}\pi$

$\Rightarrow \quad \frac{400}{9} - r^2 = \frac{2048}{3}\pi \times \frac{1}{24\pi}$

$\Rightarrow \quad r^2 = \frac{400}{9} - \frac{256}{9} = \frac{144}{9} = 16$

$\Rightarrow \quad r = 4$ cm

Hence, internal radius of each ring is 4 cm.

EXAMPLE 48 *A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Fig.14.13. What fraction of water over flows?*

SOLUTION Let the radius of the sphere be r cm.

In $\Delta VO'A$, we have

$$\tan\theta = \frac{6}{8} = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$$

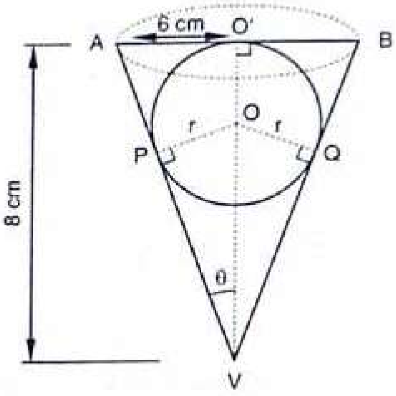

Fig. 14.13

In ΔVPO, We have

$$\sin\theta = \frac{r}{VO}$$

$\Rightarrow \quad \frac{3}{5} = \frac{r}{8-r}$

$\Rightarrow \quad 24 - 3r = 5r$

$\Rightarrow \quad 8r = 24$

$\Rightarrow \quad r = 3$ cm

$\therefore \quad V_1 = \text{Volume of the sphere} = \frac{4}{3}\pi \times 3^3 \text{ cm}^3 = 36\pi \text{ cm}^3$

$V_2 = \text{Volume of the water} = \text{Volume of the cone} = \frac{1}{3}\pi \times 6^2 \times 8 \text{ cm}^3 = 96\pi \text{ cm}^3$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1.

$\therefore \quad$ Fraction of the water that flows out $= V_1 : V_2 = 36\pi : 96\pi = 3 : 8$

ALITER In $\Delta VO'A$, we have

$VA^2 = VO'^2 + O'A^2$

$\Rightarrow \quad VA^2 = 8^2 + 6^2 = 100$

$\Rightarrow \quad VA = 10$ cm.

Now, $\quad AO' = AP \quad$ [$\because$ Tangents drawn from A to the circle are equal]

$\Rightarrow \quad AP = 6 \quad$ [$\because AO' = 6$ cm]

$\therefore \quad VP = VA - AP = (10 - 6)$ cm $= 4$ cm

Now, $\quad VO = VO' - OO' = (8 - r)$ cm

In ΔVPO, we have

$VO^2 = VP^2 + OP^2$

$\Rightarrow \quad (8 - r)^2 = 16 + r^2$

$\Rightarrow \quad 64 - 16r + r^2 = 16 + r^2 \Rightarrow 16r = 48 \Rightarrow r = 3$ cm.

Now, proceed as in the previous solution.

EXAMPLE 49 *Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank). Which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 0.95 m. The overhead tank has its radius of 60 cm and its height is 95 cm. Find the height of the water, left in the sump after the overhead tank has been completely filled with water from a sump which had been full. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$).* **[NCERT]**

SOLUTION Clearly, the volume of the water in the overhead tank is equal to the volume of the water removed from the sump.

Now,

Volume of water in the overhead tank $= 3.14 \times 0.6 \times 0.6 \times 0.95 \text{ m}^3$ $\left[\text{Using: } V = \pi r^2 h\right]$

$$= 3.14 \times 0.36 \times 0.95 \text{ m}^3$$

Volume of water in the sump when it is full of water $= 1.57 \times 1.44 \times 0.95 \text{ m}^3$

$$= 1.57 \times 4 \times 0.36 \times 0.95 \text{ m}^3$$

$$= 2 \times 3.14 \times 0.36 \times 0.95 \text{ m}^3$$

∴ Volume of water left in the sump after filling the tank

$$= (2 \times 3.14 \times 0.36 \times 0.95 - 3.14 \times 0.36 \times 0.95) \text{ m}^3$$

$$= 3.14 \times 0.36 \times 0.95 (2-1) \text{ m}^3 = 3.14 \times 0.36 \times 0.95 \text{ m}^3$$

Area of the base of the sump $= 1.57 \times 1.44 \text{ m}^2 = 1.57 \times 4 \times 0.36 \text{ m}^2 = 2 \times 3.14 \times 0.36 \text{ m}^2$

∴ Height of water in the sump $= \dfrac{3.14 \times 0.36 \times 0.95}{2 \times 3.14 \times 0.36} \text{ m} = \dfrac{0.95}{2} = 0.475 \text{ m} = 47.5 \text{ cm}$

$$\frac{\text{Capacity of tank}}{\text{Capacity of sump}} = \frac{3.14 \times 0.36 \times 0.95}{2 \times 3.14 \times 0.36 \times 0.95} = \frac{1}{2}$$

Hence, the capacity of the tank is half the capacity of the sump.

EXAMPLE 50 *If the diameter of cross-section of a wire is decreased by 5% how much percent will the length be increased so that the volume remains the same ?*

SOLUTION Let r be the radius of cross-section of wire and h be its length. Then,

Volume $= \pi r^2 h$ ...(i)

5% of diameter of cross-section $= \dfrac{5}{100} \times 2r = \dfrac{r}{10}$

New diameter $= 2r - \dfrac{r}{10} = \dfrac{19r}{10}$

⇒ New radius $= \dfrac{19r}{20}$

Let the new length be h_1. Then,

$$\text{Volume} = \pi\left(\frac{19r}{20}\right)^2 h_1 \quad ..(ii)$$

From (i) and (ii), we obtain

$$\pi r^2 h = \pi\left(\frac{19r}{20}\right)^2 h_1 \Rightarrow h = \frac{361}{400}h_1 \Rightarrow h_1 = \frac{400}{361}h$$

$\therefore$ Increase in length $= h_1 - h = \frac{400h}{361} - h = \frac{39h}{361}$

$\Rightarrow$ Percentage increase in length $= \frac{h_1 - h}{h} \times 100 = \frac{39h}{h} \times 100 = \frac{3900}{361} = 10.8\%$

Hence, the length of the wire increases by 10.8%

EXAMPLE 51 *A well, whose diameter is 7m, has been dug 22.5 m deep and the earth dugout is used to form an embankment around it. If the height of the embankment is 1.5 m, find the width of the embankment.*

SOLUTION We have, Radius of the well $= \frac{7}{2}$ m $= 3.5$ m and, Depth of the well $= 22.5$ m

$\therefore$ Volume of the earth dugout $= \pi \times (3.5)^2 \times 22.5 \text{ m}^3 = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2} \text{ m}^3$

Let the width of the embankment be r metres. Clearly, embankment forms a cylindrical shell whose inner and outer radii are 3.5 m and $(r + 3.5)$ m respectively and height 1.5 m.

$\therefore$ Volume of the embankment $= \pi\{(r+3.5)^2 - (3.5)^2\} \times 1.5 \text{ m}^3 = \pi(r+7)r \times \frac{3}{2} \text{ m}^3$

But, Volume of the embankment = Volume of the earth dugout

$\Rightarrow \quad \pi r(r+7) \times \frac{3}{2} = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{45}{2}$

$\Rightarrow \quad r(r+7) = \frac{49}{4} \times 15$

$\Rightarrow \quad 4r^2 + 28r = 735$

$\Rightarrow \quad 4r^2 + 28r - 735 = 0$

$\Rightarrow \quad r = \frac{-28 \pm \sqrt{784 + 11760}}{8}$

$\Rightarrow \quad r = \frac{-28 \pm \sqrt{12544}}{8} = \frac{-28 \pm 112}{8} = \frac{84}{8} = 10.5 \quad [\because r > 0]$

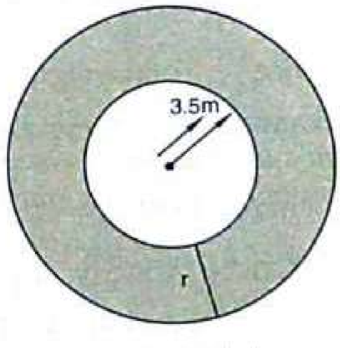

Fig. 14.14

Hence, the width of the embankment is 10.5 m.

EXAMPLE 52 *A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/cm³ and that of the graphite is 2.1 gm/cm³.*

SOLUTION We have, Diameter of the graphite cylinder $= 1 \text{ mm} = \frac{1}{10}$ cm

$\therefore$ Radius $= \frac{1}{20}$ cm

Length of the graphite cylinder $= 10$ cm

Volume of the graphite cylinder $= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10\right) \text{cm}^3$

Weight of graphite = Volume × Specific gravity

$$= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10 \times 2.1\right) \text{gm} = 0.165 \text{ gm}$$

Diameter of pencil $= 7 \text{ mm} = \frac{7}{10}$ cm

$\therefore$ Radius of pencil $= \frac{7}{20}$ cm and, Length of pencil $= 10$ cm

$\therefore$ Volume of pencil $= \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 10 \text{ cm}^3$

Volume of wood $= \left(\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 10 - \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10\right) \text{cm}^3$

$$= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10(7 \times 7 - 1) \text{ cm}^3 = \frac{11}{7} \times \frac{1}{20} \times 48 \text{ cm}^3$$

$\therefore$ Weight of wood $= \left(\frac{11}{7} \times \frac{1}{20} \times 48 \times 0.7\right) \text{gm} = \left(\frac{11}{7} \times \frac{1}{20} \times 48 \times \frac{7}{10}\right) \text{gm} = 2.64 \text{ gm}$

Hence, Total weight $= (2.64 + 0.165) \text{ gm} = 2.805$ gm.

EXAMPLE 53 *A copper wire 4 mm in diameter is evenly wound about a cylinder whose length is 24 cm and diameter 20 cm so as to cover the whole surface. Find the length and weight of the wire assuming the specific gravity to be 8.88 gm/cm*^{3}.

SOLUTION Clearly, one round of wire covers 4 mm $\left(= \frac{4}{10} \text{ cm}\right)$ in thickness of the surface of the cylinder and length of the cylinder is 24 cm.

$\therefore$ Number of rounds to cover 24 cm $= \frac{24}{4/10} = \frac{24 \times 10}{4} = 60$

Diameter of the cylinder = 20 cm

Radius of the cylinder = 10 cm

Length of the wire in completing one round $= 2\pi r = 2\pi \times 10 \text{ cm} = 20\pi$ cm.

$\therefore$ Length of the wire in covering the whole surface

= Length of the wire in completing 60 rounds

$= (20\pi \times 60) \text{ cm} = 1200\pi$ cm

Radius of copper wire = 2 mm = $\frac{2}{10}$ cm

$\therefore$ Volume of wire = $\left(\pi \times \frac{2}{10} \times \frac{2}{10} \times 1200\pi\right) \text{cm}^3 = 48\pi^2 \text{ cm}^3$

So, Weight of wire = $(48\pi^2 \times 8.88)$ gm = $426.24\,\pi^2$gm

EXAMPLE 54 *A copper wire 3 mm in diameter is wound about a cylinder whose length is 1.2 m, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of the copper wire to be 8.88 gram per cm.* **[NCERT]**

SOLUTION We have, d = Diameter of copper wire = 3 mm = $\frac{3}{10}$ cm.

and, h = Height (length) of the cylinder = 1.2 m = 120 cm.

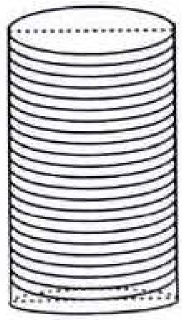

Fig. 14.15

$\therefore$ Number of rounds taken by the wire to cover the curved surface of the cylinder $= \frac{h}{d}$

$$= \frac{120}{\frac{3}{10}} = 400$$

Length of wire used in taking one round $= 2\pi r = (2 \times 3.14 \times 5)$ cm = 31.4 cm

$\therefore$ Total length of wire used in covering the curved surface of the cylinder = (31.4×400) cm

= 12560 cm = 125.6 m

Mass of the wire = Length × Density = 12560 × 8.88 gram = 111532.8 gm $\cong$ 111.533 kg

EXERCISE 14.1

LEVEL-1

1. How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm?
2. How many spherical bullets each of 5 cm in diameter can be cast from a rectangular block of metal 11 dm × 1 m × 5 dm?
3. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of the two of the balls are 1.5 cm and 2 cm respectively. Determine the diameter of the third ball.
4. 2.2 cubic dm of brass is to be drawn into a cylindrical wire 0.25 cm in diameter. Find the length of the wire.

5. What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?

6. A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel.

7. 50 circular plates each of diameter 14 cm and thickness 0.5 cm are placed one above the other to form a right circular cylinder. Find its total surface area.

8. 25 circular plates, each of radius 10.5 cm and thickness 1.6 cm, are placed one above the other to form a solid circular cylinder. Find the curved surface area and the volume of the cylinder so formed.

9. Find the number of metallic circular discs with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm. **[NCERT EXEMPLAR]**

10. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 6 cm × 42 cm × 21 cm. **[NCERT EXEMPLAR]**

11. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm. **[NCERT EXEMPLAR]**

12. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes. **[NCERT EXEMPLAR]**

13. A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed. **[CBSE 2017, NCERT EXEMPLAR]**

14. The diameter of a metallic sphere is equal to 9 cm. It is melted and drawn into a long wire of diameter 2 mm having uniform cross-section. Find the length of the wire.

15. An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is 1/4 of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

16. A copper sphere of radius 3 cm is melted and recast into a right circular cone of height 3 cm. Find the radius of the base of the cone.

17. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire. **[NCERT]**

18. The diameters of internal and external surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

19. How many coins 1.75 cm in diameter and 2 mm thick must be melted to form a cuboid 11 cm × 10 cm × 7 cm? **[NCERT]**

20. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed. (Use $\pi = 22/7$). **[CBSE 2010]**

21. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied out on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap. **[CBSE 2012, 2014]**

22. A solid metallic sphere of radius 5.6 cm is melted and solid cones each of radius 2.8 cm and height 3.2 cm are made. Find the number of such cones formed. **[CBSE 2014, 2017]**

23. A solid cuboid of iron with dimensions 53 cm × 40 cm × 15 cm is melted and recast into a cylindrical pipe. The outer and inner diameters of pipe are 8 cm and 7 cm respectively. Find the length of pipe. **[CBSE 2015]**

24. The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder. **[CBSE 2001 C]**

25. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Calculate the height of the cone.

26. A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

27. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

28. A path 2 m wide surrounds a circular pond of diameter 40 m. How many cubic metres of gravel are required to grave the path to a depth of 20 cm?

29. A 16 m deep well with diameter 3.5 m is dug up and the earth from it is spread evenly to form a platform 27.5 m by 7 m. Find the height of the platform.

30. A well of diameter 2 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to form an embankment of height 40 cm. Find the width of the embankment. **[CBSE 2015]**

31. A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment. **[CBSE 2016]**

32. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment. **[CBSE 2016]**

33. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.

34. A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

35. Rain water, which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if a rainfall of 1 cm has fallen ? [Use $\pi = 22/7$]

36. The rain water from a roof of dimensions 22 m × 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the rain water collected from the roof just fills the cylindrical vessel, then find the rain fall in cm. **[NCERT EXEMPLAR]**

37. A conical flask is full of water. The flask has base-radius r and height h. The water is poured into a cylindrical flask of base-radius mr. Find the height of water in the cylindrical flask.

38. A rectangular tank 15 m long and 11 m broad is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21 m and length 5 m. Find the least height of the tank that will serve the purpose.

39. A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl?

40. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the radius of the ball.

41. 500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m^3? **[NCERT EXEMPLAR]**

42. A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimetres?

43. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball of radius 9 cm is dropped into the tub and thus the level of water is raised by h cm. What is the value of h?

44. Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimension 16 cm × 8 cm × 8 cm when 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. [Use $\pi = 669/213$]

45. A vessel in the shape of a cuboid contains some water. If three indentical spheres are immersed in the water, the level of water is increased by 2 cm. If the area of the base of the cuboid is 160 cm^2 and its height 12 cm, determine the radius of any of the spheres.

46. 150 spherical marbles, each of diameter 1.4 cm are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel. **[CBSE 2014]**

47. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\left(\frac{2}{5}\right)^{th}$ of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant? **[CBSE 2014]**

48. 16 glass spheres each of radius 2 cm are packed into a cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box. **[NCERT EXEMPLAR]**

49. Water flows through a cylindrical pipe, whose inner radius is 1 cm, at the rate of 80 cm/sec in an empty cylindrical tank, the radius of whose base is 40 cm. What is the rise of water level in tank in half an hour? **[NCERT EXEMPLAR]**

50. Water in a canal 1.5 m wide and 6 m deep is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired? **[NCERT]**

51. A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled? **[CBSE 2014, NCERT]**

52. A cylindrical tank full of water is emptied by a pipe at the rate of 225 litres per minute. How much time will it take to empty half the tank, if the diameter of its base is 3 m and its height is 3.5 m? [Use $\pi = 22/7$] **[CBSE 2014]**

53. Water is flowing at the rate of 2.52 km/h through a cylinderical pipe into a cylindrical tank, the radius of the base is 40 cm. If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe. **[CBSE 2015]**

54. Water flows at the rate of 15 km/hr through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm? **[NCERT EXEMPLAR]**

55. A canal is 300 cm wide and 120 cm deep. The water in the canal is flowing with a speed of 20 km/h. How much area will it irrigate in 20 minutes if 8 cm of standing water is desired? **[NCERT EXEMPLAR]**

56. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 cm^2, find the volume of cylinder. (Use $\pi = 22/7$) **[CBSE 2016]**

57. A tent of height 77 dm is in the form a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at ₹ 3.50 per m^2. [Use $\pi = 22/7$]

58. The largest sphere is to be curved out of a right circular cylinder of radius 7 cm. and height 14 cm. Find the volume of the sphere.

59. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two cones so formed. Also, find their curved surfaces.

60. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of ₹ 25 per metre. [Use $\pi = 22/7$] **[CBSE 2014]**

61. The volume of a hemi-sphere is $2425\frac{1}{2}cm^3$. Find its curved surface area. (Use $\pi = 22/7$) **[CBSE 2012]**

62. The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14 cm long is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. (Use $\pi = 22/7$) **[CBSE 2010]**

63. The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25.2 cm respectively. The cost of painting 1 cm^2 of the surface is 10 paise. Find the total cost to paint the vessel all over.

64. Prove that the surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

65. If the total surface area of a solid hemisphere is 462 cm^2, find its volume (Take $\pi = 22/7$) **[CBSE 2014]**

66. Water flows at the rate of 10 m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm? **[NCERT EXEMPLAR]**

67. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone. **[NCERT EXEMPLAR]**

68. A heap of rice in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of rice. How much canvas cloth is required to cover the heap?
[NCERT EXEMPLAR, CBSE 2018]

69. A cylindrical bucket of height 32 cm and base radius 18 cm is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.
[NCERT EXEMPLAR]

70. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped bottles each of radius 1.5 cm and height 4 cm. How many bottles are needed to empty the bowl?
[NCERT EXEMPLAR]

71. A factory manufactures 120,000 pencils daily The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹ 0.05 per dm^2.
[NCERT EXEMPLAR]

72. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.
[CBSE 2017]

ANSWERS

1. 512	2. 8400	3. 5 cm	4. 448 metres
5. 79 cm	6. 42 cm	7. 1408 cm^2	
8. 2640 cm^2, 13860 cm^3		9. 450	10. 1500
11. 2541	12. 6 cm, 8 cm, 10 cm		13. 126
14. 12150 cm	15. 64 balls, 4 : 1	16. 6 cm	17. 0.67 mm
18. 14 cm	19. 400	20. 14 cm	21. 36 cm, $12\sqrt{13}$ cm
22. 28	23. 2698.18 cm	24. $\frac{8}{3}$ cm	25. 14 cm
26. 14 cm, 14.56 cm	27. 2.5 cm	28. 52.8 m^3	29. 80 cm
30. 5 m	31. 6.78 m	32. 9/8	33. 190.93 cm^3
34. 36 cm, 43.27cm	35. 191 cm	36. 2.5 cm	37. $\frac{h}{3m^2}$
38. 10.5 m	39. 54	40. 9 cm	41. 0.5 cm
42. 16	43. 6.75 cm	44. 448 cm^3	45. 2.94 cm
46. 5.6 cm	47. 440	48. 487.6 cm^3	49. 90 cm
50. 562500 m^2	51. 1 hour 40 minutes		52. 55 minutes
53. 4 cm	54. 2 hour	55. 30 hectares	56. 4620 cm^3
57. ₹ 5365. 80	58. 1437	59. 4π cm^3, 20π cm^2, 15π cm^2	
60. ₹ 2750	61. 693 cm^2	62. 5 cm, 3 cm	63. ₹ 184.34
65. 7 cm	66. 51 minutes 12 sec		67. 1.584 m^3
68. 74.25 m^3, 80.61 m^2		69. 36 cm, 43.27 cm	
70. 54	71. ₹ 2250	72. 1.5 cm.	

HINT TO SELECTED PROBLEMS

1. Number of balls $= \frac{\text{Volume of sphere of radius 8 cm}}{\text{Volume of sphere of radius 1 cm}} = \frac{\frac{4}{3}\pi \times 8^3}{\frac{4}{3}\pi \times 1^3} = 512$

19. Total number of coins $= \dfrac{11 \times 10 \times 7}{\frac{22}{7} \times \left(\frac{1.75}{2}\right)^2 \times \left(\frac{2}{10}\right)} = \dfrac{11 \times 10 \times 7}{\frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times \frac{2}{10}} = 400$

27. Let r be the radius of the third ball. Then,

$$\frac{4}{3}\pi \times (3)^3 = \frac{4}{3}\pi \times (1.5)^3 + \frac{4}{3}\pi \times (2)^3 + \frac{4}{3}\pi r^3 \Rightarrow 27 = \frac{27}{8} + 8 + r^3 \Rightarrow r^3 = \frac{125}{8} \Rightarrow r = \frac{5}{2}$$

31. Height of embankment $= \dfrac{\text{Volume of the earth dugout}}{\text{Area of the embankment}} = \dfrac{\frac{22}{7} \times 4^2 \times 14}{\frac{22}{7} \times (7^2 - 4^2)} = 6.78$ m

32. Height of the embankment $= \dfrac{\pi \times \frac{3}{2} \times \frac{3}{2} \times 14}{\pi\left(\frac{11}{2} \times \frac{11}{2} - \frac{3}{2} \times \frac{3}{2}\right)}$ m $= \dfrac{9}{8}$ m

34. Let r be the radius and 1 the slant height. Then,

Volume of the bucket = Volume of the heap $\Rightarrow \pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r^2 \times 24 \Rightarrow r = 36$

38. Let the least height be h metre. Then, $15 \times 11 \times h = \frac{1}{3}\pi \times \left(\frac{21}{2}\right)^2 \times 5.$

42. Suppose x iron spheres are required to raise the level of the oil by two cm. Then,

Volume of x iron spheres = Volume of oil raised

$$\Rightarrow x \times \frac{4}{3}\pi \times (1.5)^3 = \pi \times 6^2 \times 2 \Rightarrow x \times \frac{4}{3}\pi\left(\frac{3}{2}\right)^2 = 72\pi \Rightarrow x = 16$$

43. Let r cm be the radius of the ball. Then,

Volume of ball = Volume of water raised $\Rightarrow \frac{4}{3}\pi r^2 = \pi \times (12)^2 \times 6.75 \Rightarrow r = 9$ cm.

