

4

Quadratic Equations

Exercise 4.1 Multiple Choice Questions (MCQs)

Q. 1 Which of the following is a quadratic equation?

- (a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$
 (c) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$ (d) $x^3 - x^2 = (x - 1)^3$

Thinking Process

An equation which is of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation. So, simplify each part of the question and check whether it is in the form of $ax^2 + bx + c = 0$, $a \neq 0$ or not.

Sol. (d)

- (a) Given that, $x^2 + 2x + 1 = (4 - x)^2 + 3$
 $\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$
 $\Rightarrow 10x - 18 = 0$
 which is not of the form $ax^2 + bx + c$, $a \neq 0$. Thus, the equation is not a quadratic equation.
- (b) Given that, $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$
 $\Rightarrow -2x^2 = 10x - 2x^2 - 2 + \frac{2x}{5}$
 $\Rightarrow 50x + 2x - 10 = 0$
 $\Rightarrow 52x - 10 = 0$
 which is also not a quadratic equation.
- (c) Given that, $x^2(k + 1) + \frac{3}{2}x = 7$
 Given, $k = -1$
 $\Rightarrow x^2(-1 + 1) + \frac{3}{2}x = 7$
 $\Rightarrow 3x - 14 = 0$
 which is also not a quadratic equation.

(d) Given that, $x^3 - x^2 = (x - 1)^3$
 $\Rightarrow x^3 - x^2 = x^3 - 3x^2(1) + 3x(1)^2 - (1)^3$
 $\qquad\qquad\qquad [\because (a - b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b]$
 $\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$
 $\Rightarrow -x^2 + 3x^2 - 3x + 1 = 0$
 $\Rightarrow 2x^2 - 3x + 1 = 0$
 which represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

Q. 2 Which of the following is not a quadratic equation?

- (a) $2(x - 1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$
 (c) $(\sqrt{2}x + \sqrt{3})^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

Sol. (d)

(a) Given that, $2(x - 1)^2 = 4x^2 - 2x + 1$
 $\Rightarrow 2(x^2 + 1 - 2x) = 4x^2 - 2x + 1$
 $\Rightarrow 2x^2 + 2 - 4x = 4x^2 - 2x + 1$
 $\Rightarrow 2x^2 + 2x - 1 = 0$
 which represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

(b) Given that, $2x - x^2 = x^2 + 5$
 $\Rightarrow 2x^2 - 2x + 5 = 0$
 which also represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

(c) Given that, $(\sqrt{2} \cdot x + \sqrt{3})^2 = 3x^2 - 5x$
 $\Rightarrow 2 \cdot x^2 + 3 + 2\sqrt{6} \cdot x = 3x^2 - 5x$
 $\Rightarrow x^2 - (5 + 2\sqrt{6})x - 3 = 0$
 which also represents a quadratic equation because it has the quadratic form $ax^2 + bx + c = 0, a \neq 0$.

(d) Given that, $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$
 $\Rightarrow x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2$
 $\Rightarrow 4x^3 - 3 = 0$
 which is not of the form $ax^2 + bx + c, a \neq 0$. Thus, the equation is not quadratic. This is a cubic equation.

Q. 3 Which of the following equations has 2 as a root?

- (a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$
 (c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

Thinking Process

If α is one of the root of any quadratic equation i.e., $f(x) = ax^2 + bx + c = 0$, then $x = \alpha$ i.e., α satisfies the equation $a\alpha^2 + b\alpha + c = 0$.

Sol. (c)

(a) Substituting $x = 2$ in $x^2 - 4x + 5$, we get $(2)^2 - 4(2) + 5$
 $= 4 - 8 + 5 = 1 \neq 0$.

So, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$.

(b) Substituting $x = 2$ in $x^2 + 3x - 12$, we get
 $(2)^2 + 3(2) - 12$
 $= 4 + 6 - 12 = -2 \neq 0$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$.

(c) Substituting $x = 2$ in $2x^2 - 7x + 6$, we get
 $2(2)^2 - 7(2) + 6 = 2(4) - 14 + 6$
 $= 8 - 14 + 6 = 14 - 14 = 0$

So, $x = 2$ is root of the equation $2x^2 - 7x + 6 = 0$.

(d) Substituting $x = 2$ in $3x^2 - 6x - 2$, we get
 $3(2)^2 - 6(2) - 2$
 $= 12 - 12 - 2 = -2 \neq 0$

So, $x = 2$ is not a root of $3x^2 - 6x - 2 = 0$.

Q. 4 If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

- (a) 2 (b) -2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Thinking Process

Since, $\frac{1}{2}$ is a root of given equation, then put $x = \frac{1}{2}$ in given equation and get the value of k .

Sol. (a) Since, $\frac{1}{2}$ is a root of the quadratic equation $x^2 + kx - \frac{5}{4} = 0$.

Then, $\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$
 $\Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0 \Rightarrow \frac{1 + 2k - 5}{4} = 0$
 $\Rightarrow 2k - 4 = 0$
 $\Rightarrow 2k = 4 \Rightarrow k = 2$

Q. 5 Which of the following equations has the sum of its roots as 3?

- (a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$
(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

Thinking Process

If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, then sum of

$$\text{roots} = \alpha + \beta = (-1) \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = (-1) \cdot \frac{b}{a}$$

Sol. (b)

(a) Given that, $2x^2 - 3x + 6 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = 6$$

$$\therefore \text{Sum of the roots} = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

So, sum of the roots of the quadratic equation $2x^2 - 3x + 6 = 0$ is not 3, so it is not the answer.

