CHAPTER 12

HEIGHT AND DISTANCES

12.1 INTRODUCTION

In this chapter, we shall be applying the trigonometric results to discuss problems regarding heights and distances. We begin by defining some terms which will be used in this chapter.

12.2 ANGLES OF ELEVATION AND DEPRESSION

Let O and P be two points such that the point P is at higher level. Let OA and PB be horizontal lines through O and P respectively.

If an observer is at O and the point P is the object under consideration, then the line OP is called the *line of sight* of the point P and the angle AOP, between the line of sight and the horizontal line OA, is known as the angle of elevation of point P as seen from O.

If an observer is at P and the object under consideration is at O, then the angle BPO is known as the angle of *depression* of O as seen from P.

Obviously, the angle of elevation of a point P as seen from a point O is equal to the angle of *depression* of O as seen from P.

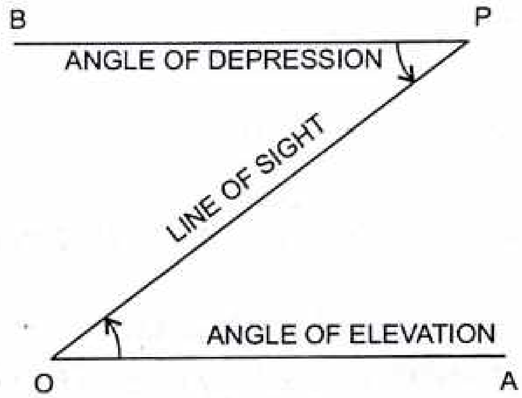

Fig. 12.1

LEVEL-1

EXAMPLE 1 *A tower is $100\sqrt{3}$ metres high. Find the angle of elevation if its top from a point 100 metres away from its foot.*

SOLUTION Let AB be the tower of height $100\sqrt{3}$ metres, and let C be a point at a distance of 100 metres from the foot of the tower.

Let θ be the angle of elevation of the top of the tower from point C.

Clearly, in ΔCAB the lengths of base AC and perpendicular AB are known. So, we will use the trigonometric ratio containing base and perpendicular. Such a ratio is tangent. Taking tangent of angle $\angle ACB$ in ΔCAB, we have

$$\tan\theta = \frac{AB}{AC}$$

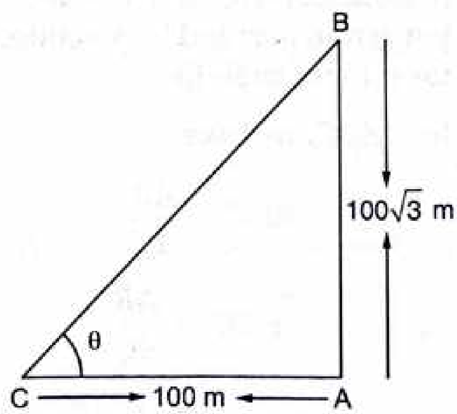

Fig. 12.2

$$\Rightarrow \quad \tan\theta = \frac{100\sqrt{3}}{100} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^\circ$$

$$\Rightarrow \quad \theta = 60^\circ$$

Hence, the angle of elevation of the top of the tower from a point 100 metres away from its foot is 60°.

EXAMPLE 2 *The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.* **[NCERT]**

SOLUTION Let AB be the tower of height h meters and C be a point on the ground such that the angle of elevation of the top A of tower AB is of 30°.

In ΔABC, we are given $\angle C = 30^\circ$ and Base $BC = 30$ m and we have to find perpendicular AB. So, we use that trigonometrical ratios which contains base and perpendicular. Clearly, such ratio is tangent. So, we take tangent of $\angle C$.

In ΔABC, taking tangent of $\angle C$, we have,

$$\tan C = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} \text{ metres} = 10\sqrt{3} \text{ metres}$$

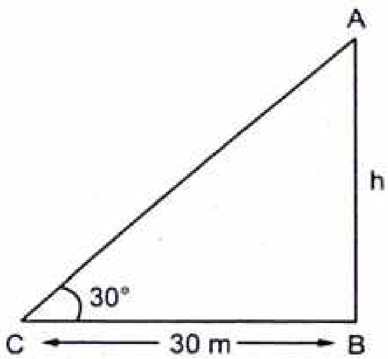

Fig. 12.3

Hence, the height of the tower is $10\sqrt{3}$ metres.

EXAMPLE 3 *A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.* **[NCERT]**

SOLUTION Let A be the kite and CA be the string attached to the kite such that its one end is tied to a point C on the ground. The inclination of the string CA with the ground is 60°.

In ΔABC, we are given that $\angle C = 60^\circ$ and perpendicular $AB = 60$ m and we have to find hypotenuse AC. So, we use the trigonometric ratio involving perpendicular and hypotenuse. Clearly, such ratio is sine. So, we take sine of angle C.

In ΔABC, we have

$$\sin C = \frac{AB}{AC}$$

$$\Rightarrow \quad \sin 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

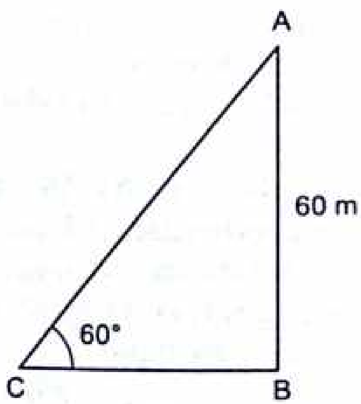

Fig. 12.4

$$\Rightarrow \qquad AC = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}.$$

Hence, the length of the string is $40\sqrt{3}$ m

EXAMPLE 4 *The string of a kite is 100 metres long and it makes an angle of 60° with the horizontal. Find the height of the kite, assuming that there is no slack in the string.*

SOLUTION Let OA be the horizontal ground, and let K be the position of the kite at a height h above the ground. Then, $AK = h$.

It is given that $OK = 100$ metres and $\angle AOK = 60°$.

Thus, in $\Delta\, OAK$, we have hypotenuse $OK = 100$ m and $\angle AOK = 60°$ and we wish to find the perpendicular AK. So, we use the trigonometric ratio involving perpendicular and hypotenuse. Clearly, sine is such a ratio. So, we take the sine of $\angle AOK$ in ΔOAK.