50. Required area $= \dfrac{1.5 \times 6 \times 10000 \times \frac{1}{2}}{\left(\frac{8}{100}\right)}\ \text{m}^2 = \dfrac{9 \times 5000 \times 25}{2}\ \text{m}^2 = 562500\ \text{m}^2$

51. Volume of cylindrical tank $= \pi \times 5^2 \times 2\ \text{m}^3 = 50\pi\ \text{m}^3$

Volume of the water that flows through the pipe in t hours

= Volume of a cylinder of radius of 10 cm and length = $3t$ km = $3000\,t$ m

$$= \left\{\pi \times \left(\frac{1}{10}\right)^2 \times 3000\,t\right\} \text{m}^3 = 30\pi t\ \text{m}^3$$

$\therefore \quad 30\pi t = 50\pi \Rightarrow t = \frac{5}{3}$ hours = 1 hour 40 minutes

64. Height of the circumscribing cylinder $= 2r$,

Radius of the circumscribing cylinder $= r$, where r is the radius of the sphere.

14.4 SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS

Uptill now we have learnt about various applications of the formulas for finding the surface areas and volumes of basic solids, namely, a right circular cone, a right circular cylinder, a sphere, a hemisphere etc. In this section, we shall apply them to find the surface areas and volumes of solids which are combinations of the basic solids. For example, a circus tent consisting of a cylindrical base surmounted by a conical roof, a toy in the form of a cone mounted on a hemisphere etc. are combinations of two or more basic solids. Following examples will illustrate the method of finding surface areas and volumes of such combinations of solids.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cubic cm of iron weighs 7.8 grams.*

SOLUTION Let r_1 cm and r_2 cm denote the radii of the base of the cylinder and cone respectively. Then,

$$r_1 = r_2 = 8\,\text{cm}$$

Let h_1 and h_2 cm be the heights of the cylinder and the cone respectively. Then,

$$h_1 = 240\text{ cm and } h_2 = 36\text{ cm}$$

$$\therefore \quad \text{Volume of the cylinder} = \pi r_1^2 h_1\text{ cm}^3$$

$$= (\pi \times 8 \times 8 \times 240)\text{ cm}^3$$

$$= (\pi \times 64 \times 240)\text{ cm}^3$$

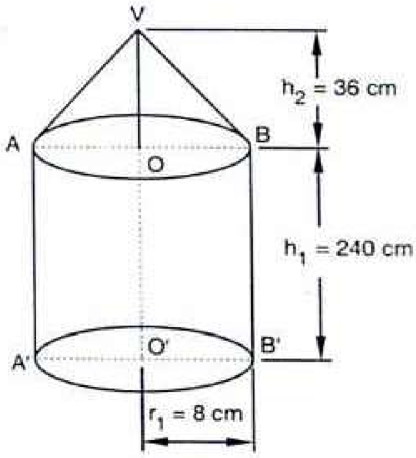

Fig. 14.16

$$\text{Volume of the cone} = \frac{1}{3}\pi r_2^2 h_2\text{ cm}^3$$

$$= \left(\frac{1}{3}\pi \times 8 \times 8 \times 36\right)\text{cm}^3$$

$$= \left(\frac{1}{3}\pi \times 64 \times 36\right)\text{cm}^3$$

$$\therefore \quad \text{Total volume of the iron} = \text{Volume of the cylinder} + \text{Volume of the cone}$$

$$= \left(\pi \times 64 \times 240 + \frac{1}{3}\pi \times 64 \times 36\right)\text{cm}^3$$

$$= \pi \times 64 \times (240 + 12)\text{ cm}^3$$

$$= \frac{22}{7} \times 64 \times 252\text{ cm}^3 = 22 \times 64 \times 36\text{ cm}^3$$

Hence, $\quad \text{Total weight of the pillar} = \text{Volume} \times \text{Weight per cm}^3$

$$= (22 \times 64 \times 36) \times 7.8\text{ gms}$$

$$= 395366.4\text{ gms} = 395.3664\text{ kg}$$

EXAMPLE 2 *The interior of a building is in the form of a right circular cylinder of diameter 4.2 m and height 4 m surmounted by a cone. The vertical height of cone is 2.1 m. Find the outer surface area and volume of the building.* (Use $\pi = 22/7$)

SOLUTION Let r_1 be the radius of base of the cylinder and h_1 m be its height. It is given that $r_1 = 2.1$ m and $h_2 = 4$ m. Let r_2 m be the radius of the base of the cone, h_2 m be its height and l_2 m and be its slant height. It is also given that $r_2 = 2.1$ m, $h_2 = 2.1$ m.

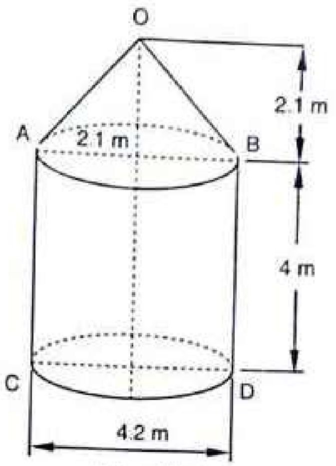

Fig. 14.17

$\therefore \quad l_2^2 = r_2^2 + h_2^2$

$\Rightarrow \quad l_2 = \sqrt{r_2^2 + h_2^2} = \sqrt{(2.1)^2 + (2.1)^2} = \sqrt{(2.1)^2 \times 2} = 2.1 \times \sqrt{2}$ m

Let S be the outer surface area and V be the volume of the building. Then,

$S =$ Curved surface area of cylinder + Curved surface area of cone

$= (2\pi r_1 h_1 + \pi r_2 l_2)\ \text{m}^2$

$= \pi(2r_1 h_1 + r_2 l_2)\ \text{m}^2$

$= \frac{22}{7}(2 \times 2.1 \times 4 + 2.1 \times 2.1 \times \sqrt{2})\ \text{m}^2$

$= \frac{22}{7} \times 2.1 \times (8 + 2.1 \times \sqrt{2})\ \text{m}^2$

$= \frac{22}{7} \times 2.1 \times (8 + 2.1 \times 1.414)\ \text{m}^2$

$= \frac{22}{7} \times 2.1 \times (8 + 2.9694)\ \text{m}^2$

$= \frac{22}{7} \times 2.1 \times 10.9694\ \text{m}^2 = 22 \times 0.3 \times 10.9694\ \text{m}^2 = 72.3980\ \text{m}^2 = 72.40\ \text{m}^2$

and, $\quad V =$ Volume of the cylinder + Volume of the cone

$= \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_2^2 h_2\right)\text{m}^3$

$= \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_1^2 h_2\right)\text{m}^3 \qquad [\because r_2 = r_1]$

$= \pi r_1^2\left(h_1 + \frac{1}{3}h_2\right)\text{m}^3$

$= \frac{22}{7} \times 2.1 \times 2.1 \times \left(4 + \frac{1}{3} \times 2.1\right)\text{m}^3$

$= \frac{22}{7} \times 2.1 \times 2.1 \times (4 + 0.7)\ \text{m}^3 = 22 \times 0.3 \times 2.1 \times 4.7\ \text{m}^3 = 65.142\ \text{m}^3$

EXAMPLE 3 *A circus tent is cylindrical upto a height of 3 m and conical above it. If the diameter of the base is 105 m and the slant height of the conical part is 53 m, find the total canvas used in making the tent.* **[CBSE 2004]**

SOLUTION The total canvas used is equal to the outer surface area of the tent.

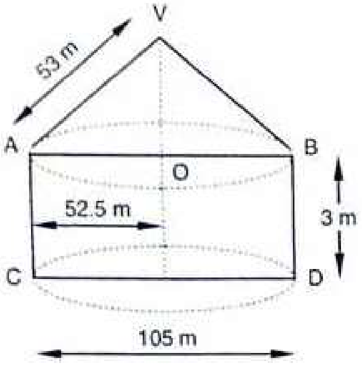

Fig. 14.18

Total canvas used = Curved surface area of cylinder + Curved surface area of cone

$$= \left(2 \times \frac{22}{7} \times 52.5 \times 3 + \frac{22}{7} \times 52.5 \times 53\right) \text{m}^2 \qquad [\because S = 2\pi rh + \pi rl]$$

$$= \frac{22}{7} \times 52.5\,(6 + 53)\ \text{m}^2 = 9735\ \text{m}^2$$

EXAMPLE 4 *A right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.* **[CBSE 2006C]**

SOLUTION We have, r = Radius of the cylinder = 6 cm, h = Height of the cylinder = 15 cm

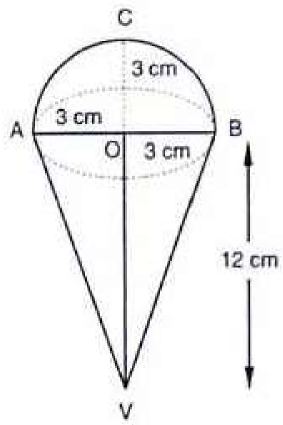

Fig. 14.19

$\therefore$ Volume of the cylinder $= \pi r^2 h = \pi \times 6^2 \times 15\ \text{cm}^3 = 540\,\pi\ \text{cm}^3$

It is given that

r_1 = Radius of the ice-cream cone = 3 cm and, h_1 = Height of the ice-cream cone = 12 cm

$\therefore$ Volume of the conical part of ice-cream cone $= \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3} \times \pi \times 3^2 \times 12\ \text{cm}^3 = 36\pi\ \text{cm}^3$

Volume of the hemispherical top of the ice-cream cone $= \frac{2}{3}\pi r_1^3 = \frac{2}{3} \times \pi \times 3^3 = 18\pi \text{ cm}^3$

Total volume of the ice-cream cone $= (36\pi + 18\pi) \text{ cm}^3 = 54\pi \text{ cm}^3$

$\therefore$ Number of ice-cream cones $= \frac{\text{Volume of the cylinder}}{\text{Total volume of ice-cream cone}} = \frac{540\pi}{54\pi} = 10$

EXAMPLE 5 *A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹2 per square metre, if the radius of the base is 14 metres.* **[CBSE 2005]**

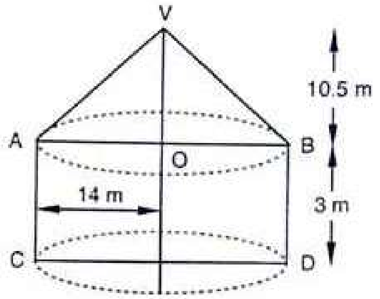

Fig. 14.20

SOLUTION Let r m be the radius of the base of the cylinder and h metres be its height. It is given that $r = 14$ m and $h = 3$ m.

Curved surface area of the cylinder $= 2\pi r h \text{ m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right) \text{m}^2 = 264 \text{ m}^2$

Let r_1 m be the radius of the base, h_1 m be the height and l_1 m be the slant height of the cone. It is given that $r_1 = 14$ m, $h_1 = (13.5 - 3)$ m $= 10.5$ m.

$$\therefore \quad l_1 = \sqrt{r_1^2 + h_1^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$$

$$\therefore \quad \text{Curved surface area of the cone} = \pi r_1 l_1 = \left(\frac{22}{7} \times 14 \times 17.5\right) \text{m}^2 = 770 \text{ m}^2$$

Let S be the total area which is to be painted. Then,

S = Curved surface area of the cylinder + Curved surface area of the cone

$\Rightarrow \quad S = (264 + 770) \text{ m}^2 = 1034 \text{ m}^2$

Hence, Cost of painting $= S \times$ Rate $=$ ₹ $(1034 \times 2) =$ ₹ 2068

EXAMPLE 6 *A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.* **[CBSE 2012]**

SOLUTION We have, $VO' = 10.2$ cm, $OA = OO' = 4.2$ cm

Let r be the radius of the hemisphere and h be the height of the conical part of the toy. Then,
$r = OA = 4.2$ cm, $h = VO = VO' - OO' = (10.2 - 4.2)$ cm $= 6$ cm

Also, radius of the base of the cone $= OA = r = 4.2$ cm

Let V be the volume of the wooden toy. Then,

V = Volume of the conical part + Volume of the hemispherical part

$$\Rightarrow \quad V = \left(\frac{1}{3}\pi r^2 h + \frac{2\pi}{3} r^3\right) \text{cm}^3$$

$$\Rightarrow \quad V = \frac{\pi r^2}{3}(h + 2r)\ \text{cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times (6 + 2 \times 4.2)\ \text{cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 14.4\ \text{cm}^3 = 266.11\ \text{cm}^3$$

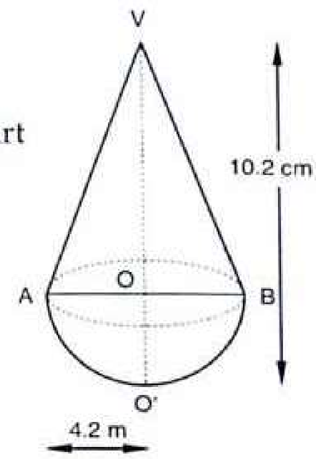

Fig. 14.21

EXAMPLE 7 *A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub.* (Use $\pi = 22/7$)

SOLUTION We have, $VO = 4$ cm, $OA = OB = OO' = 3.5$ cm.

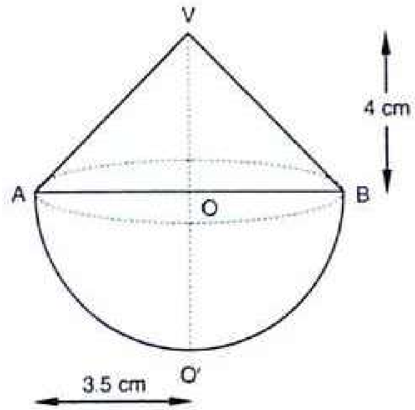

Fig. 14.22

$\therefore$ Volume of the solid = Volume of its conical part + Volume of its hemispherical part

$$= \left\{\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3\right\} \text{cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2\ [4 + 2 \times 3.5]\ \text{cm}^3 = \left\{\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$$

Clearly, when the solid is submerged in the cylindrical tub the volume of water that flows out of the cylinder is equal to the volume of the solid.

Hence,

Volume of water left in the cylinder = Volume of cylinder – Volume of the solid

$$= \left\{\frac{22}{7} \times (5)^2 \times 10.5 - \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \text{cm}^3$$

$$= \left\{\frac{22}{7} \times 25 \times \frac{21}{2} - \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11\right\} \text{cm}^3$$

$$= \left(11 \times 25 \times 3 - \frac{1}{3} \times 11 \times \frac{7}{2} \times 11\right) \text{cm}^3$$

$$= (825 - 141.16)\ \text{cm}^3 = 683.83\ \text{cm}^3$$

EXAMPLE 8 *A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.*

[CBSE 2016]

SOLUTION Let the radius of the base of the conical portion be r cm. Then, height of the conical portion $= 4r$ cm.

Let V be the volume of cone with hemispherical top. Then,

$V =$ Volume of the cone + Volume of the hemispherical top

$$= \left(\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right) \text{cm}^3 = \left(\frac{6}{3}\pi r^3\right) \text{cm}^3 = (2\pi r^3)\ \text{cm}^3$$

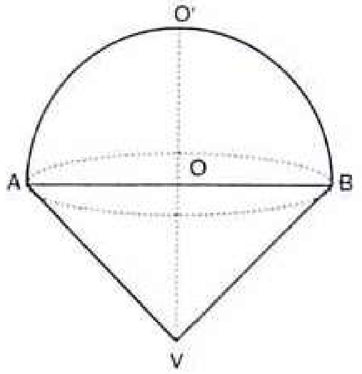

Fig. 14.23

Volume of 10 cones with hemispherical tops $= 10\ V = (10 \times 2\pi r^3)\ \text{cm}^3 = 20\pi r^3\ \text{cm}^3$

Volume of the cylindrical container $= (\pi \times 6^2 \times 15)\ \text{cm}^3 = 540\pi\ \text{cm}^3$

Clearly,

Volume of 10 cones with hemispherical tops = Volume of the cylindrical container

$\Rightarrow \quad 20\pi r^3 = 540\pi$

$\Rightarrow \quad r^3 = 27$

$\Rightarrow \quad r = 3$ cm

Hence, radius of the ice-cream cone is 3 cm.

EXAMPLE 9 *A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid (Use $\pi = 22/7$)*

SOLUTION Let r cm be the radius and h cm the height of the cylinder. Then,

$$r = \frac{7}{2}\ \text{cm and, } h = \left(19 - 2 \times \frac{7}{2}\right) \text{cm} = 12\ \text{cm}$$

Also, radius of hemisphere $= \frac{7}{2}$ cm $= r$ cm

Let V be the volume and S be the surface area of the solid. Then,

$V =$ Volume of the cylinder + Volume of two hemispheres

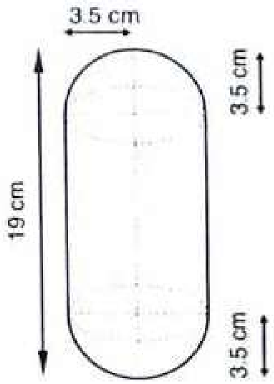

Fig. 14.24

$$\Rightarrow \quad V = \left\{ \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right) \right\} \text{cm}^3$$

$$\Rightarrow \quad V = \pi r^2 \left(h + \frac{4r}{3} \right) \text{cm}^3$$

$$\Rightarrow \quad V = \left\{ \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times \left(12 + \frac{4}{3} \times \frac{7}{2}\right) \right\} \text{cm}^3 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{50}{3} \text{cm}^3 = 641.66 \text{ cm}^3$$

and,

$S =$ Curved surface area of cylinder + Surface area of two hemispheres

$$\Rightarrow \quad S = (2\pi r h + 2 \times 2\pi r^2) \text{cm}^2$$

$$\Rightarrow \quad S = 2\pi r(h + 2r) \text{cm}^2$$

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times \frac{7}{2} \times \left(12 + 2 \times \frac{7}{2}\right) \text{cm}^2 = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 19\right) \text{cm}^2 = 418 \text{ cm}^2$$

EXAMPLE 10 *A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemispherical ends is 36 cm, find the cost of polishing the surface of the solid at the rate of 7 paise per sq. cm.* (*Use* $\pi = 22/7$).

SOLUTION We have,

$r =$ radius of the cylinder = radius of hemispherical ends = 18 cm

$h =$ height of the cylinder = 72 cm

Let S be the total surface area of the solid. Then,

$S =$ Curved surface area of the cylinder + Surface areas of hemispherical ends

$$\Rightarrow \quad S = (2\pi r h + 2 \times 2\pi r^2) \text{cm}^2$$

$$\Rightarrow \quad S = (2\pi r h + 4\pi r^2) \text{cm}^2$$

$$\Rightarrow \quad S = 2\pi r(h + 2r) \text{cm}^2$$

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 18 \times (72 + 36) \text{cm}^2 \qquad [\because r = 18 \text{ cm}, h = 72 \text{ cm}]$$

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 18 \times 108 \text{ cm}^2 = 12219.42 \text{ cm}^2$$

Rate of polishing = 7 paise per sq. cm.

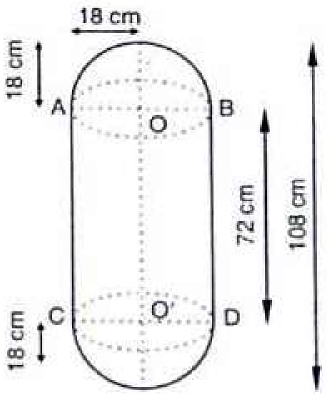

Fig. 14.25

$$\therefore \quad \text{Cost of polishing} = ₹\left(12219.42 \times \frac{7}{100}\right) = ₹\,855.36$$

EXAMPLE 11 *A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm.*

SOLUTION Let r cm be the radius and h cm the height of the cylindrical part. It is given that $r = 5$ cm and $h = 13$ cm. Clearly, radii of the spherical part and base of the conical part are also r cm. Let h_1 cm be the height, l cm be the slant height of the conical part. Then,

$$l^2 = r^2 + h_1^2$$

$$\Rightarrow \quad l = \sqrt{r^2 + h_1^2}$$

$$\Rightarrow \quad l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ cm} \quad [\because h_1 = 12 \text{ cm}, r = 5 \text{ cm}]$$

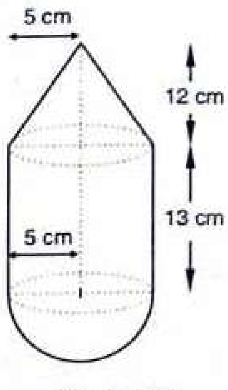

Fig. 14.26

Let S be the surface area of the toy. Then,

S = Curved surface area of the cylindrical part
+ Curved surface area of hemispherical part
+ Curved surface area of conical part

$$\Rightarrow \quad S = (2\pi rh + 2\pi r^2 + \pi r l) \text{ cm}^2$$

$$\Rightarrow \quad S = \pi r(2h + 2r + l) \text{ cm}^2$$

$$\Rightarrow \quad S = \left\{\frac{22}{7} \times 5 \times (2 \times 13 + 2 \times 5 + 13)\right\} \text{cm}^2 = \left(\frac{22}{7} \times 5 \times 49\right) \text{cm}^2 = 770 \text{ cm}^2$$

EXAMPLE 12 *A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the sphere is 14 cm and the total height of the vessel is 13 cm. Find its capacity. (Take $\pi = 22/7$)* **[NCERT, CBSE 2006C]**

SOLUTION Let r be the radius of the hemispherical bowl and h be the height of the cylinder. It is given that $r = 7$ cm and $h = 6$ cm. Let V be the total capacity of the bowl. Then,

V = Volume of the cylinder + Volume of the hemisphere

$$\Rightarrow \quad V = \left(\pi r^2 h + \frac{2}{3}\pi r^3\right) \text{cm}^3$$

$$\Rightarrow \quad V = \pi r^2\left(h + \frac{2}{3}r\right) \text{cm}^3$$

$$\Rightarrow \quad V = \frac{22}{7} \times 7^2 \times \left(6 + \frac{2}{3} \times 7\right) \text{cm}^3$$

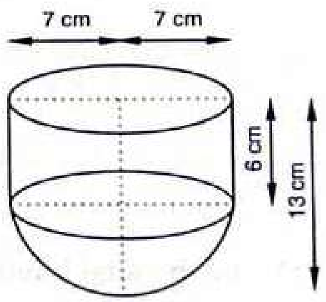

Fig. 14.27

$$\Rightarrow \quad V = 22 \times 7 \times \frac{32}{3} \text{ cm}^3 = \frac{4928}{3} \text{ cm}^3 = 1642.66 \text{ cm}^3$$

EXAMPLE 13 *A solid toy is in the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2 cm and the height of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the solid toy. (Use $\pi = 22/7$)*

[NCERT, CBSE 2002C]

SOLUTION Let V be the volume of the solid toy. Then,

V = Volume of the conical portion + Volume of the cylindrical portion
+ Volume of the hemispherical portion.