(b) Given that, $-x^2 + 3x - 3 = 0$

On compare with $ax^2 + bx + c = 0$, we get

$$a = -1, b = 3 \text{ and } c = -3$$

$$\therefore \text{Sum of the roots} = \frac{-b}{a} = \frac{-(3)}{-1} = 3$$

So, sum of the roots of the quadratic equation $-x^2 + 3x - 3 = 0$ is 3, so it is the answer.

(c) Given that, $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

$$\Rightarrow 2x^2 - 3x + \sqrt{2} = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = \sqrt{2}$$

$$\therefore \text{Sum of the roots} = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

So, sum of the roots of the quadratic equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ is not 3, so it is not the answer.

(d) Given that, $3x^2 - 3x + 3 = 0$

$$\Rightarrow x^2 - x + 1 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -1 \text{ and } c = 1$$

$$\therefore \text{Sum of the roots} = \frac{-b}{a} = \frac{-(-1)}{1} = 1$$

So, sum of the roots of the quadratic equation $3x^2 - 3x + 3 = 0$ is not 3, so it is not the answer.

Q. 6 Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0

(b) 4

(c) 8

(d) 0, 8

Thinking Process

If any quadratic equation i.e., $ax^2 + bx + c = 0$, $a \neq 0$ has two equal real roots, then its discriminant should be equal to zero. i.e., $D = b^2 - 4ac = 0$

Sol. (d) Given equation is $2x^2 - kx + k = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -k \text{ and } c = k$$

For equal roots, the discriminant must be zero.

$$\text{i.e., } D = b^2 - 4ac = 0$$

$$\Rightarrow (-k)^2 - 4(2)k = 0$$

$$\Rightarrow k^2 - 8k = 0$$

$$\Rightarrow k(k - 8) = 0$$

$$\therefore k = 0, 8$$

Hence, the required values of k are 0 and 8.

Q. 7 Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$

(b) $\frac{1}{64}$

(c) $\frac{1}{4}$

(d) $\frac{9}{64}$

Sol. (b) Given equation is $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$.

$$(3x)^2 + \frac{1}{4}(3x) - \sqrt{2} = 0$$

On putting $3x = y$, we have $y^2 + \frac{1}{4}y - \sqrt{2} = 0$

$$y^2 + \frac{1}{4}y + \left(\frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 - \sqrt{2} = 0$$

$$\left(y + \frac{1}{8}\right)^2 = \frac{1}{64} + \sqrt{2}$$

$$\left(y + \frac{1}{8}\right)^2 = \frac{1 + 64 \cdot \sqrt{2}}{64}$$

Thus, $\frac{1}{64}$ must be added and subtracted to solve the given equation.

Q. 8 The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Thinking Process

If a quadratic equation is in the form of $ax^2 + bx + c = 0$, $a \neq 0$, then

(i) If $D = b^2 - 4ac > 0$, then its roots are distinct and real.

(ii) If $D = b^2 - 4ac = 0$, then its roots are real and equal.

(iii) If $D = b^2 - 4ac < 0$, then its roots are not real or imaginary roots.

Any quadratic equation must have only two roots.

Sol. (c) Given equation is $2x^2 - \sqrt{5}x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -\sqrt{5} \text{ and } c = 1$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1) = 5 - 8 \\ = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots i.e., imaginary roots.

Q. 9 Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d) $5x^2 - 3x + 1 = 0$

Sol. (b) The given equation is $x^2 + x - 5 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 1 \text{ and } c = -5$$

The discriminant of $x^2 + x - 5 = 0$ is

$$\begin{aligned} D &= b^2 - 4ac = (1)^2 - 4(1)(-5) \\ &= 1 + 20 = 21 \end{aligned}$$

$$\Rightarrow b^2 - 4ac > 0$$

So, $x^2 + x - 5 = 0$ has two distinct real roots.

(a) Given equation is, $2x^2 - 3\sqrt{2}x + 9/4 = 0$.

On comparing with $ax^2 + bx + c = 0$

$$a = 2, b = -3\sqrt{2} \text{ and } c = 9/4$$

$$\text{Now, } D = b^2 - 4ac = (-3\sqrt{2})^2 - 4(2)(9/4) = 18 - 18 = 0$$

Thus, the equation has real and equal roots.

(c) Given equation is $x^2 + 3x + 2\sqrt{2} = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 1, b = 3 \text{ and } c = 2\sqrt{2}$$

$$\text{Now, } D = b^2 - 4ac = (3)^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} < 0$$

∴ Roots of the equation are not real.

(d) Given equation is, $5x^2 - 3x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

$$a = 5, b = -3, c = 1$$

$$\text{Now, } D = b^2 - 4ac = (-3)^2 - 4(5)(1) = 9 - 20 < 0$$

Hence, roots of the equation are not real.

Q. 10 Which of the following equations has no real roots?

(a) $x^2 - 4x + 3\sqrt{2} = 0$

(b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$

(d) $3x^2 + 4\sqrt{3}x + 4 = 0$

Sol. (a)

(a) The given equation is $x^2 - 4x + 3\sqrt{2} = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4 \text{ and } c = 3\sqrt{2}$$

The discriminant of $x^2 - 4x + 3\sqrt{2} = 0$ is

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-4)^2 - 4(1)(3\sqrt{2}) = 16 - 12\sqrt{2} = 16 - 12 \times (1.41) \\ &= 16 - 16.92 = -0.92 \end{aligned}$$

$$\Rightarrow b^2 - 4ac < 0$$

(b) The given equation is $x^2 + 4x - 3\sqrt{2} = 0$

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 4 \text{ and } c = -3\sqrt{2}$$

$$\begin{aligned} \text{Then, } D &= b^2 - 4ac = (-4)^2 - 4(1)(-3\sqrt{2}) \\ &= 16 + 12\sqrt{2} > 0 \end{aligned}$$

Hence, the equation has real roots.

