

15.1 INTRODUCTION

In class IX, we have learnt about representation of statistical data in the form of histograms and frequency polygons. We have also learnt about mean, median and mode of ungrouped data. In this chapter, we will study about the techniques for finding mean, median and mode of grouped data. We will also learn about cumulative frequency graph of a frequency distribution.

15.2 MEAN OF GROUPED DATA

If $x_1, x_2, x_3, \dots, x_n$ are n values of a variable X , then the arithmetic mean or simply mean of these values is denoted by \bar{X} and is defined as

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \text{ or, } \bar{X} = \frac{\sum_{i=1}^n x_i}{n}$$

Here, the symbol $\sum_{i=1}^n x_i$ denotes the sum $x_1 + x_2 + x_3 + \dots + x_n$.

In other words, we can say that the arithmetic mean of a set of observations is equal to their sum divided by the total number of observations.

In this section, we will study about the arithmetic mean of grouped data or a discrete frequency distribution. In a discrete frequency distribution the arithmetic mean may be computed by any one of the following methods:

- (i) Direct method,
- (ii) Short-cut method,
- (iii) Step-Deviation method.

Let us now learn about these methods one by one.

15.2.1 DIRECT METHOD

If a variate X takes values x_1, x_2, \dots, x_n with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively, then arithmetic mean of these values is given by

$$\bar{X} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n}$$

$$\text{or, } \bar{X} = \frac{\sum_{i=1}^n f_i x_i}{N}, \text{ where } N = \sum_{i=1}^n f_i = f_1 + f_2 + \dots + f_n$$

The following algorithm may be used to compute arithmetic mean by direct method.

ALGORITHM

STEP I Prepare the frequency table in such a way that its first column consists of the values of the variate and the second column the corresponding frequencies.

STEP II Multiply the frequency of each row with the corresponding values of variable to obtain third column containing $f_i x_i$.

STEP III Find the sum of all entries in column III to obtain $\sum f_i x_i$

STEP IV Find the sum of all the frequencies in column II to obtain $\sum f_i = N$

STEP V Use the formula: $\bar{X} = \frac{\sum f_i x_i}{N}$

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 Find the mean of the following distribution:

$x:$	4	6	9	10	15
$f:$	5	10	10	7	8

SOLUTION Calculation of Arithmetic Mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
$N = \sum f_i = 40$		$\sum f_i x_i = 360$

$$\therefore \text{Mean} = \bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{360}{40} = 9$$

EXAMPLE 2 Following table shows the weight of 12 students:

Weight (in kgs):	67	70	72	73	75
Number of students:	4	3	2	2	1

Find the mean weight of the students.

SOLUTION

Calculation of Arithmetic Mean

Weight (in kgs)	Frequency	$f_i x_i$
x_i	f_i	
67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
$N = \sum f_i = 12$		$\sum f_i x_i = 843$

$$\therefore \text{Mean} = \bar{X} = \frac{\sum f_i x_i}{N} = \frac{843}{12} = 70.25 \text{ kg}$$

EXAMPLE 3 Find the mean of the following distribution:

$x:$	10	30	50	70	89
$f:$	7	8	10	15	10

SOLUTION

Calculation of Mean

x_i	f_i	$f_i x_i$
10	7	70
30	8	240
50	10	500
70	15	1050
89	10	890
$N = \sum f_i = 50$		$\sum f_i x_i = 2750$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{N} = \frac{2750}{50} = 55$$

EXAMPLE 4 If the mean of the following distribution is 6, find the value of p .

$x:$	2	4	6	10	$p+5$
$f:$	3	2	3	1	2

SOLUTION

Calculation of Mean

x_i	f_i	$f_i x_i$
2	3	6
4	2	8
6	3	18
10	1	10
$p+5$	2	$2p+10$
$N = \sum f_i = 11$		$\sum f_i x_i = 2p+52$

We have, $N = \sum f_i = 11$, $\sum f_i x_i = 2p + 52$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{N}$$

$$\Rightarrow 6 = \frac{2p+52}{11} \Rightarrow 66 = 2p+52 \Rightarrow 2p = 14 \Rightarrow p = 7$$

EXAMPLE 5 Find the value of p , if the mean of the following distribution is 7.5.

$x:$	3	5	7	9	11	13
$f:$	6	8	15	p	8	4

SOLUTION

Calculation of Mean

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	p	$9p$
11	8	88
13	4	52
$N = \sum f_i = 41 + p$		$\sum f_i x_i = 303 + 9p$

We have, $\sum f_i = 41 + p$, $\sum f_i x_i = 303 + 9p$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow 7.5 = \frac{303 + 9p}{41 + p}$$

$$\Rightarrow 7.5 \times (41 + p) = 303 + 9p$$

$$\Rightarrow 307.5 + 7.5p = 303 + 9p \Rightarrow 9p - 7.5p = 307.5 - 303 \Rightarrow 1.5p = 4.5 \Rightarrow p = 3$$

LEVEL-2

EXAMPLE 6 Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 1.46.

Number of accidents (x):	0	1	2	3	4	5	Total
Frequency (f):	46	?	?	25	10	5	200

SOLUTION Let the missing frequencies be f_1 and f_2 .

Computation of Arithmetic Mean

x_i	f_i	$f_i x_i$
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
$N = 86 + f_1 + f_2$		$\sum f_i x_i = 140 + f_1 + 2f_2$

We have,

$$N = 200 \Rightarrow 200 = 86 + f_1 + f_2 \Rightarrow f_1 + f_2 = 114 \quad \dots(i)$$

Also,

$$\text{Mean} = 1.46$$

$$\Rightarrow 1.46 = \frac{\sum f_i x_i}{N}$$

$$\Rightarrow 1.46 = \frac{140 + f_1 + 2f_2}{200}$$

$$\Rightarrow 292 = 140 + f_1 + 2f_2 \Rightarrow f_1 + 2f_2 = 152 \quad \dots(ii)$$

Solving equation (i) and (ii), we get

$$f_1 = 76 \text{ and } f_2 = 38$$

EXERCISE 15.1

LEVEL-1

1. Calculate the mean for the following distribution:

$x:$	5	6	7	8	9
$f:$	4	8	14	11	3

2. Find the mean of the following data:

$x:$	19	21	23	25	27	29	31
$f:$	13	15	16	18	16	15	13

3. If the mean of the following data is 20.6. Find the value of p .

$x:$	10	15	p	25	35
$f:$	3	10	25	7	5

4. If the mean of the following data is 15, find p .

$x:$	5	10	15	20	25
$f:$	6	p	6	10	5

5. Find the value of p for the following distribution whose mean is 16.6.

$x:$	8	12	15	p	20	25	30
$f:$	12	16	20	24	16	8	4

6. Find the missing value of p for the following distribution whose mean is 12.58.

$x:$	5	8	10	12	p	20	25
$f:$	2	5	8	22	7	4	2

7. Find the missing frequency (p) for the following distribution whose mean is 7.68.

$x:$	3	5	7	9	11	13
$f:$	6	8	15	p	8	4

8. The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students

Age (in years):	15	16	17	18	19	20
No. of students:	3	8	10	10	5	4

9. Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	Not available	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

10. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

11. The arithmetic mean of the following data is 14. Find the value of k .

x_i :	5	10	15	20	25
f_i :	7	k	8	4	5.

[CBSE 2002C]

12. The arithmetic mean of the following data is 25, find the value of k .

x_i :	5	15	25	35	45
f_i :	3	k	3	6	2

[CBSE 2001]

13. If the mean of the following data is 18.75. Find the value of p .

x_i :	10	15	p	25	30
f_i :	5	10	7	8	2

[CBSE 2005]

LEVEL-2

14. Find the value of p , if the mean of the following distribution is 20.

x :	15	17	19	$20 + p$	23
f :	2	3	4	$5p$	6

15. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 50.

x :	10	30	50	70	90
f :	17	f_1	32	f_2	19

Total 120.

ANSWERS

- | | | | |
|--------------|-------------|--------------------------|----------------|
| 1. 7.025 | 2. 25 | 3. $p = 20$ | 4. 8 |
| 5. 18 | 6. 15 | 7. 9 | 8. 17.45 years |
| 9. 52 | 10. 2.47 | 11. 6 | 12. 4 |
| 13. $p = 20$ | 14. $p = 1$ | 15. $f_1 = 28, f_2 = 24$ | |

15.2.2 SHORT-CUT METHOD

If the values of x or (and) f are large, the calculation of AM by the direct method is quite tedious and time consuming, because calculations involved are lengthy. In such a case to minimize the time involved in calculation, we take deviations from an arbitrary point as discussed below.

Let x_1, x_2, \dots, x_n be values of a variable X with corresponding frequencies $f_1, f_2, f_3, \dots, f_n$ respectively. Taking deviations about an arbitrary point ' A ', we have

$$d_i = x_i - A, i = 1, 2, 3, \dots, n$$

$$\Rightarrow f_i d_i = f_i (x_i - A); i = 1, 2, 3, \dots, n$$

$$\Rightarrow \sum_{i=1}^n f_i d_i = \sum_{i=1}^n f_i (x_i - A)$$

$$\Rightarrow \sum_{i=1}^n f_i d_i = \sum_{i=1}^n f_i x_i - A \sum_{i=1}^n f_i$$

$$\Rightarrow \sum_{i=1}^n f_i d_i = \sum_{i=1}^n f_i x_i - AN \quad \left[\because N = \sum_{i=1}^n f_i \right]$$

$$\Rightarrow \frac{1}{N} \sum_{i=1}^n f_i d_i = \frac{1}{N} \sum_{i=1}^n f_i x_i - \frac{AN}{N}$$

$$\Rightarrow \frac{1}{N} \sum_{i=1}^n f_i d_i = \bar{X} - A \quad \left[\because \bar{X} = \frac{1}{N} \sum_{i=1}^n f_i x_i \right]$$

$$\Rightarrow \bar{X} = A + \frac{1}{N} \sum_{i=1}^n f_i d_i$$

Finding AM by using the above formula is known as the short-cut method.

NOTE The number ' A ' is generally known as the assumed mean and is generally chosen in such a way that the deviations are small.

Following algorithm may be used to find arithmetic mean by the short-cut method.

ALGORITHM

STEP I Prepare the frequency table in such a way that its first column consists of the values of the variable and the second column consists of the corresponding frequencies.

STEP II Choose a number ' A ' (preferable among the values in first column) and take deviations $d_i = x_i - A$ of the values x_i of variable X about A . Write these deviations against the corresponding frequencies in the third column.

STEP III Multiply the frequencies in column II with the corresponding deviations d_i in column III to prepare column IV consisting of $f_i d_i$.

STEP IV Find the sum of all entries in column III to obtain $\sum_{i=1}^n f_i d_i$ and the sum of all frequencies in column II to obtain $\sum_{i=1}^n f_i = N$.

STEP V Use the formula: $\bar{X} = A + \frac{1}{N} \left(\sum_{i=1}^n f_i d_i \right)$.

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 The following table shows the weights of 12 students:

Weight (in kg):	67	70	72	73	75
Number of students:	4	3	2	2	1

Find the mean weight by using short-cut method.

SOLUTION Let the assumed mean be $A = 72$

Calculation of Mean

weight in Kg x_i	No. of students f_i	$d_i = x_i - A = x_i - 72$	$f_i d_i$
67	4	-5	-20
70	3	-2	-6
72	2	0	2
73	2	1	2
75	1	3	3
$N = \sum f_i = 12$			$\sum f_i d_i = -21$

We have,

$$N = 12, \sum f_i d_i = -21, \text{ and } A = 72$$

$$\therefore \text{Mean} = A + \frac{1}{N} (\sum f_i d_i)$$

$$\Rightarrow \text{Mean} = 72 + \frac{(-21)}{12} = 72 - \frac{7}{4}$$

$$\Rightarrow \text{Mean} = \frac{288 - 7}{4} = \frac{281}{4} = 70.25 \text{ kg}$$

Hence, mean weight = 70.25 kg.

EXAMPLE 2 Find the mean wage from the data given below:

Wage (in ₹):	800	820	860	900	920	980	1000
No. of workers:	7	14	19	25	20	10	5

SOLUTION Let the assumed mean be $A = 900$.

Calculation of Mean

Wage (in ₹)	No. of workers	$d_i = x_i - A = x_i - 900$	$f_i d_i$
x_i	f_i		
800	7	- 100	- 700
820	14	- 80	- 1120
860	19	- 40	- 760
900	25	0	0
920	20	20	400
980	10	80	800
1000	5	100	500
$N = \sum f_i = 100$			$\sum f_i d_i = -880$

We have,

$$N = 100, \sum f_i d_i = -880 \text{ and } A = 900$$

$$\therefore \text{Mean } \bar{X} = A + \frac{\sum f_i d_i}{N}$$

$$\Rightarrow \text{Mean } \bar{X} = 900 + \frac{-880}{100} = 900 - 8.8 = 891.2$$

Hence, mean wage = ₹ 891.2.

15.2.3 STEP-DEVIATION METHOD

Sometimes, during the application of the short-cut method for finding AM, the deviations d_i are divisible by a common number h (say). In such a case the arithmetic is reduced to a great extent by taking

$$u_i = \frac{x_i - A}{h}; \quad i = 1, 2, 3, \dots, n$$

$$\Rightarrow x_i = A + h u_i, \quad i = 1, 2, 3, \dots, n$$

$$\Rightarrow f_i x_i = A f_i + h f_i u_i, \quad i = 1, 2, 3, \dots, n$$

$$\Rightarrow \sum_{i=1}^n f_i x_i = A \sum_{i=1}^n f_i + h \sum_{i=1}^n f_i u_i$$

$$\Rightarrow \frac{1}{N} \left(\sum_{i=1}^n f_i x_i \right) = \frac{AN}{N} + h \left\{ \frac{1}{N} \sum_{i=1}^n f_i u_i \right\}$$

$$\Rightarrow \bar{X} = A + h \left\{ \frac{1}{N} \sum_{i=1}^n f_i u_i \right\}$$

Finding AM by using this formula is known as the step-deviation method.

Following algorithm may be used to find the arithmetic mean by step deviation method:

ALGORITHM

STEP I Obtain the frequency distribution and prepare the frequency table in such a way that its first column consists of the values of the variable and the second column corresponding frequencies.

STEP II Choose a number 'A' (generally known as the assumed mean) and take deviations $d_i = x_i - A$ about A. Write these deviations against the corresponding frequencies in the third column.

STEP III Choose a number h, generally common factor of all d_i 's in III column, divide deviations d_i by h to get u_i . Write these u_i 's against the corresponding d_i 's in the IV column.

STEP IV Multiply the frequencies in II column with the corresponding u_i 's in IV column to prepare V column of $f_i u_i$.

STEP V Find the sum of all entries in V column to obtain $\sum_{i=1}^n f_i u_i$ and the sum of all frequencies in II column to obtain $N = \left(\sum_{i=1}^n f_i \right)$

STEP VI Use the formula: $\bar{X} = A + h \left\{ \frac{1}{N} \sum_{i=1}^n f_i u_i \right\}$

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES**LEVEL-1**

EXAMPLE 1 Find the mean wage from the following data:

Wage (in ₹) :	800	820	860	900	920	980	1000
No. of workers:	7	14	19	25	20	10	5

SOLUTION Let the assumed mean be $A = 900$ and $h = 20$.

Calculation of Mean

Wage (in ₹) x_i	No. of workers f_i	$d_i = x_i - A$ $= x_i - 900$	$u_i = \frac{x_i - 900}{20}$	$f_i u_i$
800	7	-100	-5	-35
820	14	-80	-4	-56
860	19	-40	-2	-38
900	25	0	0	0
920	20	20	1	20
980	10	80	4	40
1000	5	100	5	25
$N = \sum f_i = 100$				$\sum f_i u_i = -44$

We have,

$$N = 100, \Sigma f_i u_i = -44, A = 900 \text{ and } h = 20.$$

$$\therefore \text{Mean} = \bar{X} = A + h \left(\frac{1}{N} \Sigma f_i u_i \right)$$

$$\Rightarrow \bar{X} = 900 + 20 \times \frac{-44}{100} = 900 - 8.8 = 891.2$$

Hence, mean wage = ₹ 891.2.

EXAMPLE 2 Apply step-deviation method to find the AM of the following frequency distribution

Variate (x): 5 10 15 20 25 30 35 40 45 50

Frequency (f): 20 43 75 67 72 45 39 9 8 6

SOLUTION Let the assumed mean be $A = 25$ and $h = 5$.

Calculation of Mean

Variate	Frequency	Deviations	$u_i = \frac{x_i - 25}{5}$	$f_i u_i$
x_i	f_i	$d_i = x_i - 25$		
5	20	-20	-4	-80
10	43	-15	-3	-129
15	75	-10	-2	-150
20	67	-5	-1	-67
25	72	0	0	0
30	45	5	1	45
35	39	10	2	78
40	9	15	3	27
45	8	20	4	32
50	6	25	5	30
$N = \Sigma f_i = 384$				$\Sigma f_i u_i = -214$

We have,

$$N = 384, A = 25, h = 5 \text{ and } \Sigma f_i u_i = -214$$

$$\therefore \text{Mean} = \bar{X} = A + h \left\{ \frac{1}{N} \Sigma f_i u_i \right\}$$

$$\Rightarrow \text{Mean} = 25 + 5 \times \left(\frac{-214}{384} \right) = 25 - 2.786 = 22.214$$

EXAMPLE 3 The weights in kilograms of 60 workers in a factory are given in the following frequency table. Find the mean weight of a worker.