$\Rightarrow \quad V = \frac{1}{3}\pi \times (2.1)^2 \times 7 + \pi \times (2.1)^2 \times 12 + \frac{2}{3} \times \pi \times (2.1)^3$

$\Rightarrow \quad V = \frac{1}{3} \times \pi \times (2.1)^2\ (7 + 3 \times 12 + 2 \times 2.1)\ \text{cm}^3$

$\Rightarrow \quad V = \frac{1}{3} \times \pi \times (2.1)^2 \times 47.2\ \text{cm}^3$

$\Rightarrow \quad V = \pi \times 0.7 \times 2.1 \times 47.2\ \text{cm}^3$

$\Rightarrow \quad V = \frac{22}{7} \times 0.7 \times 2.1 \times 47.2\ \text{cm}^3 = 218.064\ \text{cm}^3$

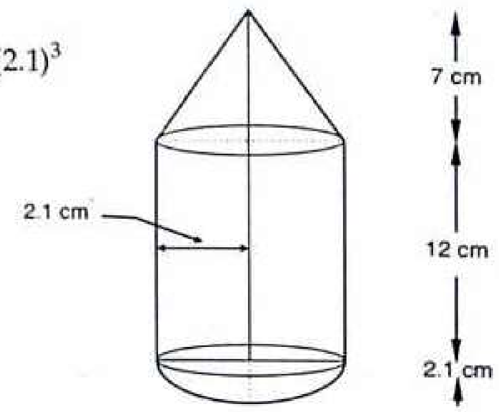

Fig. 14.28

EXAMPLE 14 *A godown building is in the form as shown in Fig. 14.29. The vertical cross-section parallel to the width side of the building is a rectangle 7 m × 3 m, mounted by a semi-circle of radius 3.5 m. The inner measurements of the cuboidal portion of the building are 10 m × 7 m × 3 m. Find the volume of the godown and the total interior surface area excluding the floor (base).* (*Take* $\pi = 22/7$)

SOLUTION Since the top of the building is in the form of half of the cylinder of radius 3.5 m, and length 10 m, split along the diameter. Let V be the volume of the godown. Then,

V = Volume of the cuboid $+\frac{1}{2}$ (Volume of the cylinder of radius 3.5 m and length 10 m)

$$\Rightarrow V = \left\{10 \times 7 \times 3 + \frac{1}{2}\left(\frac{22}{7} \times 3.5 \times 3.5 \times 10\right)\right\} \text{m}^3 = (210 + 192.5)\ \text{m}^3 = 402.5\ \text{m}^3$$

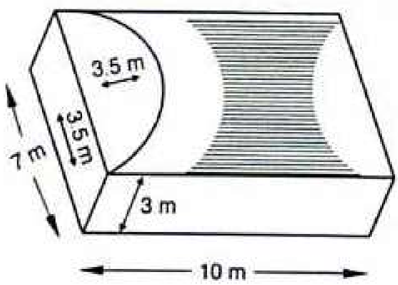

Fig. 14.29

Let S be the total interior surface area excluding the base floor. Then,

S = Area of four walls $+\frac{1}{2}$ (Curved surface area of the cylinder) + 2 (Area of the semi-circles)

$$\Rightarrow \quad S = \left[2(10 + 7) \times 3 \times \frac{1}{2}\left(2 \times \frac{22}{7} \times 3.5 \times 10\right) + 2\left(\frac{1}{2} \times \frac{22}{7} \times (3.5)^2\right)\right] \text{m}^2$$

$$\Rightarrow \quad S = (102 + 110 + 38.5)\ \text{m}^2 = 250.5\ \text{m}^2$$

EXAMPLE 15 *A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid. Find how much more space it will cover.* **[NCERT]**

SOLUTION Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere. Let $EFGH$ be the right circular cylinder circumscribing the given toy.

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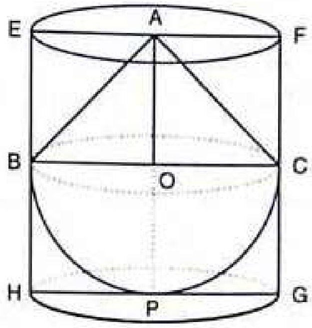

Fig. 14.30

We have,

OA = Height of the cone = 2 cm

and, BC = Diameter of the base of the cone = 4 cm

$\therefore$ BO = Radius of the hemisphere $= \frac{1}{2} BC = \frac{4}{2}$ cm = 2 cm

$\Rightarrow$ $OP = 2$ cm [$\because OP = OB$]

$\therefore$ $AP = OP + OA = (2 + 2)$ cm = 4 cm

Now, Volume of the right circular cylinder $= \pi \times 2^2 \times 4 \text{ cm}^3 = 16\pi \text{ cm}^3$

Volume of the solid toy $= \left\{\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2\right\} \text{cm}^3 = 8\pi \text{ cm}^3$

$\therefore$ Required space = Volume of the right circular cylinder – Volume of the toy

$= 16\pi \text{cm}^3 - 8\pi \text{cm}^3 = 8\pi \text{cm}^3$.

Hence, the right circular cylinder covers $8\pi \text{cm}^3$ more space than the solid toy.

EXAMPLE 16 *From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid. Also, find the whole surface area.* **[CBSE 2009]**

SOLUTION Let V be the volume of the remaining solid and S be the whole surface area. Then,

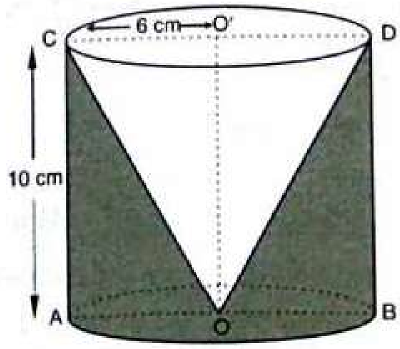

Fig. 14.31

V = Volume of the cylinder – Volume of the cone.

$\Rightarrow$ $V = \left\{\pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10\right\} \text{cm}^3 = (360\pi - 120\pi) \text{ cm}^3 = 240\pi \text{cm}^3$

Slant height of the cone $= OC = \sqrt{OO'^2 + O'C^2} = \sqrt{10^2 + 6^2} = \sqrt{136}$ cm $= 2\sqrt{34}$ cm

and,

$S =$ Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of cone

$\Rightarrow \quad S = \{2\pi \times 6 \times 10 + \pi \times 6^2 + \pi \times 6 \times 2\sqrt{34}\}\ \text{cm}^2 = (156 + 12\sqrt{34})\ \pi\ \text{cm}^2$

EXAMPLE 17 *A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 14.32. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.* **[CBSE 2014, 2018, NCERT]**

SOLUTION Let r be the radius of the base of the cylinder and h be its height. Let S be the total surface area of the article. Then,

$S =$ Curved surface area of the cylinder + 2 (Surface area of a hemisphere)

$\Rightarrow \quad S = 2\pi rh + 2(2\pi r^2)$

$\Rightarrow \quad S = 2\pi r(h + 2r)$

$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 3.5\,(10 + 2 \times 3.5)\ \text{cm}^2$

$\Rightarrow \quad S = 22 \times 17\ \text{cm}^2 = 374\ \text{cm}^2$

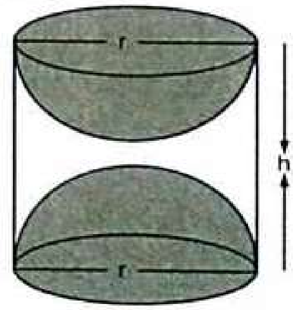

Fig. 14.32

EXAMPLE 18 *A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m, and slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of canvas of the tent at the rate of ₹ 500 per m².* **[NCERT]**

SOLUTION For conical portion, $r = 2$ m and $l = 2.8$ m. Let S_1 the curved surface area of conical portion. Then,

$S_1 = \pi rl = \pi \times 2 \times 2.8\ \text{m}^2 = 5.6\,\pi\,\text{m}^2$

For cylindrical portion, we have $r = 2$ m, $h = 2.1$ m

Let S_2 be the curved surface area of cylindrical portion. Then,

$S_2 = 2\pi rh = 2\pi \times 2 \times 2.1\ \text{m} = 8.4\pi\ \text{m}^2$

Let S be the area of the canvas used. Then,

$S = S_1 + S_2 = (5.6\pi + 8.4\pi)\ \text{m}^2 = 14 \times \frac{22}{7}\ \text{m}^2 = 44\ \text{m}^2$

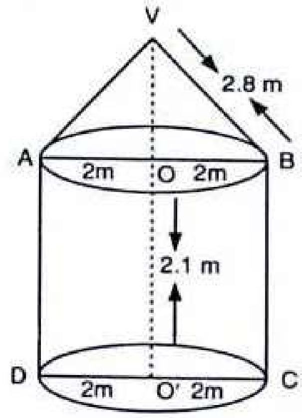

Fig. 14.33

Total cost of the canvas at the rate of ₹ 500 per m² = ₹ (500 × 44) = ₹ 22000

EXAMPLE 19 *From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².* **[NCERT, CBSE 2012, 2017]**

SOLUTION Let S be the total surface area of the remaining solid. Then,

$S =$ Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone

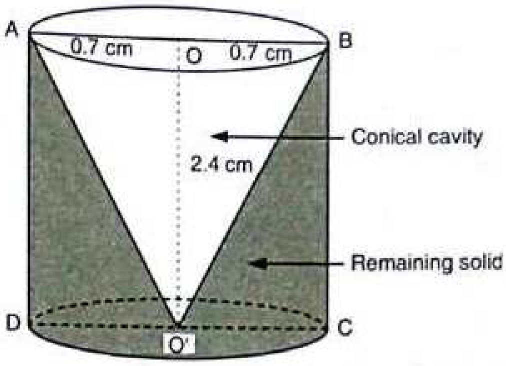

Fig. 14.34

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times (0.7)^2 + \frac{22}{7} \times 0.7 \times \sqrt{(2.4)^2 + (0.7)^2}$$

$$\left[\because l = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2}\right]$$

$$\Rightarrow \quad S = 44 \times 0.24 + 22 \times 0.1 \times 0.7 + 22 \times 0.1 \times 2.5 = 10.56 + 1.54 + 5.5 \text{ cm}^2 = 17.6 \text{ cm}^2$$

Hence, the total surface area to the nearest cm^2 is 18 cm^2

EXAMPLE 20 *A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.* **[NCERT]**

SOLUTION We have,

r_1 = Radius of the cone = 1 cm, h_1 = Height of the cone = 1 cm

and, r_2 = Radius of the hemisphere = 1 cm

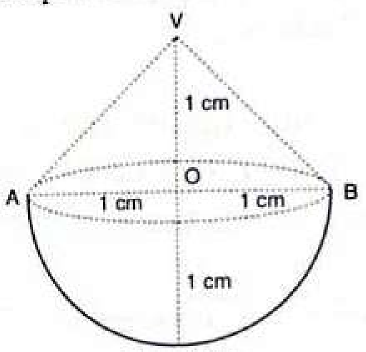

Fig. 14.35

Let V be the volume of the solid. Then,

V = Volume of the cone + Volume of the hemisphere

$$\Rightarrow \quad V = \frac{1}{3}\pi r_1^2 h_1^2 + \frac{2}{3}\pi r_2^3 = \left(\frac{1}{3}\pi \times 1^2 \times 1 + \frac{2}{3}\pi \times 1^3\right) \text{cm}^3 = \pi \text{ cm}^3$$

EXAMPLE 21 *Rachel, an engineering student was asked to make a model in her workshop, which was shaped like a cylinder with two cones attached to its two ends, using thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made.* **[NCERT]**

SOLUTION We have,

r_1 = Radius of two conical parts = 1.5 cm, h_1= Height of two conical parts = 2 cm

r_2 = Radius of the cylindrical part = 1.5 cm, h_2= Height of the cylindrical part = 8 cm

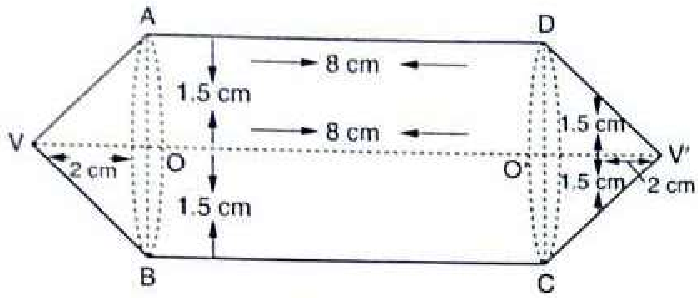

Fig. 14.36

Let V be the volume of the air contained in the model. Then,

V = Volume of the cylindrical part + Volumes of two conical parts

$$\Rightarrow \quad V = \pi r_2^2 h_2 + 2 \times \left(\frac{1}{3}\pi r_1^2 h_1\right)$$

$$\Rightarrow \quad V = \left\{\pi \times (1.5)^2 \times 8 + 2 \times \frac{1}{3} \times \pi \times (1.5)^2 \times 2\right\} \text{cm}^3$$

$$\Rightarrow \quad V = \left\{\pi \times \left(\frac{3}{2}\right)^2 \times 8 + \frac{2}{3} \times \pi \times \left(\frac{3}{2}\right)^2 \times 2\right\} \text{cm}^3$$

$$\Rightarrow \quad V = \{18\pi + 3\pi\} \text{cm}^3 = 21\pi \text{ cm}^3 = 21 \times \frac{22}{7} \text{cm}^3 = 66 \text{ cm}^3.$$

EXAMPLE 22 *A gulabjamun when completely ready for eating contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulabjamuns shaped like a cylinder with two hemispherical ends, if the complete length of each of the gulabjamun is 5 cm and its diameter is 2.8 cm.* **[NCERT]**

SOLUTION We have,

h = Length of the cylindrical part of a gulabjamun = (5 − 1.4 − 1.4) cm = 2.2 cm

r = Radius of the cylindrical part of a gulabjamun = 1.4 cm

Also, r = Radius of a hemispherical part of a gulabjamun = 1.4 cm

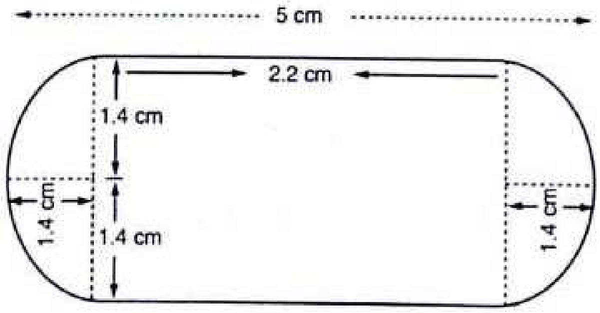

Fig. 14.37

Let V be the volume of a gulabjamun. Then,

V = Volume of two hemispherical part + Volume of cylindrical part

$$\Rightarrow \quad V = 2\left(\frac{2}{3}\pi r^3\right) + \pi r^2 h$$

$$\Rightarrow \quad V = \frac{4}{3}\pi r^3 + \pi r^2 h$$

$\Rightarrow \quad V = \pi r^2 \left(\frac{4}{3} r + h\right)$

$\Rightarrow \quad V = \frac{22}{7} \times 1.4 \times 1.4 \times \left(\frac{4}{3} \times 1.4 + 2.2\right) \text{cm}^3 = 22 \times 0.2 \times 1.4 \times \frac{12.2}{3} \text{cm}^3 = \frac{75.152}{3} \text{cm}^3$

Volume of 45 gulabjamuns $= \frac{75.152}{3} \times 45 \text{ cm}^3 = 1127.28 \text{ cm}^3$

$\therefore$ Volume of syrup = 30% of 1127.28 cm³ $= \frac{30}{100} \times 1127.28 \text{ cm}^3 = 338.184 \text{ cm}^3$

EXAMPLE 23 *A vessel in the form of inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped in the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.* **[NCERT]**

SOLUTION We have,

r = Radius of the base of the cone = 5 cm, h = Height of the cone = 8 cm

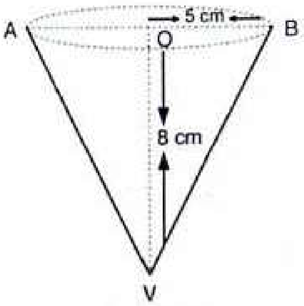

Fig. 14.38

$\therefore$ Volume of water in the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5^2 \times 8 \text{ cm}^3 = \frac{200}{3} \pi \text{ cm}^3$

Volume of the water that flows out of the cone $= \frac{1}{4} \times \frac{200\pi}{3} \text{cm}^3 = \frac{50\pi}{3} \text{cm}^3$

Volume of a spherical lead shot of radius 0.5 cm $= \frac{4}{3}\pi \times (0.5)^3 \text{ cm}^3 = \frac{4}{3}\pi \times \frac{1}{8} \text{cm}^3 = \frac{\pi}{6} \text{cm}^3$

Suppose n spherical lead shots are dropped in the vessel so that $\frac{1}{4}$th of the water contained in the vessel flows out of the vessel.

$\therefore$ Volume of n spherical lead-shots = Volume of the water that flows out

$$\Rightarrow \quad n \times \frac{\pi}{6} = \frac{50}{3}\pi$$

$$\Rightarrow \quad n = 100$$

Hence, 100 lead shots are dropped in the vessel.

EXAMPLE 24 *A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm, the radius of the hemisphere is 60 cm and height of the cone is 120 cm, assuming that the hemisphere and the cone have common base.* **[NCERT]**

SOLUTION For the cylinder: r = Radius of the base = 60 cm, h = Height = 180 cm

$\therefore$ V = Volume of water that the cylinder contains = $\pi r^2 h = \left\{\pi \times (60)^2 \times 180\right\}$ cm^3

For conical part: r = Radius of the base = 60 cm, h_1 = Height = 120 cm.

Let V_1 be the volume of the conical part. Then,

$$V_1 = \frac{1}{3}\pi r^2 h_1$$

$$\Rightarrow \quad V_1 = \frac{1}{3} \times \pi \times 60^2 \times 120 \text{ cm}^3 = \left\{\pi \times 60^2 \times 40\right\} \text{cm}^3$$

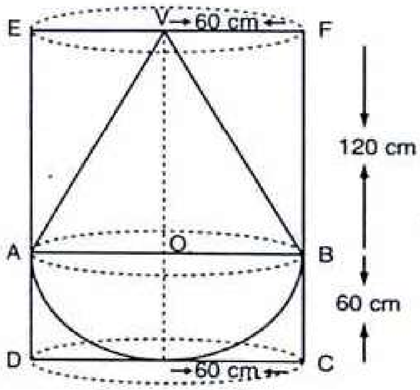

Fig. 14.39

For hemispherical part: r = Radius = 60 cm

Let V_2 be the volume of the hemisphere. Then,

$$V_2 = \left\{\frac{2}{3}\pi \times 60^3\right\} \text{cm}^3$$

$$\Rightarrow \quad V_2 = \left\{2\pi \times 20 \times 60^2\right\} \text{cm}^3 = \left\{40\pi \times 60^2\right\} \text{cm}^3$$

Let V_3 the the volume of the water left-out in the cylinder. Then,

$$V_3 = V - V_1 - V_2$$

$$\Rightarrow \quad V_3 = \left\{\pi \times 60^2 \times 180 - \pi \times 60^2 \times 40 - 40\pi \times 60^2\right\} \text{cm}^3$$

$$\Rightarrow \quad V_3 = \pi \times 60^2 \times \{180 - 40 - 40\} \text{ cm}^3$$

$$\Rightarrow \quad V_3 = \frac{22}{7} \times 3600 \times 100 \text{ cm}^3$$

$$\Rightarrow \quad V_3 = \frac{22 \times 360000}{7} \text{cm}^3 = \frac{22 \times 360000}{7 \times (100)^3} \text{m}^3 = \frac{22 \times 36}{700} \text{m}^3 = 1.1314 \text{ m}^3.$$

EXAMPLE 25 *A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass (Use π = 3.14).* **[NCERT]**

SOLUTION Let V be the total volume of iron in two cylinders. Then,

$$V = \left\{\pi \times 12^2 \times 220 + \pi \times 8^2 \times 60\right\} \text{cm}^3$$

$$\Rightarrow \quad V = \{\pi \times 144 \times 220 + \pi \times 64 \times 60\} \text{ cm}^3$$

$$\Rightarrow \quad V = 35520\pi \text{ cm}^3 = 35520 \times 3.14 \text{ cm}^3 = 111532.8 \text{ cm}^3$$

Let M be the total mass of the iron pole. Then,

$$M = 111532.8 \times 8 \text{ grams} = \frac{111532.8 \times 8}{1000} \text{kg} = 892.2624 \text{ kg}$$

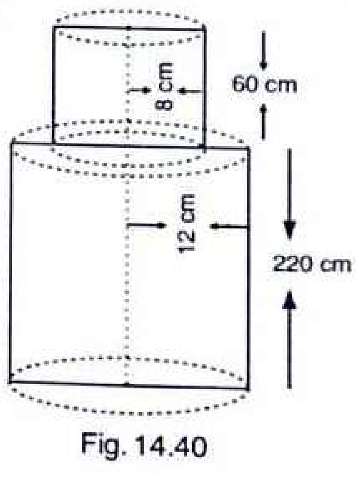

Fig. 14.40

EXAMPLE 26 *A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter: the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements and $\pi = 3.16$.* **[NCERT]**

SOLUTION We have,

h = Length of the cylindrical neck = 8 cm

r = Radius of the cylindrical neck = 1 cm

$\therefore$ Volume of the cylindrical neck $= \pi r^2 h = \pi \times 1^2 \times 8 \text{ cm}^3 = 8\pi \text{ cm}^3$

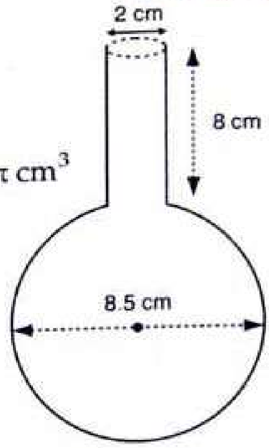

Fig. 14.41

Volume of the spherical part $= \frac{4}{3}\pi \times \left(\frac{8.5}{2}\right)^3 \text{ cm}^3$

$= \frac{4\pi}{3} \times (4.25)^3 \text{ cm}^3$

$\therefore$ Amount of water in the vessel $= \left\{8\pi + \frac{4\pi}{3} \times (4.25)^3\right\} \text{cm}^3$

$= \pi \left\{8 + \frac{4}{3} \times (4.25)^3\right\} \text{cm}^3$

$= 3.14 \times \left\{8 + \frac{4}{3} \times 4.25 \times 4.25 \times 4.25\right\} \text{cm}^3$

$= 3.14 \times (8 + 102.354) \text{ cm}^3$

$= 346.511 \text{ cm}^3 \cong 346.5 \text{ cm}^3$

Hence, the volume found by the child is not correct.