(c) Given equation is $x^2 - 4x - 3\sqrt{2} = 0$

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4 \text{ and } c = -3\sqrt{2}$$

$$\begin{aligned} \text{Then, } D &= b^2 - 4ac = (-4)^2 - 4(1)(-3\sqrt{2}) \\ &= 16 + 12\sqrt{2} > 0 \end{aligned}$$

Hence, the equation has real roots.

(d) Given equation is $3x^2 + 4\sqrt{3}x + 4 = 0$.

On comparing the equation with $ax^2 + bx + c = 0$, we get

$$a = 3, b = 4\sqrt{3} \text{ and } c = 4$$

$$\text{Then, } D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$$

Hence, the equation has real roots.

Hence, $x^2 - 4x + 3\sqrt{2} = 0$ has no real roots.

Q. 11 $(x^2 + 1)^2 - x^2 = 0$ has

- (a) four real roots (b) two real roots (c) no real roots (d) one real root

Sol. (c) Given equation is $(x^2 + 1)^2 - x^2 = 0$

$$\Rightarrow x^4 + 1 + 2x^2 - x^2 = 0 \quad [\because (a + b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow x^4 + x^2 + 1 = 0$$

$$\text{Let } x^2 = y$$

$$\therefore (x^2)^2 + x^2 + 1 = 0$$

$$y^2 + y + 1 = 0$$

On comparing with $ay^2 + by + c = 0$, we get

$$a = 1, b = 1 \text{ and } c = 1$$

$$\text{Discriminant, } D = b^2 - 4ac$$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4 = -3$$

Since, $D < 0$

$\therefore y^2 + y + 1 = 0$ i.e., $x^4 + x^2 + 1 = 0$ or $(x^2 + 1)^2 - x^2 = 0$ has no real roots.

Exercise 4.2 Very Short Answer Type Questions

Q. 1 State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$

(ii) $2x^2 + x - 1 = 0$

(iii) $2x^2 - 6x + \frac{9}{2} = 0$

(iv) $3x^2 - 4x + 1 = 0$

(v) $(x + 4)^2 - 8x = 0$

(vi) $(x - \sqrt{2})^2 - \sqrt{2}(x + 1) = 0$

(vii) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

(viii) $x(1 - x) - 2 = 0$

(ix) $(x - 1)(x + 2) + 2 = 0$

(x) $(x + 1)(x - 2) + x = 0$

Thinking Process

If $ax^2 + bx + c = 0$ $a \neq 0$ be any quadratic equation and $D = b^2 - 4ac$ be its discriminant, then

(i) $D > 0$ i.e., $b^2 - 4ac > 0 \Rightarrow$ Roots are real and distinct.

(ii) $D = 0$ i.e., $b^2 - 4ac = 0 \Rightarrow$ Roots are real and equal.

(iii) $D < 0$ i.e., $b^2 - 4ac < 0 \Rightarrow$ No real roots i.e., imaginary roots.

Sol. (i) Given equation is $x^2 - 3x + 4 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -3 \text{ and } c = 4$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac = (-3)^2 - 4(1)(4)$$

$$= 9 - 16 = -7 < 0 \text{ i.e., } D < 0$$

Hence, the equation $x^2 - 3x + 4 = 0$ has no real roots.

(ii) Given equation is, $2x^2 + x - 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = 1 \text{ and } c = -1$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac = (1)^2 - 4(2)(-1)$$

$$= 1 + 8 = 9 > 0 \text{ i.e., } D > 0$$

Hence, the equation $2x^2 + x - 1 = 0$ has two distinct real roots.

(iii) Given equation is $2x^2 - 6x + \frac{9}{2} = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -6 \text{ and } c = \frac{9}{2}$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac$$

$$= (-6)^2 - 4(2)\left(\frac{9}{2}\right) = 36 - 36 = 0 \text{ i.e., } D = 0$$

Hence, the equation $2x^2 - 6x + \frac{9}{2} = 0$ has equal and real roots.

(iv) Given equation is $3x^2 - 4x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -4 \text{ and } c = 1$$

$$\begin{aligned} \therefore \text{Discriminant, } D &= b^2 - 4ac = (-4)^2 - 4(3)(1) \\ &= 16 - 12 = 4 > 0 \text{ i.e., } D > 0 \end{aligned}$$

Hence, the equation $3x^2 - 4x + 1 = 0$ has two distinct real roots.

(v) Given equation is $(x + 4)^2 - 8x = 0$.

$$\Rightarrow x^2 + 16 + 8x - 8x = 0 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$\Rightarrow x^2 + 16 = 0$$

$$\Rightarrow x^2 + 0 \cdot x + 16 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 0 \text{ and } c = 16$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac = (0)^2 - 4(1)(16) = -64 < 0 \text{ i.e., } D < 0$$

Hence, the equation $(x + 4)^2 - 8x = 0$ has imaginary roots, i.e., no real roots.

(vi) Given equation is $(x - \sqrt{2})^2 - \sqrt{2}(x + 1) = 0$.

$$\Rightarrow x^2 + (\sqrt{2})^2 - 2x\sqrt{2} - \sqrt{2}x - \sqrt{2} = 0 \quad [\because (a - b)^2 = a^2 - 2ab + b^2]$$

$$\Rightarrow x^2 + 2 - 2\sqrt{2}x - \sqrt{2}x - \sqrt{2} = 0$$

$$\Rightarrow x^2 - 3\sqrt{2}x + (2 - \sqrt{2}) = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -3\sqrt{2} \text{ and } c = 2 - \sqrt{2}$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac$$

$$= (-3\sqrt{2})^2 - 4(1)(2 - \sqrt{2}) = 9 \times 2 - 8 + 4\sqrt{2}$$

$$= 18 - 8 + 4\sqrt{2} = 10 + 4\sqrt{2} > 0 \text{ i.e., } D > 0$$

Hence, the equation $(x - \sqrt{2})^2 - \sqrt{2}(x + 1) = 0$ has two distinct real roots.