Weight (in kg) x: 60 61 62 63 64 65

No. of workers f: 5 8 14 16 10 7

SOLUTION Let the assumed mean be $A = 63$

Calculation of Mean

Weight (in kg)	No. of workers	$d_i = x_i - 63$	$f_i d_i$
x	f		
60	5	-3	-15
61	8	-2	-16
62	14	-1	-14
63	16	0	0
64	10	1	10
65	7	2	14
$N = \sum f_i = 60$			$\sum f_i d_i = -21$

We have, $N = 60$, $A = 63$ and $\sum f_i d_i = -21$

$$\therefore \text{Mean} = A + \left\{ \frac{1}{N} \sum f_i d_i \right\}$$

$$\Rightarrow \text{Mean} = 63 + \left(\frac{-21}{60} \right) = 63 - \frac{7}{20} = 63 - 0.35 = 62.65$$

Hence, mean weight of a worker = 62.65 kg

EXAMPLE 4 The table below gives the distribution of villages under different heights from sea level in a certain region. Compute the mean height of the region:

Height (in metres):	200	600	1000	1400	1800	2200
No. of village:	142	265	560	271	89	16

SOLUTION Let the assumed mean be $A = 1400$ and $h = 400$

Calculation of Mean

Height (in metres)	No. of villages	$d_i = x_i - 1400$	$u_i = \frac{x_i - 1400}{400}$	$f_i u_i$
x_i	f_i			
200	142	-1200	-3	-426
600	265	-800	-2	-530
1000	560	-400	-1	-560
1400	271	0	0	0
1800	89	400	1	89
2200	16	800	2	32
$N = \sum f_i = 1343$			$\sum f_i u_i = -1395$	

We have, $A = 1400$, $h = 400$, $\sum f_i u_i = -1395$ and $N = 1343$

$$\therefore \text{Mean} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow \text{Mean} = 1400 + 400 \times \frac{-1395}{1343} = 1400 - 415.49 = 984.51$$

LEVEL-3

[CBSE 2006 C]

$f_i u_i$

-30

-18

0

29

34

$\Sigma f_i u_i = 15$

ure (in rupees) of

450-500

7

for 18,000 kilomet

9. Find the mean from the following frequency distribution of marks at a test in statistics :

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

ANSWERS

1. 3.54 2. 2.47 3. 3.62 (approx) 4. 2.35
 5. 26.08 6. 3.53 (approx) 7. 0.73 8. 0.83
 9. 22.075

15.2.4 ARITHMETIC MEAN OF A CONTINUOUS FREQUENCY DISTRIBUTION

Uptill now we have been discussing about various methods for computing arithmetic mean of a discrete frequency distribution. In case of a continuous frequency distribution or a frequency distribution with class intervals arithmetic mean may be computed by applying any of the methods discussed so far. The values of $x_1, x_2, x_3, \dots, x_n$ are taken as the mid-points or class-marks of the various classes. It should be noted that the mid-value or class-marks of a class interval is equal to $\frac{1}{2}$ (lower limit + upper limit).

Following examples will illustrate the procedure.

ILLUSTRATIVE EXAMPLES**LEVEL-1**

EXAMPLE 1 Find the mean of the following frequency distribution:

Class-interval:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of workers f :	7	10	15	8	10

SOLUTION

Calculation of Mean

Class-interval	Mid-values (x_i)	Frequency f_i	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 - 10	5	7	-20	-2	-14
10 - 20	15	10	-10	-1	-10
20 - 30	25	15	0	0	0
30 - 40	35	8	10	1	8
40 - 50	45	10	20	2	20
$N = \sum f_i = 50$					$\sum f_i u_i = 4$

We have,

$$A = 25, h = 10, N = 50 \text{ and } \sum f_i u_i = 4.$$

$$\therefore \text{Mean} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow \text{Mean} = 25 + 10 \times \frac{4}{50} = 25.8$$

EXAMPLE 2 Find the mean of the following frequency distribution:

Classes:	0-20	20-40	40-60	60-80	80-100
Frequency:	15	18	21	29	17

[CBSE 2006 C]

SOLUTION

Calculation of Mean

Classes	Mid-values (x_i)	Frequency (f_i)	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 - 20	10	15	-40	-2	-30
20 - 40	30	18	-20	-1	-18
40 - 60	50	21	0	0	0
60 - 80	70	29	20	1	29
80 - 100	90	17	40	2	34
$\sum f_i = 100$					$\sum f_i u_i = 15$

We have,

$$A = 50, h = 20, N = 100 \text{ and } \sum f_i u_i = 15$$

$$\therefore \text{Mean} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow \text{Mean} = 50 + 20 \times \frac{15}{100}$$

$$\Rightarrow \text{Mean} = 50 + 3 = 53$$

EXAMPLE 3 The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure: (in ₹)	100-150	150-200	200-250	250-300	300-350	350-400	400-450	450-500
Frequency:	24	40	33	28	30	22	16	7

Find the average expenditure (in ₹) per household.

SOLUTION Let the assumed mean be $A = 325$ and $h = 50$.

Calculation of Mean

Expen - diture (in ₹) x_i	Freq - uency f_i	Mid - values x_i	$d_i = x_i - A$ $= x_i - 325$	$u_i = \frac{x_i - A}{h}$ $= \frac{x_i - 325}{50}$	$f_i u_i$
100 - 150	24	125	- 200	- 4	- 96
150 - 200	40	175	- 150	- 3	- 120
200 - 250	33	225	- 100	- 2	- 66
250 - 300	28	275	- 50	- 1	- 28
300 - 350	30	325	0	0	0
350 - 400	22	375	50	1	22
400 - 450	16	425	100	2	32
450 - 500	7	475	150	3	21
$N = \sum f_i = 200$					$\sum f_i u_i = -235$

We have,

$$N = 200, A = 325, h = 50, \text{ and } \sum f_i u_i = -235$$

$$\therefore \bar{X} = A + h \frac{1}{N} \sum f_i u_i$$

$$\Rightarrow \bar{X} = 325 + 50 \times \left\{ \frac{-235}{200} \right\}$$

$$\Rightarrow \bar{X} = 325 - \frac{235}{4} = 325 - 58.75 = 266.25$$

Hence, the average expenditure is ₹ 266.25.

EXAMPLE 4 A frequency distribution of the life times of 400 T.V. picture tubes tested in a tube company is given below. Find the average life of tube.

Life time (in hrs)	Frequency	Life time (in hrs)	Frequency
300-399	14	800-899	62
400-499	46	900-999	48
500-599	58	1000-1099	22
600-699	76	1100-1199	6
700-799	68		

SOLUTION Here, the class-intervals are formed by exclusive method. If we make the series an inclusive one the mid-values remain same. So, there is no need to convert the series into an inclusive form.

Let the assumed mean be $A = 749.5$ and $h = 100$.

Calculation of Mean

Life time (in hrs):	Frequency f_i	Mid-Values x_i	$d_i = x_i - A$ $= x_i - 749.5$	$u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 749.5}{100}$	$f_i u_i$
300 - 399	14	349.5	- 400	- 4	- 56
400 - 499	46	449.5	- 300	- 3	- 138
500 - 599	58	549.5	- 200	- 2	- 116
600 - 699	76	649.5	- 100	- 1	- 76
700 - 799	68	749.5	0	0	0
800 - 899	62	849.5	100	1	62
900 - 999	48	949.5	200	2	96
1000 - 1099	22	1049.5	300	3	66
1100 - 1199	6	1149.5	400	4	24
$N = \sum f_i = 400$					$\sum f_i u_i = -138$

We have, $N = 400, A = 749.5, h = 100$ and $\sum f_i u_i = -138$

$$\therefore \bar{X} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow \bar{X} = 749.5 + 100 \times \left(\frac{-138}{400} \right) = 749.5 - \frac{138}{4} = 749.5 - 34.5 = 715$$

Hence, the average life time of a tube is 715 hours.

EXAMPLE 5 If the mean of the following distributions is 54, find the value of p :

Class:	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency:	7	p	10	9	13

[CBSE 2006C]

SOLUTION

Computation of Arithmetic Mean

Class	Mid-values (x_i)	Frequency (f_i)	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 - 20	10	7	- 40	- 2	- 14
20 - 40	30	p	- 20	- 1	- p
40 - 60	50	10	0	0	0
60 - 80	70	9	20	1	9
80 - 100	90	13	40	2	26
$\sum f_i = 39 + p$					$\sum f_i u_i = 21 - p$

We have,

$$A = 50, N = 39 + p, h = 20, \sum f_i u_i = 21 - p \text{ and } \bar{X} = 54$$

$$\begin{aligned} \therefore \text{Mean} &= A + h \left\{ \frac{1}{N} \sum f_i u_i \right\} \\ \Rightarrow 54 &= 50 + 20 \times \left\{ \frac{21 - p}{39 + p} \right\} \\ \Rightarrow 4 &= 20 \times \left\{ \frac{21 - p}{39 + p} \right\} \\ \Rightarrow 1 &= 5 \left(\frac{21 - p}{39 + p} \right) \Rightarrow 39 + p = 105 - 5p \Rightarrow 6p = 66 \Rightarrow p = 11 \end{aligned}$$

LEVEL-2

EXAMPLE 6 The following table gives weekly wages in rupees of workers in a certain commercial organization. The frequency of class 49-52 is missing. It is known that the mean of the frequency distribution is 47.2. Find the missing frequency.

Weekly wages (₹):	40 - 43	43 - 46	46 - 49	49 - 52	52 - 55
Number of workers:	31	58	60	?	27

SOLUTION Let the missing frequency be f , the assumed mean be $A = 47$ and $h = 3$.

Calculation of Mean

Class- Intervals	mid-values x_i	f_i	$d_i = x_i - 47.5$	$u_i = \frac{x_i - 47.5}{3}$	$f_i u_i$
40 - 43	41.5	31	-6	-2	-62
43 - 46	44.5	58	-3	-1	-58
46 - 49	47.5	60	0	0	0
49 - 52	50.5	f	3	1	f
52 - 55	53.5	27	6	2	54
$N = \sum f_i = 176 + f$					$\sum f_i u_i = f - 66$

We have,

$$\bar{X} = 47.2, A = 47.5 \text{ and } h = 3$$

$$\therefore \bar{X} = A + h \left\{ \frac{1}{N} \sum f_i u_i \right\}$$

$$\Rightarrow 47.2 = 47.5 + 3 \times \left\{ \frac{f - 66}{176 + f} \right\}$$

$$\Rightarrow -0.3 = 3 \times \left\{ \frac{f - 66}{176 + f} \right\}$$

$$\Rightarrow \frac{-1}{10} = \frac{f - 66}{176 + f} \Rightarrow -176 - f = 10f - 660 \Rightarrow 11f = 484 \Rightarrow f = 44$$

Hence, the missing frequency is 44.

EXAMPLE 7 The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class 20 - 40 and 60 - 80 are missing. Find the missing frequencies.

Class:	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	Total
Frequency:	17	f_1	32	f_2	19	120

[CBSE 2006C]

SOLUTION Let the assumed mean be $A = 50$ and $h = 20$.

Calculation of Mean

Class	Frequency f_i	Mid-values x_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0 - 20	17	10	-2	-34
20 - 40	f_1	30	-1	$-f_1$
40 - 60	32	50	0	0
60 - 80	f_2	70	1	f_2
80 - 100	19	90	2	38
	$N = \Sigma f_i = 68 + f_1 + f_2$			$\Sigma f_i u_i = 4 - f_1 + f_2$

We have,

$$N = \Sigma f_i = 120 \quad \text{[Given]}$$

$$\Rightarrow 68 + f_1 + f_2 = 120$$

$$\Rightarrow f_1 + f_2 = 52 \quad \dots(i)$$

Now,

$$\text{Mean} = 50$$

$$\Rightarrow A + h \left\{ \frac{1}{N} \Sigma f_i u_i \right\} = 50$$

$$\Rightarrow 50 + 20 \times \left\{ \frac{4 - f_1 + f_2}{120} \right\} = 50$$

$$\Rightarrow 50 + \frac{4 - f_1 + f_2}{6} = 50$$

$$\Rightarrow \frac{4 - f_1 + f_2}{6} = 0$$

$$\Rightarrow 4 - f_1 + f_2 = 0$$

$$\Rightarrow f_1 - f_2 = 4 \quad \dots(ii)$$

Solving equations (i) and (ii), we get $f_1 = 28$ and $f_2 = 24$.

EXAMPLE 8 Find the mean marks of students from the following cumulative frequency distribution:

Marks	Number of students	Marks	Number of students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above.	0
50 and above	43		

SOLUTION Here we have, the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is $80 - 77 = 3$.

Similarly, the number of students getting marks between 10 and 20 is $77 - 72 = 5$ and so on. Thus, we obtain the following frequency distribution.

Marks	Number of students	Marks	Number of students
0 - 10	3	50 - 60	15
10 - 20	5	60 - 70	12
20 - 30	7	70 - 80	6
30 - 40	10	80 - 90	2
40 - 50	12	90 - 100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.

Computation of Mean

Marks (x_i)	Mid-value (f_i)	Frequency	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
0 - 10	5	3	-5	-15
10 - 20	15	5	-4	-20
20 - 30	25	7	-3	-21
30 - 40	35	10	-2	-20
40 - 50	45	12	-1	-12
50 - 60	55	15	0	0
60 - 70	65	12	1	12
70 - 80	75	6	2	12
80 - 90	85	2	3	6
90 - 100	95	8	4	32
Total		$\Sigma f_i = 80$		$\Sigma f_i u_i = -26$

We have,

$$N = \Sigma f_i = 80, \Sigma f_i u_i = -26, A = 55 \text{ and } h = 10$$

$$\therefore \bar{X} = A + h \left\{ \frac{1}{N} \Sigma f_i u_i \right\}$$

$$\Rightarrow \bar{X} = 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75 \text{ Marks.}$$

EXAMPLE 9 Find the mean marks of the students from the following cumulative frequency distribution:

Marks:	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
Number of students:	5	9	17	29	45	60	70	78	83	85

SOLUTION Here we have, cumulative frequency distribution less than type. First we convert it into an ordinary frequency distribution. We observe that the number of students getting marks less than 10 is 5 and 9 students have secured marks less than 20. Therefore, number of students getting marks between 10 and 20 (inclusive 0 and exclusive 10) is $9 - 5 = 4$. Similarly, the number of students getting marks between 20 and 30 is $17 - 9 = 8$ and so on. Thus, we have the following frequency distribution:

Marks:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of students:	5	4	8	12	16	15	10	8	5	2

Let us now compute arithmetic mean by taking 55 and the assumed mean.

Computation of Mean

Marks	Mid-value	Frequency	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$
0 - 10	5	5	-5	-25
10 - 20	10	4	-4	-16
20 - 30	25	8	-3	-24
30 - 40	35	12	-2	-24
40 - 50	45	16	-1	-16
50 - 60	55	15	0	0
60 - 70	65	10	1	10
70 - 80	75	8	2	16
80 - 90	85	5	3	15
90 - 100	95	2	4	8
Total		$N = \Sigma f_i = 85$		$\Sigma f_i u_i = -56$

We have,

$$N = \Sigma f_i = 85, \Sigma f_i u_i = -56, h = 10 \text{ and } A = 55$$

$$\therefore \bar{X} = A + h \left\{ \frac{1}{N} \Sigma f_i u_i \right\}$$

$$\Rightarrow \bar{X} = 55 + 10 \times \frac{-56}{85} = 55 - 6.59 = 48.41 \text{ Marks}$$

Hence, mean marks scored by the students = 48.41.

EXERCISE 15.3

LEVEL-1

1. The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees) (x_i)	Frequency (f_i)	Expenditure (in rupees) (x_i)	Frequency (f_i)
100-150	24	300-350	30
150-200	40	350-400	22
200-250	33	400-450	16
250-300	28	450-500	7

Find the average expenditure (in rupees) per household.

2. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants:	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of houses:	1	2	1	5	6	2	3

[NCERT]

Which method did you use for finding the mean, and why?

3. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in ₹).	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers:	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarised as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute:	65 - 68	68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women:	2	4	3	8	7	4	2

[NCERT]

Find the mean of each of the following frequency distributions: (5 - 14)

5. Class interval: 0 - 6 6 - 12 12 - 18 18 - 24 24 - 30
Frequency: 6 8 10 9 7
6. Class interval: 50 - 70 70 - 90 90 - 110 110 - 130 130 - 150 150 - 170
Frequency: 18 12 13 27 8 22

7. Class interval: 0 - 8 8 - 16 16 - 24 24 - 32 32 - 40
Frequency: 6 7 10 8 9
8. Class interval: 0 - 6 6 - 12 12 - 18 18 - 24 24 - 30
Frequency: 7 5 10 12 6
9. Class interval: 0 - 10 10 - 20 20 - 30 30 - 40 40 - 50
Frequency: 9 12 15 10 14
10. Class interval: 0 - 8 8 - 16 16 - 24 24 - 32 32 - 40
Frequency: 5 9 10 8 8
11. Class interval: 0 - 8 8 - 16 16 - 24 24 - 32 32 - 40
Frequency: 5 6 4 3 2
12. Class interval: 10 - 30 30 - 50 50 - 70 70 - 90 90 - 110 110 - 130
Frequency: 5 8 12 20 3 2
13. Class interval: 25 - 35 35 - 45 45 - 55 55 - 65 65 - 75
Frequency: 6 10 8 12 4
14. Classes: 25 - 29 30 - 34 35 - 39 40 - 44 45 - 49 50 - 54 55 - 59
Frequency: 14 22 16 6 5 3 4
[CBSE 2006C]

15. For the following distribution, calculate mean using all suitable methods:

Size of item:	1 - 4	4 - 9	9 - 16	16 - 27
Frequency:	6	12	26	20

16. The weekly observations on cost of living index in a certain city for the year 2004 - 2005 are given below. Compute the weekly cost of living index.

Cost of living Index	Number of Students	Cost of living Index	Number of Students
1400 - 1500	5	1700 - 1800	9
1500 - 1600	10	1800 - 1900	6
1600 - 1700	20	1900 - 2000	2

17. The following table shows the marks scored by 140 students in an examination of a certain paper:

Marks:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of students:	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

18. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency f_1 and f_2 .