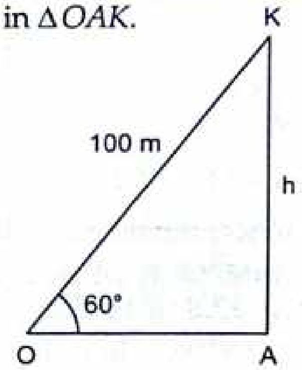

Fig. 12.5

In ΔAOK, we have

$$\sin 60° = \frac{AK}{OK}$$

$$\Rightarrow \qquad \sin 60° = \frac{h}{100}$$

$$\Rightarrow \qquad h = 100 \sin 60°$$

$$\Rightarrow \qquad h = 100\frac{\sqrt{3}}{2} = 50\sqrt{3} = 86.60 \text{ metres}.$$

Hence, the height of the kite is 86.60 metres.

EXAMPLE 5 *A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is 30°. Calculate the distance covered by the artist in climbing to the top of the pole.*

SOLUTION Clearly, distance covered by the artist is equal to the length of the rope AC. Let AB be the vertical pole of height 12 m.

It is given that $\angle ACB = 30°$.

Thus, in right-angled triangle ABC, we have

Perpendicular $AB = 12$ m, $\angle ACB = 30°$ and we wish to find hypotenuse AC.

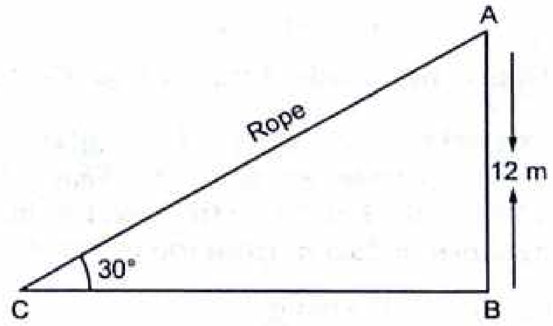

Fig. 12.6

$$\therefore \qquad \sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \qquad \frac{1}{2} = \frac{12}{AC}$$

$$\Rightarrow \qquad AC = 24 \text{ m}$$

Hence, the distance covered by the circus artist is 24 m.

EXAMPLE 6 *A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.*

SOLUTION Let AB be the vertical pole and CA be the 20 m long rope such that its one end is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.

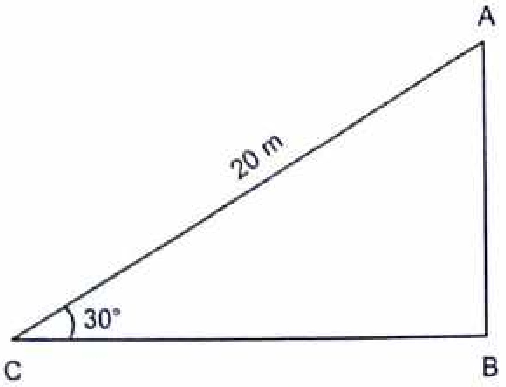

Fig. 12.7

In ΔABC, we have

$$\sin 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{AC}$$

$$\Rightarrow \quad AB = 10 \text{ m}.$$

Hence, the height of the pole is 10 m.

EXAMPLE 7 *A bridge across a river makes an angle of 45° with the river bank as shown in Fig. 12.8. If the length of the bridge across the river is 150 m, what is the width of the river?*

SOLUTION In right triangle ABC, we have

$$\sin 45^\circ = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$\Rightarrow \quad BC = \frac{150}{\sqrt{2}}$$

$$\Rightarrow \quad BC = 75\sqrt{2} \text{ m}$$

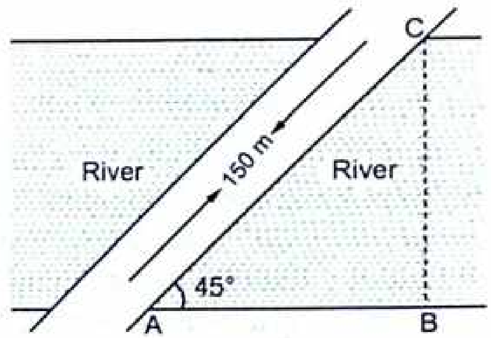

Fig. 12.8

Hence, the width of the river is $75\sqrt{2}$ metres.

EXAMPLE 8 *An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of the tower from her eyes is 45°. What is the height of the tower?* **[NCERT]**

SOLUTION Let AB be the tower of height h and CD be the observer of height 1.5 m at a distance of 28.5 m from the tower AB.

In ΔAED, we have

$$\tan 45^\circ = \frac{AE}{DE}$$

$$\Rightarrow \quad 1 = \frac{AE}{28.5}$$

$$\Rightarrow \quad AE = 28.5 \text{ m}$$

$$\therefore \quad h = AE + BE = AE + DC$$

$$= (28.5 + 1.5) \text{ m} = 30 \text{ m}$$

Hence, the height of the tower is 30 m.

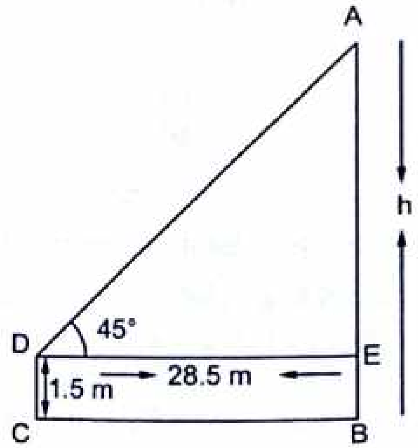

Fig. 12.9

EXAMPLE 9 *An electrician has to repair an electric fault on a pole of height 4 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use which when inclined at an angle of 60° to the horizontal would enable him to reach the required position?* **[NCERT]**

SOLUTION Let AC be the electric pole of height 4 m. Let B be a point 1.3 m below the top A of the pole AC.

Then, $BC = AC - AB = (4 - 1.3)\text{ m} = 2.7\text{ m}$

Let BD be the ladder inclined at an angle of 60° to the horizontal.

In ΔBCD, we have

$$\sin 60° = \frac{BC}{BD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{2.7}{BD} \qquad [\because BC = 2.7\text{ m}]$$

$$\Rightarrow \quad BD = \frac{2 \times 2.7}{\sqrt{3}}\text{ m} = \frac{5.4}{\sqrt{3}}\text{ m} = \frac{5.4 \times \sqrt{3}}{3}\text{ m}$$

$$\Rightarrow \quad BD = (1.8)\sqrt{3}\text{ m} = \frac{9}{5}\sqrt{3}\text{ m}$$

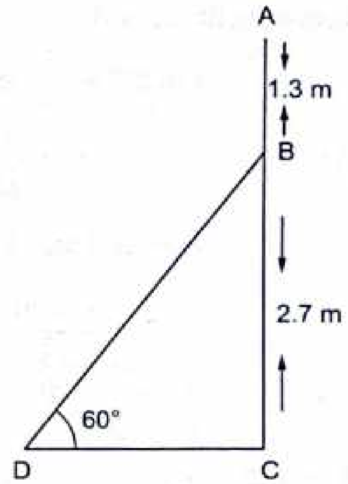

Fig. 12.10

Hence, the length of the ladder should be $\frac{9\sqrt{3}}{5}$ m.