EXAMPLE 27 *Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour.* (Take $\pi = 22/7$). **[NCERT]**

SOLUTION We have,

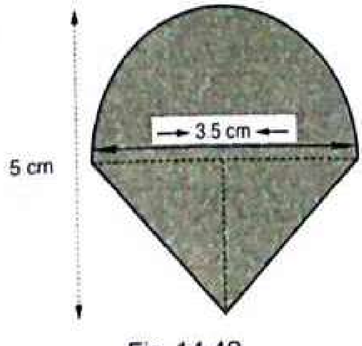

Fig. 14.42

r = Radius of hemispherical portion of the lattu $= \frac{3.5}{2} \text{ cm} = \frac{7}{4} \text{cm}$

r = Radius of the conical portion $= \frac{3.5}{2} = \frac{7}{4}$ cm

h = Height of the conical portion $= \left(5 - \frac{3.5}{2}\right) \text{cm} = \frac{13}{4}$ cm

Let l be the slant height of the conical part. Then,

$$l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{7}{4}\right)^2 + \left(\frac{13}{4}\right)^2} = \sqrt{\frac{49 + 169}{4}} \text{ cm} = \sqrt{\frac{218}{4}} \text{ cm} = 3.69 \text{ cm} \simeq 3.7 \text{ cm}$$

Let S be the total surface area of the top. Then,

$$S = 2\pi r^2 + \pi r l$$

$$\Rightarrow \quad S = \pi r(2r + l)$$

$$\Rightarrow \quad S = \frac{22}{7} \times \frac{7}{4}\left(2 \times \frac{7}{4} + 3.7\right) \text{cm}^2$$

$$\Rightarrow \quad S = \frac{11}{2}(3.5 + 3.7) \text{ cm}^2 = \frac{11}{2} \times 7.2 \text{ cm}^2 = 11 \times 3.6 \text{ cm}^2 = 39.6 \text{ cm}^2$$

EXAMPLE 28 *A decorative block shown in Fig. 14.43 is made of two solids — a cube and a hemi-sphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter 4.2 cm. Find the total surface area of the block (Take $\pi = 22/7$).*

[NCERT, CBSE 2009, 2016]

SOLUTION Let S be the total surface area of the decorative block. Then,

S = Total surface area of the cube – Base area of hemisphere + Curved surface area of hemisphere

$$\Rightarrow \quad S = (6 \times 5 \times 5 - \pi r^2 + 2\pi r^2) \text{ cm}^2$$

$$\Rightarrow \quad S = (150 + \pi r^2) \text{ cm}^2$$

$$\Rightarrow \quad S = \left\{150 + \frac{22}{7} \times (2.1)^2\right\} \text{cm}^2$$

$$\Rightarrow \quad S = \{150 + 22 \times 0.3 \times 2.1\} \text{ cm}^2$$

$$\Rightarrow \quad S = (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$$

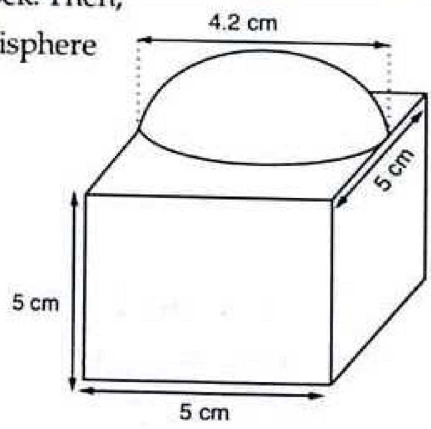

Fig. 14.43

EXAMPLE 29 *A cubical block of side 7 cm is surmounded by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the total surface area of the solid.* **[NCERT]**

SOLUTION Clearly, greatest diameter of the hemisphere is equal to the length of an edge of the cube i.e. 7 cm.

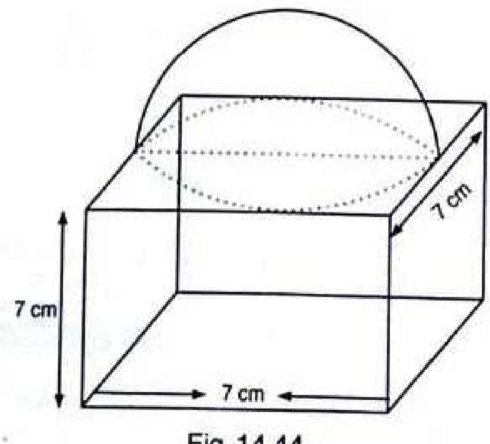

Fig. 14.44

$\therefore$ Radius of the hemisphere $= \frac{7}{2}$ cm

Let S be the total surface area of the solid. Then,

S = Surface area of the cube + Curved surface area of hemisphere − Area of the base of the hemisphere

$$\Rightarrow \quad S = \left\{6 \times 7^2 + 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 - \frac{22}{7} \times \left(\frac{7}{2}\right)^2\right\} \text{cm}^2$$

$$\Rightarrow \quad S = \left\{294 + 77 - \frac{77}{2}\right\} \text{cm}^2 = \left(294 + \frac{77}{2}\right) \text{cm}^2 = 332.5 \text{ cm}^2$$

EXAMPLE 30 *A hemispherical depression is cut-out from one face of the cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.* **[NCERT, CBSE 2010, 2014]**

SOLUTION It is given that a hemisphere of radius $\frac{l}{2}$ is cut-out from the top face of the cuboidal wooden box. Let S be the surface area of the remaining solid. Then,

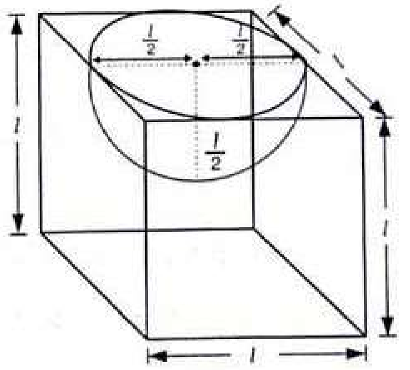

Fig. 14.45

S = Surface area of the cuboidal box whose each edge is of length l − Area of the top of the hemispherical part + Curved surface area of the hemispherical part

$$\Rightarrow \quad S = 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2 = 6l^2 - \frac{\pi l^2}{4} + \frac{\pi l^2}{2} = 6l^2 + \frac{\pi l^2}{4} = \frac{l^2}{4}(24 + \pi) \text{ sq. units}$$

EXAMPLE 31 *A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends as shown in Fig. 14.46. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.* **[NCERT]**

SOLUTION For cylindrical part, we have

$$r = \text{Radius} = \frac{5}{2} \text{ mm} = 2.5 \text{ mm}, \quad h = \text{length} = \{14 - (2.5 + 2.5)\} \text{ mm} = 9 \text{ mm}$$

Let S_1 be the curved surface area of the cylindrical part. Then,

$\Rightarrow \quad S_1 = 2\pi rh = 2 \times \frac{22}{7} \times 2.5 \times 9 \text{ mm}^2$

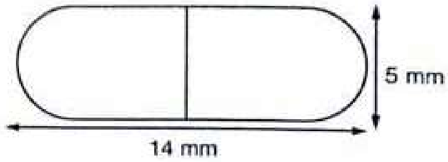

Fig. 16.46

For two hemispherical parts, we have

r = Radius = 2.5 mm

Let S_2 be the curved surface area of two hemispherical parts. Then,

$$S_2 = 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times (2.5)^2 \text{ mm}^2$$

Let S be the surface area of the capsule. Then,

$$S = S_1 + S_2 = \left\{2 \times \frac{22}{7} \times 2.5 \times 9 + 4 \times \frac{22}{7} \times (2.5)^2\right\} \text{mm}^2$$

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 2.5\,(9 + 2 \times 2.5) \text{ mm}^2$$

$$\Rightarrow \quad S = 2 \times \frac{22}{7} \times 2.5 \times 14 \text{ mm}^2 = 2 \times 22 \times 2.5 \times 2 \text{ mm}^2 = 220 \text{ mm}^2$$

EXAMPLE 32 *Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half-cylinder. If the base of the shed is 7 m by 15 m, and height of the cuboidal portion is 8 m, find the volume of the air that the shed can hold. If the industry requires machinery which would occupy a total space of 300 m³ and there are 20 workers each of whom would occupy 0.08 m³ space on an average, how much air would be in the shed when it is working? (Take π = 22/7).* **[NCERT]**

SOLUTION Clearly, volume of air inside the shed (when there is no people or machinery) is equal to the volume of air inside the cuboid and inside the half-cylinder taken together.

For cuboidal part, we have

length = 15 m, breadth = 7 m and height = 8 m

$\therefore$ Volume of cuboidal part $= 15 \times 7 \times 8 \text{ m}^2 = 840 \text{ m}^3$

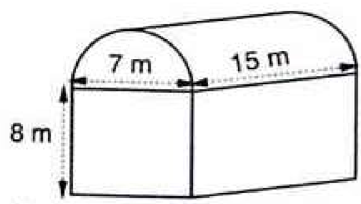

Fig. 14.47

Clearly,

$$r = \text{Radius of half-cylinder} = \frac{1}{2}\text{(Width of the cuboid)} = \frac{7}{2} \text{ m}$$

and,

$$h = \text{Height (length) of half-cylinder} = \text{Length of cuboid} = 15 \text{ m}$$

$$\therefore \quad \text{Volume of half-cylinder} = \frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15 \text{ m}^3 = \frac{1155}{4} \text{ m}^3 = 288.75 \text{ m}^3$$

Volume of air inside the shed when there is no people or machinery $= (840 + 288.75) \text{ m}^3$

$= 1128.75 \text{ m}^3$

Now,

Total space occupied by 20 workers $= 20 \times 0.08 \text{ m}^3 = 1.6 \text{ m}^3$

Total space occupied by the machinery $= 300 \text{ m}^3$

$\therefore$ Volume of the air inside the shed when there are machine and workers in side it

$= (1128.75 - 1.6 - 300) \text{ m}^3 = 827.15 \text{ m}^3$

EXAMPLE 33 *A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find what the apparent capacity of the glass was and what the actual capacity was (Use $\pi = 3.14$).* **[NCERT, CBSE 2009]**

SOLUTION We have, Inner diameter of the glass = 5 cm, Height of the glass = 10 cm

$$\therefore \text{ Apparent capacity of the glass} = 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 \text{ cm}^3$$

$$= 3.14 \times \frac{25}{4} \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$$

Fig. 14.48

$$\text{Volume of hemispherical part} = \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 \text{ cm}^3$$

$$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3 = 32.71 \text{ cm}^3$$

$\therefore$ Actual capacity of glass = Apparent capacity of glass – Volume of hemispherical part

$$= (196.25 - 32.71) \text{ cm}^3 = 163.54 \text{ cm}^3$$

EXAMPLE 34 *A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.*

SOLUTION Let V_1 be the volume of ice-cream in the container shaped like a right circular cylinder having radius 6 cm and height 15 cm. Then,

$$V_1 = \pi \times 6^2 \times 15 \text{ cm}^3$$

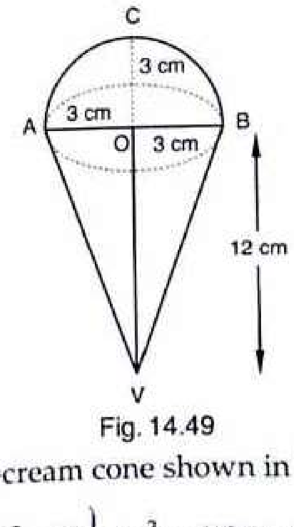

Fig. 14.49

Let V_2 be the volume of one ice-cream cone shown in Fig. 14.48. Then,

$$V_2 = \left\{\frac{2}{3}\pi \times 3^3 + \frac{1}{3}\pi \times 3^2 \times 12\right\} \text{ cm}^3 = (18\pi + 36\pi) \text{ cm}^3 = 54\pi \text{ cm}^3$$

Let the total number of cones that can be filled with the ice-cream given in the container be n. Then,

Volume of ice-cream in n cones = Volume of ice-cream in the container

$$\Rightarrow \quad n V_2 = V_1$$

$$\Rightarrow \quad 54\pi \times n = \pi \times 36 \times 15 \Rightarrow n = \frac{\pi \times 36 \times 15}{54\pi} = 10.$$

LEVEL-2

EXAMPLE 35 *The interior of a building is in the form of cylinder of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building.* (Take $\pi = 3.14$).

SOLUTION We have,

$$r_1 = \text{Radius of the base of the cylinder} = \frac{4.3}{2}\text{ m} = 2.15\text{ m}$$

$\therefore$ r_2 = Radius of the base of the cone = 2.15 m, h_1 = Height of the cylinder = 3.8 m

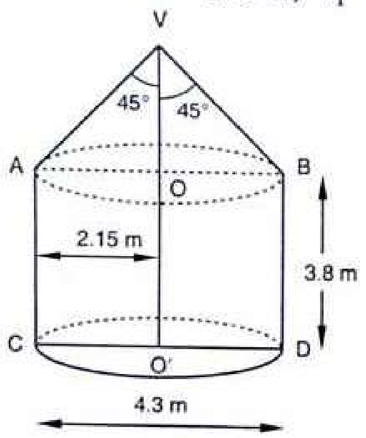

Fig. 14.50

In ΔVOA, we have

$$\sin 45° = \frac{OA}{VA} \Rightarrow \frac{1}{\sqrt{2}} = \frac{2.15}{VA} \Rightarrow VA = (\sqrt{2} \times 2.15)\text{ m} = (1.414 \times 2.15)\text{ m} = 3.04\text{ m}$$

Clearly, ΔVOA is an isosceles triangle. Therefore, $VO = OA = 2.15$ m

Thus, we have

h_2 = Height of the cone = VO = 2.15 m, l_2 = Slant height of the cone = VA = 3.04 m

Let S be the Surface area of the building. Then,

S = Surface area of the cylinder + Surface area of cone

$$\Rightarrow \quad S = (2\pi r_1 h_1 + \pi r_2 l_2)\text{ m}^2$$

$$\Rightarrow \quad S = (2\pi r_1 h_1 + \pi r_1 l_2)\text{ m}^2 \qquad [\because r_1 = r_2 = 2.15\text{ m}]$$

$$\Rightarrow \quad S = \pi r_1 (2h_1 + l_2)\text{ m}^2$$

$$\Rightarrow \quad S = 3.14 \times 2.15 \times (2 \times 3.8 + 3.04)\text{ m}^2 = 3.14 \times 2.15 \times 10.64\text{ m}^2 = 71.83\text{ m}^2$$

Let V be the volume of the building. Then,

V = Volume of the cylinder + Volume of the cone

$$\Rightarrow \quad V = \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_2^2 h_2\right)\text{m}^3$$

$$\Rightarrow \quad V = \left(\pi r_1^2 h_1 + \frac{1}{3}\pi r_1^2 h_2\right)\text{m}^3 \quad [\because r_2 = r_1]$$

$$\Rightarrow \quad V = \pi r_1^2 \left(h_1 + \frac{1}{3} h_2\right)\text{m}^3$$

$\Rightarrow \quad V = 3.14 \times 2.15 \times 2.15 \times \left(3.8 + \frac{2.15}{3}\right) m^3$

$\Rightarrow \quad V = [3.14 \times 2.15 \times 2.15 \times (3.8 + 0.7166)] m^3$

$\Rightarrow \quad V = (3.14 \times 2.15 \times 2.15 \times 4.5166) m^3 = 65.55 m^3$

EXAMPLE 36 *Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.*

SOLUTION The base of the largest right circular cone will be the circle inscribed in a face of the cube and its height will be equal to an edge of the cube.

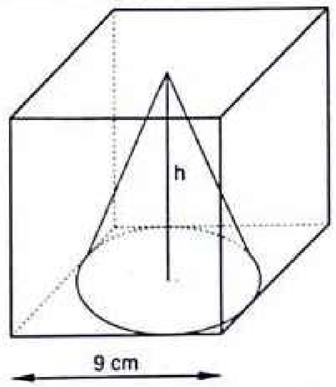

Fig. 14.51

$\therefore \quad r$ = Radius of the base of the cone $= \frac{9}{2}$ cm [$\because$ edge = 9 cm]

and, $\quad h$ = Height of cone = 9 cm

$\therefore \quad$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 9 \text{ cm}^3 = \frac{2673}{14} \text{ cm}^3 = 190.93 \text{ cm}^3$

EXAMPLE 37 *A right triangle, whose sides are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Use π = 3.14)*

SOLUTION Let ABC be the right angled triangle such that $AB = 15$ cm and $AC = 20$ cm. Using Pythagoras theorem, we have

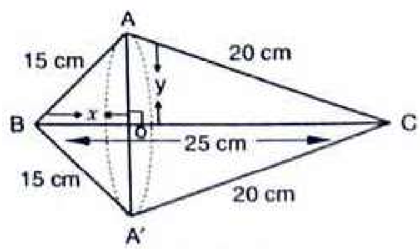

Fig. 14.52

$BC^2 = AB^2 + AC^2$

$\Rightarrow \quad BC^2 = 15^2 + 20^2$

$\Rightarrow \quad BC^2 = 225 + 400 = 625$

$\Rightarrow \quad BC = 25$ cm

Let $OB = x$ and $OA = y$.

Applying Pythagoras theorems in triangles OAB and OAC, we have

$AB^2 = OB^2 + OA^2$ and $AC^2 = OA^2 + OC^2$

$\Rightarrow \quad 15^2 = x^2 + y^2$ and $20^2 = y^2 + (25 - x)^2$

$\Rightarrow \quad x^2 + y^2 = 225$ and $(25 - x)^2 + y^2 = 400$

$\Rightarrow \quad \{(25 - x)^2 + y^2\} - \{x^2 + y^2\} = 400 - 225$

$\Rightarrow \quad (25-x)^2 - x^2 = 175$

$\Rightarrow \quad (25-x-x)(25-x+x) = 175$

$\Rightarrow \quad (25-2x)\times 25 = 175 \Rightarrow 25-2x = 7 \Rightarrow 2x = 18 \Rightarrow x = 9$

Putting $x = 9$ in $x^2 + y^2 = 225$, we get

$$81 + y^2 = 225 \Rightarrow y^2 = 144 \Rightarrow y = 12.$$

Thus, we have $OA = 12$ cm and $OB = 9$ cm.

Let V be the volume and S be the surface area of the double cone. Then,

$$V = \text{Vol. of cone } CAA' + \text{Vol. of cone } BAA'$$

$$\Rightarrow \quad V = \frac{1}{3}\pi(OA^2)\times OC + \frac{1}{3}\pi(OA^2)\times OB$$

$$\Rightarrow \quad V = \frac{1}{3}\pi\times 12^2\times 16 + \frac{1}{3}\pi\times 12^2\times 9$$

$$\Rightarrow \quad V = \frac{1}{3}\pi\times 144(16+9)$$

$$\Rightarrow \quad V = \frac{1}{3}\times 3.14\times 144\times 25\text{ cm}^3 = 3768\text{ cm}^3$$

and, $\quad S =$ Curved surface area of cone CAA' + Curved surface area of cone BAA'

$$\Rightarrow \quad S = \pi\times OA\times AC + \pi\times OA\times AB$$

$$\Rightarrow \quad S = (\pi\times 12\times 20 + \pi\times 12\times 15)\text{ cm}^2$$

$$\Rightarrow \quad S = 420\,\pi\text{cm}^2 = 420\times 3.14\text{ cm}^2 = 1318.8\text{ cm}^2$$

EXAMPLE 38 *A cone made of paper has height 3h and vertical angle 2α. It contains two other cones of height 2h and h and vertical angles 4α and 6α respectively. Find the ratio of the two volumes in between the cones.*

SOLUTION Let U, V, and W be the volumes of cones VAB, $V_1A_1B_1$ and $V_2A_2B_2$ respectively.

For cone VAB, we have

$$VO = 3h \text{ and } OA = 3h\tan\alpha$$

$$\therefore \quad U = \frac{1}{3}\pi(3h\tan\alpha)^2\times 3h = \frac{27\pi}{3}h^2\tan^2\alpha$$

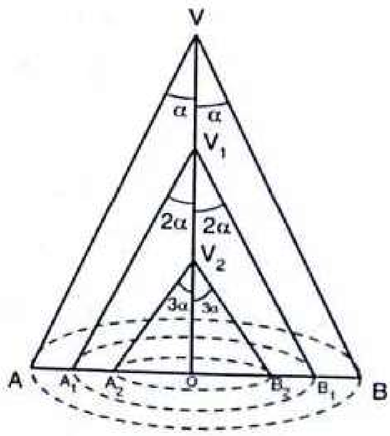

Fig. 14.53

From cone $V_1A_1B_1$, we have

$$V_1O = 2h \text{ and } OA_1 = 2h\tan 2\alpha$$

$$\therefore \quad V = \frac{1}{3}\pi(2h\tan 2\alpha)^2\times 2h = \frac{8}{3}\pi h^3\tan^2 2\alpha$$

For cone $V_2A_2B_2$, we have

$$V_2O = h \text{ and } OA_2 = h\tan 3\alpha$$

$$\therefore \quad W = \frac{1}{3}\pi(h\tan 3\alpha)^2\times h = \frac{1}{3}\pi h^3\tan^2 3\alpha$$

Now, $U - V = \frac{27\pi}{3} h^3 \tan^2 \alpha - \frac{8\pi}{3} h^3 \tan^2 2\alpha = \frac{\pi h^3}{3}(27 \tan^2 \alpha - 8 \tan^2 2\alpha)$

and, $V - W = \frac{8}{3}\pi h^3 \tan^2 2\alpha - \frac{\pi}{3} h^3 \tan^2 3\alpha = \frac{\pi h^3}{3}(8 \tan^2 2\alpha - \tan^2 3a)$

$\therefore$ Required ratio $= (U - V) : (V - W)$

$= (27 \tan^2 \alpha - 8 \tan^2 2\alpha) : (8 \tan^2 2\alpha - \tan^2 3\alpha)$.

EXAMPLE 39 *A golf ball has diameter equal to 4.1 cm. Its surface has 150 dimples each of radius 2 mm. Calculate total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.*

SOLUTION We observe that:

$$\text{Surface area of the ball} = 4\pi \times \left(\frac{4.1}{2}\right)^2 \text{cm}^2 = 16.81\,\pi\,\text{cm}^2$$

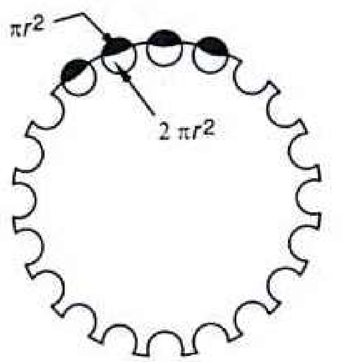

Fig. 14.54

In case of each dimple, surface area equal to πr^2 (r is the radius of each dimple) is removed from the surface of the ball where as the surface area of hemisphere i.e. $2\pi r^2$ is exposed to the surroundings. Let S be the total surface area exposed to the surroundings. Then,

$$S = \text{Surface area of the ball} - 150 \times \pi r^2 + 150 \times 2\pi r^2$$

$$\Rightarrow \quad S = 16.81\pi + 150\pi r^2$$

$$\Rightarrow \quad S = \left\{16.81\pi + 150\pi \times \left(\frac{2}{10}\right)^2\right\} \text{cm}^2$$

$$\Rightarrow \quad S = (16.81\pi + 6\pi)\,\text{cm}^2 = 22.81\pi\,\text{cm}^2 = 22.81 \times \frac{22}{7}\,\text{cm}^2 = 71.68\,\text{cm}^2$$

EXAMPLE 40 *A wooden toy rocket is in the shape of a cone mounted on a cylinder as shown in Fig. 14.55. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take $\pi = 3.14$)*

[NCERT]

SOLUTION Let r be the radius of the base of the cone and its slant height be l. Further, let r_1 be the radius of the cylinder and h_1 be its height. It is given that

$$r = 2.5\text{ cm},\ h = 6\text{ cm},\ r_1 = 1.5\text{ cm} \text{ and } h_1 = 20\text{ cm}$$

$\therefore \quad l = \sqrt{r^2 + h^2} \quad = \sqrt{(2.5)^2 + 6^2} = 6.5 \text{ cm} = \sqrt{4.25 + 36} = \sqrt{40.25}$

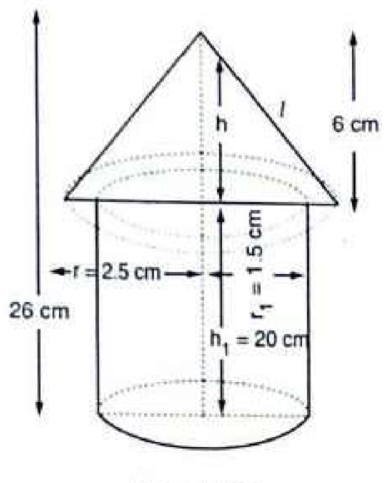

Fig. 14.55

Let S_1 and S_2 be the areas to be painted orange and yellow respectively.