Class:	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency:	5	f_1	10	f_2	7	8

[CBSE 2004]

19. The following distribution shows the daily pocket allowance given to the children of a multistorey building. The average pocket allowance is ₹ 18.00. Find out the missing frequency.

Class interval:	11-13	13-15	15-17	17-19	19-21	21-23	23-25	
Frequency:	7	6	9	13	-	5	4	[NCERT]

20. If the mean of the following distribution is 27, find the value of p .

Class:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	
Frequency:	8	p	12	13	10	[CBSE 2006C]

21. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes:	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of boxes:	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose? [NCERT]

22. The table below shows the daily expenditure on food of 25 households in a locality

Daily expenditure (in ₹):	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Number of households:	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method. [NCERT]

23. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Find the mean concentration of SO_2 in the air. [NCERT]

24. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days:	0 - 6	6 - 10	10 - 14	14 - 20	20 - 28	28 - 38	38 - 40
Number of students:	11	10	7	4	4	3	1

[NCERT]

25. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %):	45 - 55	55 - 65	65 - 75	75 - 85	85 - 95
Number of cities:	3	10	11	8	3

[NCERT]

26. The following is the cumulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age.

Age below (in years):	30	40	50	60	70	80
Number of persons:	100	220	350	750	950	1000

[NCERT EXEMPLAR]

27. If the mean of the following frequency distribution is 18, find the missing frequency.
- | | | | | | | | |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| Class interval: | 11-13 | 13-15 | 15-17 | 17-19 | 19-21 | 21-23 | 23-25 |
| Frequency: | 3 | 6 | 9 | 13 | f | 5 | 4 |

[NCERT EXEMPLAR, CBSE 2018]

28. Find the missing frequencies in the following distribution, if the sum of the frequencies is 120 and the mean is 50.

Class:	0-20	20-40	40-60	60-80	80-100
Frequency:	17	f_1	32	f_2	19

[NCERT EXEMPLAR]

29. The daily income of a sample of 50 employees are tabulated as follows:

Income (in ₹):	1-200	201-400	401-600	601-800
No. of employees:	14	15	14	7

Find the mean daily income of employees.

[NCERT EXEMPLAR]

ANSWERS

- | | | | |
|--------------------------|---------------|--------------------------|-------------|
| 1. 266.25 | 2. 8.1 plants | 3. 145.20 | 4. 75.9 |
| 5. 15.45 | 6. 112.20 | 7. 21.4 | 8. 15.75 |
| 9. 26.333 | 10. 21 | 11. 16.4 | 12. 65.6 |
| 13. 49.5 | 14. 36.357 | 16. 1663.3 | 17. 25.857 |
| 18. $f_1 = 8, f_2 = 12.$ | 19. 20 | 20. $p = 7$ | 21. 57.19 |
| 22. ₹ 211 | 23. 0.099 ppm | 24. 12.475 days | 25. 69.43% |
| 26. 51.3 years | 27. 8 | 28. $f_1 = 28, f_2 = 24$ | 29. ₹ 356.5 |

15.3 MEDIAN

The median is the middle value of a distribution i.e., median of a distribution is the value of the variable which divides it into two equal parts. It is the value of the variable such that the number of observations above it is equal to the number of observations below it.

In class IX, we have studied about the method for finding median of individual observations. If $x_1, x_2, x_3, \dots, x_n$ are n values of a variable X , then to find the median we use the following algorithm.

ALGORITHM

STEP I Arrange the observations x_1, x_2, \dots, x_n in ascending or descending order of magnitude.

STEP II Determine the total number of observations, say, n

STEP III If n is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation.

If n is even, then median is the AM of the values of $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ observations.

ILLUSTRATION 1 (i) The following are the marks of 9 students in a class. Find the median

34, 32, 48, 38, 24, 30, 27, 21, 35

(ii) Find the median of the daily wages of ten workers from the following data:

₹20, 25, 17, 18, 8, 15, 22, 11, 9, 14.

SOLUTION (i) Arranging the data in ascending order of magnitude, we have

21, 24, 27, 30, 32, 34, 35, 38, 48

Since there are 9 i.e., an odd number of items. Therefore, median is the value of

$\left(\frac{9+1}{2}\right)^{\text{th}}$ observation i.e., 32.

(ii) Arranging the wages in ascending order of magnitude, we have

8, 9, 11, 14, 15, 17, 18, 20, 22, 25

Since there are 10 observations Therefore, median is the arithmetic mean of

$\left(\frac{10}{2}\right)^{\text{th}}$ and $\left(\frac{10}{2} + 1\right)^{\text{th}}$ observations.

Hence, Median = $\frac{15 + 17}{2} = 16$

15.3.1 MEDIAN OF DISCRETE FREQUENCY DISTRIBUTION

In case of a discrete frequency distribution $x_i / f_i; i = 1, 2, \dots, n$ we calculate the median by using the following algorithm.

ALGORITHM

STEP I Find the cumulative frequencies (c.f.)

STEP II Find $\frac{N}{2}$, where $N = \sum_{i=1}^n f_i$

STEP III See the cumulative frequency (c.f.) just greater than $\frac{N}{2}$ and determine the corresponding value of the variable.

STEP IV The value obtained in step III is the median.

ILLUSTRATION 2 Obtain the median for the following frequency distribution:

$x:$	1	2	3	4	5	6	7	8	9
$f:$	8	10	11	16	20	25	15	9	6

SOLUTION

Calculation of Median

x	f	cf
1	8	8
2	10	18
3	11	29
4	16	45
5	20	65
6	25	90
7	15	105
8	9	114
9	6	120

$N = 120$

Here, $N = 120 \Rightarrow \frac{N}{2} = 60$

We find that the cumulative frequency just greater than $\frac{N}{2}$ i.e., 60 is 65 and the value of x corresponding to 65 is 5. Therefore, Median = 5.

15.3.2 MEDIAN OF A GROUPED OR CONTINUOUS FREQUENCY DISTRIBUTION

In order to calculate the median of a grouped or continuous frequency distribution, we use the following algorithm.

ALGORITHM

STEP I Obtain the frequency distribution.

STEP II Prepare the cumulative frequency column and obtain $N = \sum f_i$.

STEP III Find $N/2$.

STEP IV See the cumulative frequency just greater than $N/2$ and determine the corresponding class. This class is known as the median class.

STEP V Use the following formula:

$$\text{Median} = l + \left\{ \frac{\frac{N}{2} - F}{f} \right\} \times h$$

where, l = lower limit of the median class

f = frequency of the median class

h = with (size) of the median class

F = Cumulative frequency of the class preceding the median class $N = \sum f_i$

ILLUSTRATION 3 Calculate the median from the following distribution:

Class: 5-10 10-15 15-20 20-25 25-30 30-35 35-40 40-45

Frequency: 5 6 15 10 5 4 2 2

SOLUTION First we prepare the following cumulative table to compute the median

Class	Frequency	Cumulative frequency
5-10	5	5
10-15	6	11
15-20	15	26
20-25	10	36
25-30	5	41
30-35	4	45
35-40	2	47
40-45	2	49
$N = 49$		

We have, $N = 49$

$$\therefore \frac{N}{2} = \frac{49}{2} = 24.5$$

The cumulative frequency just greater than $N/2$ is 26 and the corresponding class is 15 - 20. Thus, 15 - 20 is the median class such that

$$l = 15, f = 15, F = 11 \text{ and } h = 5$$

$$\therefore \text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h = 15 + \frac{24.5 - 11}{15} \times 5 = 15 + \frac{13.5}{3} = 19.5$$

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 The number of students absent in a school was recorded every day for 147 days and the raw data was presented in the form of the following frequency table.

No. of students absent:	5	6	7	8	9	10	11	12	13	15	18	20
No. of days:	1	5	11	14	16	13	10	70	4	1	1	1

Obtain the median and describe what information it conveys.

SOLUTION

Calculation of median

x_i	f_i	cf
5	1	1
6	5	6
7	11	17
8	14	31
9	16	47
10	13	60
11	10	70
12	70	140
13	4	144
15	1	145
18	1	146
20	1	147

$$N = \sum f_i = 147$$

We have,

$$N = 147 \Rightarrow \frac{N}{2} = \frac{147}{2} = 73.5$$

The cumulative frequency just greater than $N/2$ is 140 and the corresponding value of variable x is 12.

Hence, median = 12. This means that for about half the number of days, more than 12 students were absent.

EXAMPLE 2 Calculate the median from the following data:

Marks:	0-10	10-30	30-60	60-80	80-90
No. of students:	5	15	30	8	2

SOLUTION Here, the class intervals are of unequal width. If the class intervals are of unequal width the frequencies need not be adjusted to make the class intervals equal.

Calculation of Median

Marks	No. of students (Frequency)	Cumulative frequency
0 - 10	5	5
10 - 30	15	20
30 - 60	30	50
60 - 80	8	58
80 - 90	2	60

$N = \Sigma f_i = 60$

Here, $N = 60 \therefore N/2 = 30$

The cumulative frequency just greater than $N/2 = 30$ is 50 and the corresponding class is 30 - 60. Hence, 30 - 60 is the median class.

$$\therefore l = 30, f = 30, F = 20, h = 30$$

$$\text{Now, Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \text{Median} = 30 + \frac{30 - 20}{30} \times 30 = 40$$

EXAMPLE 3 If the median of the following frequency distribution is 46, find the missing frequencies.

Variable:	10-20	20-30	30-40	40-50	50-60	60-70	70-80	Total
Frequency:	12	30	?	65	?	25	18	229

SOLUTION Let the frequency of the class 30 - 40 be f_1 and that of the class 50 - 60 be f_2 . The total frequency is 229.

$$12 + 30 + f_1 + 65 + f_2 + 25 + 18 = 229 \Rightarrow f_1 + f_2 = 79$$

It is given that the median is 46.

Clearly, 46 lies in the class 40 - 50. So, 40 - 50 is the median class.

$$\therefore l = 40, h = 10, f = 65 \text{ and } F = 12 + 30 + f_1 = 42 + f_1, N = 229$$

Now,

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 46 = 40 + \frac{\frac{229}{2} - (42 + f_1)}{65} \times 10$$

$$\Rightarrow 46 = 40 + \frac{145 - 2f_1}{13}$$

$$\Rightarrow 6 = \frac{145 - 2f_1}{13} \Rightarrow 2f_1 = 67 \Rightarrow f_1 = 33.5 \text{ or } 34 \text{ (say)}$$

Since $f_1 + f_2 = 79$. Therefore, $f_2 = 45$.

Hence, $f_1 = 34$ and $f_2 = 45$.

LEVEL-2

EXAMPLE 4 Find the median of the following frequency distribution:

Weekly wages (in ₹):	60-69	70-79	80-89	90-99	100-109	110-119
No. of days:	5	15	20	30	20	8

SOLUTION Here, the frequency table is given in inclusive form. So, we first transform it into exclusive form by subtracting and adding $h/2$ to the lower and upper limits respectively of each class, where h denotes the difference of lower limit of a class and the upper limit of the previous class.

Transforming the above table into exclusive form and preparing the cumulative frequency table, we get

Weekly wages (in ₹)	No. of workers	Cumulative frequency
59.5 – 69.5	5	5
69.5 – 79.5	15	20
79.5 – 89.5	20	40
89.5 – 99.5	30	70
99.5 – 109.5	20	90
109.5 – 119.5	8	98

$N = \sum f_i = 98$

We have, $N = 98$. $\therefore N/2 = 49$

The cumulative frequency just greater than $N/2$ is 70 and the corresponding class is 89.5 – 99.5. So, 89.5 – 99.5 is the median class.

$\therefore l = 89.5, h = 10, f = 30$ and $F = 40$

$$\text{Now, Median} = l + \frac{\frac{N}{2} - f}{f} \times h$$

$$\Rightarrow \text{Median} = 89.5 + \frac{49 - 40}{30} \times 10 = 92.5$$

EXAMPLE 5 Compute the median from the following data:

Mid-value:	115	125	135	145	155	165	175	185	195
Frequency:	6	25	48	72	116	60	38	22	3

SOLUTION Here, we are given the mid-values. So, should first find the upper and lower limits of the various classes. The difference between two consecutive values is $h = 125 - 115 = 10$.

\therefore Lower limit of a class = Mid-value $- h/2$, Upper limit = Mid-value $+ h/2$.

Calculation of Median

Mid-value	Class groups	Frequency	Cumulative Frequency
115	110 - 120	6	6
125	120 - 130	25	31
135	130 - 140	48	79
145	140 - 150	72	151
155	150 - 160	116	267
165	160 - 170	60	327
175	170 - 180	38	365
185	180 - 190	22	387
195	190 - 200	3	390

$N = \sum f_i = 390$

We have,

$$N = 390 \therefore \frac{N}{2} = \frac{390}{2} = 195$$

The cumulative frequency just greater than $N/2$ i.e., 195 is 267 and the corresponding class is 150 - 160. So, 150 - 160 is the median class.

$$\therefore l = 150, f = 116, h = 10, F = 151$$

Now,

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \text{Median} = 150 + \frac{195 - 151}{116} \times 10 = 153.80$$

EXAMPLE 6 Compute the median for the following cumulative frequency distribution:

Less than 0	Less than 4	Less than 16	Less than 30	Less than 46	Less than 66	Less than 82	Less than 92	Less than 100
-------------	-------------	--------------	--------------	--------------	--------------	--------------	--------------	---------------

SOLUTION We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

Class intervals	Frequency (f)	Cumulative frequency (c.f.)
20 - 30	4	4
30 - 40	12	16
40 - 50	14	30
50 - 60	16	46
60 - 70	20	66
70 - 80	16	82
80 - 90	10	92
90 - 100	8	100

$N = \sum f_i = 100$

Here, $N = \sum f_i = 100 \therefore \frac{N}{2} = 50$

We observe that the cumulative frequency just greater than $\frac{N}{2} = 50$ is 66 and the corresponding class is 60 - 70.

So, 60 - 70 is the median class.

$$\therefore l = 60, f = 20, F = 46 \text{ and } h = 10$$

$$\text{Now, Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow \text{Median} = 60 + \frac{50 - 46}{20} \times 10 = 62$$

EXAMPLE 7 The median of the following data is 525. Find the values of x and y , if the total frequency is 100 [NCERT]

Class interval	Frequency
0 - 100	2
100 - 200	5
200 - 300	x
300 - 400	12
400 - 500	17
500 - 600	20
600 - 700	y
700 - 800	9
800 - 900	7
900 - 1000	4

SOLUTION

Computation of Median

Class intervals	Frequency (f)	Cumulative frequency (cf)
0 - 100	2	2
100 - 200	5	7
200 - 300	x	$7 + x$
300 - 400	12	$19 + x$
400 - 500	17	$36 + x$
500 - 600	20	$56 + x$
600 - 700	y	$56 + x + y$
700 - 800	9	$65 + x + y$
800 - 900	7	$72 + x + y$
900 - 1000	4	$76 + x + y$
		Total = 100

We have,

$$N = \Sigma f_i = 100$$

$$\Rightarrow 76 + x + y = 100 \Rightarrow x + y = 24$$

It is given that the median is 525. Clearly, it lies in the class 500 - 600

$$\therefore l = 500, h = 100, f = 20, F = 36 + x \text{ and } N = 100$$

Now,

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$\Rightarrow 525 - 500 = (14 - x) \times 5$$

$$\Rightarrow 25 = 70 - 5x \Rightarrow 5x = 45 \Rightarrow x = 9$$

Putting $x = 9$ in $x + y = 24$, we get $y = 15$.

Hence, $x = 9$ and $y = 15$.

EXAMPLE 8 If the median of the distribution given below is 28.5, find the value of x and y .

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60
No. of students:	5	x	20	15	y	5

Total 60
[NCERT]

SOLUTION

Computation of Median

Class intervals	Frequency (f)	Cumulative frequency (cf)
0-10	5	5
10-20	x	$5+x$
20-30	20	$25+x$
30-40	15	$40+x$
40-50	y	$40+x+y$
50-60	5	$45+x+y$
$\Sigma f_1 = 60$		

We have,

$$\text{Median} = 28.5$$

Clearly, it lies in the class interval 20 - 30. So, 20 - 30 is the median class.

$$\therefore l = 20, h = 10, f = 20, F = 5 + x \text{ and } N = 60$$

Now,

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$\Rightarrow 28.5 = 20 + \frac{30 - (5 + x)}{20} \times 10$$

$$\Rightarrow 28.5 = 20 + \frac{25 - x}{2}$$

$$\Rightarrow 8.5 = \frac{25 - x}{2} \Rightarrow 25 - x = 17 \Rightarrow x = 8$$

We have,

$$N = 60$$

$$\therefore 45 + x + y = 60 \Rightarrow x + y = 15$$

Putting $x = 8$ in $x + y = 15$, we get $y = 7$

Hence, $x = 8$ and $y = 7$.

EXERCISE 15.4

LEVEL-1

- Following are the lives in hours of 15 pieces of the components of aircraft engine. Find the median:
715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719.
- The following is the distribution of height of students of a certain class in a certain city:
Height (in cms): 160 - 162 163 - 165 166 - 168 169 - 171 172 - 174
No. of students: 15 118 142 127 18
Find the median height.
- Following is the distribution of I.Q. of 100 students. Find the median I.Q.
I.Q.: 55-64 65-74 75-84 85-94 95-104 105-114 115-124 125-134 135-144
No of
Students: 1 2 9 22 33 22 8 2 1
- Calculate the median salary of the following data giving salaries of 280 persons:
Salary (in thousands): 5-10 10-15 15-20 20-25 25-30 30-35 35-40 40-45 45-50
No. of persons: 49 133 63 15 6 7 4 2 1
[CBSE 2018]
- Calculate the median from the following data:
Marks below: 10 20 30 40 50 60 70 80
No. of students: 15 35 60 84 96 127 198 250
- Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.
Age in years: 0 - 10 10 - 20 20 - 30 30 - 40 40 - 50
No. of persons: 5 25 ? 18 7
- The following table gives the frequency distribution of married women by age at marriage:

Age (in years)	Frequency	Age (in years)	Frequency
15-19	53	40-44	9
20-24	140	45-49	5
25-29	98	50-54	3
30-34	32	55-59	3
35-39	12	60 and above	2

Calculate the median and interpret the results.