EXAMPLE 10 *From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The angle of elevation of the top of a water tank (on the top of the tower) is 45°. Find the (i) height of the tower (ii) the depth of the tank.* **[NCERT]**

SOLUTION Let BC be the tower of height h metre and CD be the water tank of height h_1 metre. Let A be a point on the ground at a distance of 40 m away from the foot B of the tower.

In $\Delta\, ABD$, we have

$$\tan 45° = \frac{BD}{AB}$$

$$\Rightarrow \quad 1 = \frac{h + h_1}{40}$$

$$\Rightarrow \quad h + h_1 = 40\text{ m} \qquad \dots (i)$$

In $\Delta\, ABC$, we have

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$$

$$\Rightarrow \quad h = \frac{40}{\sqrt{3}}\text{ m} = \frac{40\sqrt{3}}{3}\text{ m} = 23.1\text{ m}$$

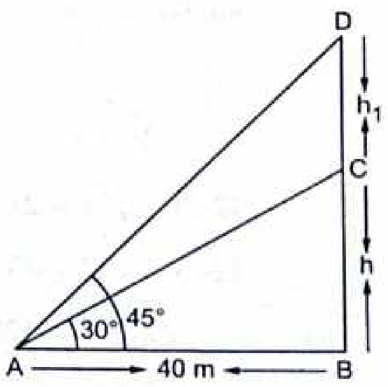

Fig. 12.11

Substituting the value of h in (i), we have

$$23.1 + h_1 = 40$$

$$\Rightarrow \quad h_1 = (40 - 23.1)\text{ m} = 16.9\text{ m}$$

Hence, the height of the tower is $h = 23.1$ m and the depth of the tank is $h_1 = 16.9$ m.

EXAMPLE 11 *A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°. When he retreates 20 m from the bank, he finds the angle to be 30°. Find the height of the tree and the breadth of the river.*

SOLUTION Let AB be the width of the river and BC be the tree which makes an angle of 60° at a point A on the opposite bank. Let D be the position of the person after retreating 20 m from the bank. Let $AB = x$ metres and $BC = h$ metres.

From right angled triangles ABC and DBC, we have

$$\tan 60^\circ = \frac{BC}{AB} \text{ and } \tan 30^\circ = \frac{BC}{DB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \text{ and } \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\Rightarrow \quad h = x\sqrt{3} \text{ and } h = \frac{x+20}{\sqrt{3}}$$

$$\Rightarrow \quad x\sqrt{3} = \frac{x+20}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = x + 20$$

$$\Rightarrow \quad x = 10 \text{ m}$$

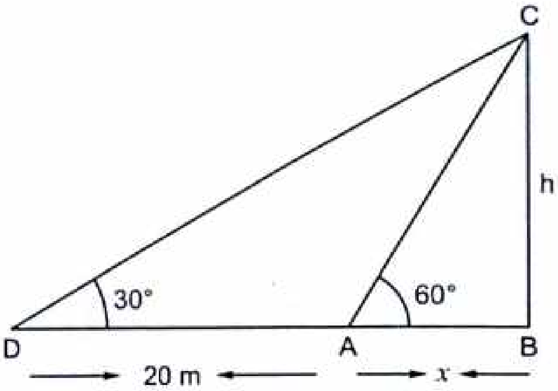

Fig. 12.12

Putting $x = 10$ in $h = \sqrt{3}x$, we get

$$h = 10\sqrt{3} = 17.32 \text{ m}$$

Hence, height of the tree is 17.32 m and the breadth of the river is 10 m.

EXAMPLE 12 *A tree 12 m high, is broken by the wind in such a way that its top touches the ground and makes an angle 60° with the ground. At what height from the bottom the tree is broken by the wind?*

SOLUTION Let AB be the tree of height 12 metres. Suppose the tree is broken by the wind at point C and the part CB assumes the position CO and meets the ground at O.

Let $AC = x$. Then, $CO = CB = 12 - x$. It is given that $\angle AOC = 60°$

In $\Delta\, OAC$, we have

$$\sin 60^\circ = \frac{AC}{OC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{x}{12-x}$$

$$\Rightarrow \quad 12\sqrt{3} - \sqrt{3}x = 2x$$

$$\Rightarrow \quad 12\sqrt{3} = x(2+\sqrt{3})$$

$$\Rightarrow \quad x = \frac{12\sqrt{3}}{2+\sqrt{3}} = \frac{12\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 12\sqrt{3}(2-\sqrt{3})$$

$$\Rightarrow \quad x = 24\sqrt{3} - 36 = 5.569 \text{ metres}$$

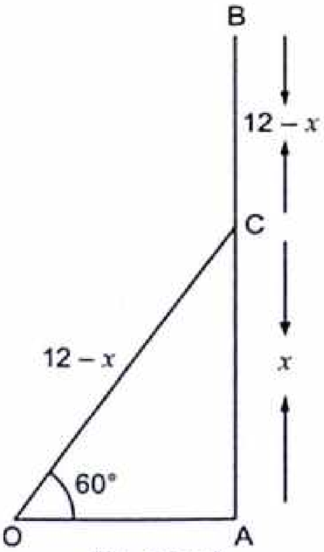

Fig. 12.13

Hence, the tree is broken at a height of 5.569 metres from the ground.

EXAMPLE 13 *A tree is broken by the wind. The top struck the ground at an angle of 30° and at a distance of 30 metres from the root. Find the whole height of the tree.*

SOLUTION Let AB be the tree broken at a point C such that the broken part CB takes the position CO and strikes the ground at O. It is given that $OA = 30$ metres and $\angle AOC = 30°$.

Let $AC = x$ and $CB = y$. Then, $CO = y$.