S_1 = Curved surface area of the cone + Base area of the cone – Base area of the cylinder

$\Rightarrow \quad S_1 = \pi r l + \pi r^2 - \pi r_1^2$

$\Rightarrow \quad S_1 = \pi \{ rl + r^2 - r_1^2 \}$

$\Rightarrow \quad S_1 = \pi \{ 2.5 \times 6.5 + (2.5)^2 - (1.5)^2 \} \text{ cm}^2$

$\Rightarrow \quad S_1 = 3.14 \, (16.25 + 6.25 - 2.25) \text{ cm}^2 = 3.14 \times 20.25 \text{ cm}^2 = 63.585 \text{ cm}^2$

and,

S_2 = Curved surface area of the cylinder + Area of the base of the cylinder

$\Rightarrow \quad S_2 = 2\pi r_1 h_1 + \pi r_1^2$

$\Rightarrow \quad S_2 = \pi r_1 (2h_1 + r_1)$

$\Rightarrow \quad S_2 = 3.14 \times 1.5 \, (2 \times 20 + 1.5) \text{ cm}^2$

$\Rightarrow \quad S_2 = 3.14 \times 1.5 \times 41.5 \text{ cm}^2 = 4.71 \times 41.5 \text{ cm}^2 = 195.465 \text{ cm}^2$

EXAMPLE 41 *Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical dipression at one end as shown in Fig. 14.56. The height of the hollow cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.* (*Take* $\pi = 22/7$)

[NCERT]

SOLUTION Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder. Then, $r = 30$ cm and $h = 1.45$ m $= 145$ cm. Let S be the total surface area of the bird-bath. Then,

S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow \quad S = 2\pi r h + 2\pi r^2 = 2\pi r \, (h + r)$

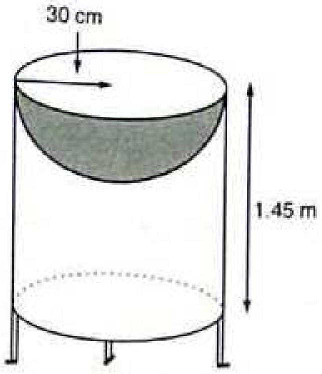

Fig. 14.56

$\Rightarrow \qquad S = 2 \times \frac{22}{7} \times 30\,(145 + 30)\text{ cm}^2 = 33000\text{ cm}^2 = 3.3\text{ m}^2$

EXAMPLE 42 *A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The diameter of each of the depression is 1 cm and the depth is 1.4 cm. Find the volume of the wood in the entire stand.*

[NCERT, CBSE 2017]

SOLUTION It is given that the dimensions of the cuboidal part are 15 cm by 10 cm by 3.5 cm

$\therefore \qquad$ Volume of the cuboid $= (15 \times 10 \times 3.5)\text{ cm}^3 = 525\text{ cm}^3$

It is given that there are four conical depressions such that the radius of each depression is 0.5 cm and the depth is 1.4 cm. Let V be the volume of wood taken out to make four cavities. Then,

$V = 4 \times$ Volume of a cone of base radius 0.5 cm and height 1.4 cm

$\Rightarrow \qquad V = 4 \times \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4\text{ cm}^3$

$\Rightarrow \qquad V = 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{14}{10}\text{ cm}^3$

$\Rightarrow \qquad V = \frac{22}{15}\text{ cm}^3 = 1.47\text{ cm}^3$ (approximately)

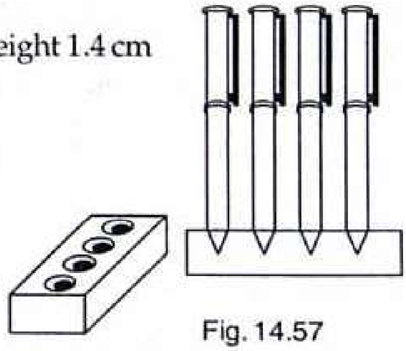
Fig. 14.57

Hence, Volume of the wood in the entire stand $= (525 - 1.47)\text{ cm}^3 = 523.53\text{ cm}^3$

EXAMPLE 43 *A cistern, internally measuring 150 cm × 120 cm × 110 cm has 129600 cm³ of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one seventeenth of its own volume of water. How many bricks can be put in without the water overflowing, each brick being 22.5 cm × 7.5 cm × 6.5 cm?* **[NCERT]**

SOLUTION We have,

Volume of cistern $= 150 \times 120 \times 110\text{ cm}^3 = 1980000\text{ cm}^3$

Volume of water in cistern $= 129600\text{ cm}^3$

Volume of one brick $= 22.5 \times 7.5 \times 6.5\text{ cm}^3 = 1096.875\text{ cm}^3$

Volume of water absorbed by one brick $= \frac{1}{17} \times 1096.875\text{ cm}^3$

Let n be the total number of bricks which can be put in the cistern without water overflowing. Then, Volume of water absorbed by n bricks $= n \times \frac{1}{17} \times 1096.875 \text{ cm}^3$

$\therefore$ Volume of water left in the cistern $= \left(129600 - \frac{n}{17} \times 1096.875\right) \text{cm}^3$

Since the cistern is filled upto the brim. Therefore,

Volume of water left in the cistern + Volume of bricks = Volume of the cistern

$$\Rightarrow \quad 129600 - \frac{n}{17} \times 1096.875 + n \times 1096.875 = 1980000$$

$$\Rightarrow \quad n \times 1096.875 - \frac{n}{17} \times 1096.875 = 1980000 - 129600$$

$$\Rightarrow \quad 1096.875 \times \left(n - \frac{n}{17}\right) = 1850400$$

$$\Rightarrow \quad 1096.875 \times \frac{16n}{17} = 1850400$$

$$\Rightarrow \quad 17550 \times \frac{n}{17} = 1850400 \Rightarrow n = \frac{1850400 \times 17}{17550} = 1792.41 \cong 1792$$

EXERCISE 14.2

LEVEL-1

1. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.
2. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area and the volume of the rocket.
3. A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at ₹ 3.50 per m^2 (Use $\pi = 22/7$).
4. A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy. (Use $\pi = 3.14$)
5. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of the cylindrical and conical portions are 10 cm. and 6 cm, respectively. Find the total surface area of the solid. (Use $\pi = 22/7$)
6. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm. **[CBSE 2002]**
7. A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in the tub. If the radius of the hemisphere is immersed in the tub. If the radius of the hemisphere is 3.5 cm and height of the cone outside the hemisphere is 5 cm, find the volume of the water left in the tub. (Take $\pi = 22/7$) **[CBSE 2000C]**

8. A circus tent has cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 m. The heights of the cylindrical and conical portions are 4.2 m and 2.1 m respectively. Find the volume of the tent.
9. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank.
10. A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and the base radius of the cone are also the same. Find the whole surface and volume of the remaining cylinder.
11. A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. Find the capacity of the tent and the cost of the canvas at ₹ 100 per square metre.
12. A boiler is in the form of a cylinder 2 m long with hemispherical ends each of 2 metre diameter. Find the volume of the boiler.
13. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $\frac{14}{3}$ m and the diameter of hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.
14. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of ₹ 10 per dm^2. **[CBSE 2006C]**
15. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between the two vessels is filled with cork dust for heat insulation purposes. How many cubic centimeters of cork dust will be required?
16. A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making the roller is 9 cm. Find the mass of the roller, if 1 cm^3 of iron has 7.8 gm mass. (Use $\pi = 3.14$)
17. A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. **[CBSE 2013]**
18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. **[CBSE 2013]**
19. The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 m^2. If the pipe is made of 99 cm^3 of metal, find the outer and inner radii of the pipe.
20. A right circular cylinder having diameter 12 cm and height 15 cm is full ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.
21. A solid iron pole having cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high Find the mass of the pole, given that the mass of 1 cm^3 of iron is 8 gm.
22. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover.
23. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottoms. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

24. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the value of water (i) displaced out of the cylinder. (ii) left in the cylinder. (Take $\pi = 22/7$) **[CBSE 2009]**

25. A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Determine the volume and total surface area of the remaining block. **[CBSE 2010]**

26. A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and the sufrace area of the toy. (Use $\pi = 22/7$). **[CBSE 2010]**

27. A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. (Use $\pi = 22/7$). **[CBSE 2012]**

28. An wooden toy is made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. (Use $\pi = 22/7$). **[CBSE 2013]**

29. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. (Use $\pi = 22/7$). **[CBSE 2014]**

30. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 22/7$). **[CBSE 2014]**

31. The largest cone is curved out from one face of solid cube of side 21 cm. Find the volume of the remaining solid. **[CBSE 2015]**

32. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}\text{ cm}^3$. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 10 per cm^2. (Take $\pi = 22/7$). **[CBSE 2015]**

33. In Fig. 14.58, from a cuboidal solid metalic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. (Take $\pi = 22/7$). **[CBSE 2015]**

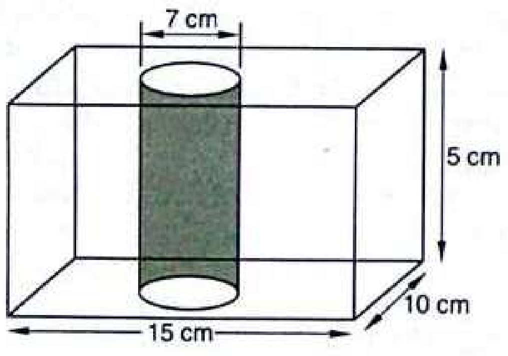

Fig. 14.58

34. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}\text{ m}^3$ of air. If the internal diameter of dome is equal to its total height above the floor, find the height of the building? **[NCERT EXEMPLAR]**

35. A pen stand made of wood is in the shape of a cuboid with four conical depressions and a cubical depression to hold the pens and pins, respectively. The dimension of the cuboid are 10 cm × 5 cm × 4 cm. The radius of each of the conical depression is 0.5 cm and the depth is 2.1 cm. The edge of the cubical depression is 3 cm. Find the volume of the wood in the entire stand. **[NCERT EXEMPLAR]**

36. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air. [NCERT EXEMPLAR]

37. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy. [NCERT EXEMPLAR]

38. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If their common diameter is 56 m, the height of the cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area oif the canvas used in making the tent. [CBSE 2017]

ANSWERS

1. 1320 m^2
2. 412.5 m^2, 461.77 cm^3
3. ₹ 5365.80
4. 103. 62 cm^2
5. 372. 56 cm^2
6. 770 cm^2
7. 616 cm^3
8. 6160 m^3
9. 8316 cm^3
10. Volume = 200π cm^3, Surface area = 210π cm^2
11. 500π m^3, ₹ 55000
12. $\frac{220}{21}$ m^3
13. 56.15 m^3, $70\frac{7}{12}$ m^2
14. ₹ 457.60
15. 1980 cm^3
16. 1388.7 kg
17. 572 cm^2
18. 214.5 cm^2
19. $\frac{5}{2}$ cm and 2 cm
20. 10
21. 102.188 kg
22. 8π cm^3
23. 1.131 m^3
24. (i) 77 cm^3 (ii) 748 cm^3
25. 2992.50 cm^2, 9030 cm^3
26. 28 cm, 5082 cm^2
27. 166.83 cm^3
28. 205.33 cm^3
29. 163.33 cm^3
30. 73.92 cm^2
31. 4410 cm^3
32. 9.5 cm, ₹770
33. 583 cm^2
34. 4 m
35. 170.8 cm^2
36. 6 m
37. 310.86 cm^3, 171.68 cm^2
38. 4136 m^2

HINTS TO SELECTED PROBLEMS

19. Let outer and inner radii of the pipe be R and r respectively. Then,

$$2 \times \frac{22}{7} \times (R - r) \times 14 = 44 \text{ and, } \frac{22}{7} \times (R^2 - r^2) \times 14 = 99$$

$$\Rightarrow \quad R - r = \frac{1}{2} \text{ and } R^2 - r^2 = \frac{9}{4} \Rightarrow R - r = \frac{1}{2} \text{ and, } (R + r) \times \frac{1}{2} = \frac{9}{4}$$

$$\Rightarrow \quad R - r = \frac{1}{2} \text{ and } R + r = \frac{9}{2} \Rightarrow \quad R = \frac{5}{2} \text{ and } r = 2$$

20. Number of ice-cream cones $= \dfrac{\text{Volume of the circular cylinder}}{\text{Volume of one ice-cream cone}}$

$$= \frac{\pi \times 6 \times 6 \times 15}{\frac{1}{3} \times \pi \times 3 \times 3 \times 12 + \frac{2\pi}{3} \times 3^3} = \frac{\pi \times 36 \times 15}{\frac{\pi}{3} \times (108 + 54)} = 10$$

21. Mass of the pole $= \left\{\pi \times 6^2 \times 110 + \frac{\pi}{3} \times 6^2 \times 9\right\} \times \frac{8}{1000}$ kg

22. Required space = Volume of the cylinder – Volume of the toy

$$=\left[\pi \times 2^2 \times 4 - \left\{\frac{2}{3}\times\pi\times 2^3 + \frac{1}{3}\times\pi\times 2^2 \times 2\right\}\right] = 8\pi \text{ cm}^3$$

14.5 FRUSTUM OF A RIGHT CIRCULAR CONE

In the previous section, we have learnt about surface areas and volumes of combina-tions of two or more basic solids like right circular cone, right circular cylinder, sphere etc. In this section, we shall learn about a solid which is a part of a right circular cone when it is cut by a plane parallel to the base of the cone. Such a solid is called a frustum as defined below.

FRUSTUM *If a right circular cone is cut off by a plane parallel to its base, then the portion of the cone between the cutting plane and the base of the cone is called a frustum of the cone.*

In Fig. 14.59 (i) right circular cone VAB is cut by a plane parallel to its circular base with centre O and diameter AB. The portion containing the vertex V is removed (Fig. 14.59 (ii)). The left out portion $ABB'A'$ shown in Fig. 14.59 (iii) is the frustum of the cone VAB. The circular faces AOB and $A'O'B'$ are called the circular ends of the frustum.

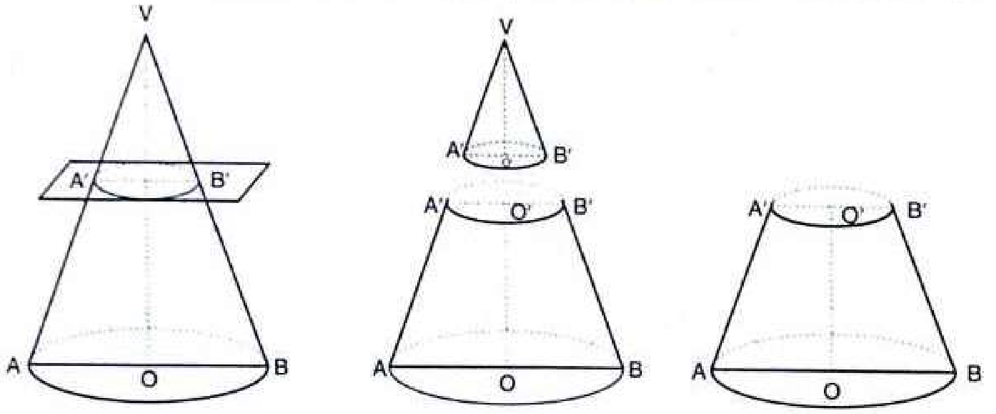

Fig. 14.59 (i), (ii), (iii)

Clearly, a frustum of a right circular cone has two unequal flat circular bases and a curved surface.

Let us now define some other terms like height, lateral (slant) height etc. related to a frustum.

HEIGHT *The height or thickness of a frustum is the perpendicular distance between its two circular bases.*

Clearly, the line segment OO' joining the centres of two circular bases (Fig. 14.59 (iii)) is perpendicular to them. So, OO' is the height of the frustum.

Also, $OO' = VO - VO'$

Thus, the height of the frustum $ABB'A'$ is equal to the difference between the heights of the cones VAB and $VA'B'$.

SLANT HEIGHT *The slant height of a frustum of a right circular cone is the length of the line segment joining the extremities of two parallel radii, drawn in the same direction, of the two circular bases.*

In Fig. 14.59, slant height of the frustum $ABA'B' = AA' = BB'$.

Clearly, $AA' = VA - VA'$ and $BB' = VB - VB'$

Thus, the slant height of the frustum equals the difference between the slant heights of the cones VAB and $VA'B'$.

14.6 VOLUME AND SURFACE AREA OF A FRUSTUM OF A RIGHT CIRCULAR CONE

Let h be the height, l the slant height and r_1 and r_2 the radii of the circular bases of the frustum $ABB'A'$ shown in Fig. 14.60 such that $r_1 > r_2$. Clearly, this frustum can be viewed as the difference of the two right circular cones VAB and $VA'B'$.

Let the height of the cone VAB be h_1 and its slant height be l_1 i.e., $VO = h_1$ and $VA = VB = l_1$

$\therefore \quad VA' = VA - AA' = l_1 - l$ and $VO' = VO - OO' = h_1 - h$

Clearly, right triangles VOA and $VO'A'$ are similar.

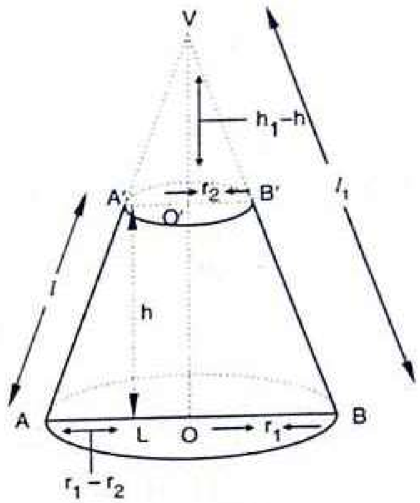

Fig. 14.60

$$\therefore \quad \frac{VO}{VO'} = \frac{OA}{O'A'} = \frac{VA}{VA'}$$

$$\Rightarrow \quad \frac{h_1}{h_1 - h} = \frac{r_1}{r_2} = \frac{l_1}{l_1 - l}$$

$$\Rightarrow \quad \frac{h_1 - h}{h_1} = \frac{r_2}{r_1} = \frac{l_1 - l}{l_1}$$

$$\Rightarrow \quad 1 - \frac{h}{h_1} = \frac{r_2}{r_1} = 1 - \frac{l}{l_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = 1 - \frac{r_2}{r_1} \text{ and } \frac{l}{l_1} = 1 - \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{h}{h_1} = \frac{r_1 - r_2}{r_1} \text{ and } \frac{l}{l_1} = \frac{r_1 - r_2}{r_1}$$

$$\Rightarrow \quad h_1 = \frac{hr_1}{r_1 - r_2} \text{ and } l_1 = \frac{lr_1}{r_1 - r_2} \qquad \ldots\text{(i)}$$

Now,

Height of the cone $VA'B' = h_1 - h = \frac{hr_1}{r_1 - r_2} - h = \frac{hr_2}{r_1 - r_2}$ ...(ii)

Slant height of the cone $VA'B' = l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2}$ ...(iii)

Let V be the volume of the frustum of cone. Then,

V = Volume of cone VAB – Volume of cone $VA'B'$

$$\Rightarrow \quad V = \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{h_1 r_1^2 - (h_1 - h) r_2^2\right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{\left(\frac{hr_1^3}{r_1 - r_2}\right) - \left(\frac{hr_2^3}{r_1 - r_2}\right)\right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{\frac{h}{r_1 - r_2}(r_1^3 - r_2^3)\right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}\left\{\frac{h}{r_1 - r_2}(r_1 - r_2)(r_1^2 + r_1 r_2 + r_2^2)\right\}$$

$$\Rightarrow \quad V = \frac{\pi}{3}h\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

Thus, the volume V of the frustum of the cone is given by $V = \frac{1}{3}\pi\left(r_1^2 + r_1 r_2 + r_2^2\right)h$.