- The following table gives the distribution of the life time of 400 neon lamps:

Life time: (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life.

[NCERT]

9. The distribution below gives the weight of 30 students in a class. Find the median weight of students:

Weight (in kg):	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students:	2	3	8	6	6	3	2 [NCERT]

LEVEL-2

10. Find the missing frequencies and the median for the following distribution if the mean is 1.46.

No. of accidents:	0	1	2	3	4	5	Total
Frequency (No. of days):	46	?	?	25	10	5	200

11. An incomplete distribution is given below:

Variable:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	12	30	-	65	-	25	18

You are given that the median value is 46 and the total number of items is 230.

- (i) Using the median formula fill up missing frequencies.
 (ii) Calculate the AM of the completed distribution.
12. If the median of the following frequency distribution is 28.5 find the missing frequencies:

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency:	5	f_1	20	15	f_2	5	60

13. The median of the following data is 525. Find the missing frequency, if it is given that there are 100 observations in the data:

Class interval	Frequency	Class interval	Frequency
0 - 100	2	500 - 600	20
100 - 200	5	600 - 700	f_2
200 - 300	f_1	700 - 800	9
300 - 400	12	800 - 900	7
400 - 500	17	900 - 1000	4

14. If the median of the following data is 32.5, find the missing frequencies.

Class interval:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	Total
Frequency:	f_1	5	9	12	f_2	3	2	40

15. Compute the median for each of the following data:

(i) Marks	No. of students	(ii) Marks	No. of students
Less than 10	0	More than 150	0
Less than 30	10	More than 140	12
Less than 50	25	More than 130	27
Less than 70	43	More than 120	60
Less than 90	65	More than 110	105
Less than 110	87	More than 100	124
Less than 130	96	More than 90	141
Less than 150	100	More than 80	150

16. A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

Height in cm	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height.

[NCERT]

17. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

[NCERT]

18. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table:

Length (in mm):	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves:	3	5	9	12	5	4	2

Find the mean length of leaf.

[NCERT]

19. An incomplete distribution is given as follows:

Variable:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency:	10	20	?	40	?	25	15

You are given that the median value is 35 and the sum of all the frequencies is 170. Using the median formula, fill up the missing frequencies.

20. The median of the distribution given below is 14.4. Find the values of x and y , if the total frequency is 20.

Class interval:	0-6	6-12	12-18	18-24	24-30
Frequency:	4	x	5	y	1

[NCERT EXEMPLAR]

21. The median of the following data is 50. Find the values of p and q , if the sum of all the frequencies is 90.

Marks:	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency:	p	15	25	20	q	8	10

[NCERT EXEMPLAR]

ANSWERS

1. 716 2. 167.13 cms 3. 99.35 4. ₹13421 5. 59.35 6. 25

7. Median = 24.5 years.

Nearly half the women were married between the ages 15 and 24.5 years.

8. Median life = 3406.98 hours 9. 56.67 kg

10. Missing Frequencies 76 and 38, Median = 1.

11. Missing frequencies 34 and 46, Mean = 45.87.

12. $f_1 = 8, f_2 = 7$ 13. $f_1 = 9, f_2 = 15$ 14. $f_1 = 3, f_2 = 6$

15. (i) 76.36 (ii) 116.67 16. 149.03 cm 17. 35.76 years

18. 146.75 mm 19. Class 20 - 30 40 - 50
Frequency 35 25

20. $x = 4, y = 6$ 21. $p = 5, q = 7$

15.4 MERITS AND DEMERITS OF MEDIAN

The following are some merits and demerits of median:

MERITS

- (i) It is easy to compute and understand.
- (ii) It is well defined an ideal average should be.
- (iii) It can also be computed in case of frequency distribution with open ended classes.
- (iv) It is not affected by extreme values.
- (v) It can be determined graphically.
- (vi) It is proper average for qualitative data where items are not measured but are scored.

DEMERITS

- (i) For computing median data needs to be arranged in ascending or descending order.
- (ii) It is not based on all the observations of the data.
- (iii) It cannot be given further algebraic treatment.
- (iv) It is affected by fluctuations of sampling.
- (v) It is not accurate when the data is not large.
- (vi) In some cases median is determined approximately as the mid-point of two observations whereas for mean this does not happen.

15.5 MODE

In earlier classes, we have studied about the computation of mode of raw data. In this section, we shall learn about the computation of mode of a discrete frequency distribution and frequency distribution with class intervals. But, let us first recall the definition of mode.

MODE *The mode or modal value of a distribution is that value of the variable for which the frequency is maximum.*

Thus, the mode of a distribution is that value of the variable around which the values of the variable are clustered densely.

15.5.1 COMPUTATION OF MODE OF A SERIES OF INDIVIDUAL OBSERVATIONS

In order to compute the mode of a series of individual observations, we first convert it into a discrete series frequency distribution by preparing a frequency table. From the frequency table, we identify the value having maximum frequency. The value of variable so obtained is the mode or modal value.

Following examples will illustrate the procedure.

ILLUSTRATIVE EXAMPLES**LEVEL-1**

EXAMPLE 1 Find the mode of the following data:

120, 110, 130, 110, 120, 140, 130, 120, 140, 120

SOLUTION Let us first form the frequency table for the given data as given below:

Value x_i :	110	120	130	140
Frequency f_i :	2	4	2	2

We observe that the value 120 has the maximum frequency.

Hence, the mode or modal value is 120.

EXAMPLE 2 Find the mode of the following data:

25, 16, 19, 48, 19, 20, 34, 15, 19, 20, 21, 24, 19, 16, 22, 16, 18, 20, 16, 19

SOLUTION The frequency table of the given data is as given below:

Value (x_i):	15	16	18	19	20	21	22	24	25	34	48
Frequency (f_i):	1	4	1	5	3	1	1	1	1	1	1

We observe that the value 19 has the maximum frequency i.e. it occurs maximum number of times. Therefore, mode of the given data is 19.

EXAMPLE 3 Find the value of x , if the mode of the following data is 25:

15, 20, 25, 18, 14, 15, 25, 15, 18, 16, 20, 25, 20, x , 18

SOLUTION The frequency table of the given data is as given below:

Value (x_i):	14	15	16	18	20	25	x
Frequency (f_i):	1	3	1	3	3	3	1

It is given that the mode of the given data is 25. So, it must have the maximum frequency. That is possible only when $x = 25$.

Hence, $x = 25$.

15.5.2 COMPUTATION OF MODE BY GROUPING

Sometimes there are two or more values having the same frequency. In such cases one cannot say which is modal value and hence mode is said to be ill-defined. Such a frequency distribution is also known as bimodal or multimodal distribution. For such frequency distribution mode is computed by grouping method.

Consider the following frequency distribution:

x:	5	6	7	8	9	10	11	12	13
f:	8	12	13	14	13	11	7	4	3

From the above frequency distribution, we can clearly say that modal value is 8, because the value 8 of variable x has occurred the maximum number of times i.e. 14. But, we find that the difference between the maximum frequency and the frequencies of the values of the variable on both sides of 8 which are very close to 8 is very small. This means that the values of variable are heavily concentrated on either side of 8. Therefore, if we find mode just by inspection, an error is possible. In such cases, we prepare a grouping table and an analysis table to find the mode. These tables help us in determining the correct value of mode. The grouping table consists of six columns which are constructed by using the following algorithm.

ALGORITHM

- STEP I** Obtain the discrete frequency distribution.
- STEP II** Take the column of frequencies as column I and encircle the maximum frequency in it.
- STEP III** Construct column II, containing the sum of the frequencies taken two at a time and encircle the maximum frequency in it.
- STEP IV** Leave the first frequency and construct column III, containing the sum of the frequencies taken two at a time. Encircle the maximum frequency in column III.
- STEP V** Construct column IV, containing the sum of three frequencies at a time and encircle the maximum frequency in it.
- STEP VI** Exclude the first frequency and compute the sum of the frequencies taken three at a time to construct column V. Encircle the maximum frequency in this column.
- STEP VII** Exclude the first two frequencies and compute the sum of the frequencies taken three at a time to construct column VI. Encircle the maximum frequency in this column.

After preparing the grouping table, we prepare an analysis table by using the following algorithm.

ALGORITHM

- STEP I** Prepare a table in which in the top most row write all values of the variable and in the left most column write column numbers from I to VI.
- STEP II** See the maximum frequency in the first column of the grouping table and obtain the corresponding value of the variable. Now, mark a bar (|) in the first row of the analysis table against the value of the variable having the maximum frequency. Continue the same procedure for the remaining five columns.
- STEP III** Find the total number of bars corresponding to each value of the variable. That value of the variable which has the maximum number of bars is the mode of the frequency distribution.

Following illustration will illustrate the grouping and analysis tables.

ILLUSTRATION 4 Compute the modal value for the following frequency distribution:

$x:$	95	105	115	125	135	145	155	165	175
$y:$	4	2	18	22	21	19	10	3	2

SOLUTION It is clear from the frequency distribution that the difference between the maximum frequency and frequency succeeding it is very small and values of the variable x are closely concentrated on its either side. So, we compute the modal value by grouping method.

Grouping Table

x	f					
	Col. I	Col. II	Col. III	Col. IV	Col. V	Col. VI
95	4					
105	2	6		24		
115	18		20			
125	(22)	(40)			45	
135	21		(43)	(62)		(61)
145	19	(40)				
155	10		29		(50)	
165	3	13		15		32
175	2		5			

Analysis Table

Col.No.	95	105	115	125	135	145	155	165	175
I									
II									
III									
IV									
V									
VI									
Total			2	5	4	2	1		

From the analysis table, it is clear that the value 125 has the maximum number of bars. So, Modal value is 125.

15.5.3 COMPUTATION OF MODE FOR A CONTINUOUS FREQUENCY DISTRIBUTION

In case of a grouped or continuous frequency distribution with equal class intervals, we use the following algorithm to compute the mode.

ALGORITHM

STEP I Obtain the continuous frequency distribution.

STEP II Determine the class of maximum frequency either by inspection or by grouping method. This class is called the modal class.

STEP III Obtain the values of the following from the frequency distribution:

l = lower limit of the modal class, f = frequency of the modal class

h = width of the modal class,

f_1 = frequency of the class preceding the modal class,

f_2 = frequency of the class following the modal class.

STEP IV Substitute the values obtained in step III in the following formula:

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

Following examples will illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 Compute the mode for the following frequency distribution:

Size of items: 0-4 4-8 8-12 12-16 16-20 20-24 24-28 28-32 32-36 36-40

Frequency: 5 7 9 17 12 10 6 3 1 0

SOLUTION Here, the maximum frequency is 17 and the corresponding class is 12-16. So, 12-16 is the modal class such that $l = 12$, $h = 4$, $f = 17$, $f_1 = 9$ and $f_2 = 12$.

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 12 + \frac{17 - 9}{34 - 9 - 12} \times 4 = 12 + \frac{8}{13} \times 4 = 12 + \frac{32}{13} = 12 + 2.46 = 14.46$$

EXAMPLE 2 For the following grouped frequency distribution find the mode:

Class: 3-6 6-9 9-12 12-15 15-18 18-21 21-24

Frequency: 2 5 10 23 21 12 3

SOLUTION We observe that the class 12-15 has maximum frequency. Therefore, this is the modal class such that $l = 12$, $h = 3$, $f = 23$, $f_1 = 10$ and $f_2 = 21$.

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 12 + \frac{23 - 10}{46 - 10 - 21} \times 3 = 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6$$

EXAMPLE 3 Compute the value of mode for the following frequency distribution.

Class: 100-110 110-120 120-130 130-140 140-150 150-160 160-170

Frequency: 4 6 20 32 33 8 2

SOLUTION Clearly, the difference between the maximum frequency and the frequency preceding is very small. So, we shall determine the modal class by grouping method.

Grouping Table

Class	Frequency					
	Col. I	Col. II	Col. III	Col. IV	Col. V	Col. VI
100-110	4					
110-120	6	10		30		
120-130	20		26		(58)	
130-140	32	(52)				
140-150	(33)		(65)	(73)		(85)
150-160	8	41			43	
160-170	2		10			

Analysis Table

Col. No.	100-110	110-120	120-130	130-140	140-150	150-160	160-170
I							
II							
III							
IV							
V							
VI							
Total		1	3	5	4	1	

Clearly, class 130-140 has maximum number of bars. So, 130-140 is the modal class.

$$\therefore l = 130, h = 10, f = 32, f_1 = 20, f_2 = 33$$

$$\text{Now, Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 130 + \frac{32 - 20}{64 - 20 - 33} \times 10$$

$$\Rightarrow \text{Mode} = 130 + \frac{12}{9} \times 10 = 140.9$$

LEVEL-2

EXAMPLE 4 The following data gives the distribution of total household expenditure (in rupees) of manual workers in a city:

Expenditure (in ₹)	Frequency	Expenditure (in ₹)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

Find the average expenditure which is being done by the maximum number of manual workers.

[NCERT]

SOLUTION We know that the mode is the value of the variable which occurs maximum number of times in a frequency distribution. So, the average expenditure done by the maximum number of workers is the modal value. We observe that the class 1500-2000 has the maximum frequency 40. So, it is the modal class such that $l = 1500$, $h = 500$, $f = 40$, $f_1 = 24$ and $f_2 = 33$.

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500 = 1500 + \frac{16}{23} \times 500 = 1847.826$$

EXAMPLE 5 Calculate the value of mode for the following frequency distribution:

Class:	1-4	5-8	9-12	13-16	17-20	21-24	25-28	29-32	33-26	37-40
Frequency:	2	5	8	9	12	14	14	15	11	13

SOLUTION Here, the classes are not in the inclusive form. So, we first convert them in inclusive form by subtracting $h/2$ from the lower limit and adding $h/2$ to the upper limit of each class, where h is the difference between the lower limit of a class and the upper limit of the preceding class.

Class	Frequency	Class	Frequency
0.5-4.5	24	20.5-24.5	30
4.5-8.5	40	24.5-28.5	22
8.5-12.5	40	28.5-32.5	22
12.5-16.5	33	32.5-36.5	16
16.5-20.5	28	36.5-36.5	7

To find the modal class, we use the grouping method:

Grouping Table

Class:	Frequency:					
	Col I	Col II	Col III	Col IV	Col V	Col VI
0.5-4.5	2					
4.5-8.5	5	7		15		
8.5-12.5	8		13		22	29
12.5-16.5	9	17				
16.5-20.5	12		21	35		
20.5-24.5	14	26			(40)	
24.5-28.5	14	(29)	(28)			(43)
28.5-32.5	(15)			(40)	39	
32.5-36.5	11		26			
36.5-40.5	13	24				

Analysis Table

Col. No.	0.5-4.5	4.5-8.5	8.5-12.5	12.5-16.5	16.5-20.5	20.5-24.5	24.5-28.5	28.5-32.5	32.5-36.5	36.5-40.5
I										
II										
III										
IV										
V										
VI										
Total					1	3	5	4	1	

Since 24.5-28.5 has the maximum number of bars. So, 24.5-28.5 is the modal class.

$$\therefore l = 24.5, h = 4, f = 14, f_1 = 14, f_2 = 15$$

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 24.5 + \frac{14 - 14}{28 - 14 - 15} \times 4$$

$$\Rightarrow \text{Mode} = 24.5 + 0 = 24.5$$

EXAMPLE 6 The following table shows the age distribution of cases of a certain disease admitted during a year in a particular hospital.

Age (in years):	5 - 14	15 - 24	25 - 34	35 - 44	45 - 54	55 - 64
No. of cases:	6	11	21	23	14	5

Find the average age for which maximum cases occurred.

SOLUTION Here, class intervals are not in inclusive form. So, we first convert them in inclusive form by subtracting $h/2$ from the lower limit and adding $h/2$ to the upper limit of each class, where h is the difference between the lower limit of a class and the upper limit of the preceding class. The given frequency distribution in inclusive form is as follows.

Age (in years):	4.5 - 14.5	14.5 - 24.5	24.5 - 34.5	34.5 - 44.5	44.5 - 54.5	54.5 - 64.5
No. of cases:	6	11	21	23	14	5

We observe that the class 34.5 - 44.5 has the maximum frequency. So, it is the modal class such that

$$l = 34.5, h = 10, f = 23, f_1 = 21 \text{ and } f_2 = 14$$

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 34.5 + \frac{23 - 21}{46 - 21 - 14} \times 10 = 34.5 + \frac{2}{11} \times 10 = 36.31$$

EXERCISE 15.5

LEVEL-1

1. Find the mode of the following data:

- (i) 3, 5, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4
- (ii) 3, 3, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4
- (iii) 15, 8, 26, 25, 24, 15, 18, 20, 24, 15, 19, 15

2. The shirt sizes worn by a group of 200 persons, who bought the shirt from a store, are as follows:

Shirt size:	37	38	39	40	41	42	43	44
Number of persons:	15	25	39	41	36	17	15	12

Find the modal shirt size worn by the group.