In ΔOAC, we have

$$\tan 30^\circ = \frac{AC}{OA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{30}$$

$$\Rightarrow \quad x = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

Again in $\Delta\, OAC$, we have

$$\cos 30^\circ = \frac{OA}{OC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{30}{y}$$

$$\Rightarrow \quad y = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

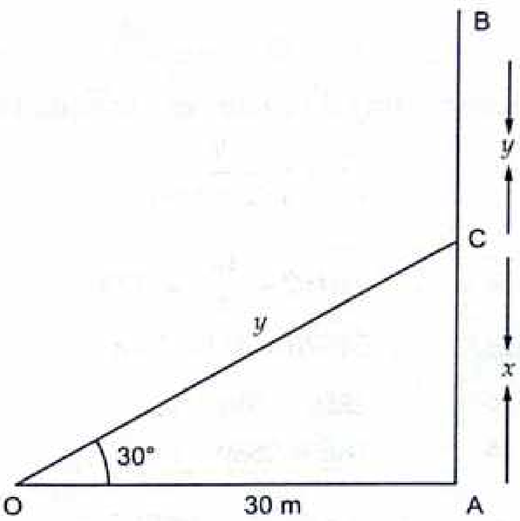

Fig. 12.14

$\therefore$ Height of the tree $= (x + y)$ metres

$$= (10\sqrt{3} + 20\sqrt{3}) \text{ metres}$$

$$= 30\sqrt{3} \text{ metres} = 30 \times 1.732 \text{ metres} = 51.96 \text{ metres}.$$

EXAMPLE 14 *At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is 5/12. On walking 192 metres towards the tower, the tangent of the angle of elevation is 3/4. Find the height of the tower.*

SOLUTION Let AB be the tower and let the angle of elevation of its top at C be α. Let D be a point at a distance of 192 metres from C such that the angle of elevation of the top of the tower at D be β. Let h be the height of the tower and $AD = x$.

It is given that

$$\tan\alpha = \frac{5}{12} \text{ and } \tan\beta = \frac{3}{4}$$

In $\Delta\, CAB$, we have

$$\tan\alpha = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{h}{x + 192} \quad \text{... (i)}$$

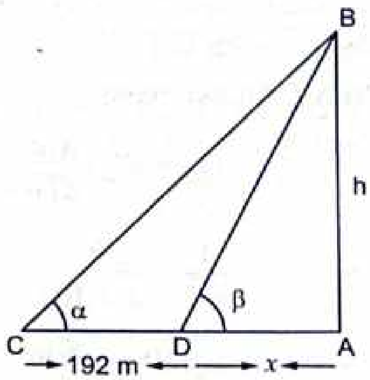

Fig. 12.15

In $\Delta\, DAB$, we have

$$\tan\beta = \frac{AB}{AD}$$

$$\Rightarrow \quad \tan\beta = \frac{h}{x}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{h}{x} \quad \text{... (ii)}$$

We have to find h. This means that we have to eliminate x from equations (i) and (ii). From equation (ii), we have

$$3x = 4h \Rightarrow x = \frac{4h}{3}$$

Substituting this value of x in equation (i), we get

$$\frac{5}{12} = \frac{h}{192 + 4h/3}$$

$\Rightarrow \quad 5\left(192 + \frac{4h}{3}\right) = 12h$

$\Rightarrow \quad 5(576 + 4h) = 36\,h$

$\Rightarrow \quad 2880 + 20h = 36h$

$\Rightarrow \quad 16h = 2880$

$\Rightarrow \quad h = \frac{2880}{16} = 180$

Hence, the height of the tower is 180 metres.

EXAMPLE 15 *The shadow of a vertical tower on level ground increases by 10 metres, when the altitude of the sun changes from angle of elevation 45° to 30°. Find the height of the tower, correct to one place of decimal. (Take $\sqrt{3} = 1.73$)*

SOLUTION Let AB be the tower and AC and AD be its shadows when the angles of elevation of the sun are 45° and 30° respectively. Then, $CD = 10$ metres. Let h be the height of the tower and let $AC = x$ metres.

In $\Delta\, CAB$, we have

$$\tan 45° = \frac{AB}{AC}$$

$\Rightarrow \quad 1 = \frac{h}{x}$

$\Rightarrow \quad x = h \quad$ ...(i)

In $\Delta\, DAB$, we have

$$\tan 30° = \frac{AB}{AD}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x + 10}$

$\Rightarrow \quad x + 10 = \sqrt{3}\,h \quad$ ...(ii)

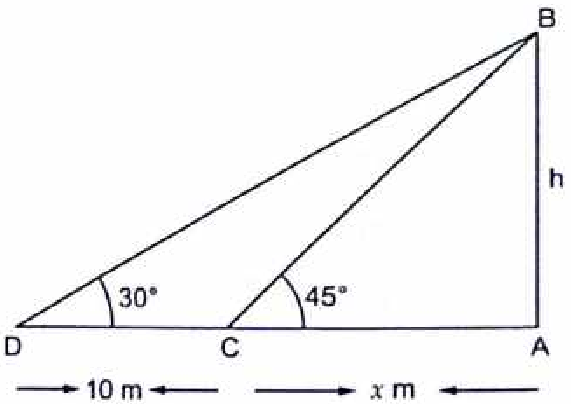

Fig. 12.16

Substituting the value of x obtained from equation (i) and (ii), we get

$$h + 10 = \sqrt{3}\,h$$

$\Rightarrow \quad h(\sqrt{3} - 1) = 10$

$\Rightarrow \quad h = \frac{10}{\sqrt{3} - 1} = \frac{10}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = 10\left(\frac{\sqrt{3} + 1}{2}\right) = 5(\sqrt{3} + 1)$

$\Rightarrow \quad h = 5(1.73 + 1) = 13.65$ metres

Hence, the height of the tower is 13.65 metres.

EXAMPLE 16 *From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 30° and 45°. Find the height of the hill.*

SOLUTION Let AB be the hill of height h km. Let C and D be two stones due east of the hill at a distance of 1 km from each other such that the angles of depression of C and D be 45° and 30° respectively. Let $AC = x$ km.

In ΔCAB, we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad 1 = \frac{h}{x} \Rightarrow h = x \qquad \text{...(i)}$$

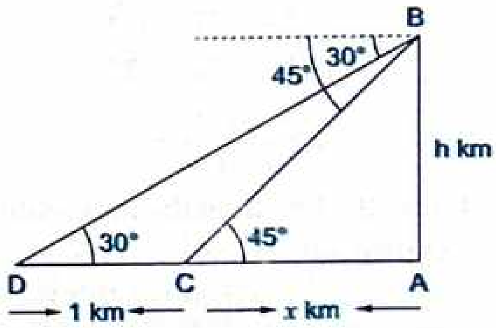

Fig. 12.17

In ΔDAB, we have

$$\tan 30^\circ = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$$\Rightarrow \quad \sqrt{3}h = x + 1 \qquad \text{...(ii)}$$

Substituting the value of x from equation (i) in equation (ii), we get

$$\sqrt{3}h = h + 1$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 1$$

$$\Rightarrow \quad h = \frac{1}{\sqrt{3} - 1} = \frac{\sqrt{3} + 1}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$\Rightarrow \quad h = \frac{\sqrt{3} + 1}{2} = \frac{2.73}{2} = 1.365$$

Hence, the height of the hill is 1.365 km.