(vii) Given, equation is $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = \sqrt{2}, b = -\frac{3}{\sqrt{2}} \text{ and } c = \frac{1}{\sqrt{2}}$$

$$\therefore \text{Discriminant, } D = b^2 - 4ac$$

$$= \left(-\frac{3}{\sqrt{2}}\right)^2 - 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{9}{2} - 4 = \frac{9-8}{2} = \frac{1}{2} > 0 \text{ i.e., } D > 0$$

Hence, the equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ has two distinct real roots.

(viii) Given equation is $x(1 - x) - 2 = 0$.

$$\Rightarrow x - x^2 - 2 = 0$$

$$\Rightarrow x^2 - x + 2 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -1 \text{ and } c = 2$$

\therefore Discriminant, $D = b^2 - 4ac$

$$= (-1)^2 - 4(1)(2) = 1 - 8 = -7 < 0 \text{ i.e., } D < 0$$

Hence, the equation $x(1 - x) - 2 = 0$ has imaginary roots *i.e.*, no real roots.

(ix) Given equation is

$$(x - 1)(x + 2) + 2 = 0$$

$$\Rightarrow x^2 + x - 2 + 2 = 0$$

$$\Rightarrow x^2 + x + 0 = 0$$

On comparing the equation with $ax^2 + bx + c = 0$, We have

$$a = 1, b = 1 \text{ and } c = 0$$

\therefore Discriminant, $D = b^2 - 4ac$

$$= 1 - 4(1)(0) = 1 > 0 \text{ i.e., } D > 0$$

Hence, equation has two distinct real roots.

(x) Given equation is $(x + 1)(x - 2) + x = 0$

$$\Rightarrow x^2 + x - 2x - 2 + x = 0$$

$$\Rightarrow x^2 - 2 = 0$$

$$\Rightarrow x^2 + 0 \cdot x - 2 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 0 \text{ and } c = -2$$

\therefore Discriminant, $D = b^2 - 4ac = (0)^2 - 4(1)(-2) = 0 + 8 = 8 > 0$

Hence, the equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots.

Q. 2 Write whether the following statements are true or false. Justify your answers.

- (i) Every quadratic equation has exactly one root.
- (ii) Every quadratic equation has atleast one real root.
- (iii) Every quadratic equation has atleast two roots.
- (iv) Every quadratic equation has atmost two roots.
- (v) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
- (vi) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

- Sol.**
- (i) *False*, since a quadratic equation has two and only two roots.
 - (ii) *False*, for example $x^2 + 4 = 0$ has no real root.
 - (iii) *False*, since a quadratic equation has two and only two roots.
 - (iv) *True*, because every quadratic polynomial has atmost two roots.
 - (v) *True*, since in this case discriminant is always positive, so it has always real roots, *i.e.*, $ac < 0$ and so, $b^2 - 4ac > 0$.
 - (vi) *True*, since in this case discriminant is always negative, so it has no real roots *i.e.*, if $b = 0$, then $b^2 - 4ac \Rightarrow -4ac < 0$ and $ac > 0$.

Q. 3 A quadratic equation with integral coefficient has integral roots. Justify your answer.

Sol. No, consider the quadratic equation $2x^2 + x - 6 = 0$ with integral coefficient. The roots of the given quadratic equation are -2 and $\frac{3}{2}$ which are not integers.

Q. 4 Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

Sol. Yes, consider the quadratic equation $2x^2 + x - 4 = 0$ with rational coefficient. The roots of the given quadratic equation are $\frac{-1 + \sqrt{33}}{4}$ and $\frac{-1 - \sqrt{33}}{4}$ are irrational.

Q. 5 Does there exist a quadratic equation whose coefficient are all distinct irrationals but both the roots are rationals? why?

Sol. Yes, consider the quadratic equation with all distinct irrationals coefficients i.e., $\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$. The roots of this quadratic equation are 3 and 4, which are rationals.

Q. 6 Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify your answer.

Sol. No, since 0.2 does not satisfy the quadratic equation i.e., $(0.2)^2 - 0.4 = 0.04 - 0.4 \neq 0$.

Q. 7 If $b = 0$, $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify your answer.

Sol. Given that, $b = 0$ and $c < 0$ and quadratic equation,

$$x^2 + bx + c = 0 \quad \dots(i)$$

Put $b = 0$ in Eq. (i), we get

$$x^2 + 0 + c = 0$$

\Rightarrow

$$x^2 = -c$$

$$\left[\begin{array}{l} \text{here } c < 0 \\ \therefore -c > 0 \end{array} \right]$$

\therefore

$$x = \pm \sqrt{-c}$$

So, the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign.

Exercise 7.3 Short Answer Type Questions

Q. 1 Find the roots of the quadratic equations by using the quadratic formula in each of the following

- | | |
|---|----------------------------------|
| (i) $2x^2 - 3x - 5 = 0$ | (ii) $5x^2 + 13x + 8 = 0$ |
| (iii) $-3x^2 + 5x + 12 = 0$ | (iv) $-x^2 + 7x - 10 = 0$ |
| (v) $x^2 + 2\sqrt{2}x - 6 = 0$ | (vi) $x^2 - 3\sqrt{5}x + 10 = 0$ |
| (vii) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$ | |

Thinking Process

Compare the given quadratic equation with $ax^2 + bx + c = 0$ and get the values of a, b and c . Now, use the quadratic formula for finding the roots of quadratic equation.

$$\text{i.e., } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sol. (i) Given equation is $2x^2 - 3x - 5 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -3 \text{ and } c = -5$$

By quadratic formula,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} = \frac{3 \pm \sqrt{9 + 40}}{4} \\ &= \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4} = \frac{10}{4}, \frac{-4}{4} = \frac{5}{2}, -1 \end{aligned}$$

So, $\frac{5}{2}$ and -1 are the roots of the given equation.