Let S denote the curved surface area of the frustum of cone. Then,

S = Lateral (curved) surface area of cone VAB – Curved surface area of cone $VA'B'$

$\Rightarrow \quad S = \pi r_1 l_1 - \pi r_2 (l_1 - l)$

$\Rightarrow \quad S = \pi r_1 \cdot \dfrac{l r_1}{r_1 - r_2} - \pi r_1 \cdot \dfrac{l r_2}{r_1 - r_2}$ [Using (i) and (iii)]

$\Rightarrow \quad S = \pi \left(\dfrac{r_1^2 - r_2^2}{r_1 - r_2} \right) l$

$\Rightarrow \quad S = \pi (r_1 + r_2)$

Thus, Curved surface area of a the frustum $= \pi (r_1 + r_2) l$

Total surface area of the frustum

= Lateral (curved) surface area + Surface area of circular bases

$= \pi (r_1 + r_2) l + \pi r_1^2 + \pi r_2^2$

$= \pi \{(r_1 + r_2) l + r_1^2 + r_2^2\}$

REMARK 1 *If A_1 and A_2 denote the surface areas of circular bases, with centres O and O' respectively, of the frustum. Then, $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$*

$\therefore$ Volume of the frustum of cone $= \dfrac{\pi}{3} (r_1^2 + r_1 r_2 + r_2^2) h$

$\Rightarrow$ Volume of the frustum of cone $= \dfrac{h}{3} \left\{ \pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2 \times \pi r_2^2} \right\} l$

$\Rightarrow$ Volume of the frustum of cone $= \dfrac{h}{3} (A_1 + A_2 + \sqrt{A_1 A_2})$

REMARK 2 *In right triangle ALA' (Fig. 14.60), we have*

$AA'^2 = AL^2 + A'L^2 \Rightarrow l^2 = (r_1 - r_2)^2 + h^2 \Rightarrow l = \sqrt{(r_1 - r_2)^2 + h^2}$

$\therefore$ Slant height of the frustum of cone $= \sqrt{(r_1 - r_2)^2 + h^2}$

Height of the cone of which the frustum is a part $= \dfrac{h r_1}{r_1 - r_2}$

Slant height of the cone of which the frustum is a part $= \dfrac{l r_1}{r_1 - r_2}$

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *If the radii of the circular ends of a conical bucket which is 45 cm high, are 28 cm and 7 cm, find the capacity of the bucket* (Use $\pi = 22/7$). [CBSE 2004, 2005]

SOLUTION Clearly, bucket forms a frustum of a cone such that the radii of its circular ends are $r_1 = 28$ cm, $r_2 = 7$ cm and height $h = 45$ cm. Let V be the capacity of the bucket. Then,

V = Volume of the frustum

$\Rightarrow \quad V = \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$

$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 45\,(28^2 + 7^2 + 28 \times 7)$

$\Rightarrow \quad V = 22 \times 15 \times (28 \times 4 + 7 + 28) = 330 \times 147 \text{ cm}^3 = 48510 \text{ cm}^3$

EXAMPLE 2 *The radii of the circular ends of a frustum of height 6 cm are 14 cm and 6 cm respectively. Find the lateral surface area and total surface area of the frustum.*

SOLUTION We have, $r_1 = 14$ cm, $r_2 = 6$ cm and $h = 6$ cm. Let l be the slant height of the frustum. Then,

$$l = \sqrt{h^2 + (r_1 - r_2)^2} \Rightarrow l = \sqrt{36 + (14-6)^2} = \sqrt{36 + 64} = 10 \text{ cm}$$

Let LSA and TSA respectively be the lateral surface area and total surface area of the frustum. Then,

$$\therefore \quad LSA = \pi(r_1 + r_2)l = \frac{22}{7} \times (14+6) \times 10 \text{ cm}^2 = \frac{22}{7} \times 200 \text{ cm}^2 = 628.57 \text{ cm}^2$$

$$TSA = \pi\left\{r_1^2 + r_2^2 + (r_1 + r_2)l\right\} = \frac{22}{7} \times (196 + 36 + 20 \times 10) \text{ cm}^2 = \frac{22}{7} \times 432 \text{ cm}^2 = 1357.71 \text{ cm}^2$$

EXAMPLE 3 *The perimeters of the ends of a frustum are 48 cm and 36 cm. If the height of the frustum be 11 cm, find its volume.*

SOLUTION Let r_1 and r_2 be the radii of the circular ends of the frustum and h be its height. Then,

$$2\pi r_1 = 48,\ 2\pi r_2 = 36 \text{ and } h = 11 \text{ cm} \Rightarrow r_1 = \frac{24}{\pi},\ r_2 = \frac{18}{\pi} \text{ and } h = 11 \text{ cm}$$

Let V be the volume of the frustum. Then,

$$V = \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)\,h$$

$$\Rightarrow \quad V = \frac{1}{3} \times \pi \times 11 \times \left\{\left(\frac{24}{\pi}\right)^2 + \left(\frac{18}{\pi}\right)^2 + \frac{24}{\pi} \times \frac{18}{\pi}\right\}$$

$$\Rightarrow \quad V = \frac{1}{3} \times \pi \times 11 \times \frac{(576 + 324 + 432)}{\pi^2} \text{ cm}^3$$

$$\Rightarrow \quad V = \frac{11}{3} \times \frac{1332}{\pi} \text{cm}^3 = \frac{11}{3} \times \frac{1332}{22} \times 7 \text{ cm}^3 = 1554 \text{ cm}^3$$

EXAMPLE 4 *The slant height of the frustum of a cone is 4 cm, and the perimeter of its circular bases are 18 cm and 6 cm respectively. Find the curved surface area of the frustum.*

SOLUTION Let r_1 and r_2 be the radii of the circular bases of the frust, l be the slant height and h be the height. It is given that

$$l = 4 \text{ cm},\ 2\pi r_1 = 18 \text{ and } 2\pi r_2 = 6 \Rightarrow l = 4 \text{ cm},\ r_1 = \frac{9}{\pi} \text{ and } r_2 = \frac{3}{\pi}$$

$$\therefore \quad \text{Curved surface area} = \pi(r_1 + r_2)l = \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4 \text{ cm}^2 = 48 \text{ cm}^2$$

EXAMPLE 5 *A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.*

[CBSE 2012, 2014]

SOLUTION Let the height of the bucket be h cm. We have, $r_1 = 28$ cm, $r_2 = 21$ cm

and, V = Volume of the bucket = 28.490 litres = 28.490×1000 cm³ = 28490 cm³

Now,

$$V = 28490 \text{ cm}^3$$

$$\Rightarrow \quad \frac{1}{3}\pi h\,(r_1^2 + r_1 r_2 + r_2^2) = 28490$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times h \times (28^2 + 28 \times 21 + 21^2) = 28490$$

$$\Rightarrow \quad \frac{22}{21} \times h \times (784 + 588 + 441) = 28490$$

$$\Rightarrow \quad \frac{22}{21} \times h \times 1813 = 28490 \Rightarrow h = \frac{28490 \times 21}{22 \times 1813}\text{ cm} \Rightarrow h = 15 \text{ cm}$$

Thus, height of the bucket = 15 cm.

EXAMPLE 6 *A friction clutch is in the form of a frustum of a cone, the diameter of the ends being 32 cm and 20 cm and length 8 cm. Find its bearing surface and volume.*

SOLUTION Let $ABB'A'$ be the friction clutch of slant height l cm.

We have,

$$r_1 = 16 \text{ cm}, r_2 = 10 \text{ cm and } h = 8 \text{ cm}$$

$$\therefore \quad l^2 = h^2 + (r_1 - r_2)^2$$

$$\Rightarrow \quad l^2 = 64 + 36 \Rightarrow l = 10 \text{ cm}$$

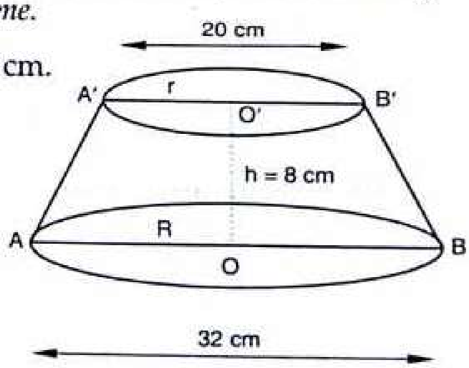

Fig. 14.61

Let S be the bearing surface and V be the volume of the clutch. Then,

$$S = \text{Lateral surface of the frustum}$$

$$\Rightarrow \quad S = \pi(r_1 + r_2)l = \frac{22}{7} \times (16 + 10) \times 10 \text{ cm}^2 = 817.14 \text{ cm}^2$$

and,

$$V = \frac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times (16^2 + 16 \times 10 + 10^2) \text{ cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times (256 + 160 + 100) \text{ cm}^3 = \frac{176}{21} \times 516 \text{ cm}^3 = 4324.57 \text{ cm}^3$$

EXAMPLE 7 *The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?* **[CBSE 2016, 2017]**

SOLUTION Let VAB be a cone of height 30 cm and base radius r cm. Suppose it is cut off by a plane parallel to the base at a height h from the base of the cone.

Clearly, $\Delta VOA \sim \Delta VO'A'$

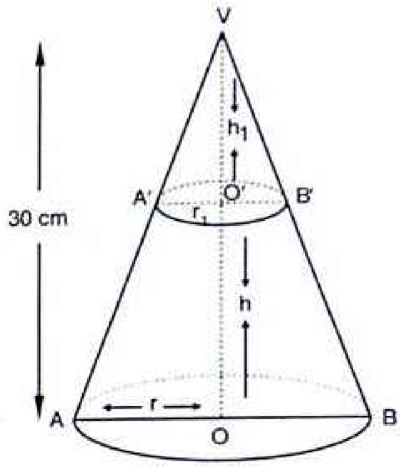

Fig. 14.62

$$\therefore \quad \frac{VO}{VO'} = \frac{OA}{O'A'} \Rightarrow \frac{30}{h_1} = \frac{r}{r_1} \quad \ldots(i)$$

It is given that

$$\text{Volume of cone } VA'B' = \frac{1}{27} \text{ Volume of cone } VAB$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{27} \times \frac{1}{3}\pi r^2 \times 30$$

$$\Rightarrow \quad \left(\frac{r_1}{r}\right)^2 h_1 = \frac{10}{9}$$

$$\Rightarrow \quad \left(\frac{h_1}{30}\right)^2 h_1 = \frac{10}{9} \quad \text{[Using (i)]}$$

$$\Rightarrow \quad h_1^3 = 1000$$

$$\Rightarrow \quad h_1 = 10 \text{ cm}$$

$$\therefore \quad h = 30 - h_1 = (30 - 10) \text{ cm} = 20 \text{ cm}$$

Hence, the section is made at a height of 20 cm from the base of the cone.

EXAMPLE 8 *A container, open from the top, made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹15 per litre and the cost of metal sheet used, if the costs ₹5 per 100 cm². (Use $\pi = 3.14$)* **[CBSE 2008, 2014, 2016]**

SOLUTION Let l be the slant height of the frustum. It is given that $r_1 = 20$ cm, $r_2 = 8$ cm and $h = 16$ cm.

$$\therefore \quad l = \sqrt{(r_1 - r_2)^2 + h^2} \Rightarrow l = \sqrt{(20-8)^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20 \text{ cm}$$

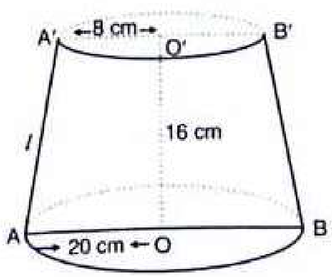

Fig. 14.63

Let V be the volume of the container. Then,

$$\therefore \quad V = \frac{\pi}{3}\left\{r_1^2 + r_2^2 + r_1 r_2\right\} h$$

$\Rightarrow \quad V = \frac{\pi}{3}\{20^2 + 8^2 + 20 \times 8\} \times 16 \text{ cm}^3$

$\Rightarrow \quad V = \frac{3.14 \times 624 \times 16}{3} \text{ cm}^3 = 10449.92 \text{ cm}^3 = \frac{10449.92}{1000} \text{ litres} = 10.45 \text{ litres approx.}$

$\therefore$ Cost of milk at the rate of ₹ 15 per litre = ₹ (10.45 × 15) = ₹ 156.75

Let S be the surface area of the frustum. Then,

$S = \pi(r_1 + r_2)l + \pi r_2^2$ [∵ Top is open]

$\Rightarrow \quad S = \{3.14(20 + 8) \times 20 + 3.14 \times 8^2\} \text{ cm}^2$

$\Rightarrow \quad S = 3.14 \times (560 + 64) \text{ cm}^2 = 3.14 \times 624 \text{ cm}^2 = 1959.36 \text{ cm}^2$

$\therefore$ Cost of metal used = ₹ $\left(\frac{1959.36 \times 5}{100}\right)$ = ₹ 97.96 (Approx)

EXAMPLE 9 *A tent is made in the form of a conic frustum surmounted by a cone. The diameters of the base and the top of the frustum are 20 m and 6 m respectively and the height is 24 m. If the height of the tent is 28 m, find the quantity of canvas required.*

SOLUTION Let h be the height of the frustum and r_1 and r_2 be the radii of its circular bases. It is given that $h = 24$ m, $r_1 = 10$ m and $r_2 = 3$ m. Let l be the slant height of the frustum. Then,

$$l = \sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(10-3)^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} \text{ m} = 25 \text{ m}$$

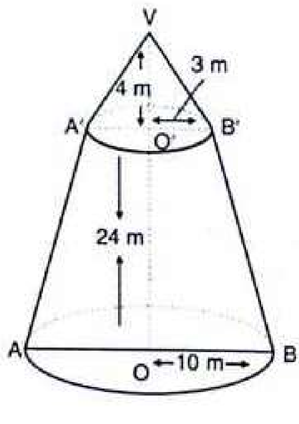

Fig. 14.64

Let l_2 be the slant height of the cone $VA'B'$. Then,

$$l_2 = \sqrt{O'B'^2 + VO'^2} = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

Let S be the quantity of canvas required. Then,

S = Lateral surface area of frustum + Lateral surface area of cone $VA'B'$

$\Rightarrow \quad S = \pi(r_1 + r_2)l + \pi r_2 l_2$

$\Rightarrow \quad S = \{\pi(10 + 3) \times 25 + \pi \times 3 \times 5\} \text{ m}^2 = (325\pi + 15\pi) \text{ m}^2 = 340\pi \text{ m}^2$

EXAMPLE 10 *An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height be 22 cm, diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm, find the area of the tin required to make the funnel.*

[NCERT]

SOLUTION Let l be the slant height of the frustum part of the funnel. Then,

$$l = \sqrt{(9-4)^2 + 12^2} = \sqrt{25+144} \text{ cm} = 13 \text{ cm}$$

Let S be the area of the tin required to make the funnel. Then,

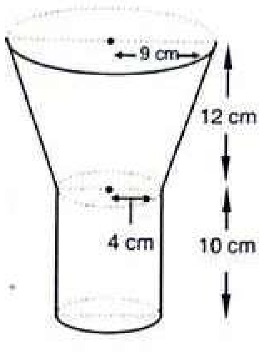

Fig. 14.65

S = Curved surface area of cylindrical portion + Curved surface area of frustum portion

$\Rightarrow \quad S = 2\pi r_1 h + \pi(r_1 + r_2) l$

$\Rightarrow \quad S = \{2\pi \times 4 \times 10 + \pi(4+9) \times 13\} \text{ cm}^2 = (80\pi + 169\pi) \text{ cm}^2 = 249\pi \text{ cm}^2$

EXAMPLE 11 *A solid metallic right circular cone 20 cm high with vertical angle 60° is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained, be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.* **[CBSE 2014]**

SOLUTION Let VAB be the solid metallic right circular cone of height 20 cm. Suppose this cone is cut by a plane parallel to its base at a point O' such that $VO' = O'O$ i.e. O' is the mid-point of VO. Let r_1 and r_2 be the radii of circular ends of the frustum $ABB'A'$.

In triangles VOA and $VO'A'$, we have

$$\tan 30° = \frac{OA}{VO} \text{ and } \tan 30° = \frac{O'A'}{VO'}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{r_1}{20} \text{ and } \frac{1}{\sqrt{3}} = \frac{r_2}{10}$

$\Rightarrow \quad r_1 = \frac{20}{\sqrt{3}} \text{ cm and } r_2 = \frac{10}{\sqrt{3}} \text{ cm}$

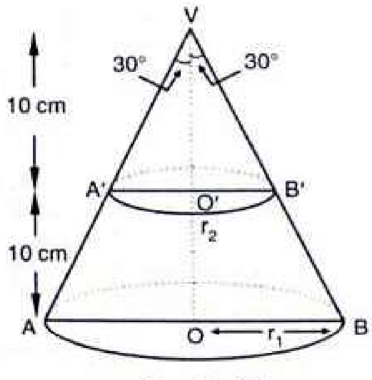

Fig. 14.66

Let V be the volume of the frustum. Then,

$$V = \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$$

$\Rightarrow \quad V = \frac{\pi}{3}\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) \times 10 \text{ cm}^2 = \frac{7000}{9}\pi \text{ cm}^2$

Let the length of the wire of $\frac{1}{16}$ cm diameter be l cm and V_1 be the volume of the metal used in the wire.

$V_1 = \pi \times \left(\frac{1}{32}\right)^2 \times l \text{ cm}^2 \qquad \left[\because \text{ radius} = \frac{1}{32} \text{ cm}\right]$

$\Rightarrow \quad V_1 = \frac{\pi l}{1024} \text{ cm}^2$

The frustum is recast into a wire of length l cm and diameter $\frac{1}{16}$ cm.

$\therefore$ Volume of the metal used in wire = Volume of the frustum

$\Rightarrow \quad V_1 = V$

$\Rightarrow \quad \frac{\pi l}{1024} = \frac{7000\pi}{9}$

$\Rightarrow \quad l = \frac{7000\pi}{9} \times \frac{1024}{\pi} \text{ cm} = \frac{7000}{9} \times 1024 \text{ cm} = 7964.4 \text{ m}$

EXAMPLE 12 *A bucket of height 8 cm and made up of copper sheet is in the form of frustum of a right circular cone with radii of its lower and upper ends as 3 cm and 9 cm respectively. Calculate.*

(i) the height of the cone of which the bucket is a part.

(ii) the volume of water which can be filled in the bucket.

(iii) the area of copper sheet required to make the bucket. **[CBSE 2014]**

SOLUTION Let h be the height, l the slant height and r_1 and r_2 the radii of the circular bases of a frustum of a cone. It is given that $h = 8$ cm, $r_1 = 9$ cm and $r_2 = 3$ cm

(i) Let h_1 be the height of the cone of which the bucket is a part. Then,

$$h_1 = \frac{hr_1}{r_1 - r_2} \Rightarrow h_1 = \left(\frac{8 \times 9}{9 - 3}\right) \text{cm} = 12 \text{ cm}$$

(ii) Let V be the volume of the water which can be filled in the bucket. Then,

V = Volume of the frustum

$$\Rightarrow \quad V = \frac{\pi}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)h = \frac{\pi}{3}(9^2 + 9 \times 3 + 3^2) \times 8 \text{ cm}^3 = 312\,\pi \text{ cm}^3$$

(iii) Let S be the area of the copper sheet required to make the bucket. Then,

$S = \pi(r_1 + r_2)\,l + \pi r_2^2$, where l is the slant height of the frustum

$$\Rightarrow \quad S = \pi(9 + 3) \times \sqrt{(9-3)^2 + 8^2} + \pi \times 3^2 \qquad \left[\because l = \sqrt{(r_1 - r_2)^2 + h^2}\right]$$

$$\Rightarrow \quad S = 129\pi \text{ cm}^2$$

EXAMPLE 13 *An open metallic bucket is in the shape of a frustum of a cone mounted on hollow cylindrical base made of metallic sheet. If the diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 30 cm and that of the cylindrical portion is 6 cm, find the area of the metallic sheet used to make the bucket. Also, find the volume of the water it can hold.* (*Take* $\pi = 22/7$). **[NCERT]**

SOLUTION h = Height of the frustum of the cone = $(30 - 6)$ cm = 24 cm

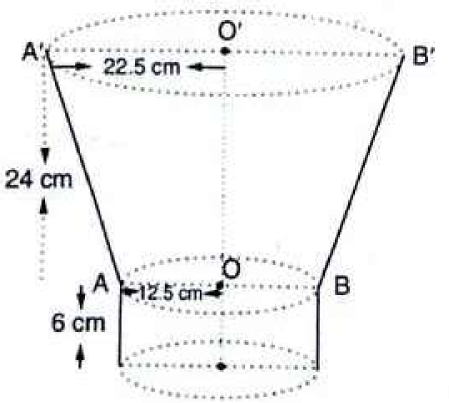

Fig. 14.67

Radii of the circular ends are $r_1 = 22.5$ cm and $r_2 = 12.5$ cm.
Let l be the slant height of the frustum. Then,

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + (22.5 - 12.5)^2} = \sqrt{576 + 100} = 26 \text{ cm}$$

Let A be the area of metallic sheet used. Then,

A = Curved surface area of the frustum of cone + Area of circular base + Curved surface area of cylinder.

$$\Rightarrow \quad A = \pi(r_1 + r_2)l + \pi r_2^2 + 2\pi r_2 h_2, \text{ where } h_2 = \text{height of the base} = 6 \text{ cm}$$

$$\Rightarrow \quad A = \pi\left[(22.5 + 12.5) \times 26 + 12.5^2 + 2 \times 12.5 \times 6\right] \text{cm}^2$$

$$\Rightarrow \quad A = \pi \times (1216.25) \text{ cm}^2 = \frac{22}{7} \times 1216.25 \text{ cm}^2 = 3822.5 \text{ cm}^2$$

Let V be the volume of water that the bucket can hold. Then,

$$V = \frac{1}{3} \times \pi \times \left(r_1^2 + r_2^2 + r_1 r_2\right) \times h$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times \left\{22.5^2 + 12.5^2 + 22.5 \times 12.5\right\} \times 24 \text{ cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times \left\{(9 \times 2.5)^2 + (5 \times 2.5)^2 + (9 \times 2.5) \times (5 \times 2.5)\right\} \times 24 \text{ cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2\left(9^2 + 5^2 + 9 \times 5\right) \times 24 \text{ cm}^3$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times (151) \times 24 \text{cm}^3 = 23728.57 \text{ cm}^3 = 23.728 \text{ litres}$$

So, the bucket can hold 23.728 litres of water.

EXAMPLE 14 *A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere as shown in Fig. 14.68. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find its external surface area.*

SOLUTION We have,

r_1 = Radius of the lower end of the frustum = 1 cm

r_2 = Radius of the upper end of the frustum = 2.5 cm

h = Height of the frustum = 6 cm.

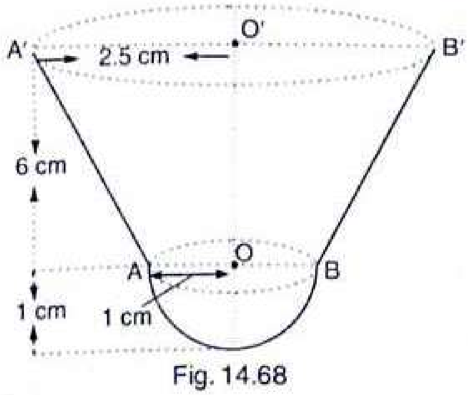

Fig. 14.68

Let l be the slant height of the frustum. Then,

$$l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{36 + (2.5 - 1)^2} = \sqrt{38.25} \text{ cm} = 6.18 \text{ cm}$$

Let S be the external surface area of shuttle cock. Then,

$$S = \text{Curved surface area of the frustum} + \text{Surface area of hemisphere}$$

$$\Rightarrow \quad S = \pi(r_1 + r_2)l + 2\pi r_1^2$$

$$\Rightarrow \quad S = \left\{\pi(1 + 2.5) \times 6.18 + 2 \times \pi \times 1^2\right\} \text{cm}^2$$

$$\Rightarrow \quad S = \left\{\frac{22}{7} \times 3.5 \times 6.18 + 2 \times \frac{22}{7}\right\} \text{cm}^2 = (67.98 + 6.28)\text{ cm}^2 = 74.26\text{ cm}^2$$

EXAMPLE 15 *Hanumappa and his wife Gangavva are busy making Jaggery out of sugar-cane. They have processed the sugarcane juice to make the molasses which is poured into moulds of the shape shown in Fig. 14.69. It will be cooled to solidify in this shape to be sent to the market. Each mould is in the shape of a frustum of a cone having the diameters of its two circular ends as 30 cm and 35 cm and the height of the mould is 14 cm. If each cm^3 of molasses weighs about 1.2 gm, find the weight of molasses that can be poured into each mould (take $\pi = 22/7$).* **[NCERT]**

SOLUTION Clearly, the mould is in the shape of a frustum of a cone with radii of two circular ends as $r_1 = \frac{30}{2}$ cm = 15 cm, $r_2 = \frac{35}{2}$ cm = 17.5 cm and height $h = 14$ cm.

Let V be the volume of molasses that can be poured into the mould. Then,

$$V = \text{Volume of the mould}$$

$$\Rightarrow \quad V = \frac{\pi}{3}h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 14\left(15^2 + 17.5^2 + 15 \times 17.5\right)\text{cm}^3$$

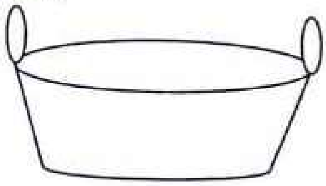

Fig. 14.69

$$\Rightarrow \quad V = \frac{44}{3}(225 + 306.25 + 262.5)\text{ cm}^3 = \frac{44}{3} \times 793.75\text{ cm}^3 = \frac{34925}{3}\text{ cm}^3$$

Let M be the mass of molasses that can be poured into each mould.