EXAMPLE 17 *Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15°. (Use* $\tan 15^\circ = 0.27$*)*

SOLUTION Let AB be the mountain of height h kilometres. Let C be a point at a distance of x km. from the base of the mountain such that the angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that the angle of elevation at D is of 15°.

In ΔCAB, we have

$$\tan 30^\circ = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h \qquad \text{...(i)}$$

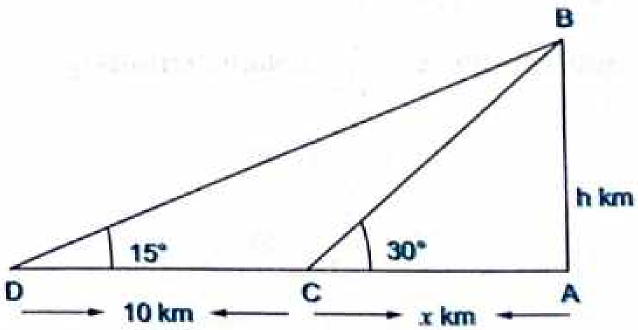

Fig. 12.18

In ΔDAB, we have

$$\tan 15^\circ = \frac{AB}{AD}$$

$$\Rightarrow \quad 0.27 = \frac{h}{x + 10}$$

$$\Rightarrow \quad (0.27)(x + 10) = h \quad \ldots \text{(ii)}$$

Substituting $x = \sqrt{3}\, h$ obtained from equation (i) in equation (ii), we get

$$0.27\,(\sqrt{3}\, h + 10) = h$$

$$\Rightarrow \quad 0.27 \times 10 = h - 0.27 \times \sqrt{3}\, h$$

$$\Rightarrow \quad h\,(1 - 0.27 \times \sqrt{3}) = 2.7$$

$$\Rightarrow \quad h\,(1 - 0.46) = 2.7$$

$$\Rightarrow \quad h = \frac{2.7}{0.54} = 5$$

Hence, the height of the mountain is 5 km.

EXAMPLE 18 *A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.*

SOLUTION Let AB be the tree of height h metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that $BC = x$ metres. Let D be the new position of the man. It is given that $CD = 40$ metres and the angles of elevation of the top of the tree at C and D are $60°$ and $30°$ respectively i.e., $\angle ACB = 60°$ and $\angle ADB = 30°$.

In $\Delta\, CBA$, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad \ldots \text{(i)}$$

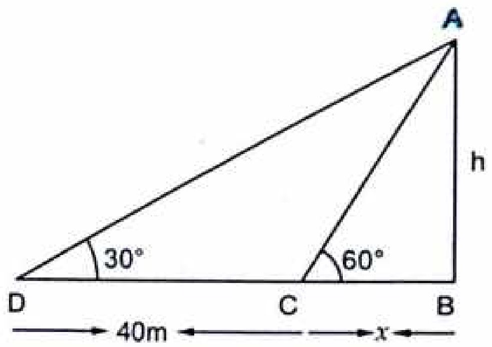

Fig. 12.19

In $\Delta\, DBA$, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x + 40}$$

$$\Rightarrow \quad \sqrt{3}h = x + 40 \quad \ldots \text{(ii)}$$

Substituting $x = \dfrac{h}{\sqrt{3}}$ obtained from equation (i) in equation (ii), we get

$$\sqrt{3}\, h = \frac{h}{\sqrt{3}} + 40$$

$$\Rightarrow \quad \sqrt{3}\, h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \quad \frac{3h - h}{\sqrt{3}} = 40 \Rightarrow \frac{2h}{\sqrt{3}} = 40 \Rightarrow h = 20\sqrt{3} = 20 \times 1.73 = 34.64 \text{ metres}$$

Substituting h in equation (i), we get $x = \frac{20\sqrt{3}}{\sqrt{3}} = 20$ metres

Hence, the height of the tree is 34.64 metres and width of the river is 20 metres.

EXAMPLE 19 *An aeroplane at an altitude of 1200 metres finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two ships.*

SOLUTION Let the aeroplane be at B and let the two ships be at C and D such that their angles of depression from B are 30° and 60° respectively.

We have, $AB = 1200$ metres. Let $AC = x$ and $CD = y$.

In ΔCAB, we have

$$\tan 60° = \frac{AB}{CA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1200}{x}$$

$$\Rightarrow \quad x = \frac{1200}{\sqrt{3}} = 400\sqrt{3}$$

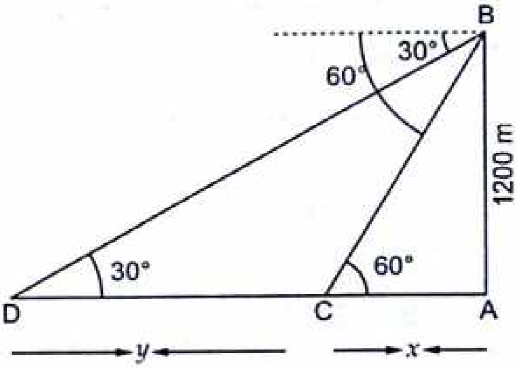

Fig. 12.20

In ΔBAD, we have

$$\tan 30° = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1200}{x+y}$$

$$\Rightarrow \quad x + y = 1200\sqrt{3}$$

$$\Rightarrow \quad y = 1200\sqrt{3} - x$$

$$\Rightarrow \quad y = 1200\sqrt{3} - 400\sqrt{3} = 800\sqrt{3} = 800 \times 1.732 = 1385.6$$

Hence, the distance between the two ships is 1385.6 metres.

EXAMPLE 20 *The shadow of a flag-staff is three times as long as the shadow of the flag-staff when the sun rays meet the ground at an angle of 60°. Find the angle between the sun rays and the ground at the time of longer shadow.*

SOLUTION Let AB be the flag-staff and let $x = AC$ be the length of its shadow when the sun rays meet the ground at an angle of 60°. Let θ be the angle between the sun rays and the ground when the length of the shadow of the flag-staff is $AD = 3x$. Let h be the height of the flag-staff.