(ii) Given equation is $5x^2 + 13x + 8 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 5, b = 13 \text{ and } c = 8$$

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned} &= \frac{-(13) \pm \sqrt{(13)^2 - 4(5)(8)}}{2(5)} \\ &= \frac{-13 \pm \sqrt{169 - 160}}{10} = \frac{-13 \pm \sqrt{9}}{10} \\ &= \frac{-13 \pm 3}{10} = -\frac{10}{10}, -\frac{16}{10} = -1, -\frac{8}{5} \end{aligned}$$

(iii) Given equation is $-3x^2 + 5x + 12 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -3, b = 5 \text{ and } c = 12$$

$$\begin{aligned}
 \text{By quadratic formula, } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(5) \pm \sqrt{(5)^2 - 4(-3)(12)}}{2(-3)} \\
 &= \frac{-5 \pm \sqrt{25 + 144}}{-6} = \frac{-5 \pm \sqrt{169}}{-6} \\
 &= \frac{-5 \pm 13}{-6} = \frac{8}{-6}, \frac{-18}{-6} = -\frac{4}{3}, 3
 \end{aligned}$$

So, $-\frac{4}{3}$ and 3 are two roots of the given equation.

(iv) Given equation is $-x^2 + 7x - 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -1, b = 7 \text{ and } c = -10$$

$$\begin{aligned}
 \text{By quadratic formula, } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(7) \pm \sqrt{(7)^2 - 4(-1)(-10)}}{2(-1)} = \frac{-7 \pm \sqrt{49 - 40}}{-2} \\
 &= \frac{-7 \pm \sqrt{9}}{-2} = \frac{-7 \pm 3}{-2} = \frac{-4}{-2}, \frac{-10}{-2} = 2, 5
 \end{aligned}$$

So, 2 and 5 are two roots of the given equation.

(v) Given equation is $x^2 + 2\sqrt{2}x - 6 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 2\sqrt{2} \text{ and } c = -6$$

$$\begin{aligned}
 \text{By quadratic formula, } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(1)(-6)}}{2(1)} = \frac{-2\sqrt{2} \pm \sqrt{8 + 24}}{2} \\
 &= \frac{-2\sqrt{2} \pm \sqrt{32}}{2} = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2} \\
 &= \frac{-2\sqrt{2} + 4\sqrt{2}}{2}, \frac{-2\sqrt{2} - 4\sqrt{2}}{2} = \sqrt{2}, -3\sqrt{2}
 \end{aligned}$$

So, $\sqrt{2}$ and $-3\sqrt{2}$ are the roots of the given equation.

(vi) Given equation is $x^2 - 3\sqrt{5}x + 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we have

$$a = 1, b = -3\sqrt{5} \text{ and } c = 10$$

$$\begin{aligned}
 \text{By quadratic formula, } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-3\sqrt{5}) \pm \sqrt{(-3\sqrt{5})^2 - 4(1)(10)}}{2(1)} \\
 &= \frac{3\sqrt{5} \pm \sqrt{45 - 40}}{2} = \frac{3\sqrt{5} \pm \sqrt{5}}{2} \\
 &= \frac{3\sqrt{5} + \sqrt{5}}{2}, \frac{3\sqrt{5} - \sqrt{5}}{2} = 2\sqrt{5}, \sqrt{5}
 \end{aligned}$$

So, $2\sqrt{5}$ and $\sqrt{5}$ are the roots of the given equation.

(vii) Given equation is $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = \frac{1}{2}, b = -\sqrt{11} \text{ and } c = 1$$

∴ By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned} &= \frac{-(-\sqrt{11}) \pm \sqrt{(-\sqrt{11})^2 - 4 \times \frac{1}{2} \times 1}}{2 \left(\frac{1}{2}\right)} \\ &= \frac{\sqrt{11} \pm \sqrt{11-2}}{1} = \sqrt{11} \pm \sqrt{9} \\ &= \sqrt{11} \pm 3 = 3 + \sqrt{11}, \sqrt{11} - 3 \end{aligned}$$

So, $3 + \sqrt{11}$ and $\sqrt{11} - 3$ are the roots of the given equation.

Q. 2 Find the roots of the following quadratic equations by the factorisation method.

(i) $2x^2 + \frac{5}{3}x - 2 = 0$

(ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

(iv) $3x^2 + 5\sqrt{5}x - 10 = 0$

(v) $21x^2 - 2x + \frac{1}{21} = 0$

Thinking Process

If any coefficient of the quadratic equation of the form $ax^2 + bx + c = 0$, is in fractional form, then make all the coefficients in integral form. Then, use the factorisation method i.e., by splitting the middle term, we get the required roots of the given quadratic equation.