3. Find the mode of the following distribution.

- (i) Class-interval: 0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80
Frequency: 5 8 7 12 28 20 10 10
- (ii) Class-interval: 10-15 15-20 20-25 25-30 30-35 35-40
Frequency: 30 45 75 35 25 15
- (iii) Class-interval: 25-30 30-35 35-40 40-45 45-50 50-60
Frequency: 25 34 50 42 38 14

4. Compare the modal ages of two groups of students appearing for an entrance test:

Age (in years):	16-18	18-20	20-22	22-24	24-26
Group A:	50	78	46	28	23
Group B:	54	89	40	25	17

5. The marks in science of 80 students of class X are given below: Find the mode of the marks obtained by the students in science.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency:	3	5	16	12	13	20	5	4	1	1

6. The following is the distribution of height of students of a certain class in a certain city:

Height (in cms):	160-162	163-165	166-168	169-171	172-174
No. of students:	15	118	142	127	18

Find the average height of maximum number of students.

7. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years):	5-15	15-25	25-35	35-45	45-55	55-65
No. of students:	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency. **[NCERT]**

8. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours):	0-20	20-40	40-60	60-80	80-100	100-120
No. of components:	10	35	52	61	38	29

Determine the modal lifetimes of the components. **[NCERT]**

9. The following table gives the daily income of 50 workers of a factory:

Daily income (in ₹)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers:	12	14	8	6	10

Find the mean, mode and median of the above data. **[CBSE 2009]**

10. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret, the two measures: **[NCERT]**

<i>Number of students per Teacher</i>	<i>Number of States/U.T.</i>	<i>Number of students per Teacher</i>	<i>Number of States/U.T.</i>
15 - 20	3	35 - 40	3
20 - 25	8	40 - 45	0
25 - 30	9	45 - 50	0
30 - 35	10	50 - 55	2

[NCERT]

11. Find the mean, median and mode of the following data:

Classes:	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency:	2	3	5	6	5	3	1

[CBSE 2008]

12. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	7	14	13	12	20	11	15	8

[NCERT]

13. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption: (in units) 65-85 85-105 105-125 125-145 145-165 165-185 185-205

No. of consumers: 4 5 13 20 14 8 4

14. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters: 1-4 4-7 7-10 10-13 13-16 16-19

Number surnames: 6 30 40 16 4 4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

15. Find the mean, median and mode of the following data:

Classes: 0-20 20-40 40-60 60-80 80-100 100-120 120-140

Frequency: 6 8 10 12 6 5 3 [CBSE 2008]

LEVEL-2

16. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure: [NCERT]

Expenditure (in ₹)	Frequency	Expenditure (in ₹)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

17. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsman	Runs scored	Number of batsman
3000-4000	4	7000-8000	6
4000-5000	18	8000-9000	3
5000-6000	9	9000-10000	1
6000-7000	7	10000-11000	1

Find the mode of the data.

18. The frequency distribution table of agriculture holdings in a village is given below:

Area of land (in hectares): 1-3 3-5 5-7 7-9 9-11 11-13

Number of families: 20 45 80 55 40 12

Find the modal agriculture holdings of the village.

[NCERT EXEMPLAR]

19. The monthly income of 100 families are given as below:

Income in (in ₹)	Number of families
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income.

[NCERT EXEMPLAR]

ANSWERS

1. (i) 5 (ii) 3 (iii) 15 2. 40
 3. (i) 46.67 (ii) 22.14 (iii) 38.33
 4. Group A: 18.93 years, Group B: 18.83 years 5. 53.17 6. 167.35
 7. Mode = 36.8 years, Mean = 35.37 years 8. 65.625 hours
 9. Mean = 145.20, Median = 138.57, Mode = 125
 10. Mode = 30.6, Mean = 29.2 11. Median = 170.83, Mean = 169, Mode = 175
 12. Mode = 44.7 cars
 13. Median = 137 units, Mean = 137.05 units, Mode = 135.76 units
 14. Median = 8.05, Mean = 8.32, Modal size = 7.88
 15. Median = 61.66, Mean = 62.4, Mode = 65
 16. ₹ 1847.83, ₹ 2662.50 17. Mode = 4608.7 runs
 18. 6.2 hectares 19. ₹ 11875

15.6 MERITS, DEMERITS AND USES OF MODE

The following are some merits and demerits of mode :

MERITS

- It is readily comprehensible and easy to compute. In some case it can be computed merely by inspection.
- It is not affected by extreme values. It can be obtained even if the extreme values are not known.
- Mode can be determined in distributions with open classes.
- Mode can be located on graph also.

DEMERITS

- It is ill-defined. It is not always possible to find a clearly defined mode. In some cases, we may come across distributions with two modes. Such distributions are called bimodal. If a distribution has more than two modes, it is said to be multimodal.
- It is not based upon all the observation.
- Mode can be calculated by various formulae as such the value may differ from one to other. Therefore, it is not rigidly defined.
- It is affected to a greater extent by fluctuations of sampling.

USES OF MODE

Mode is used by the manufacturers of readymade garments, shoes and accessories in common use etc. The readymade garment manufacturers made those sizes more which are used by most of the persons than other sizes. Similarly, the makers of shoes will make that size maximum which the majority people use and others in less quantity.

15.7 RELATIONSHIP AMONG MEAN, MEDIAN AND MODE

We have learnt about three measures of central value, namely, arithmetic mean, median and mode. These three measures are closely connected by the following relations.

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

or,
$$\text{Median} = \text{Mode} + \frac{2}{3}(\text{Mean} - \text{Mode})$$

or,
$$\text{Mean} = \text{Mode} + \frac{3}{2}(\text{Median} - \text{Mode})$$

15.8 CUMULATIVE FREQUENCY POLYGON CURVE (AN OGIVE)

In class IX, we have learnt about graphical representation of frequency distributions by using bargraphs, histograms and frequency polygons. In this section, we will learn about the construction of cumulative frequency polygon and cumulative frequency curves or ogives. The technique of drawing the cumulative frequency polygons and cumulative frequency curves or ogives is more or less the same. The only difference is that in case of simple frequency curves and polygons the frequencies are plotted against class marks of the class intervals whereas in case of a cumulative frequency polygon or curves the cumulative frequencies are plotted against the lower or upper limits of the class intervals depending upon the manner in which the series has been cumulated. There are two methods of constructing a frequency polygon and an ogive. Let us now discuss the two methods.

(i) Less than method

(ii) More than method.

15.8.1 LESS THAN METHOD

To construct a cumulative frequency polygon and an ogive by less than method, we use the following algorithm

ALGORITHM

STEP I Start with the upper limits of class intervals and add class frequencies to obtain the cumulative frequency distribution.

STEP II Mark upper class limits along X-axis on a suitable scale.

STEP III Mark cumulative frequencies along Y-axis on a suitable scale.

STEP IV Plot the points (x_i, f_i) , where x_i is the upper limit of a class and f_i is corresponding cumulative frequency.

STEP V Join the points obtained in step IV by a free hand smooth curve to get the ogive and to get the cumulative frequency polygon join the points obtained in step IV by line segments.

15.8.2 MORE THAN METHOD

To construct a cumulative frequency polygon and an ogive by more than method, we use the following algorithm.

ALGORITHM

- STEP I** Start with the lower limits of the class intervals and from the total frequency subtract the frequency of each class to obtain the cumulative frequency distribution.
- STEP II** Mark the lower class limits along X-axis on a suitable scale.
- STEP III** Mark the cumulative frequencies along Y-axis on a suitable scale.
- STEP IV** Plot the points (x_i, f_i) , where x_i is the lower limit of a class and f_i is the corresponding cumulative frequency.
- STEP V** Join the points obtained in step IV by a free hand smooth curve to get the ogive and to get the cumulative frequency polygon join these points by line segments.

Following examples illustrate the above algorithm.

ILLUSTRATIVE EXAMPLES**LEVEL-1**

EXAMPLE 1 Draw an ogive and the cumulative frequency polygon for the following frequency distribution by less than method.

Marks:	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of students:	7	10	23	51	6	3

SOLUTION We first prepare the cumulative frequency distribution table by less than method as given below:

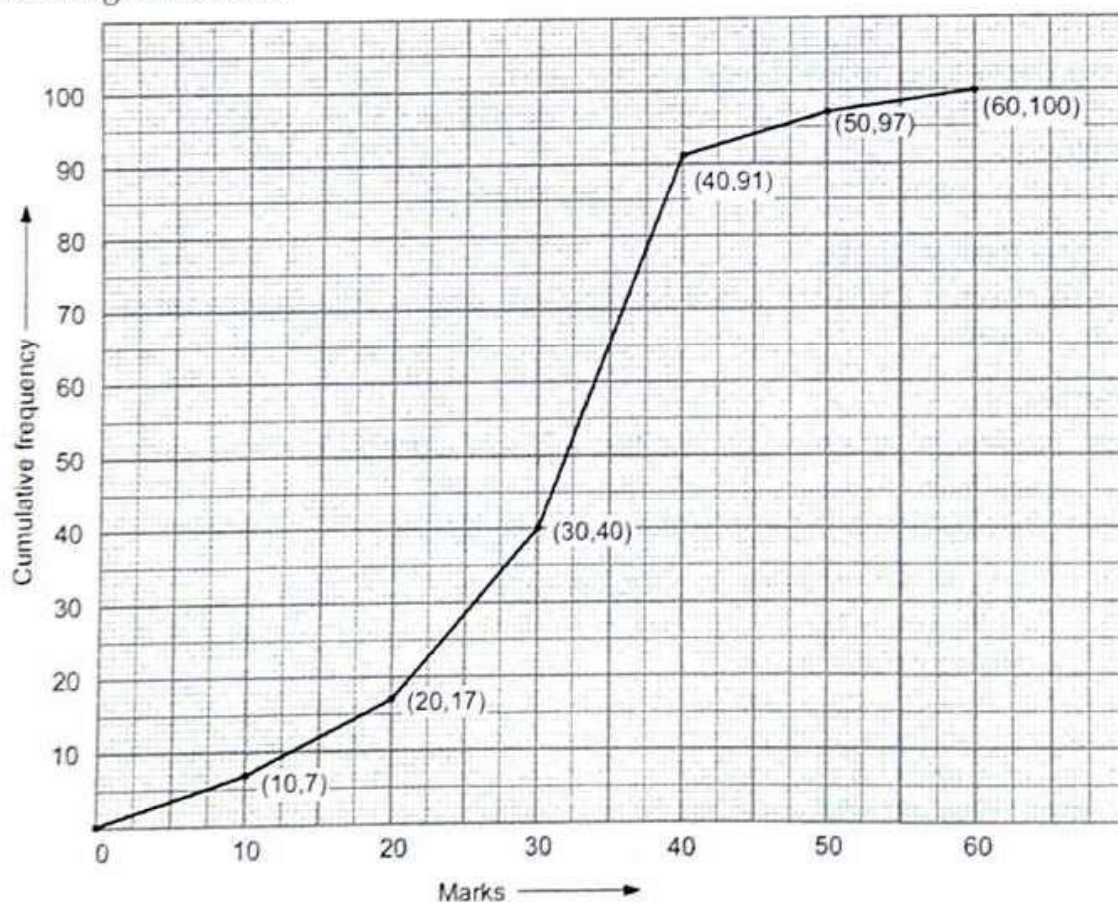


Fig. 15.1 Cumulative frequency curve or ogive

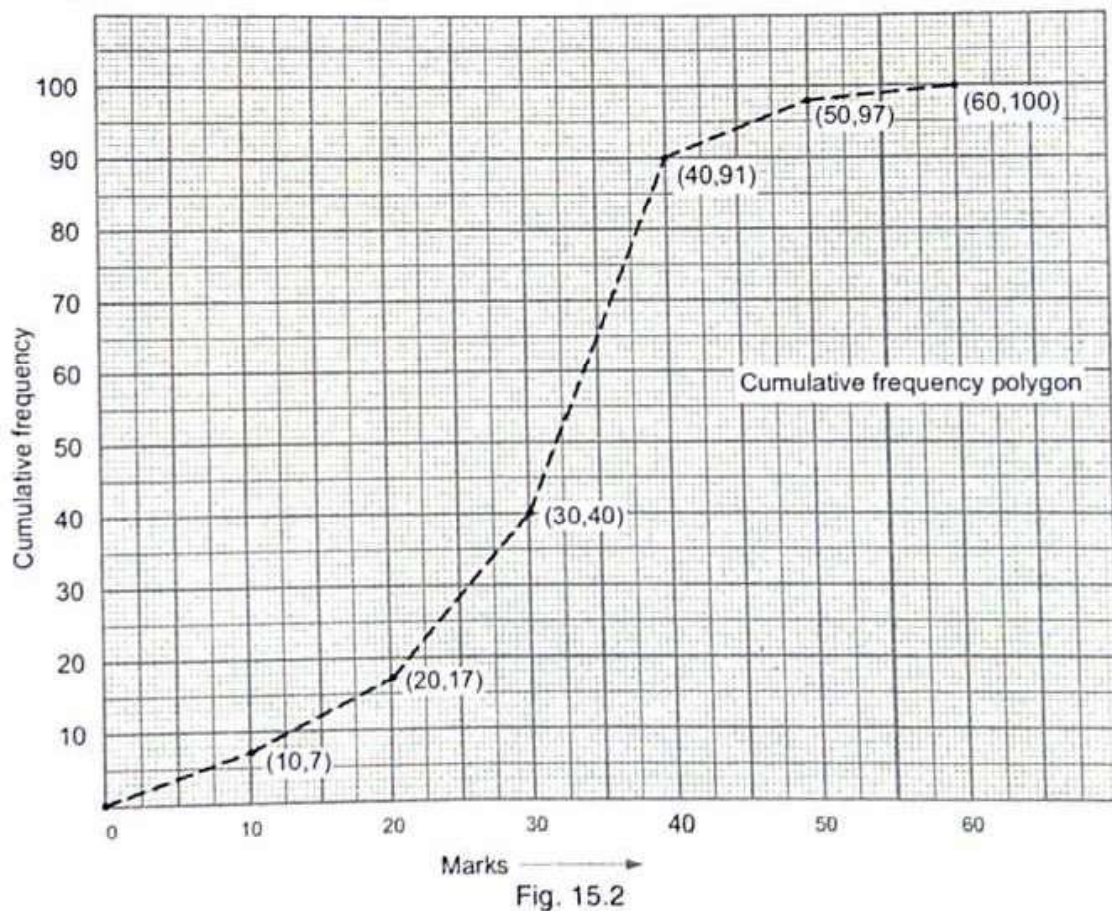
Marks	No. of Students	Marks less than	Cumulative Frequency
0 - 10	7	10	7
10 - 20	10	20	17
20 - 30	23	30	40
30 - 40	51	40	91
40 - 50	6	50	97
50 - 60	2	60	100

Other than the given class intervals, we assume a class - 10 - 0 before the first class interval 0 - 10 with zero frequency.

Now, we mark the upper class limits (including the imagined class) along X-axis on a suitable scale and the cumulative frequencies along Y-axis on a suitable scale.

Thus, we plot the points (0, 0), (10, 7), (20, 17), (30, 40), (40, 91), (50, 97) and (60, 100).

Now, we join the plotted points by a free hand curve to obtain the required ogive as shown in Fig. 15.1. In order to obtain the cumulative frequency polygon, we join the plotted points by line segments as shown in Fig. 15.2.



EXAMPLE 2 Draw a cumulative frequency curve and cumulative frequency polygon for the following frequency distribution by less than method.

Age (in years):	0 - 9	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69
No. of Persons:	5	15	20	23	17	11	9

SOLUTION The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under:

Age (in years)	Frequency	Age less than	Cumulative Frequency
-0.5 - 9.5	5	9.5	5
9.5 - 19.5	15	19.5	20
19.5 - 29.5	20	29.5	40
29.5 - 39.5	23	39.5	63
39.5 - 49.5	17	49.5	80
49.5 - 59.5	11	59.5	91
59.5 - 69.5	9	69.5	100

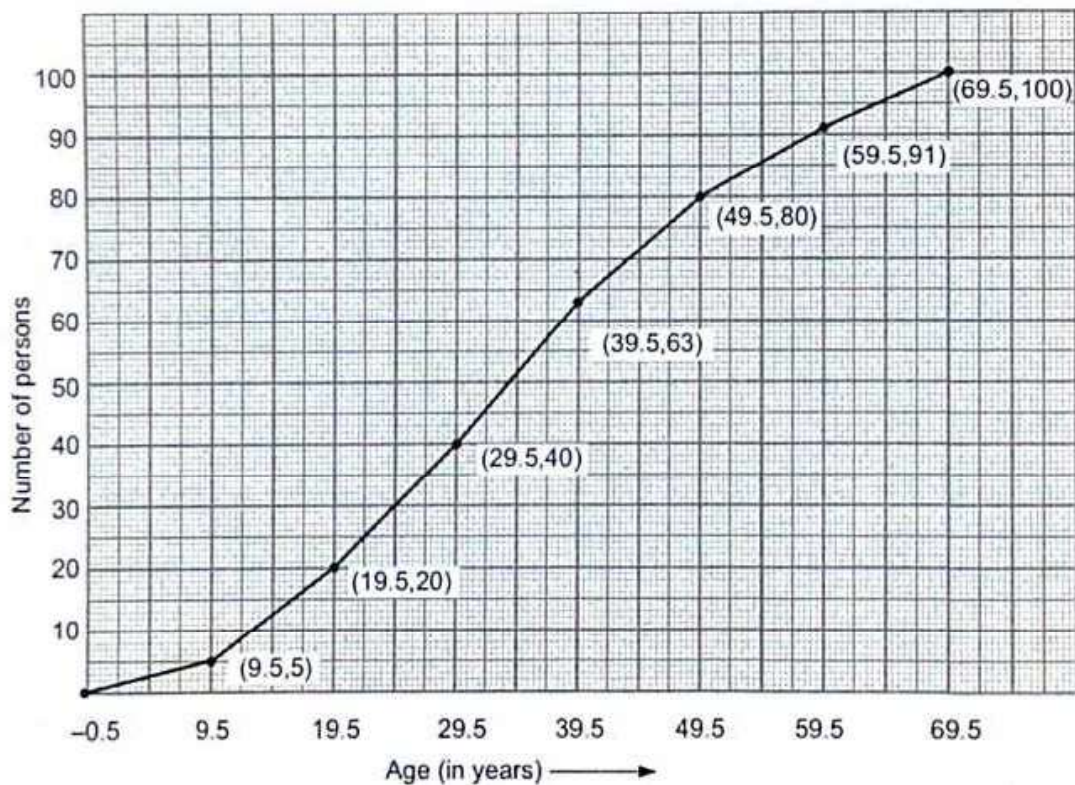


Fig. 15.3 Cumulative frequency curve

Now, we plot points $(9.5, 5)$, $(19.5, 20)$, $(29.5, 40)$, $(39.5, 63)$, $(49.5, 80)$, $(59.5, 91)$ and $(69.5, 100)$ and join them by a free hand smooth curve to obtain the required ogive as shown in Fig. 15.3. The cumulative frequency polygon is obtained by joining these points by line segments as shown in Fig. 15.4.