In ΔCAB, we have

$$\tan 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$$

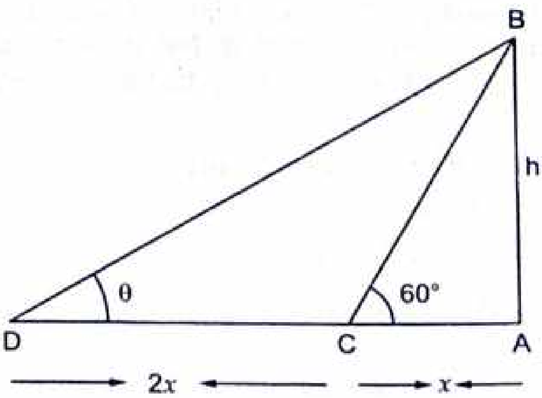

Fig. 12.21

In ΔDAB, we have

$$\tan \theta = \frac{AB}{AD}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{3x}$$

$$\Rightarrow \quad \tan\theta = \frac{\sqrt{3}x}{3x} \qquad [\because h = \sqrt{3}\,x]$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

Thus, the angle between the sun rays and the ground is 30° at the time of longer shadow.

EXAMPLE 21 *An aeroplane at an altitude of 200 metres observes the angles of depression of opposite points on the two banks of a river to be 45° and 60°. Find the width of the river.*

SOLUTION Let P be the position of the aeroplane and let A and B be two points on the two banks of a river such that the angles of depression at A and B are 60° and 45° respectively. Let $AM = x$ metres and $BM = y$ metres. We have to find AB.

In $\Delta\, AMP$, we have

$$\tan 60^\circ = \frac{PM}{AM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{200}{x}$$

$$\Rightarrow \quad 200 = \sqrt{3}\,x$$

$$\Rightarrow \quad x = \frac{200}{\sqrt{3}} \qquad \ldots \text{(i)}$$

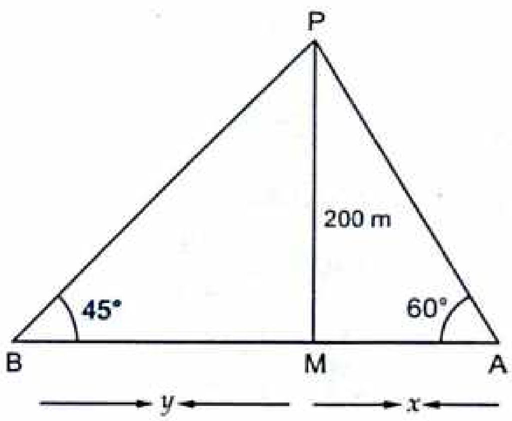

Fig. 12.22

In $\Delta\, BMP$, we have

$$\tan 45^\circ = \frac{PM}{BM}$$

$$\Rightarrow \quad 1 = \frac{200}{y}$$

$$\Rightarrow \quad y = 200 \qquad \ldots \text{(ii)}$$

From equation (i) and (ii), we get

$$AB = x + y = \frac{200}{\sqrt{3}} + 200 = 200\left(\frac{1}{\sqrt{3}} + 1\right) = 315.4 \text{ metres}.$$

Hence, the width of the river is 315.4 metres.

EXAMPLE 22 *Two pillars of equal height and on either side of a road, which is 100 m wide. The angles of elevation of the top of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars and the height of each pillar.*

[NCERT, CBSE 2005, 2013]

SOLUTION Let AB and CD be two pillars, each of height h metres. Let P be a point on the road such that $AP = x$ metres. Then, $CP = (100 - x)$ metres. It is given that $\angle APB = 60^\circ$ and $\angle CPD = 30^\circ$.

In $\Delta\, PAB$, we have

$$\tan 60^\circ = \frac{AB}{AP}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}x$ ...(i)

In $\Delta\, PCD$, we have

$$\tan 30° = \frac{CD}{PC}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$

$\Rightarrow \quad h\sqrt{3} = 100 - x$ ...(ii)

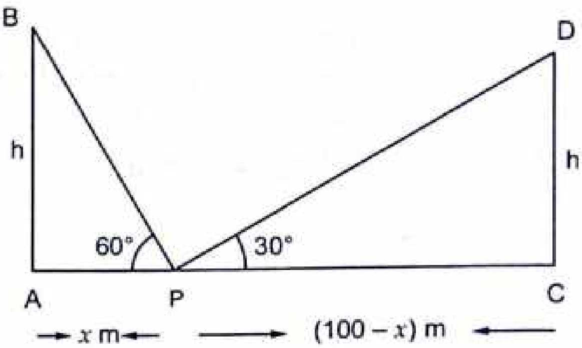

Fig. 12.23

Eliminating h between equation (i) and (ii), we get

$$3x = 100 - x \Rightarrow 4x = 100 \Rightarrow x = 25$$

Substituting $x = 25$ in equation (i), we get

$$h = 25\sqrt{3} = 25 \times 1.732 = 43.3$$

Thus, the required point is at a distance of 25 metres from the first pillar and 75 metres from the second pillar. The height of the pillars is 43.3 metres.

EXAMPLE 23 *As observed from the top of a light house, 100 m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during the period of observation.* **[CBSE 2004, 2018]**

SOLUTION Let A and B be the two positions of the ship. Let d be the distance travelled by the ship during the period of observation i.e. $AB = d$ metres.

Let the observer be at O, the top of the light house PO.

It is given that $PO = 100$ m and the angles of depression from O of A and B are 30° and 45° respectively.

$\therefore \quad \angle OAP = 30°$ and $\angle OBP = 45°$

In $\Delta\, OPB$, we have

$$\tan 45° = \frac{OP}{BP}$$

$\Rightarrow \quad 1 = \frac{100}{BP}$

$\Rightarrow \quad BP = 100$ m

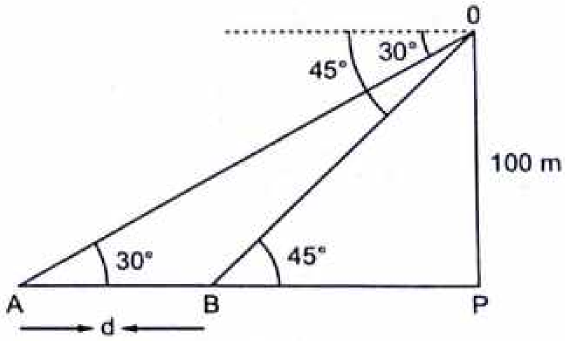

Fig. 12.24

In $\Delta\, OPA$, we have

$\Rightarrow \quad \tan 30° = \frac{OP}{AP}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{d + BP}$

$\Rightarrow \quad d + BP = 100\sqrt{3}$

$\Rightarrow \quad d + 100 = 100\sqrt{3}$ $[\because BP = 100 \text{ m}]$

$\Rightarrow \quad d = 100\sqrt{3} - 100$

$\Rightarrow \quad d = 100(\sqrt{3} - 1) = 100(1.732 - 1) = 73.2$ m

Hence, the distance travelled by the ship from A to B is 73.2 m.