It is given that 1 cm^3 of molasses has mass 1.2 gm.

$$\therefore \quad M = V \times 1.2 = \frac{34925}{3} \times 1.2\text{ gm} = \frac{34925 \times 0.4}{1000}\text{ kg} = \frac{13970}{1000}\text{ kg} = 13.97\text{ kg}$$

EXAMPLE 16 *A fez, the headgear cap used by the trucks is shaped like the frustum of a cone. If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.* **[NCERT]**

SOLUTION Clearly, the fez is in the shape of a frustum of a cone with radii of two bases as $r_1 = 10$ cm, $r_2 = 4$ cm and slant height $l = 15$ cm. Let A be the area of the material used. Then,

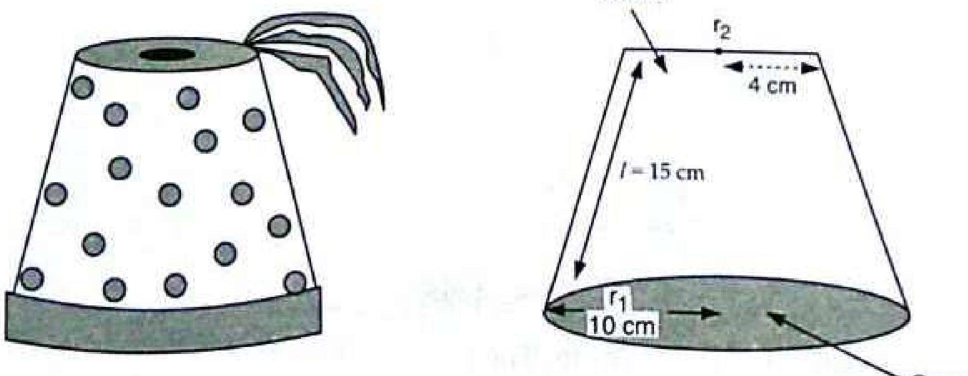

Fig. 14.70

$A =$ Curved surface area + Area of the closed base

$\Rightarrow \quad A = \pi(r_1 + r_2)\,l + \pi r_2^2$

$\Rightarrow \quad A = \left\{\frac{22}{7} \times (10 + 4) \times 15 + \frac{22}{7} \times 4^2\right\} \text{cm}^2$

$\Rightarrow \quad A = \left(352 \times 2 \times 15 + \frac{352}{7}\right) \text{cm}^2 = \left(660 + \frac{352}{7}\right) \text{cm}^2 = 660 + 50.28 \text{ cm}^2 = 710.28 \text{ cm}^2.$

LEVEL-2

EXAMPLE 17 *A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.* **[CBSE 2004]**

SOLUTION Let VAB be a hollow cone of height H, slant height L and base radius R. Suppose this cone is cut by a plane parallel to the base such that O' is the centre of the circular section of the cone. Let h be the height, l be the slant height and r be the base radius of the smaller cone $VA'B'$.

Clearly, $\Delta VO'A' \sim \Delta VOA$

$\therefore \quad \frac{VO'}{VO} = \frac{O'A'}{OA} = \frac{VA'}{VA} \Rightarrow \frac{h}{H} = \frac{r}{R} = \frac{l}{L}$ ...(i)

It is given that

Curved surface area of the frustum $ABB'A' = \frac{8}{9} \times$ Curved surface area of the cone

$\Rightarrow \quad \pi(R + r)(L - l) = \frac{8}{9}\pi RL$

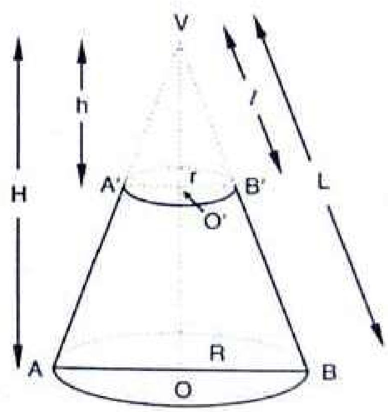

Fig. 14.71

$\Rightarrow \quad (R + r)(L - l) = \frac{8}{9}RL$

$\Rightarrow \quad \left(\frac{R + r}{R}\right)\left(\frac{L - l}{L}\right) = \frac{8}{9}$

$\Rightarrow \quad \left(1 + \frac{r}{R}\right)\left(1 - \frac{l}{L}\right) = \frac{8}{9}$

$\Rightarrow \quad \left(1 + \frac{h}{H}\right)\left(1 - \frac{h}{H}\right) = \frac{8}{9}$ [Using (i)]

$\Rightarrow \quad 1 - \frac{h^2}{H^2} = \frac{8}{9}$

$\Rightarrow \quad \frac{h^2}{H^2} = 1 - \frac{8}{9}$

$$\Rightarrow \quad \frac{h^2}{H^2} = \frac{1}{9} \Rightarrow \frac{h}{H} = \frac{1}{3} \Rightarrow h = \frac{H}{3}$$

Hence, required ratio $= \dfrac{h}{H-h} = \dfrac{\frac{H}{3}}{H - \frac{H}{3}} = \dfrac{1}{2}$

EXAMPLE 18 *The height of a right circular cone is trisected by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1 : 7 : 19.*

SOLUTION Let VAB be a right circular cone of height $3h$ and base radius r. This cone is cut by planes parallel to its base at points O' and L such that $VL = LO' = h$.

Since triangles VOA and $VO'A'$ are similar.

$$\therefore \quad \frac{VO}{VO'} = \frac{OA}{O'A'} \Rightarrow \frac{r}{r_1} = \frac{3h}{2h} \Rightarrow r_1 = \frac{2r}{3}$$

Also, $\quad \Delta VOA \sim \Delta VLC$

$$\therefore \quad \frac{VO}{VL} = \frac{OA}{LC} \Rightarrow \frac{3h}{h} = \frac{r}{r_2} \Rightarrow r_2 = \frac{r}{3}$$

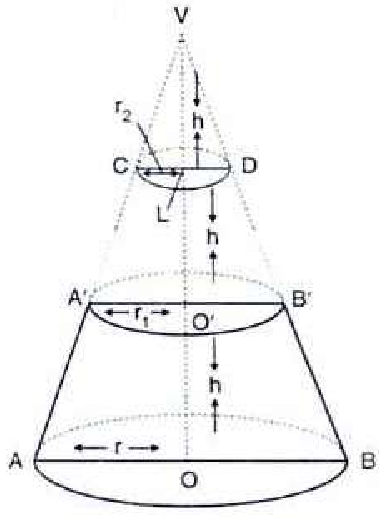

Fig. 14.72

Let V_1 be the volume of cone VCD. Then,

$$V_1 = \frac{1}{3}\pi r_2^2 h = \frac{1}{3}\pi\left(\frac{r}{3}\right)^2 h = \frac{1}{27}\pi r^2 h.$$

Let V_2 be the volume of the frustum $A'B'DC$. Then,

$$V_2 = \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$$

$$\Rightarrow \quad V_2 = \frac{1}{3}\pi\left(\frac{4r^2}{9} + \frac{r^2}{9} + \frac{2r^2}{9}\right)h \qquad \left[\because r_1 = \frac{2r}{3} \text{ and } r_2 = \frac{r}{3}\right]$$

$$\Rightarrow \quad V_2 = \frac{7}{27}\pi r^2 h$$

Let V_3 be the volume of the frustum $ABB'A'$. Then,

$$\Rightarrow \quad V_3 = \frac{1}{3}\pi\left(r^2 + r_1^2 + r_1 r\right)h$$

$$\Rightarrow \quad V_3 = \frac{1}{3}\pi\left(r^2 + \frac{4r^2}{9} + \frac{2r^2}{3}\right)h = \frac{19\pi}{27}r^2 h$$

$$\therefore \quad \text{Required ratio} = V_1 : V_2 : V_3 = \frac{1}{27}\pi r^2 h : \frac{7}{27}\pi r^2 h : \frac{19\pi}{27}r^2 h = 1 : 7 : 19.$$

EXAMPLE 19 *The radius of the base of a right circular cone is r. It is cut by a plane parallel to the base at a height h from the base. The distance of the boundary of the upper surface from the centre of the base of the frustum is* $\sqrt{h^2 + \frac{r^2}{9}}$. *Show that the volume of the frustum is* $\frac{13}{27}\pi r^2 h$.

SOLUTION We have, $OA = r, OO' = h$ and $OB' = \sqrt{h^2 + \frac{r^2}{9}}$

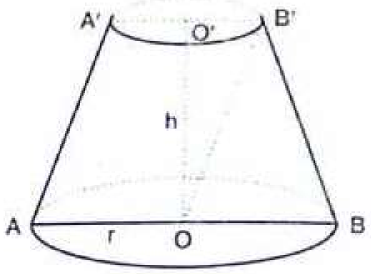

Fig. 14.73

Using Pythagoras theorem in $\Delta OO'B'$, we obtain

$$OB'^2 = OO'^2 + O'B'^2$$

$$\Rightarrow \quad h^2 + \frac{r^2}{9} = h^2 + O'B'^2$$

$$\Rightarrow \quad O'B' = \frac{r}{3}$$

Let V be the volume of the frustum. Then,

$$\therefore \quad V = \frac{1}{3}\pi\left\{r^2 + \left(\frac{r}{3}\right)^2 + r \times \frac{r}{3}\right\}h = \frac{1}{3}\pi\left\{r^2 + \frac{r^2}{9} + \frac{r^2}{3}\right\}h = \frac{13}{27}\pi r^2 h.$$

EXAMPLE 20 *A right circular cone is divided by a plane parallel to its base in two equal volumes. In what ratio will the plane divide the axis of the cone?*

SOLUTION Let VAB be a cone of height h and base radius r. Suppose it is cut by a plane parallel to the base of the cone at point O'. Let $O'A' = r_1$ and $VO' = h_1$.

Clearly, $\Delta VO'A' \sim \Delta VOA$

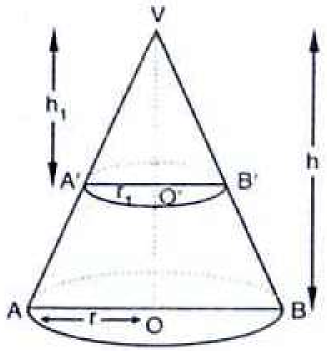

Fig. 14.74

$$\therefore \quad \frac{VO}{VO'} = \frac{OA}{O'A'} \Rightarrow \frac{h}{h_1} = \frac{r}{r_1}$$

It is given that

Volume of cone $VA'B'$ = Volume of the frustum $ABB'A'$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi\left(r^2 + r_1^2 + rr_1\right)(h - h_1)$$

$$\Rightarrow \quad r_1^2 h_1 = (r^2 + r_1^2 + rr_1)(h - h_1)$$

$$\Rightarrow \quad \frac{r_1^2 h_1}{r_1^2 h_1} = \left(\frac{r^2 + r_1^2 + rr_1}{r_1^2}\right)\left(\frac{h - h_1}{h_1}\right) \qquad \text{[Dividing both sides by } r_1^2 h_1]$$

$$\Rightarrow \quad 1 = \left\{\left(\frac{r}{r_1}\right)^2 + 1 + \left(\frac{r}{r_1}\right)\right\}\left(\frac{h}{h_1} - 1\right)$$

$$\Rightarrow \quad 1 = \left\{\left(\frac{h}{h_1}\right)^2 + 1 + \left(\frac{h}{h_1}\right)\right\}\left(\frac{h}{h_1} - 1\right) \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad 1 = \left\{\left(\frac{h}{h_1}\right)^2 + \left(\frac{h}{h_1}\right) + 1\right\}\left(\frac{h}{h_1} - 1\right)$$

$$\Rightarrow \quad 1 = \left(\frac{h}{h_1}\right)^3 - 1^3 \qquad [\because (a^2 + a + 1)(a - 1) = a^3 - 1]$$

$$\Rightarrow \quad \left(\frac{h}{h_1}\right)^3 = 2 \Rightarrow \frac{h}{h_1} = 2^{1/3}$$

Hence, required ratio $= \dfrac{h_1}{h - h_1} = \dfrac{1}{\left(\dfrac{h}{h_1} - 1\right)} = \dfrac{1}{2^{1/3} - 1}$

ALITER We have,

Volume of cone $VA'B'$ = Volume of frustum $ABB'A'$

$\Rightarrow$ Volume of cone $VA'B' = \frac{1}{2}$(Volume of cone VAB)

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{2} \times \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad r_1^2 h_1 = \frac{1}{2} r^2 h^2$$

$$\Rightarrow \quad \left(\frac{r_1}{r}\right)^2 \left(\frac{h_1}{h}\right) = \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{h_1}{h}\right)^2 \left(\frac{h_1}{h}\right) = \frac{1}{2} \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad \frac{h_1}{h} = \left(\frac{1}{2}\right)^{1/3} \Rightarrow \frac{h}{h_1} = 2^{1/3}$$

Hence, required ratio $= \dfrac{h_1}{h - h_1} = \dfrac{1}{\dfrac{h}{h_1} - 1} = \dfrac{1}{2^{1/3} - 1}$

EXERCISE 14.3

LEVEL-1

1. A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of ₹ 1.20 per dm^2. (Use $\pi = 3.14$)
2. A frustum of a right circular cone has a diameter of base 20 cm, of top 12 cm, and height 3 cm. Find the area of its whole surface and volume.
3. The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.
4. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.
5. If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket. (Use $\pi = 22/7$). [CBSE 2000]
6. The height of a cone is 20 cm. A small cone is cut off from the top by a plane parallel to the base. If its volume be $1/125$ of the volume of the original cone, determine at what height above the base the section is made.
7. If the radii of the circular ends of a bucket 24 cm high are 5 cm and 15 cm respectively, find the surface area of the bucket.

8. The radii of the circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 12 cm. Find the total surface area and the volume of the frustum.

9. A tent consists of a frustum of a cone capped by a cone. If the radii of the ends of the frustum be 13 m and 7 m, the height of the frustum be 8 m and the slant height of the conical cap be 12 m, find the canvas required for the tent. (Take : $\pi = 22/7$)

10. A milk container of height 16 cm is made of metal sheet in the form of a frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of ₹ 44 per litre which the container can hold. [NCERT EXEMPLAR]

11. A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the bucket. Also, find the cost of milk which can completely fill the cointainer, at the rate of ₹ 25 per litre. (Use $\pi = 3.14$) [NCERT EXEMPLAR]

12. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. (Use $\pi = 3.14$). [CBSE 2006C, 2016]

13. A bucket made of aluminium sheet is of height 20 cm and its upper and lower ends are of radius 25 cm and 10 cm respectively. Find the cost of making the bucket if the aluminium sheet costs ₹ 70 per 100 cm^2. (Use $\pi = 3.14$). [CBSE 2006C]

14. The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area. [CBSE 2005]

15. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also, find the cost of the bucket if the cost of metal sheet used is ₹ 20 per 100 cm^2. (Use $\pi = 3.14$) [CBSE 2008, 2013]

16. A solid is in the shape of a frustum of a cone. The diameters of the two circular ends are 60 cm and 36 cm and the leight is 9 cm. Find the area of its whole surface and the volume. [CBSE 2010]

17. A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7}\text{ cm}^3$ The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of ₹ 1.40 per cm^2. (Use $\pi = 22.7$) [CBSE 2010]

18. A solid cone of base radius 10 cm is cut into two parts through the mid-point of its height, by a plane parallel to its base. Find the ratio in the volumes of two parts of the cone. [CBSE 2013, 2017]

19. A bucket open at the top, and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of ₹ 10 per 100 cm^2. (Use $\pi = 3.14$). [CBSE 2013]

20. In Fig. 14.75, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. (Use $\pi = 22/7$ and $\sqrt{5} = 2.236$). [CBSE 2015]

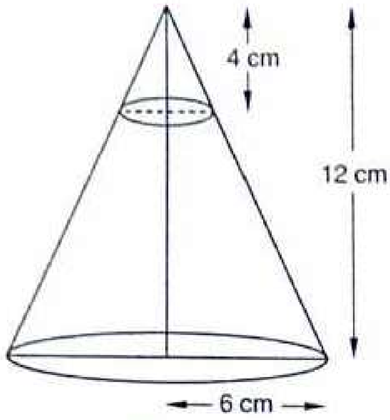

Fig. 14.75

21. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of two parts.

[CBSE 2017]

22. A bucket, made of metal sheet, is in the form of a cone whose height is 35 cm and radii of circular ends are 30 cm and 12 cm. How many litres of milk it contains if it is full to the brim? If the milk is sold at ₹ 40 per litre, find the amount received by the person.

[CBSE 2017]

23. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm,
    (i) Find the area of the metal sheet used to make the bucket.
    (ii) Why we should avoid the bucket made by ordinary plastic? (use $\pi = 3.14$)

[CBSE 2018]

LEVEL-2

24. A reservoir in the form of the frustum of a right circular cone contains 44×10^7 litres of water which fills it completely. The radii of the bottom and top of the reservoir are 50 metres and 100 metres respectively. Find the depth of water and the lateral surface area of the reservoir. (Take: $\pi = 22/7$)

ANSWERS

1. 8800 cm^3, ₹ 21.40
2. 678.85 cm^2, 616 cm^3
3. 48 cm^2
4. 1900 cm^3, 619.65 cm^2, 860.275 cm^2
5. 48510 cm^3
6. 16 cm
7. 545π cm^2
8. 378π cm^2, 756π cm^3
9. 892.57 m^2
10. ₹ 460.24
11. Capacity = 21.980 litres, Surface area = 3292.6 cm^2, Cost of milk = ₹ 549.50
12. Height = 15 cm, Area = 2160.32 cm^2
13. ₹ 2143.05
14. 7599.42 cm^2
15. 10449.92 cm^2, ₹ 2089.98
16. 1944π cm^2, 5292π cm^3
17. ₹ 4224
18. 1 : 7
19. 171.13
20. 350.59 cm^2
21. 1 : 7
22. 51.48 litres, ₹ 2059.20
23. (i) 1711.30 cm^3
24. 24 m, 26145.9 m^2

REVISION EXERCISE

1. A metallic sphere 1 dm in diameter is beaten into a circular sheet of uniform thickness equal to 1 mm. Find the radius of the sheet.
2. Three solid spheres of radii 3, 4 and 5 cm respectively are melted and converted into a single solid sphere. Find the radius of this sphere.

3. A spherical shell of lead, whose external diameter is 18 cm, is melted and recast into a right circular cylinder, whose height is 8 cm and diameter 12 cm. Determine the internal diameter of the shell.
4. A well with 10 m inside diameter is dug 8.4 m deep. Earth taken out of it is spread all around it to a width of 7.5 m to form an embankment. Find the height of the embankment.
5. In the middle of a rectangular field measuring $30 \text{ m} \times 20 \text{ m}$, a well of 7 m diameter and 10 m depth is dug. The earth so removed is evenly spread over the remaining part of the field. Find the height through which the level of the field is raised.
6. The inner and outer radii of a hollow cylinder are 15 cm and 20 cm, respectively. The cylinder is melted and recast into a solid cylinder of the same height. Find the radius of the base of new cylinder.
7. Two cylindrical vessels are filled with oil. Their radii are 15 cm, 12 cm and heights 20 cm, 16 cm respectively. Find the radius of a cylindrical vessel 21 cm in height, which will just contain the oil of the two given vessels.
8. A cylindrical bucket 28 cm in diameter and 72 cm high is full of water. The water is emptied into a rectangular tank 66 cm long and 28 cm wide. Find the height of the water level in the tank.
9. A cubic cm of gold is drawn into a wire 0.1 mm in diameter, find the length of the wire.
10. A well of diameter 3 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.
11. A conical vessel whose internal radius is 10 cm and height 48 cm is full of water. Find the volume of water. If this water is poured into a cylindrical vessel with internal radius 20 cm, find the height to which the water level rises in it.
12. The vertical height of a conical tent is 42 dm and the diameter of its base is 5.4 m. Find the number of persons it can accommodate if each person is to be allowed 29.16 cubic dm.
13. A right circular cylinder and a right circular cone have equal bases and equal heights. If their curved surfaces are in the ratio 8 : 5, determine the ratio of the radius of the base to the height of either of them.
14. A sphere of diameter 5 cm is dropped into a cylindrical vessel partly filled with water. The diameter of the base of the vessel is 10 cm. If the sphere is completely submerged, by how much will the level of water rise?
15. A spherical ball of iron has been melted and made into smaller balls. If the radius of each smaller ball is one-fourth of the radius of the original one, how many such balls can be made?
16. Find the depth of a cylindrical tank of radius 28 m, if its capacity is equal to that of a rectangular tank of size $28 \text{ m} \times 16 \text{ m} \times 11 \text{ m}$.
17. A hemispherical bowl of internal radius 15 cm contains a liquid. The liquid is to be filled into cylindrical-shaped bottles of diameter 5 cm and height 6 cm. How many bottles are necessary to empty the bowl? [CBSE 2001 C]
18. In a cylindrical vessel of diameter 24 cm, filled up with sufficient quantity of water, a solid spherical ball of radius 6 cm is completely immersed. Find the increase in height of water level.
19. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

20. A solid metallic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3 cm. Find the number of cones so formed.
CBSE 2004]
21. The diameter of a copper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.
22. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.
23. A metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained. [CBSE 2004]
24. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1 : 2 : 3.
25. A hollow sphere of internal and external diameters 4 and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.
26. The largest sphere is carved out of a cube of side 10.5 cm. Find the volume of the sphere.
27. Find the weight of a hollow sphere of metal having internal and external diameters as 20 cm and 22 cm, respectively if 1 m^3 of metal weighs 21 g.
28. A solid sphere of radius 'r' is melted and recast into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm, its height 24 cm and thickness 2 cm, find the value of 'r'.
29. Lead spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and water rises by 40 cm. find the number of lead spheres dropped in the water.
30. The height of a solid cylinder is 15 cm and the diameter of its base is 7 cm. Two equal conical holes each of radius 3 cm and height 4 cm are cut off. Find the volume of the remaining solid.
31. A solid is composed of a cylinder with hemispherical ends. If the length of the whole solid is 108 cm. and the diameter of the cylinder is 36 cm, find the cost of polishing the surface at the rate of 7 paise per cm^2. [Use $\pi = 3.1416$]
32. The surface area of a sphere is the same as the curved surface area of a cone having the radius of the base as 120 cm and height 160 cm. Find the radius of the sphere.
33. A right circular cylinder and a right circular cone have equal bases and equal heights. If their curved surfaces are in the ratio 8 : 5, determine the ratio of the radius of the base to the height of either of them.
34. A rectangular vessel of dimensions 20 cm × 16 cm × 11 cm. is full of water. This water is poured into a conical vessel. The top of the conical vessel has its radius 10 cm. If the conical vessel is filled completely, determine its height. [Use $\pi = 22/7$]
35. If r_1 and r_2 be the radii of two solid metallic spheres and if they are melted into one solid sphere, prove that the radius of the new sphere is $(r_1^3 + r_2^3)^{1/3}$.
36. A solid metal sphere of 6 cm diameter is melted and a circular sheet of thickness 1 cm is prepared Determine the diameter of the sheet.
37. A hemispherical tank full of water is emptied by a pipe at the rate of $\frac{25}{7}$ litres per second. How much time will it take to half-empty the tank, if the tank is 3 metres in diameter?