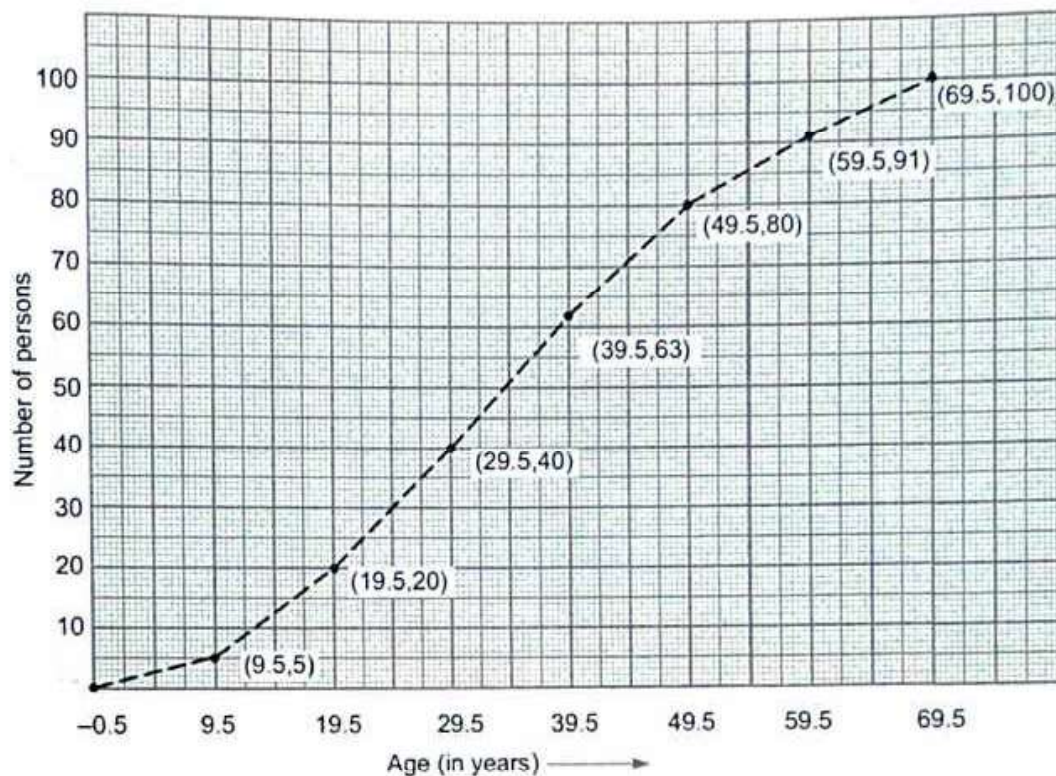


Fig. 15.4 Cumulative frequency polygon

LEVEL-2

EXAMPLE 3 The frequency distribution of scores obtained by 230 candidates in a medical entrance test is as follows.

Scores:	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
Number of Candidates:	20	35	40	32	24	27	18	24

Draw cumulative frequency curves by less than and more than method on the same axes. Also, draw the two types of cumulative frequency polygons.

SOLUTION *Less than method:* We first prepare the cumulative frequency table by less than method as given below:

Scores	Number of Candidates	Scores less than	Cumulative Frequency
400 - 450	20	450	20
450 - 500	35	500	55
500 - 550	40	550	95
550 - 600	32	600	127
600 - 650	24	650	151
650 - 700	27	700	178
700 - 750	18	750	196
750 - 800	34	800	230

Other than the given class intervals, we assume a class interval 350 - 400 prior the first class interval 400 - 500 with zero frequency.

SOLUTION The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency distribution as under:

<i>Age (in years)</i>	<i>Frequency</i>	<i>Age less than</i>	<i>Cumulative Frequency</i>
-0.5 - 9.5	5	9.5	5
9.5 - 19.5	15	19.5	20
19.5 - 29.5	20	29.5	40
29.5 - 39.5	23	39.5	63
39.5 - 49.5	17	49.5	80
49.5 - 59.5	11	59.5	91
59.5 - 69.5	9	69.5	100

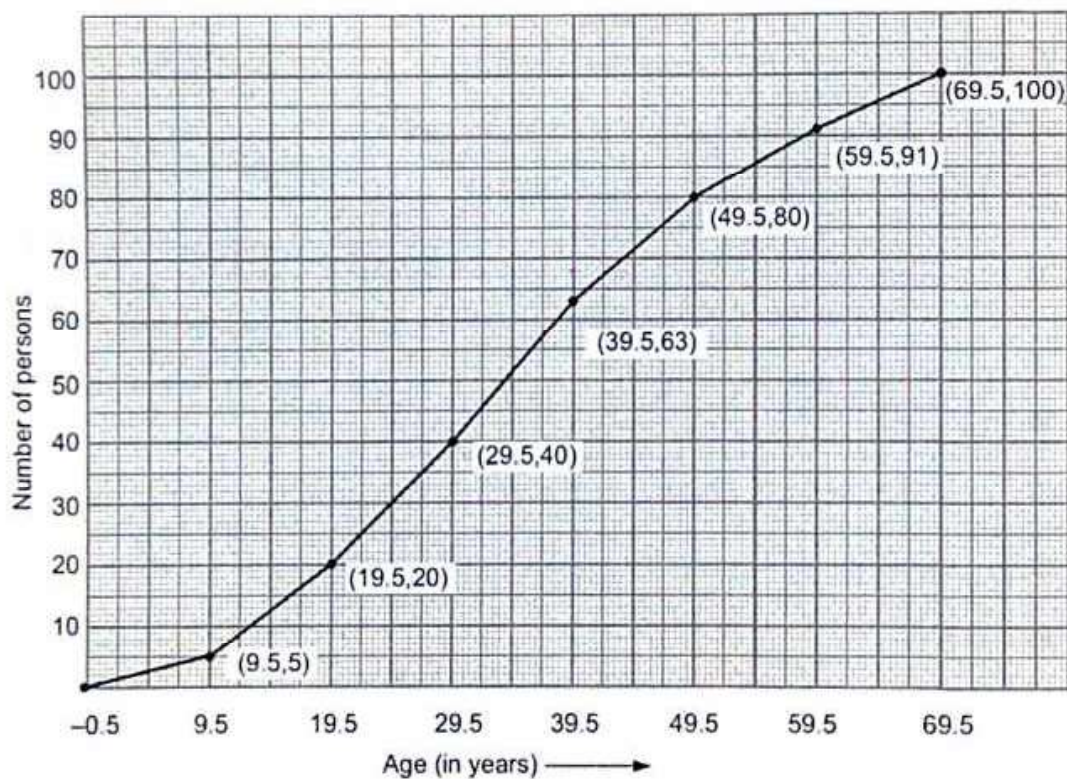


Fig. 15.3 Cumulative frequency curve

Now, we plot points $(9.5, 5)$, $(19.5, 20)$, $(29.5, 40)$, $(39.5, 63)$, $(49.5, 80)$, $(59.5, 91)$ and $(69.5, 100)$ and join them by a free hand smooth curve to obtain the required ogive as shown in Fig. 15.3. The cumulative frequency polygon is obtained by joining these points by line segments as shown in Fig. 15.4.

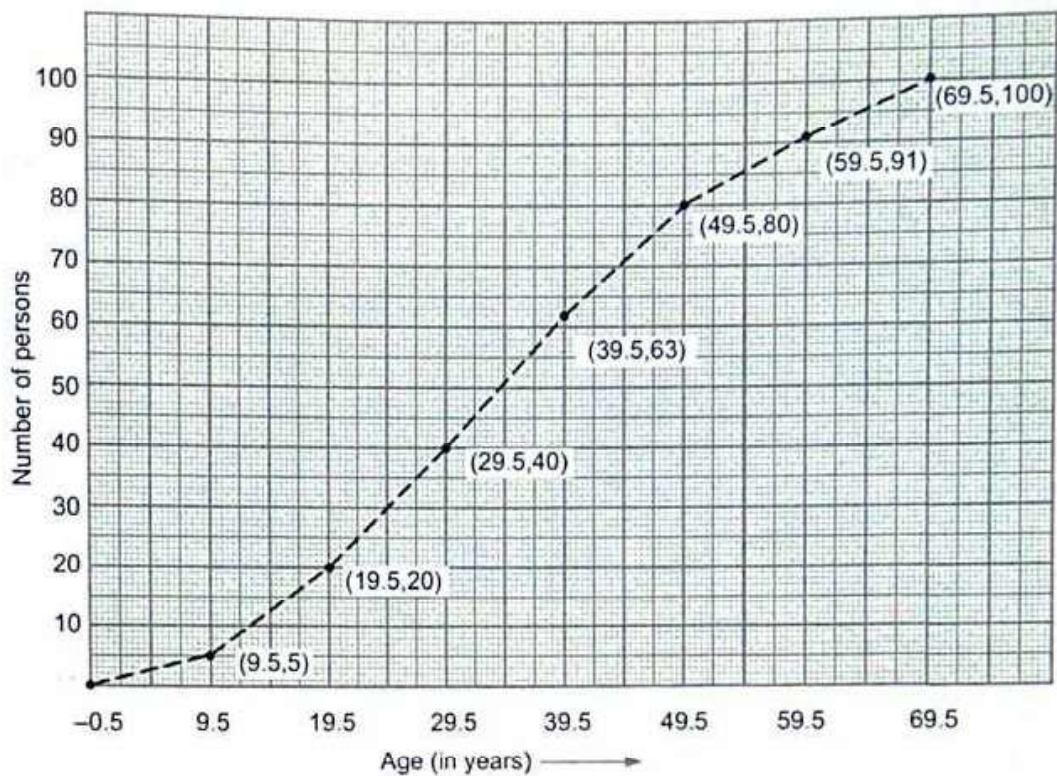


Fig. 15.4 Cumulative frequency polygon

LEVEL-2

EXAMPLE 3 The frequency distribution of scores obtained by 230 candidates in a medical entrance test is as follows.

Scores: 400-450 450-500 500-550 550-600 600-650 650-700 700-750 750-800
 Number of Candidates: 20 35 40 32 24 27 18 24

Draw cumulative frequency curves by less than and more than method on the same axes. Also, draw the two types of cumulative frequency polygons.

SOLUTION *Less than method:* We first prepare the cumulative frequency table by less than method as given below:

Scores	Number of Candidates	Scores less than	Cumulative Frequency
400 - 450	20	450	20
450 - 500	35	500	55
500 - 550	40	550	95
550 - 600	32	600	127
600 - 650	24	650	151
650 - 700	27	700	178
700 - 750	18	750	196
750 - 800	34	800	230

Other than the given class intervals, we assume a class interval 350 - 400 prior the first class interval 400 - 500 with zero frequency.

Now, we mark the upper class limits on X-axis and the cumulative frequencies along Y-axis on suitable scales.

Thus, we plot the points $(400,0)$, $(450, 20)$, $(500, 55)$, $(550, 95)$, $(600, 127)$, $(650, 151)$, $(700, 178)$, $(750, 196)$ and $(800, 230)$.

Join these points by a free hand smooth curve to obtain an ogive by less than method as shown in Fig. 15.5.

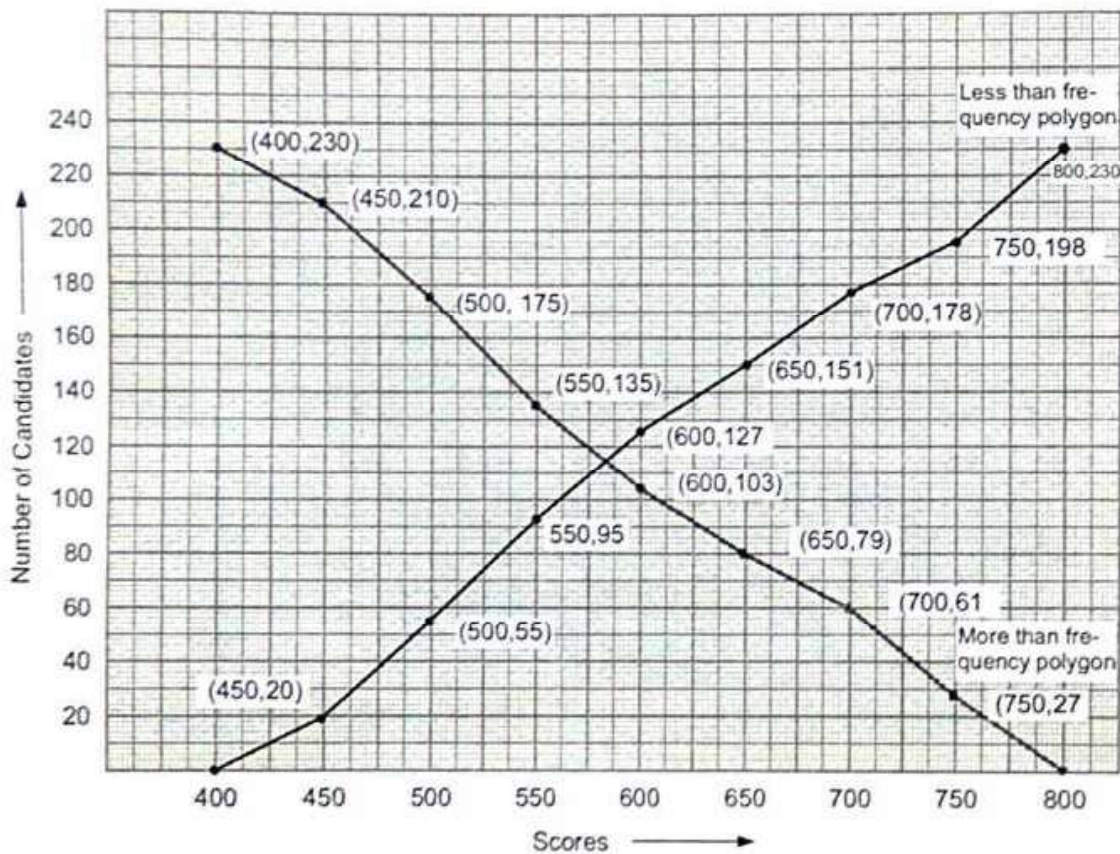


Fig. 15.5

In order to obtain the cumulative frequency polygon by less than method join these points by line segments as given in Fig. 15.6.

Other than the given class intervals, we assume class interval 800 - 850 after the last class interval 750 - 800 with zero frequency.

Now, we mark the lower class limits on X-axis and the cumulative frequencies along Y-axis on suitable scales.

Thus, we plot the points $(400, 230)$, $(450, 210)$, $(500, 175)$, $(550, 135)$, $(600, 103)$, $(650, 79)$, $(700, 61)$, $(750, 27)$ and $(800, 0)$.

By joining these points by a free hand smooth curve, we obtain an ogive by more than method as shown in Fig. 15.3.

The cumulative frequency polygon by more than method is obtained by joining these points by line segments as shown in Fig. 15.4.

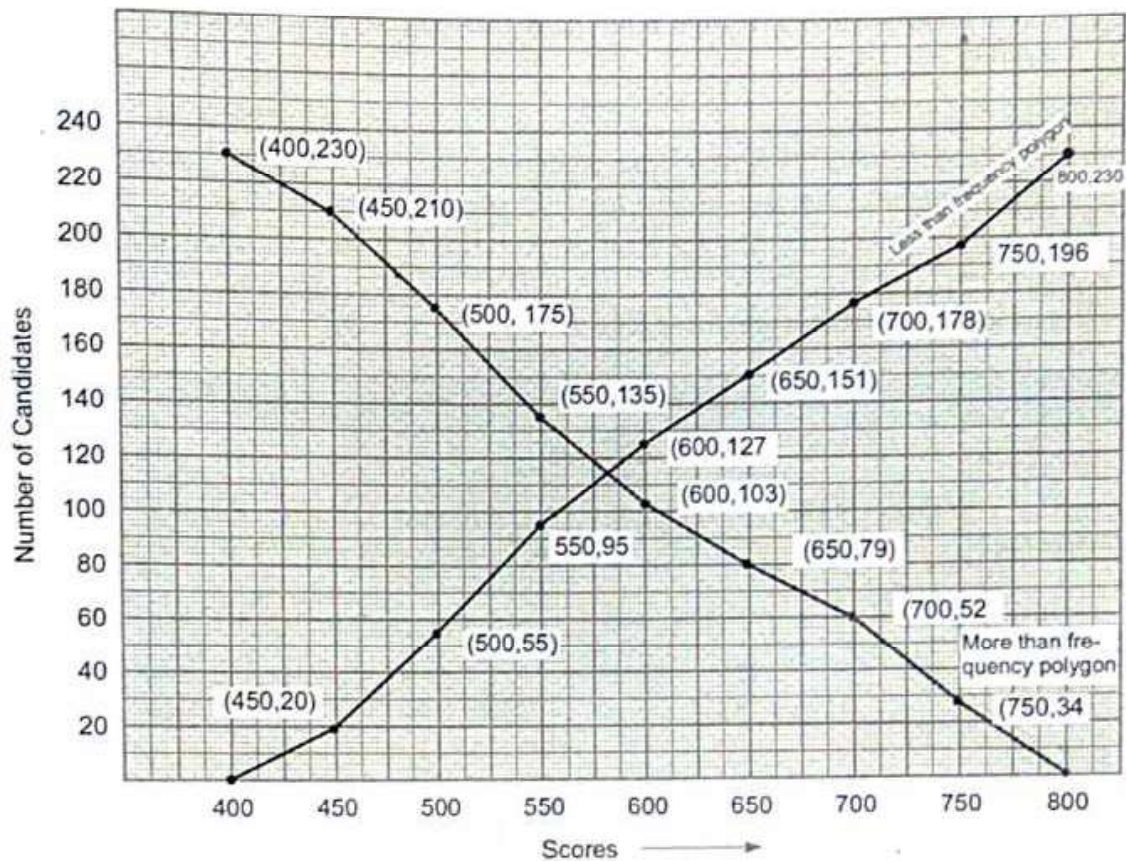


Fig. 15.6

More than method: Let us first prepare the cumulative frequency table by more than method as given below:

Scores	Number of Candidates	Scores more than	Cumulative Frequency
400 - 450	20	400	230
450 - 500	35	450	210
500 - 550	40	500	175
550 - 600	32	550	135
600 - 650	24	600	103
650 - 700	27	650	79
700 - 750	18	700	52
750 - 800	34	750	34

15.8.3 SOME APPLICATIONS OF OGIVES

Ogives can be used to find the median of a frequency distribution. In order to determine the same, we may use the following algorithms:

ALGORITHM 1

STEP I Draw any one of the two types of frequency curves on the graph paper.