EXAMPLE 24 *The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°. Find the height of the tower PQ and the distance XQ.*

SOLUTION In Δ YRQ, we have

$$\tan 45^\circ = \frac{QR}{YR}$$

$$\Rightarrow \quad 1 = \frac{x}{YR}$$

$$\Rightarrow \quad YR = x$$

$$\Rightarrow \quad XP = x \qquad [\because YR = XP]$$

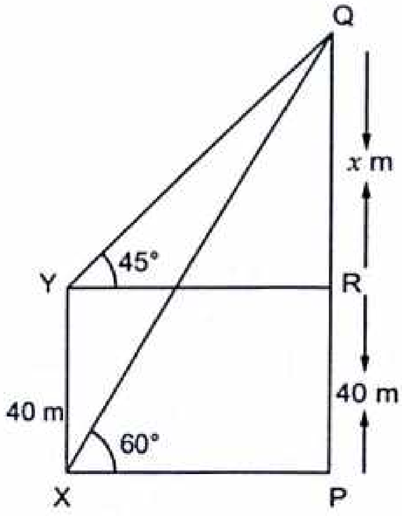

Fig. 12.25

In Δ XPQ, we have

$$\tan 60^\circ = \frac{PQ}{PX}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x+40}{x} \qquad [\because PX = x]$$

$$\Rightarrow \quad \sqrt{3}x = x + 40$$

$$\Rightarrow \quad x(\sqrt{3}-1) = 40$$

$$\Rightarrow \quad x = \frac{40}{\sqrt{3}-1}$$

$$\Rightarrow \quad x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 20(\sqrt{3}+1) = 54.64$$

So, height of the tower $PQ = x + 40 = 54.64 + 40 = 94.64$ metres

In Δ XPQ, we have

$$\sin 60^\circ = \frac{PQ}{XQ}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{94.64}{XQ}$$

$$\Rightarrow \quad XQ = \frac{94.64 \times 2}{\sqrt{3}}$$

$$\Rightarrow \quad XQ = \frac{94.64 \times 2 \times \sqrt{3}}{3} = 109.3 \text{ metres.}$$

EXAMPLE 25 *From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively show that the height of the opposite house is 23.66 metres* (*Take* $\sqrt{3} = 1.732$)

[CBSE 2006]

SOLUTION Let the window be at P at a height of 15 metres above the ground and CD be the house on the opposite side of the street such that the angles of devation of the top D of house CD as seen from P is of 30° and the angle of depression of the foot C of house CD as seen from P is of 45°.

Let h metres be the height of the house CD.

We have,

$$QD = CD - CQ = CD - AP = (h-15) \text{ metres.}$$

In ΔPQC, we have

$$\tan 45° = \frac{QC}{PQ} \Rightarrow 1 = \frac{15}{PQ} \Rightarrow PQ = 15 \text{ metres.}$$

In ΔPQD, we have

$$\tan 30° = \frac{QD}{PQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-15}{15} \Rightarrow h - 15 = \frac{15}{\sqrt{3}} \Rightarrow h - 15 = 5\sqrt{3}$$

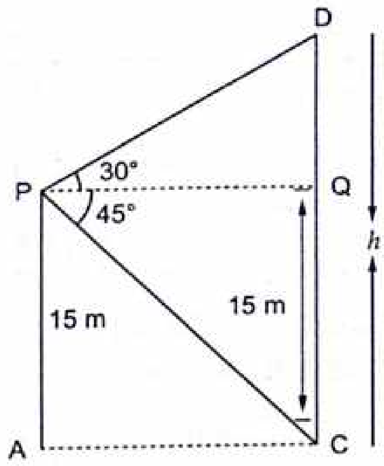

Fig. 12.26

$$\Rightarrow \quad h = 15 + 5 \times 1.732 = 23.66 \text{ metres,}$$

Hence, the height of the opposite house is 23.66 metres

EXAMPLE 26 *From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.* **[CBSE 2005]**

SOLUTION Let AB be the building and CD be the tower. Let $CD = h$ metres. Let DE be horizontal from D. It is given that the angles of depression of the top D and the bottom C of the tower CD are 30° and 60° respectively.

$$\therefore \quad \angle EDB = 30° \text{ and } \angle ACB = 60°$$

Let $AC = DE = x$.

In ΔDEB, we have

$$\tan 30° = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60-h}{x}$$

$$\Rightarrow \quad x = (60-h)\sqrt{3} \quad \text{...(i)}$$

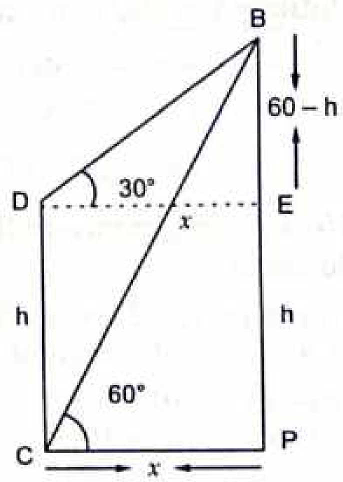

Fig. 12.27

In ΔCAB, we have

$$\tan 60° = \frac{AB}{CA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}} \quad \text{...(ii)}$$

From equations (i) and (ii), we have

$$(60-h)\sqrt{3} = \frac{60}{\sqrt{3}}$$

$$\Rightarrow \quad 3(60-h) = 60$$

$$\Rightarrow \quad 60 - h = 20$$

$$\Rightarrow \quad h = 40$$

Thus, the height of the tower is 40 metres.

EXAMPLE 27 *A man standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.*

[CBSE 2004, 2005, 2010]

SOLUTION Suppose the man is standing on the deck of a ship at point A and let CD be the hill. It is given that the angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°. Then, $\angle EAD = 60°, \angle BCA = 30°$.