Sol. (i) Given equation is $2x^2 + \frac{5}{3}x - 2 = 0$

On multiplying by 3 on both sides, we get

$$6x^2 + 5x - 6 = 0$$

$$\Rightarrow 6x^2 + (9x - 4x) - 6 = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow 6x^2 + 9x - 4x - 6 = 0$$

$$\Rightarrow 3x(2x + 3) - 2(2x + 3) = 0$$

$$\Rightarrow (2x + 3)(3x - 2) = 0$$

Now, $2x + 3 = 0$

$$\Rightarrow x = -\frac{3}{2}$$

and $3x - 2 = 0$

$$\Rightarrow x = \frac{2}{3}$$

Hence, the roots of the equation $6x^2 + 5x - 6 = 0$ are $-\frac{3}{2}$ and $\frac{2}{3}$.

(ii) Given equation is $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$.

On multiplying by 5 on both sides, we get

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$2x^2 - (6x - x) - 3 = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow 2x^2 - 6x + x - 3 = 0$$

$$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$$

$$\Rightarrow (x - 3)(2x + 1) = 0$$

Now, $x - 3 = 0 \Rightarrow x = 3$

and $2x + 1 = 0$

$$\Rightarrow x = -\frac{1}{2}$$

Hence, the roots of the equation $2x^2 - 5x - 3 = 0$ are $-\frac{1}{2}$ and 3.

(iii) Given equation is $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$.

$$3\sqrt{2}x^2 - (6x - x) - \sqrt{2} = 0 \quad \text{[by splitting the middle term]}$$

$$3\sqrt{2}x^2 - 6x + x - \sqrt{2} = 0$$

$$3\sqrt{2}x^2 - 3\sqrt{2} \cdot \sqrt{2}x + x - \sqrt{2} = 0$$

$$\Rightarrow 3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$$

$$\Rightarrow (x - \sqrt{2})(3\sqrt{2}x + 1) = 0$$

Now, $x - \sqrt{2} = 0 \Rightarrow x = \sqrt{2}$

and $3\sqrt{2}x + 1 = 0$

$$\Rightarrow x = -\frac{1}{3\sqrt{2}} = \frac{-\sqrt{2}}{6}$$

Hence, the roots of the equation $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ are $-\frac{\sqrt{2}}{6}$ and $\sqrt{2}$.

(iv) Given equation is $3x^2 + 5\sqrt{5}x - 10 = 0$.

$$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 2\sqrt{5} \cdot \sqrt{5} = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$$

$$3x^2 + 6\sqrt{5}x - \sqrt{5}x - 2\sqrt{5} \cdot \sqrt{5} = 0$$

$$\Rightarrow 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$

$$\Rightarrow (x + 2\sqrt{5})(3x - \sqrt{5}) = 0$$

Now, $x + 2\sqrt{5} = 0$

$$\Rightarrow x = -2\sqrt{5} \text{ and } 3x - \sqrt{5} = 0$$

$$\Rightarrow x = \frac{\sqrt{5}}{3}$$

Hence, the roots of the equation $3x^2 + 5\sqrt{5}x - 10 = 0$ are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{3}$.

(v) Given equation is $21x^2 - 2x + \frac{1}{21} = 0$.

On multiplying by 21 on both sides, we get

$$441x^2 - 42x + 1 = 0$$

$$441x^2 - (21x + 21x) + 1 = 0 \quad \text{[by splitting the middle term]}$$

(ii) Given equation is $-2x^2 + 3x + 2 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = -2, b = 3 \text{ and } c = 2$$

$$\begin{aligned} \therefore \text{Discriminant, } D &= b^2 - 4ac \\ &= (3)^2 - 4(-2)(2) \\ &= 9 + 16 = 25 > 0 \end{aligned}$$

Therefore, the equation $-2x^2 + 3x + 2 = 0$ has two distinct real roots because we know that if the equation $ax^2 + bx + c = 0$ has its discriminant greater than zero, then it has two distinct real roots.

$$\begin{aligned} \text{Roots, } x &= \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{25}}{2(-2)} \\ &= \frac{-3 \pm 5}{-4} = \frac{-3 + 5}{-4}, \frac{-3 - 5}{-4} \\ &= \frac{2}{-4}, \frac{-8}{-4} = -\frac{1}{2}, 2 \end{aligned}$$

(iii) Given equation is $5x^2 - 2x - 10 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 5, b = -2 \text{ and } c = -10$$

$$\begin{aligned} \therefore \text{Discriminant, } D &= b^2 - 4ac \\ &= (-2)^2 - 4(5)(-10) \\ &= 4 + 200 = 204 > 0 \end{aligned}$$

Therefore, the equation $5x^2 - 2x - 10 = 0$ has two distinct real roots.

$$\begin{aligned} \text{Roots, } x &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{-(-2) \pm \sqrt{204}}{2 \times 5} = \frac{2 \pm 2\sqrt{51}}{10} \\ &= \frac{1 \pm \sqrt{51}}{5} = \frac{1 + \sqrt{51}}{5}, \frac{1 - \sqrt{51}}{5} \end{aligned}$$

(iv) Given equation is $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

$$\Rightarrow \frac{x-5+2x-3}{(2x-3)(x-5)} = 1$$

$$\Rightarrow \frac{3x-8}{2x^2-5x-10x+25} = 1$$

$$\Rightarrow \frac{3x-8}{2x^2-15x+25} = 1$$

$$\Rightarrow 3x-8 = 2x^2-15x+25$$

$$\Rightarrow 2x^2-15x-3x+25+8=0$$

$$\Rightarrow 2x^2-18x+33=0$$

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -18 \text{ and } c = 33$$

$$\begin{aligned} \therefore \text{Discriminant, } D &= b^2 - 4ac \\ &= (-18)^2 - 4 \times 2 \text{ (33)} \\ &= 324 - 264 = 60 > 0 \end{aligned}$$

Therefore, the equation $2x^2 - 18x + 33 = 0$ has two distinct real roots.

$$\begin{aligned} \text{Roots, } x &= \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-18) \pm \sqrt{60}}{2 \text{ (2)}} \\ &= \frac{18 \pm 2\sqrt{15}}{4} = \frac{9 \pm \sqrt{15}}{2} \\ &= \frac{9 + \sqrt{15}}{2}, \frac{9 - \sqrt{15}}{2} \end{aligned}$$

(v) Given equation is $x^2 + 5\sqrt{5}x - 70 = 0$.

On comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 5\sqrt{5} \text{ and } c = -70$$

$$\begin{aligned} \therefore \text{Discriminant, } D &= b^2 - 4ac = (5\sqrt{5})^2 - 4(1)(-70) \\ &= 125 + 280 = 405 > 0 \end{aligned}$$

Therefore, the equation $x^2 + 5\sqrt{5}x - 70 = 0$ has two distinct real roots.

$$\begin{aligned} \text{Roots, } x &= \frac{-b \pm \sqrt{D}}{2a} \\ &= \frac{-5\sqrt{5} \pm \sqrt{405}}{2 \text{ (1)}} = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2} \\ &= \frac{-5\sqrt{5} + 9\sqrt{5}}{2}, \frac{-5\sqrt{5} - 9\sqrt{5}}{2} \\ &= \frac{4\sqrt{5}}{2}, -\frac{14\sqrt{5}}{2} = 2\sqrt{5}, -7\sqrt{5} \end{aligned}$$

Q. 2 Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Thinking Process

Firstly, we make a quadratic equation by using the given condition after that we solve the equation by factorisation equation. Finally, method to obtain the desired number.

Sol. Let n be a required natural number.

Square of a natural number diminished by 84 = $n^2 - 84$

and thrice of 8 more than the natural number = $3(n + 8)$

Now, by given condition,

$$\begin{aligned} & n^2 - 84 = 3(n + 8) \\ \Rightarrow & n^2 - 84 = 3n + 24 \\ \Rightarrow & n^2 - 3n - 108 = 0 \\ \Rightarrow & n^2 - 12n + 9n - 108 = 0 \quad \text{[by splitting the middle term]} \\ \Rightarrow & n(n - 12) + 9(n - 12) = 0 \\ \Rightarrow & (n - 12)(n + 9) = 0 \\ \Rightarrow & n = 12 \quad \text{[}\because n \neq -9 \text{ because } n \text{ is a natural number]} \end{aligned}$$

Hence, the required natural number is 12.

Q. 3 A natural number, when increased by 12, equals 160 times its reciprocal.
Find the number.

Sol. Let the natural number be x .
According to the question,

$$x + 12 = \frac{160}{x}$$

On multiplying by x on both sides, we get

$$\Rightarrow x^2 + 12x - 160 = 0$$

$$x^2 + (20x - 8x) - 160 = 0$$

$$\Rightarrow x^2 + 20x - 8x - 160 = 0 \quad \text{[by factorisation method]}$$

$$\Rightarrow x(x + 20) - 8(x + 20) = 0$$

$$\Rightarrow (x + 20)(x - 8) = 0$$

Now, $x + 20 = 0 \Rightarrow x = -20$ which is not possible because natural number is always greater than zero and $x - 8 = 0 \Rightarrow x = 8$.

Hence, the required natural number is 8.

Q. 4 A train, travelling at a uniform speed for 360 km, would have taken 48 min less to travel the same distance, if its speed were 5 km/h more.
Find the original speed of the train.

Sol. Let the original speed of the train = x km/h

Then, the increased speed of the train = $(x + 5)$ km/h

[by given condition]

and distance = 360 km

According to the question,

$$\frac{360}{x} - \frac{360}{x + 5} = \frac{4}{5}$$

$$\Rightarrow \frac{360(x + 5) - 360x}{x(x + 5)} = \frac{4}{5}$$

$$\Rightarrow \frac{360x + 1800 - 360x}{x^2 + 5x} = \frac{4}{5}$$

$$\Rightarrow \frac{1800}{x^2 + 5x} = \frac{4}{5}$$

$$\Rightarrow x^2 + 5x = \frac{1800 \times 5}{4} = 2250$$

$$\Rightarrow x^2 + 5x - 2250 = 0$$

$$x^2 + (50x - 45x) - 2250 = 0$$

$$\Rightarrow x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow x(x + 50) - 45(x + 50) = 0$$

$$\Rightarrow (x + 50)(x - 45) = 0$$

Now, $x + 50 = 0 \Rightarrow x = -50$

which is not possible because speed cannot be negative and $x - 45 = 0 \Rightarrow x = 45$.

Hence, the original speed of the train = 45 km/h

$$\left[\begin{array}{l} \therefore \text{time} = \frac{\text{Distance}}{\text{Speed}} \\ \text{and } 48 \text{ min} = \frac{48}{60} \text{ h} = \frac{4}{5} \text{ h} \\ \therefore 48 \text{ min} = \frac{48}{60} \text{ h} = \frac{4}{5} \text{ h} \end{array} \right]$$

[by factorisation method]

Q. 5 If Zeba were younger by 5 yr than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age, what is her age now?

Sol. Let the actual age of Zeba = x yr.

Her age when she was 5 yr younger = $(x - 5)$ yr.

Now, by given condition,

Square of her age = 11 more than five times her actual age

$$(x - 5)^2 = 5 \times \text{actual age} + 11$$

$$\Rightarrow (x - 5)^2 = 5x + 11$$

$$\Rightarrow x^2 + 25 - 10x = 5x + 11$$

$$\Rightarrow x^2 - 15x + 14 = 0$$

$$\Rightarrow x^2 - 14x - x + 14 = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow x(x - 14) - 1(x - 14) = 0$$

$$\Rightarrow (x - 1)(x - 14) = 0$$

$$\Rightarrow x = 14$$

[here, $x \neq 1$ because her age is $x - 5$. So, $x - 5 = 1 - 5 = -4$ i.e., age cannot be negative]

Hence, required Zeba's age now is 14 yr.