STEP II Compute $\frac{N}{2}$ ($N = \sum f_i$) and mark the corresponding point on y-axis.

STEP III Draw a line parallel to x-axis, from the point marked in step II, cutting the cumulative frequency curve at a point P (say).

STEP IV Draw perpendicular PM from P on the X-axis. The x-coordinate of point M gives the median.

ALGORITHM 2

STEP I Draw less than type and greater than type cumulative frequency curves on the graph paper.

STEP II Mark the point of intersection of the two curves drawn in step I. Let this point be P

STEP III Draw perpendicular PM from P on the X-axis. The x-coordinate of point M gives the Median.

Following examples will illustrate these algorithms.

ILLUSTRATIVE EXAMPLES

LEVEL-2

EXAMPLE 1 Following is the age distribution of a group of students. Draw the cumulative frequency polygon, cumulative frequency curve (less than type) and hence obtain the median value.

Age	Frequency	Age	Frequency
5 - 6	40	11 - 12	92
6 - 7	56	12 - 13	80
7 - 8	60	13 - 14	64
8 - 9	66	14 - 15	44
9 - 10	84	15 - 16	20
10 - 11	96	16 - 17	8

SOLUTION We first prepare the cumulative frequency table by less than method as given below.

Age	Frequency	Age less than	Cumulative frequency
5-6	40	6	40
6-7	56	7	96
7-8	60	8	156
8-9	66	9	222
9-10	84	10	306
10-11	96	11	402
11-12	92	12	494
12-13	80	13	574
13-14	64	14	638
14-15	44	15	682
15-16	20	16	702
16-17	8	17	710

Other than the given class intervals, we assume a class 4-5 before the first class-interval 5-6 with zero frequency.

Now, we mark the upper class limits (including the imagined class) along X-axis on a suitable scale and the cumulative frequencies along Y-axis on a suitable scale.

Thus, we plot the points (5, 0), (6, 40), (7, 96), (8, 156), (9, 222), (10, 306), (11, 402), (12, 494), (13, 574), (14, 638), (15, 682), (16, 702) and (17, 710). These points are marked and joined by line segments to obtain the cumulative frequency polygon shown in Fig. 15.7.

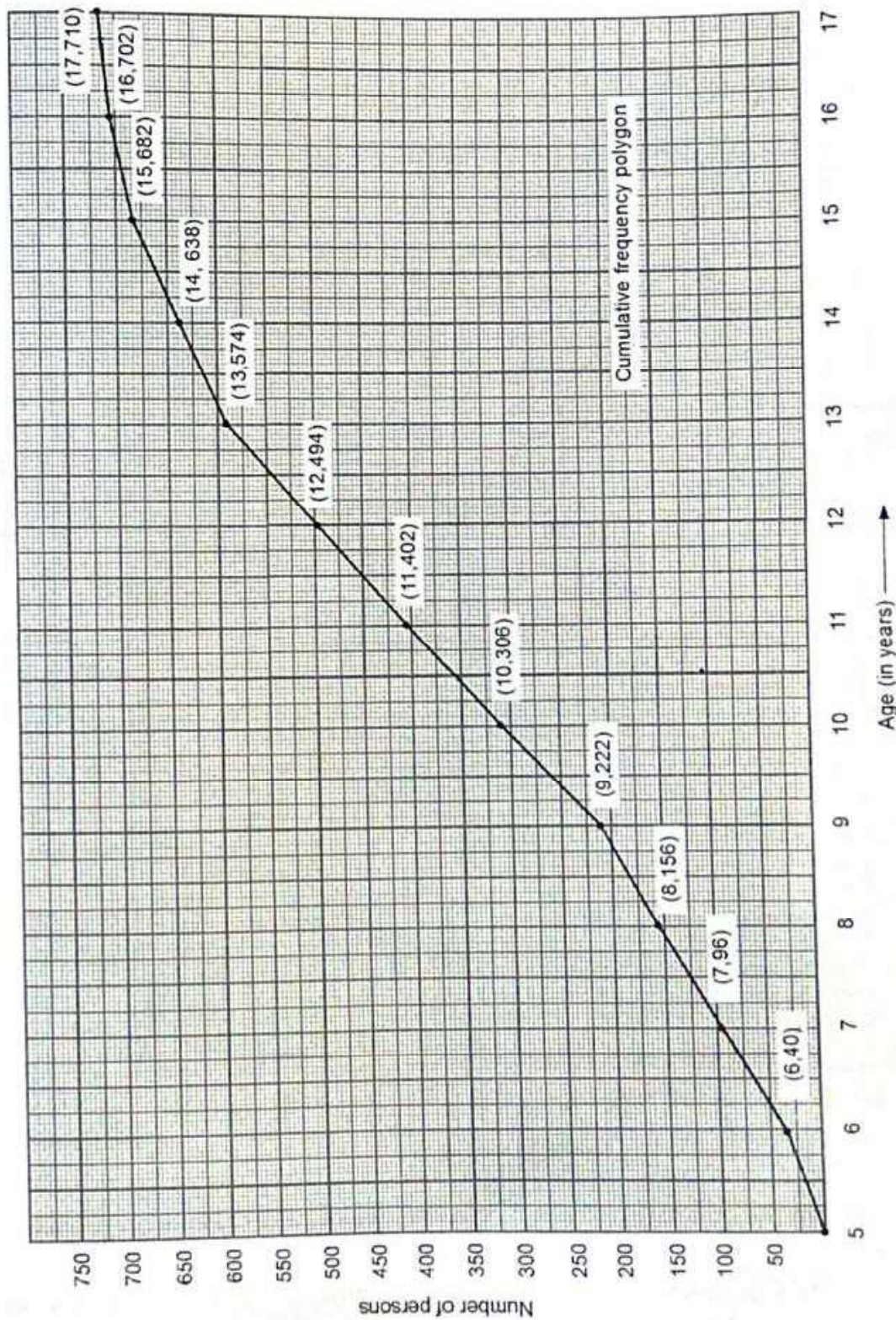


Fig. 15.7

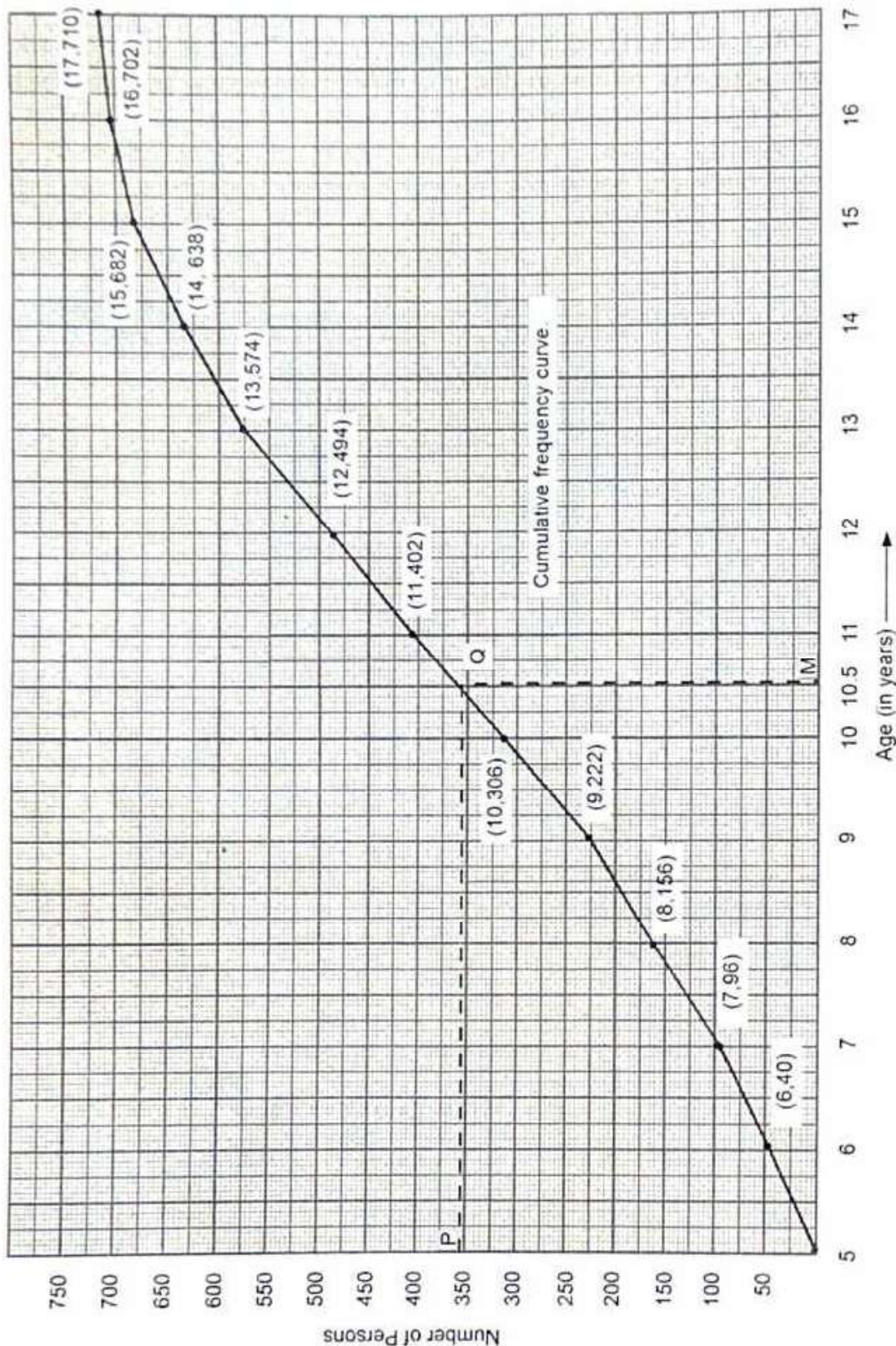


Fig. 15.8

In order to obtain the cumulative frequency curve, we draw a smooth curve passing through the points discussed above.

The graph (Fig. 15.8) shows the total number of students as 710. The median is the age corresponding to $\frac{N}{2} = \frac{710}{2} = 355$ students. In order to find the median, we first locate the

point corresponding to 355th student on Y-axis. Let the point be P . From this point draw a line parallel to the X-axis cutting the curve at Q . From this point Q draw a line parallel to Y-axis and meeting X-axis at the point M . The x-coordinate of M is 10.5 (See Fig. 15.8). Hence, median is 10.5.

EXAMPLE 2 The following observations relate to the height of a group of persons. Draw the two types of cumulative frequency polygons and cumulative frequency curves and determine the median.

Height in cms	Frequency
140-143	3
143-146	9
146-149	26
149-152	31
152-155	45
155-158	64
158-161	78
161-164	85
164-167	96
167-170	72
170-173	60
173-176	43
176-179	20
179-182	6

SOLUTION *Less than method:* We first prepare the cumulative frequency table by less than method as given below:

Height in cms	Frequency	Height less than	Cumulative frequency
140-143	3	143	3
143-146	9	146	12
146-149	26	149	38
149-152	31	152	69
152-155	45	155	114
155-158	64	158	178
158-161	78	161	256
161-164	85	164	341
164-167	96	167	437
167-170	72	170	509
170-173	60	173	569
173-176	43	176	612
176-179	20	179	632
179-182	06	182	638

Other than the given class intervals, we assume a class interval 137-140 prior to the first class interval 140-143 with zero frequency.

Now, we mark the upper class limits on X-axis and cumulative frequencies along Y-axis on a suitable scale. We plot the points (140, 0), (143, 3), (146, 12), (149, 38), (152, 69), (155, 114), (158, 178), (161, 256), (164, 341), (167, 437), (170, 509), (173, 569), (176, 612), (179, 632) and (182, 638).

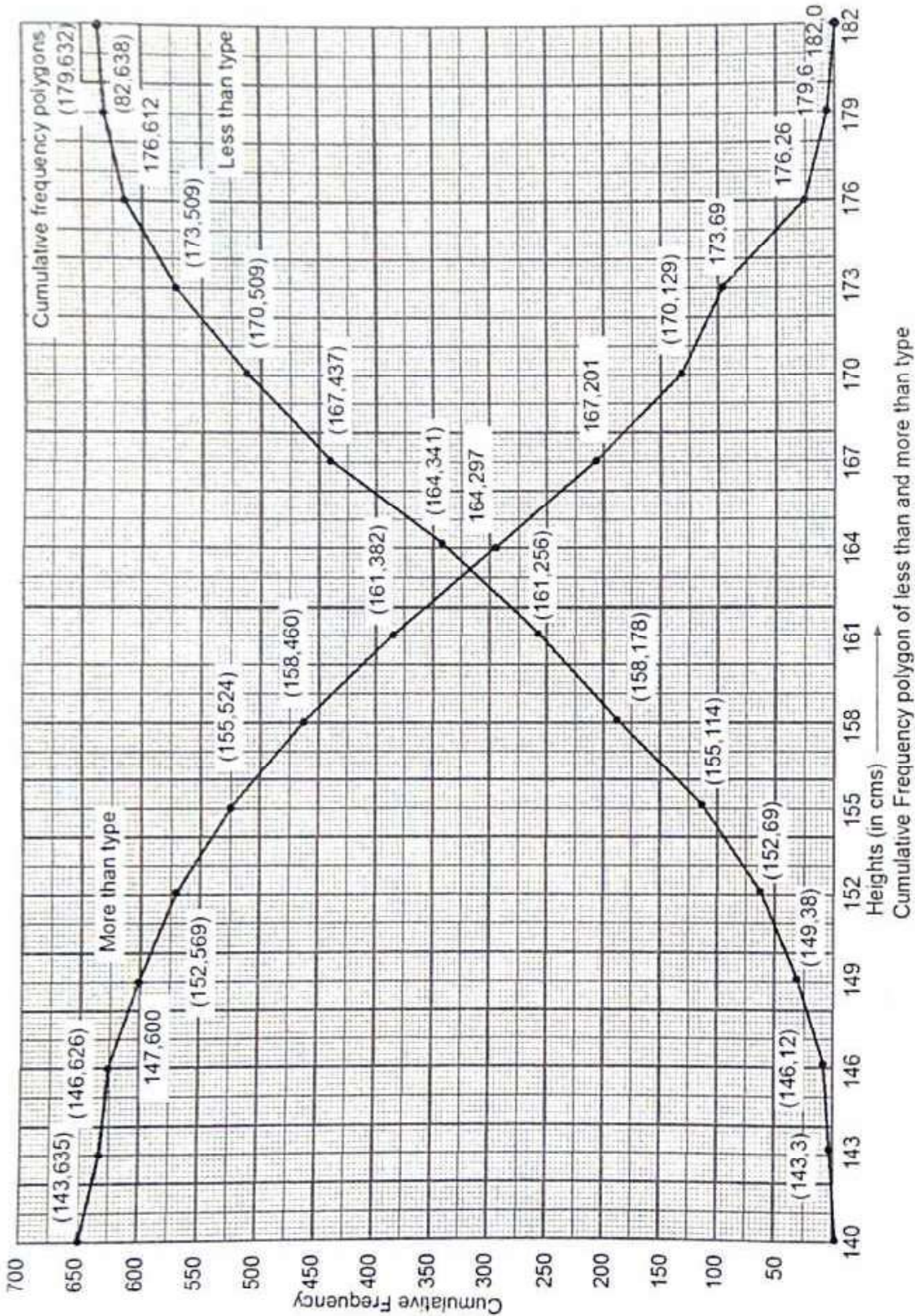


Fig. 15.9

These points are joined by line segments to obtain the cumulative frequency polygon as shown in Fig. 15.9 and by a free hand smooth curve to obtain an ogive by less than method as shown in fig. 15.10.