38. Find the number of coins, 1.5 cm is diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

39. The radius of the base of a right circular cone of semi-vertical angle α is r. Show that its volume is $\frac{1}{3}\pi r^3 \cot \alpha$ and curved surface area is $\pi r^2 \operatorname{cosec} \alpha$.

40. An iron pillar consists of a cylindrical portion 2.8 m high and 20 cm in diameter and a cone 42 cm high is surmounting it. Find the weight of the pillar, given that 1 cubic cm of iron weighs 7.5 gm.

41. A circus tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 5 m wide to make the required tent.

42. Height of a solid cylinder is 10 cm and diameter 8 cm. Two equal conical hole have been made from its both ends. If the diameter of the holes is 6 cm and height 4 cm, find (i) volume of the cylinder, (ii) volume of one conical hole, (iii) volume of the remaining solid.

43. The height of a solid cylinder is 15 cm. and the diameter of its base is 7 cm. Two equal conical holes each of radius 3 cm, and height 4 cm are cut off. Find the volume of the remaining solid.

44. A solid is composed of a cylinder with hemispherical ends. If the length of the whole solid is 108 cm and the diameter of the cylinder is 36 cm, find the cost of polishing the surface at the rate of 7 paise per cm^2. (Use $\pi = 3.1416$)

45. The largest sphere is to be curved out of a right circular cylinder of radius 7 cm. and height 14 cm. Find the volume of the sphere.

46. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the base of the cylinder or the cone is 24 m. The height of the cylinder is 11 m. If the vertex of the cone is 16 m above the ground, find the area of the canvas required for making the tent. (Use $\pi = 22/7$)

47. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm find the total surface area and volume of the toy.
[CBSE 2000, 2002]

48. A cylindrical container is filled with ice-cream, whose diameter is 12 cm and height is 15 cm. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, find the diameter of the ice-cream.

49. Find the volume of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 2.7 m and the diameter of each hemispherical end is 0.7 m.

50. A tent of height 8.25 m is in the form of a right circular cylinder with diameter of base 30 m and height 5.5 m, surmounted by a right circular cone of the same base. Find the cost of the canvas of the tent at the rate of ₹ 45 per m^2.

51. An iron pole consisting of a cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that 1 cm^3 of iron has 8 gram mass approximately. (Use: $\pi = 355/115$)

52. The interior of a building is in the form of a cylinder of base radius 12 m and height 3.5 m, surmounted by a cone of equal base and slant height 12.5 m. Find the internal curved surface area and the capacity of the building.

53. A right angled triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of the double cone thus generated.

54. A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. (Use $\pi = 3.14$)

55. Find the mass of a 3.5 m long lead pipe, if the external diameter of the pipe is 2.4 cm, thickness of the metal is 2 mm and the mass of 1 cm^3 of lead is 11.4 grams.

56. A solid is in the form of a cylinder with hemispherical ends. Total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and total surface area of the solid.

57. A golf ball has diameter equal to 4.2 cm. Its surface has 200 dimples each of radius 2 mm. Calculate the total surface area which is exposed to the surroundings assuming that the dimples are hemispherical.

58. The radii of the ends of a bucket of height 24 cm are 15 cm and 5 cm. Find its capacity. (Take $\pi = 22/7$).

59. The radii of the ends of a bucket 30 cm high are 21 cm and 7 cm. Find its capacity in litres and the amount of sheet required to make this bucket.

60. The radii of the ends of a frustum of a right circular cone are 5 metres and 8 metres and its lateral height is 5 metres. Find the lateral surface and volume of the frustum.

61. A frustum of a cone is 9 cm thick and the diameters of its circular ends are 28 cm and 4 cm. Find the volume and lateral surface area of the frustum. (Take $\pi = 22/7$).

62. A bucket is in the form of a frustum of a cone and holds 15.25 litres of water. The diameters of the top and bottom are 25 cm and 20 cm respectively. Find its height and area of tin used in its construction.

63. If a cone of radius 10 cm is divided into two parts by drawing a plane through the mid-point of its axis, parallel to its base. Compare the volumes of the two parts. **[CBSE 2000 C]**

64. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 metres.

65. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height be 22 cm, the diameter of the cylindrical portion 8 cm and the diameter of the top of the funnel 18 cm, find the area of the tin required. (Use : $\pi = 22/7$). **[NCERT]**

66. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys with the shape of a right circular cone mounted on a hemisphere of radius 3 cm. If the height of the toy is 12 cm, find the number of toys so formed. **[CBSE 2006 C]**

67. A container open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹ 21 per litre. (Use $\pi = 22/7$) **[CBSE 2014]**

68. A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the cone and of the remaining solid left out after the cone carved out.

[NCERT EXEMPLAR]

69. A cone of radius 4 cm is divided into two parts by drawing a plane through the mid point of its axis and parallel to its base. Compare the volumes of two parts.

[NCERT EXEMPLAR]

70. A wall 24 m, 0.4 m thick and 6 m high is constructed with the bricks each of dimensions 25 cm × 16 cm × 10 cm. If the mortar occupies $\frac{1}{10}$th of the volume of the wall, then find the number of bricks used in constructing the wall.

[NCERT EXEMPLAR]

71. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

[NCERT EXEMPLAR]

72. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker of diameter 7 cm containing some water. Find the number of marbles that should be dropped into the beaker so that the water level rises by 5.6 cm. **[NCERT EXEMPLAR]**

73. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape formed. **[NCERT EXEMPLAR]**

74. From a solid cube of side 7 cm, a conical cavity of height 7 cm and radius 3 cm is hollowed out. Find the volume of the remaining solid. **[NCERT EXEMPLAR]**

75. Two solid cones A and B are placed in a cylindrical tube as shown in Fig. 14.76. The ratio of their capacities are 2 : 1. Find the heights and capacities of the cones. Also, find the volume of the remaining portion of the cylinder. **[NCERT EXEMPLAR]**

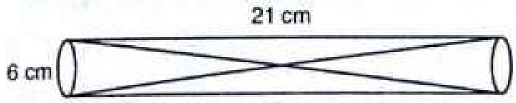

Fig. 14.76

76. An icecream cone full of icecream having radius 5 cm and height 10 cm as shown in Fig. 14.77. Calculate the volume of icecream, provided that its 1/6 part is left unfilled with icecream. **[NCERT EXEMPLAR]**

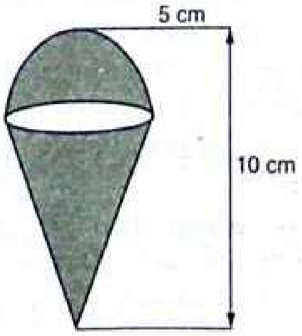

Fig. 14.77

ANSWERS

1. 4.08 cm	2. 6 cm	3. $6(19)^{1/3}$	4. 1.6 cm
5. 68.6 cm	6. 13.2 cm	7. 18 cm	8. 24 cm
9. 127.3 m	10. 1.125 m	11. 5024 cm^3, 4 cm	12. 11
13. 3 : 4	14. 5/6 cm	15. 64	16. 2 m
17. 60	18. 2 cm	19. 3.74 cm	20. 126
21. 0.6 cm	22. 3.74 cm	23. 126	25. 14 cm
26. 606.375	27. 29.13 kg	28. 6 cm	29. 90
30. 502.1 cm^3	31. ₹ 855. 02	32. 77.46 cm	33. 3 : 4
34. 33.6 cm	36. 12 cm	37. 16.5 minutes	38. 450
40. 693 kg	41. 1947 m	42. 160π cm^3, 12π cm^3, 136π cm^3	
43. 502.1 cm^3	44. ₹ 855. 02	45. 1437	46. 1320 m^2
47. 214.5 cm^2, 243.83 cm^3		48. 6 cm	49. 0.95 m (appr.)
50. ₹ 55687.50	51. 102.24 kg	52. 735.43 m^2, 2112 m^3	
53. $30\frac{6}{35}$ cm^3	54. 103.62 cm^2	55. 5.518 kg	
56. 641.67 cm^3, 418 cm^2		57. 80.58 cm^2	58. 8171.42 cm^2
59. 20.02 litres, 3069 cm^2		60. 204.28 m^2, 540.56 cm^3	
61. 684π cm^3, 240π cm^2		62. 38.18 cm, 3017 cm^2	
63. 1 : 7	64. ₹ 2068	65. 249π cm^2	66. 12
67. 329.47	68. $154(\sqrt{5}+1)$ cm^2, $(1022+154\sqrt{5})$ cm^2		69. 1 : 7
70. 12960	71. 15 cm	72. 150	73. 855 cm^2 (Approx.)
74. 277 cm^3	75. 14 cm, 7 cm, 132 cm^3, 66 cm^3, 396 cm^3		76. 327.4 cm^2

VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

1. The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. What is the ratio of their volumes?
2. If the heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4, what is the ratio of their volumes?
3. If a cone and a sphere have equal radii and equal volumes. What is the ratio of the diameter of the sphere to the height of the cone?
4. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?
5. The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio 2 : 3. What is the ratio of their curved surface areas?
6. Two cubes have their volumes in the ratio 1 : 27. What is the ratio of their surface areas?
7. Two right circular cylinders of equal volumes have their heights in the ratio 1 : 2. What is the ratio of their radii?

8. If the volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5, then write the ratio of their weights.
9. A sphere and a cube have equal surface areas. What is the ratio of the volume of the sphere to that of the cube?
10. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?
11. What is the ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height?
12. A sphere of maximum volume is cut-out from a solid hemisphere of radius r. What is the ratio of the volume of the hemisphere to that of the cut-out sphere?
13. A metallic hemisphere is melted and recast in the shape of a cone with the same base radius R as that of the hemisphere. If H is the height of the cone, then write the value of H/R.
14. A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and height are in the ratio 5 : 12, write the ratio of the total surface area of the cylinder to that of the cone.
15. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio of their volumes?
16. The radii of two cones are in the ratio 2 : 1 and their volumes are equal. What is the ratio of their heights?
17. Two cones have their heights in the ratio 1 : 3 and radii 3 : 1. What is the ratio of their volumes?
18. A hemisphere and a cone have equal bases. If their heights are also equal, then what is the ratio of their curved surfaces?
19. If r_1 and r_2 denote the radii of the circular bases of the frustum of a cone such that $r_1 > r_2$, then write the ratio of the height of the cone of which the frustum is a part to the height of the frustum.
20. If the slant height of the frustum of a cone is 6 cm and the perimeters of its circular bases are 24 cm and 12 cm respectively. What is the curved surface area of the frustum?
21. If the areas of circular bases of a frustum of a cone are $4\ \text{cm}^2$ and $9\ \text{cm}^2$ respectively and the height of the frustum is 12 cm. What is the volume of the frustum?
22. The surface area of a sphere is $616\ \text{cm}^2$. Find its radius. **[CBSE 2008]**
23. A cylinder and a cone are of the same base radius and of same height. Find the ratio of the value of the cylinder to that of the cone **[CBSE 2009]**
24. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum. **[CBSE 2010]**
25. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? **[CBSE 2017]**

ANSWERS

1. 9 : 8	2. 9 : 32	3. 1 : 2	4. 1 : 2 : 3	5. 2 : 5	6. 1 : 9
7. $\sqrt{2} : 1$	8. 25 : 64	9. $\sqrt{\frac{6}{\pi}}$	10. $6 : \pi$	11. 3 : 1 : 2	12. 4 : 1
13. 2	14. 17 : 9	15. 3 : 1 : 2	16. 1 : 4	17. 3 : 1	18. $\sqrt{2} : 1$
19. $\frac{r_1}{r_1 - r_2}$	20. $108\ \text{cm}^2$	21. $44\ \text{cm}^2$	22. 7 cm	23. 3 : 1	24. 3 cm
25. 9 units					

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is
   (a) 12 m (b) 18 m (c) 36 m (d) 66 m
2. A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. The number of such cones is
   (a) 63 (b) 126 (c) 21 (d) 130
3. A solid is hemispherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of its radius and the height of its conical part is
   (a) $1 : 3$ (b) $1 : \sqrt{3}$ (c) $1 : 1$ (d) $\sqrt{3} : 1$
4. A solid sphere of radius r is melted and cast into the shape of a solid cone of height r, the radius of the base of the cone is
   (a) $2r$ (b) $3r$ (c) r (d) $4r$
5. The material of a cone is converted into the shape of a cylinder of equal radius. If height of the cylinder is 5 cm, then height of the cone is
   (a) 10 cm (b) 15 cm (c) 18 cm (d) 24 cm
6. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m, the total area of the canvas required in m^2 is
   (a) 1760 (b) 2640 (c) 3960 (d) 7920
7. The number of solid spheres, each of diameter 6 cm that could be moulded to form a solid metal cylinder of height 45 cm and diameter 4 cm, is
   (a) 3 (b) 4 (c) 5 (d) 6
8. A sphere of radius 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 8 cm. If the sphere is submerged completely, then the surface of the water rises by
   (a) 4.5 cm (b) 3 cm (c) 4 cm (d) 2 cm
9. If the radii of the circular ends of a bucket of height 40 cm are of lengths 35 cm and 14 cm, then the volume of the bucket in cubic centimeters, is
   (a) 60060 (b) 80080 (c) 70040 (d) 80160
10. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is
    (a) $1 : 2$ (b) $1 : 4$ (c) $1 : 6$ (d) $1 : 8$
11. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, then the height above the base at which the section has been made, is
    (a) 10 cm (b) 15 cm (c) 20 cm (d) 25 cm
12. A solid consists of a circular cylinder with an exact fitting right circular cone placed at the top. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, then the height of the circular cylinder is
    (a) $2h$ (b) $\frac{2h}{3}$ (c) $\frac{3h}{2}$ (d) $4h$

13. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. If it is 6 m deep, then its capacity is
(a) 176 m^3 (b) 196 m^3 (c) 200 m^3 (d) 110 m^3

14. Water flows at the rate of 10 metre per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 24 cm?
(a) 48 minutes 15 sec (b) 51 minutes 12 sec
(c) 52 minutes 1 sec (d) 55 minutes

15. A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, the radius of its base is
(a) 12 cm (b) 24 cm (c) 36 cm (d) 48 cm

16. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is
(a) $60\pi \text{ cm}^2$ (b) $68\pi \text{ cm}^2$ (c) $120\pi \text{ cm}^2$ (d) $136\pi \text{ cm}^2$

17. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. The volume of the cone so formed is
(a) $12\pi \text{ cm}^3$ (b) $15\pi \text{ cm}^3$ (c) $16\pi \text{ cm}^3$ (d) $20\pi \text{ cm}^3$

18. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. The ratio of its diameter to its height is
(a) 3 : 7 (b) 7 : 3 (c) 6 : 7 (d) 7 : 6

19. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. The radius of the cone is
(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

20. The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is
(a) 1 : 2 (b) 2 : 3 (c) 9 : 16 (d) 16 : 9

21. If three metallic spheres of radii 6 cm, 8 cm and 10 cm are melted to form a single sphere, the diameter of the sphere is
(a) 12 cm (b) 24 cm (c) 30 cm (d) 36 cm

22. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. The radius of the sphere is
(a) 3 cm (b) 4 cm (c) 6 cm (d) 12 cm

23. The volume of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is
(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

24. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by
(a) $\frac{2}{9}$ cm (b) $\frac{4}{9}$ cm (c) $\frac{9}{4}$ cm (d) $\frac{9}{2}$ cm

25. 12 spheres of the same size are made from melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is

(a) $\sqrt{3}$ cm (b) 2 cm (c) 3 cm (d) 4 cm

26. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is

(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm

27. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. The height of the cone is

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

28. A solid piece of iron of dimensions $49 \times 33 \times 24$ cm is moulded into a sphere. The radius of the sphere is

(a) 21 cm (b) 28 cm (c) 35 cm (d) None of these

29. The ratio of lateral surface area to the total surface area of a cylinder with base diameter 1.6 m and height 20 cm is

(a) 1 : 7 (b) 1 : 5 (c) 7 : 1 (d) 5 : 1

30. A solid consists of a circular cylinder surmounted by a right circular cone. The height of the cone is h. If the total height of the solid is 3 times the volume of the cone, then the height of the cylinder is

(a) $2h$ (b) $\frac{3h}{2}$ (c) $\frac{h}{2}$ (d) $\frac{2h}{3}$

31. The maximum volume of a cone that can be carved out of a solid hemisphere of radius r is

(a) $3\pi r^2$ (b) $\frac{\pi r^3}{3}$ (c) $\frac{\pi r^2}{3}$ (d) $3\pi r^3$

32. The radii of two cylinders are in the ratio 3 : 5. If their heights are in the ratio 2 : 3, then the ratio of their curved surface areas is

(a) 2 : 5 (b) 5 : 2 (c) 2 : 3 (d) 3 : 5

33. A right circular cylinder of radius r and height h ($h = 2r$) just encloses a sphere of diameter

(a) h (b) r (c) $2r$ (d) $2h$

34. The radii of the circular ends of a frustum are 6 cm and 14 cm. If its slant height is 10 cm, then its vertical height is

(a) 6 cm (b) 8 cm (c) 4 cm (d) 7 cm

35. The height and radius of the cone of which the frustum is a part are h_1 and r_1 respectively. If h_2 and r_2 are the heights and radius of the smaller base of the frustum respectively and $h_2 : h_1 = 1 : 2$, then $r_2 : r_1$ is equql to

(a) 1 : 3 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

36. The diameters of the ends of a frustum of a cone are 32 cm and 20 cm. If its slant height is 10 cm, then its lateral surface area is

(a) 321π cm^2 (b) 300π cm^2 (c) 260π cm^2 (d) 250π cm^2

37. A solid frustum is of height 8 cm. If the radii of its lower and upper ends are 3 cm and 9 cm respectively, then its slant height is

(a) 15 cm (b) 12 cm (c) 10 cm (d) 17 cm

38. The radii of the ends of a bucket 16 cm high are 20 cm and 8 cm. The curved surface area of the bucket is
(a) 1760 cm^2 (b) 2240 cm^2 (c) 880 cm^2 (d) 3120 cm^2

39. The diameters of the top and the bottom portions of a bucket are 42 cm and 28 cm respectively. If the height of the bucket is 24 cm, then the cost of painting its outer surface at the rate of 50 paise/cm^2 is
(a) ₹ 1582.50 (b) ₹ 1724.50 (c) ₹ 1683 (d) ₹ 1642

40. If four times the sum of the areas of two circular faces of a cylinder of height 8 cm is equal to twice the curve surface area, then diameter of the cylinder is
(a) 4 cm (b) 8 cm (c) 2 cm (d) 6 cm

41. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1 [CBSE 2012

42. A metalic solid cone is melted to form a solid cylinder of equal radius. If the height of the cylinder is 6 cm, then the height of the cone was
(a) 10 cm (b) 12 cm (c) 18 cm (d) 24 cm [CBSE 2014]

43. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is
(a) 3.5 (b) 7 (c) 80/7 (d) 5 [CBSE 2014]

44. The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm is
(a) 3 (b) 5 (c) 4 (d) 6 [CBSE 2014]

45. Volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

46. A right circular cylinder of radius r and height h ($h > 2r$) just encloses a sphere of diameter
(a) r (b) $2r$ (c) h (d) $2h$

47. In a right circular cone, the cross-section made by a plane parallel to the base is a
(a) circle (b) frustyum of a cone (c) sphere (d) hemisphare

48. If two solid-hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(a) $4\pi r^2$ (b) $6\pi r^2$ (c) $3\pi r^2$ (d) $8\pi r^2$

49. The diameters of two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35 cm. The capacity of the bucket is
(a) 32.7 litres (b) 33.7 litres (c) 34.7 litres (d) 31.7 litres

50. A spherical ball of radius r is melted to make 8 new identical balls each of radius r_1. Then $r : r_1 =$
(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

ANSWERS

1. (c)	2. (b)	3. (b)	4. (a)	5. (b)	6. (d)
7. (c)	8. (a)	9. (b)	10. (d)	11. (c)	12. (b)
13. (a)	14. (b)	15. (c)	16. (d)	17. (a)	18. (b)

(i) Volume of the frustum $= \frac{\pi}{3}\left(r_1^2 + r_1r_2 + r_2^2\right)h$

(ii) Lateral surface area $= \pi(r_1 + r_2)l$

(iii) Total surface area $= \pi\{(r_1 + r_2)l + r_1^2 + r_2^2\}$

(iv) Slant height of the frustum $= \sqrt{h^2 + (r_1 - r_2)^2}$

(v) Height of the cone of which the frustum is a part $= \dfrac{hr_1}{r_1 - r_2}$

(vi) Slant height of the cone of which the frustum is a part $= \dfrac{lr_1}{r_1 - r_2}$

(vii) Volume of the frustum $= \frac{h}{3}\left\{A_1 + A_2 + \sqrt{A_1A_2}\right\}$, where A_1 and A_2 denote the areas of circular bases of the frustum.

19. (d)	20. (d)	21. (b)	22. (c)	23. (a)	24. (c)
25. (d)	26. (b)	27. (b)	28. (a)	29. (b)	30. (d)
31. (b)	32. (a)	33. (c)	34. (a)	35. (b)	36. (c)
37. (c)	38. (a)	39. (c)	40. (b)	41. (d)	42. (c)
43. (a)	44. (b)	45. (d)	46. (b)	47. (a)	48. (a)
49. (a)	50. (a)				

SUMMARY

1. If l, b and h denote respectively the length, breadth and height of a cuboid, then
   (i) Total surface area of the cuboid $= 2(lb + bh + lh)$ square units.
   (ii) Volume of the cuboid $=$ Area of the base $\times$ height $= lbh$ cubic units.
   (iii) Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$ units.
   (iv) Area of four walls of a room $= 2(l + b)\,h$ sq. units.
2. If the length of each edge of a cube is 'a' units, then
   (i) Total surface area of the cube $= 6\,a^2$ sq. units
   (ii) Volume of the cube $= a^3$ cubic units
   (iii) Diagonal of the cube $= \sqrt{3}\,a$ units.
3. If r and h denote respectively the radius of the base and height of a right circular cylinder, then
   (i) Area of each end $= \pi r^2$
   (ii) Curved surface area $= 2\pi rh$
   (iii) Total surface area $= 2\pi r\,(h + r)$ sq. units
   (iv) Volume $= \pi r^2 h =$ Area of the base $\times$ height
4. If R and r denote respectively the external and internal radii of a hollow right circular cylinder, then
   (i) Area of each end $= \pi\,(R^2 - r^2)$
   (ii) Curved surface area of hollow cylinder $= 2\pi\,(R + r)\,h$
   (iii) Total surface area $= 2\pi\,(R + r)\,(R + h - r)$
   (iv) Volume of material $= \pi h\,(R^2 - r^2)$
5. If r, h and l denote respectively the radius of base, height and slant height of a right circular cone, then
   (i) $l^2 = r^2 + h^2$
   (ii) Curved surface area $= \pi r l$
   (iii) Total surface area $= \pi r^2 + \pi r l$
   (iv) Volume $= \dfrac{1}{3}\pi r^2 h$
6. For a sphere of radius r, we have
   (i) Surface area $= 4\pi r^2$
   (ii) Volume $= \dfrac{4}{3}\pi r^3$
7. If h is the height, l the slant height and r_1 and r_2 the radii of the circular bases of a frustum of a cone, then