Q. 6 At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age. Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

Sol. Let Nisha's present age be x yr.

Then, Asha's present age = $x^2 + 2$ [by given condition]

Now, when Nisha grows to her mother's present age i.e., after $\{(x^2 + 2) - x\}$ yr. Then, Asha's age also increased by $[(x^2 + 2) - x]$ yr.

Again by given condition,

Age of Asha = One year less than 10 times the present age of Nisha

$$(x^2 + 2) + \{(x^2 + 2) - x\} = 10x - 1$$

$$\Rightarrow 2x^2 - x + 4 = 10x - 1$$

$$\Rightarrow 2x^2 - 11x + 5 = 0$$

$$\Rightarrow 2x^2 - 10x - x + 5 = 0$$

$$\Rightarrow 2x(x - 5) - 1(x - 5) = 0$$

$$\Rightarrow (x - 5)(2x - 1) = 0$$

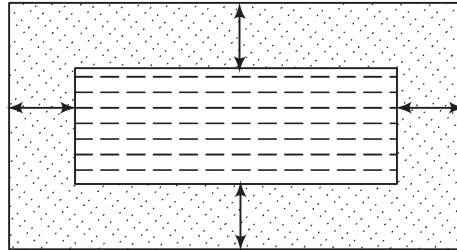
$$\therefore x = 5$$

[here, $x = \frac{1}{2}$ cannot be possible, because at $x = \frac{1}{2}$, Asha's age is $2\frac{1}{4}$ yr which is not possible]

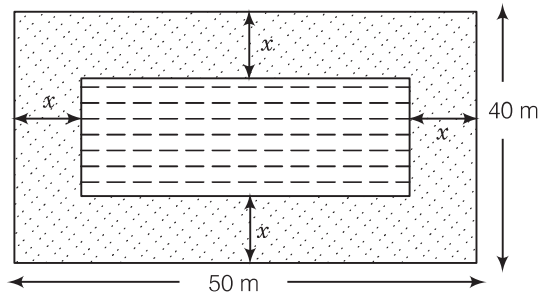
Hence, required age of Nisha = 5 yr

and required age of Asha = $x^2 + 2 = (5)^2 + 2 = 25 + 2 = 27$ yr

- Q. 7** In the centre of a rectangular lawn of dimensions $50\text{m} \times 40\text{m}$, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m^2 [see figure]. Find the length and breadth of the pond.



- Sol.** Given that a rectangular pond has to be constructed in the centre of a rectangular lawn of dimensions $50\text{ m} \times 40\text{ m}$. So, the distance between pond and lawn would be same around the pond. Say $x\text{ m}$.



Now, length of rectangular lawn (l_1) = 50 m

and breadth of rectangular lawn (b_1) = 40 m

\therefore Length of rectangular pond (l_2) = $50 - (x + x) = 50 - 2x$

and breadth of rectangular pond (b_2) = $40 - (x + x) = 40 - 2x$

Also, area of the grass surrounding the pond = 1184 m^2

\therefore Area of rectangular lawn – Area of rectangular pond

= Area of grass surrounding the pond

$$l_1 \times b_1 - l_2 \times b_2 = 1184 \quad [\because \text{area of rectangle} = \text{length} \times \text{breadth}]$$

$$\Rightarrow 50 \times 40 - (50 - 2x)(40 - 2x) = 1184$$

$$\Rightarrow 2000 - (2000 - 80x - 100x + 4x^2) = 1184$$

$$\Rightarrow 80x + 100x - 4x^2 = 1184$$

$$\Rightarrow 4x^2 - 180x + 1184 = 0$$

$$\Rightarrow x^2 - 45x + 296 = 0$$

$$\Rightarrow x^2 - 37x - 8x + 296 = 0$$

[by splitting the middle term]

$$\Rightarrow x(x - 37) - 8(x - 37) = 0$$

$$\Rightarrow (x - 37)(x - 8) = 0$$

∴ $x = 8$
 [At $x = 37$, length and breadth of pond are -24 and -34 , respectively but length and breadth cannot be negative. So, $x = 37$ cannot be possible]
 ∴ Length of pond = $50 - 2x = 50 - 2(8) = 50 - 16 = 34$ m
 and breadth of pond = $40 - 2x = 40 - 2(8) = 40 - 16 = 24$ m
 Hence, required length and breadth of pond are 34 m and 24 m, respectively.

Q. 8 At t min past 2 pm, the time needed by the minute hand of a clock to show 3 pm was found to be 3 min less than $\frac{t^2}{4}$ min. Find t .

Sol. We know that, the time between 2 pm to 3 pm = 1 h = 60 min
 Given that, at t min past 2pm, the time needed by the min hand of a clock to show 3 pm was found to be 3 min less than $\frac{t^2}{4}$ min i.e.,

$$t + \left(\frac{t^2}{4} - 3\right) = 60$$

$$\Rightarrow 4t + t^2 - 12 = 240$$

$$\Rightarrow t^2 + 4t - 252 = 0$$

$$\Rightarrow t^2 + 18t - 14t - 252 = 0 \quad \text{[by splitting the middle term]}$$

$$\Rightarrow t(t + 18) - 14(t + 18) = 0 \quad \text{[since, time cannot be negative, so } t \neq -18\text{]}$$

$$\Rightarrow (t + 18)(t - 14) = 0$$

$$\therefore t = 14 \text{ min}$$

Hence, the required value of t is 14 min.