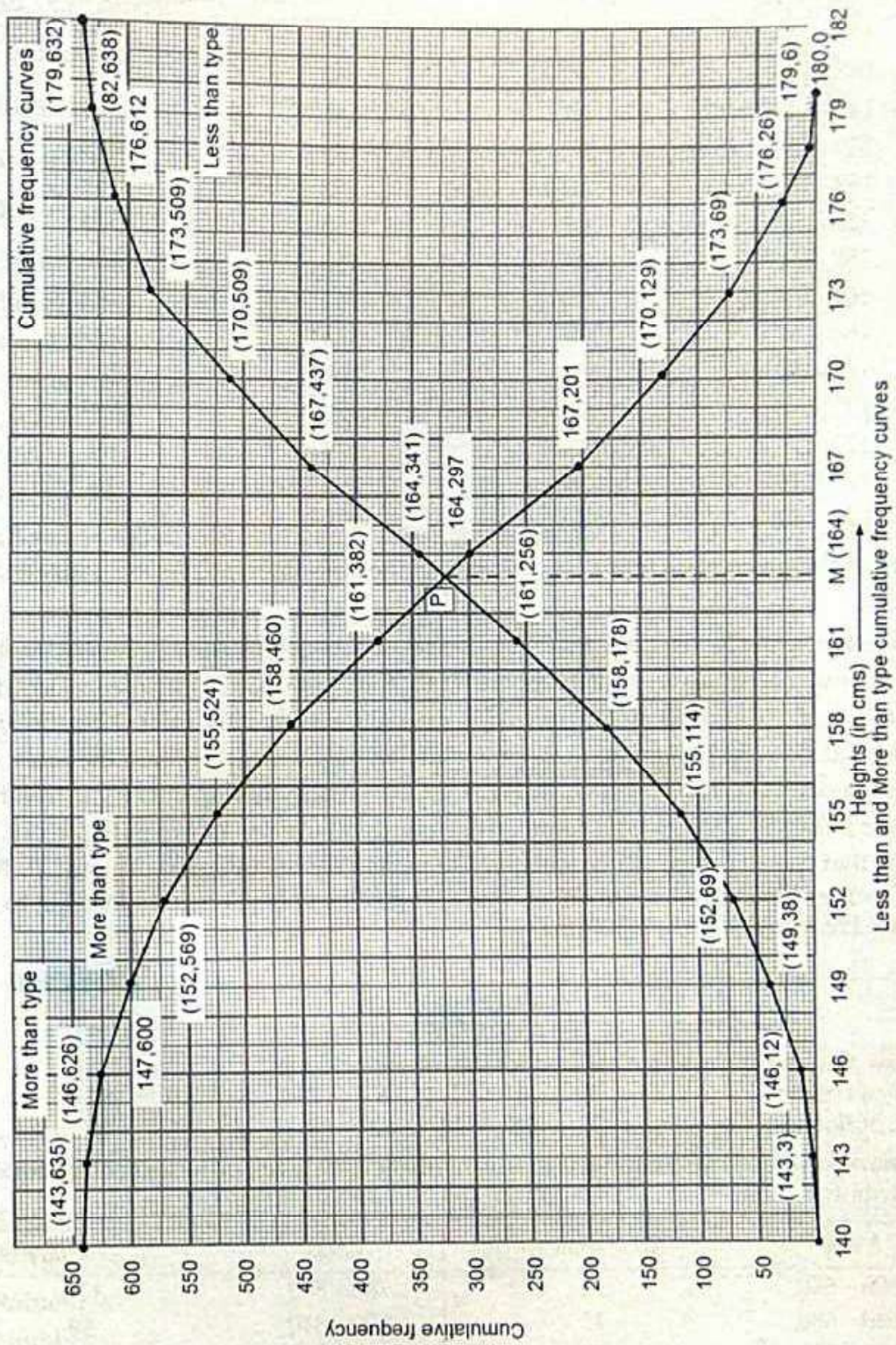


Fig. 15.10

More than method: We prepare the cumulative frequency table by more than method as given below:

Height in cms	Frequency	Height more than	Cumulative frequency
140 - 143	3	140	638
143 - 146	9	143	635
146 - 149	26	146	626
149 - 152	31	149	600
152 - 155	45	152	569
155 - 158	64	155	524
158 - 161	78	158	460
161 - 164	85	161	382
164 - 167	96	164	297
167 - 170	72	167	201
170 - 173	60	170	129
173 - 176	43	173	69
176 - 179	20	176	26
179 - 182	6	179	6

Other than the given class intervals, we assume the class interval 182-185 with zero frequency.

Now, we mark the lower class limits on X-axis and the cumulative frequencies along Y-axis on suitable scales to plot the points (140, 638), (143, 635), (146, 626), (149, 600), (152, 569), (155, 524), (158, 460), (161, 382), (164, 297), (167, 201), (170, 129), (173, 69), (176, 26) and (179, 6). By joining these points by line segments, we obtain the more than type frequency polygon as shown in Fig. 15.9. By joining these points by a free hand curve, we obtain more than type cumulative frequency curve as shown in Fig. 15.10.

We find that the two types of cumulative frequency curves intersect at point P . From point P perpendicular PM is drawn on X-axis. The value of height corresponding to M is 163.2 cm. Hence, median is 163.2 cm.

EXERCISE 15.6

LEVEL-1

1. Draw an ogive by less than method for the following data:

No. of rooms:	1	2	3	4	5	6	7	8	9	10
No. of houses:	4	9	22	28	24	12	8	6	5	2

2. The marks scored by 750 students in an examination are given in the form of a frequency distribution table:

Marks	No. of students	Marks	No. of students
600 - 640	16	760 - 800	172
640 - 680	45	800 - 840	59
680 - 720	156	840 - 880	18
720 - 760	284		

Prepare a cumulative frequency table by less than method and draw an ogive.

3. Draw an ogive to represent the following frequency distribution:

Class-interval:	0 - 4	5 - 9	10 - 14	15 - 19	20 - 24
No. of students:	2	6	10	5	3

4. The monthly profits (in ₹) of 100 shops are distributed as follows:

Profits per shop:	0 - 50	50 - 100	100 - 150	150 - 200	200 - 250	250 - 300
No. of shops:	12	18	27	20	17	6

Draw the frequency polygon for it.

5. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in ₹):	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers:	12	14	8	6	10

Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive. [CBSE 2018]

6. The following table gives production yield per hectare of wheat of 100 farms of a village:

Production yield in kg per hectare:	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms:	2	8	12	24	38	16

Draw 'less than' ogive and 'more than' ogive.

7. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula. [CBSE 2009]

8. The annual rainfall record of a city for 66 days is given in the following table:

Rainfall (in cm):	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of days:	22	10	8	15	5	6

Calculate the median rainfall using ogives of more than type and less than type.

[NCERT EXEMPLAR]

10. In the graphical representation of a frequency distribution, if the distance between mode and mean is k times the distance between median and mean, then write the value of k .
11. Find the class marks of classes 10-25 and 35-55. [CBSE 2008]
12. Write the median class of the following distribution:
- | | | | | | | | | |
|------------|--------|---------|---------|---------|---------|---------|---------|-------------|
| Classes: | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | |
| Frequency: | 4 | 4 | 8 | 10 | 12 | 8 | 4 | [CBSE 2009] |

ANSWERS

2. zero 3. Median 4. 4 5. Mode = 3 Median - 2 Mean 6. Median
7. 20 - 25 8. 50 9. 40 - 50 10. 3 11. 17.5, 45 12. 30 - 40

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

- Which of the following is not a measure of central tendency?
(a) Mean (b) Median (c) Mode (d) Standard deviation
- The algebraic sum of the deviations of a frequency distribution from its mean is
(a) always positive (b) always negative (c) 0 (d) a non-zero number
- The arithmetic mean of 1, 2, 3, ..., n is
(a) $\frac{n+1}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n}{2}$ (d) $\frac{n}{2} + 1$
- For a frequency distribution, mean, median and mode are connected by the relation
(a) Mode = 3 Mean - 2 Median (b) Mode = 2 Median - 3 Mean
(c) Mode = 3 Median - 2 Mean (d) Mode = 3 Median + 2 Mean
- Which of the following cannot be determined graphically?
(a) Mean (b) Median (c) Mode (d) None of these
- The median of a given frequency distribution is found graphically with the help of
(a) Histogram (b) Frequency curve (c) Frequency polygon (d) Ogive
- The mode of a frequency distribution can be determined graphically from
(a) Histogram (b) Frequency polygon
(c) Ogive (d) Frequency curve
- Mode is
(a) least frequent value (b) middle most value
(c) most frequent value (d) None of these
- The mean of n observations is \bar{X} . If the first item is increased by 1, second by 2 and so on, then the new mean is
(a) $\bar{X} + n$ (b) $\bar{X} + \frac{n}{2}$ (c) $\bar{X} + \frac{n+1}{2}$ (d) None of these
- One of the methods of determining mode is
(a) Mode = 2 Median - 3 Mean (b) Mode = 2 Median + 3 Mean
(c) Mode = 3 Median - 2 Mean (d) Mode = 3 Median + 2 Mean

11. If the mean of the following distribution is 2.6, then the value of y is

Variable (x):	1	2	3	4	5
Frequency:	4	5	y	1	2

- (a) 3 (b) 8 (c) 13 (d) 24

12. The relationship between mean, median and mode for a moderately skewed distribution is

- (a) Mode = 2 Median - 3 Mean (b) Mode = Median - 2 Mean
 (c) Mode = 2 Median - Mean (d) Mode = 3 Median - 2 mean

13. The mean of a discrete frequency distribution $x_i / f_i; i = 1, 2, \dots, n$ is given by

- (a) $\frac{\sum f_i x_i}{\sum f_i}$ (b) $\frac{1}{n} \sum_{i=1}^n f_i x_i$ (c) $\frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n x_i}$ (d) $\frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n i}$

14. If the arithmetic mean of $x, x + 3, x + 6, x + 9,$ and $x + 12$ is 10, the $x =$

- (a) 1 (b) 2 (c) 6 (d) 4

15. If the median of the data: 24, 25, 26, $x + 2, x + 3, 30, 31, 34$ is 27.5, then $x =$

- (a) 27 (b) 25 (c) 28 (d) 30

16. If the median of the data: 6, 7, $x - 2, x, 17, 20$, written in ascending order, is 16. Then $x =$

- (a) 15 (b) 16 (c) 17 (d) 18

17. The median of first 10 prime numbers is

- (a) 11 (b) 12 (c) 13 (d) 14

18. If the mode of the data: 64, 60, 48, $x, 43, 48, 43, 34$ is 43, then $x + 3 =$

- (a) 44 (b) 45 (c) 46 (d) 48

19. If the mode of the data: 16, 15, 17, 16, 15, $x, 19, 17, 14$ is 15, then $x =$

- (a) 15 (b) 16 (c) 17 (d) 19

20. The mean of 1, 3, 4, 5, 7, 4 is m . The numbers 3, 2, 2, 4, 3, 3, p have mean $m - 1$ and median q . Then, $p + q =$

- (a) 4 (b) 5 (c) 6 (d) 7

21. If the mean of a frequency distribution is 8.1 and $\sum f_i x_i = 132 + 5k, \sum f_i = 20$, then $k =$

- (a) 3 (b) 4 (c) 5 (d) 6

22. If the mean of 6, 7, $x, 8, y, 14$ is 9, then

- (a) $x + y = 21$ (b) $x + y = 19$ (c) $x - y = 19$ (d) $x - y = 21$

23. The mean of n observations is \bar{x} . If the first observation is increased by 1, the second by 2, the third by 3, and so on, then the new mean is

- (a) $\bar{x} + (2n + 1)$ (b) $\bar{x} + \frac{n + 1}{2}$ (c) $\bar{x} + (n + 1)$ (d) $\bar{x} - \frac{n + 1}{2}$

24. If the mean of first n natural numbers is $\frac{5n}{9}$, then $n =$

- (a) 5 (b) 4 (c) 9 (d) 10

25. The arithmetic mean and mode of a data are 24 and 12 respectively, then its median is
 (a) 25 (b) 18 (c) 20 (d) 22
26. The mean of first n odd natural number is
 (a) $\frac{n+1}{2}$ (b) $\frac{n}{2}$ (c) n (d) n^2
27. The mean of first n odd natural numbers is $\frac{n^2}{81}$, then $n =$
 (a) 9 (b) 81 (c) 27 (d) 18
28. If the difference of mode and median of a data is 24, then the difference of median and mean is
 (a) 12 (b) 24 (c) 8 (d) 36
29. If the arithmetic mean of 7, 8, x , 11, 14 is x , then $x =$
 (a) 9 (b) 9.5 (c) 10 (d) 10.5
30. If mode of a series exceeds its mean by 12, then mode exceeds the median by
 (a) 4 (b) 8 (c) 6 (d) 10
31. If the mean of first n natural number is 15, then $n =$
 (a) 15 (b) 30 (c) 14 (d) 29
32. If the mean of observations x_1, x_2, \dots, x_n is \bar{x} , then the mean of $x_1 + a, x_2 + a, \dots, x_n + a$ is
 (a) $a\bar{x}$ (b) $\bar{x} - a$ (c) $\bar{x} + a$ (d) $\frac{\bar{x}}{a}$
33. Mean of a certain number of observations is \bar{x} . If each observation is divided by m ($m \neq 0$) and increased by n , then the mean of new observation is
 (a) $\frac{\bar{x}}{m} + n$ (b) $\frac{\bar{x}}{n} + m$ (c) $\bar{x} + \frac{n}{m}$ (d) $\bar{x} + \frac{m}{n}$
34. If $u_i = \frac{x_i - 25}{10}$, $\sum f_i u_i = 20$, $\sum f_i = 100$, then $\bar{x} =$
 (a) 23 (b) 24 (c) 27 (d) 25
35. If 35 is removed from the data: 30, 34, 35, 36, 37, 38, 39, 40, then the median increases by
 (a) 2 (b) 1.5 (c) 1 (d) 0.5
36. While computing mean of grouped data, we assume that the frequencies are
 (a) evenly distributed over all the classes.
 (b) centred at the class marks of the classes.
 (c) centred at the upper limit of the classes.
 (d) centred at the lower limit of the classes.
37. In the formula $\bar{X} = a + h \left(\frac{1}{N} \sum f_i u_i \right)$, for finding the mean of grouped frequency distribution $u_i =$
 (a) $\frac{x_i + a}{h}$ (b) $h(x_i - a)$ (c) $\frac{x_i - a}{h}$ (d) $\frac{a - x_i}{h}$

38. For the following distribution:

Class:	0-5	5-10	10-15	15-20	20-25
Frequency:	10	15	12	20	9

the sum of the lower limits of the median and modal class is

- (a) 15 (b) 25 (c) 30 (d) 35

39. For the following distribution:

Below:	10	20	30	40	50	60
Number of students:	3	12	27	57	75	80

the modal class is

- (a) 10-20 (b) 20-30 (c) 30-40 (d) 50-60

40. Consider the following frequency distribution:

Class:	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency:	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

- (a) 0 (b) 19 (c) 20 (d) 38

41. In the formula $\bar{X} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data d_i 's are deviations from a of

- (a) lower limits of classes (b) upper limits of classes
(c) mid-points of classes (d) frequency of the class marks

42. The abscissa of the point of intersection of less than type and of the more than type cumulative frequency curves of a grouped data gives its

- (a) mean (b) median (c) mode (d) all the three above

43. Consider the following frequency distribution:

Class:	0-5	6-11	12-17	18-23	24-29
Frequency:	13	10	15	8	11

The upper limit of the median class is

- (a) 17 (b) 17.5 (c) 18 (d) 18.5

ANSWERS

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (a) | 4. (c) | 5. (a) | 6. (d) |
| 7. (a) | 8. (c) | 9. (c) | 10. (c) | 11. (b) | 12. (d) |
| 13. (a) | 14. (d) | 15. (b) | 16. (c) | 17. (b) | 18. (c) |
| 19. (a) | 20. (d) | 21. (d) | 22. (b) | 23. (b) | 24. (c) |
| 25. (c) | 26. (c) | 27. (b) | 28. (a) | 29. (c) | 30. (b) |
| 31. (d) | 32. (c) | 33. (a) | 34. (c) | 35. (d) | 36. (b) |
| 37. (c) | 38. (b) | 39. (c) | 40. (c) | 41. (c) | 42. (b) |
| 43. (b) | | | | | |

SUMMARY

- Three measures of central value are:
 - Mean
 - Median and
 - Mode
- Mean is computed by following methods:
 - Direct method
 - Short-cut Method
 - Step-deviation method.
- If a variate X takes values x_1, x_2, \dots, x_n with corresponding frequencies f_1, f_2, \dots, f_n respectively, then the arithmetic mean of these values is given by

$$\bar{X} = \frac{1}{N} \sum_{i=1}^n f_i x_i, \text{ where } N = \sum_{i=1}^n f_i$$

$$\text{Also, } \bar{X} = A + \frac{1}{N} \sum_{i=1}^n f_i d_i, \text{ where } d_i = x_i - A.$$

The number A is called the assumed mean.

If $u_i = \frac{x_i - A}{h}, i = 1, 2, \dots, n$. Then,

$$\bar{X} = A + h \left\{ \frac{1}{N} \sum_{i=1}^n f_i u_i \right\}$$

- The median is the middle value of a distribution i.e. median of a distribution is the value of the variable which divides it into two equal parts.

The median of a grouped or continuous frequency distribution may be computed by using the following formula:

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h, \text{ where}$$

l = lower limit of the median class

f = frequency of the median class

h = width of the median class.

F = cumulative frequency of the class preceding the median class.

$$\text{and, } N = \sum_{i=1}^n f_i$$

- Mode is the value of the variable which has the maximum frequency. The mode of a continuous or grouped frequency distribution may be computed by using the following formula:

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h, \text{ where}$$

l = lower limit of the modal class.

f = frequency of the modal class

h = width of the modal class

f_1 = frequency of the class preceding the modal class.

f_2 = frequency of the class following the modal class.

- Three measures of central value are connected by the following relation:

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

- Ogive(s) can be used to find the median of a frequency distribution.