Also, $AB = 10$ m

In ΔAED, we have

$$\tan 60° = \frac{DE}{EA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \quad \text{...(i)}$$

In ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x} \Rightarrow x = 10\sqrt{3} \quad \text{...(ii)}$$

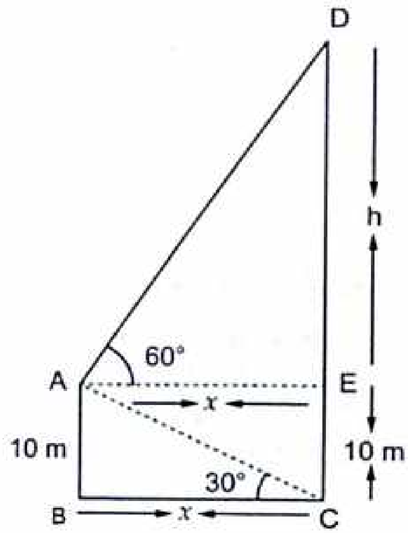

Fig. 12.28

Putting $x = 10\sqrt{3}$ in equation (i), we get

$$h = \sqrt{3} \times 10\sqrt{3} = 30$$

$$\Rightarrow \quad DE = 30 \text{ m}$$

$$\therefore \quad CD = CE + ED = 10 + 30 = 40 \text{ metres}$$

Hence, the distance of the hill from the ship is $10\sqrt{3}$ metres and the height of the hill is 40 metres.

EXAMPLE 28 *The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane.* **[CBSE 2008, 2014]**

SOLUTION Let P and Q be the two positions of the plane and let A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30° respectively.

$\therefore \quad \angle PAB = 60°, \angle QAB = 30°$. It is also given that $PB = 3600\sqrt{3}$ metres

In ΔABP, we have

$$\tan 60° = \frac{BP}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3600\sqrt{3}}{AB}$$

$$\Rightarrow \quad AB = 3600 \text{ m}$$

In ΔACQ, we have

$$\tan 30° = \frac{CQ}{AC}$$

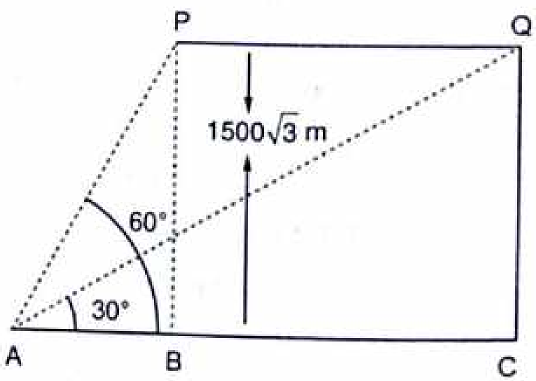

Fig. 12.29

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$$

$$\Rightarrow \quad AC = 3600 \times 3 = 10800 \text{ m}$$

$$\therefore \quad PQ = BC = AC - AB = 10800 - 3600 = 7200 \text{ m}$$

Thus, the plane travels 7200 m in 30 seconds.

Hence, Speed of plane $= \frac{7200}{30} = 240 \text{ m/sec} = \frac{240}{1000} \times 60 \times 60 = 864 \text{ km/hr}$

EXAMPLE 29 *There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. P and Q are points directly opposite to each other on two banks and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 30° and 45°, find the height of the tree.*

SOLUTION Let OA be the tree of height h metre.
In triangle POA and QOA, we have

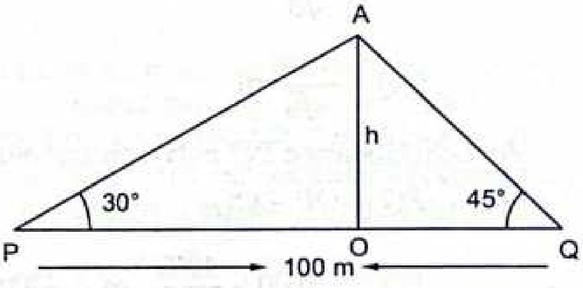

Fig. 12.30

$$\tan 30° = \frac{OA}{OP} \text{ and } \tan 45° = \frac{OA}{OQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{OP} \text{ and } 1 = \frac{h}{OQ}$$

$$\Rightarrow \quad OP = \sqrt{3}\,h \text{ and } OQ = h$$

$$\Rightarrow \quad OP + OQ = \sqrt{3}\,h + h$$

$$\Rightarrow \quad PQ = (\sqrt{3} + 1)\,h$$

$$\Rightarrow \quad 100 = (\sqrt{3} + 1)\,h \qquad [\because PQ = 100 \text{ m}]$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3} + 1} \text{ m} = \frac{100(\sqrt{3} - 1)}{2} \text{ m} = 50(1.732 - 1) \text{ m} = 36.6 \text{ m}$$

Hence, the height of the tree is 36.6 m.

EXAMPLE 30 *The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, find the height of the first tower.*

SOLUTION Let AB and CD be two towers of height h metres and 60 metres respectively such that the distance AC between them is 140 m. The angle of elevation of top B of tower AB as seen from D (top of tower CD) is 30°.

In $\Delta\, DEB$, we have

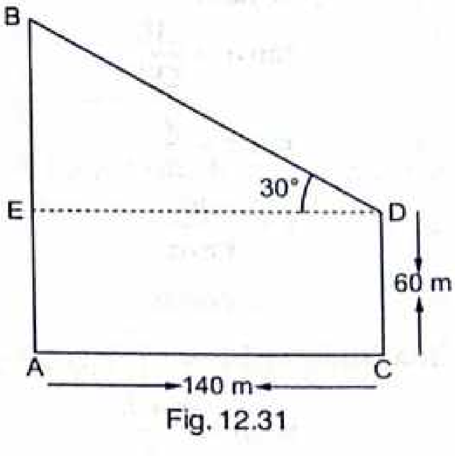

Fig. 12.31

$$\tan 30° = \frac{BE}{DE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BE}{140} \qquad [\because DE = AC = 140 \text{ m}]$$

$$\Rightarrow \quad BE = \frac{140}{\sqrt{3}} \text{ m} = \frac{140}{1.732} \text{ m} = 80.83 \text{ m}$$

$$\therefore \quad AB = AE + BE = CD + BE = 60 + 80.83 \text{ m} = 140.83 \text{ m}$$

Hence, the height of the second tower is 140.83 m.

EXAMPLE 31 *An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant.* **[CBSE 2008, 2009, 2016]**

SOLUTION Let P and Q be the positions of two aeroplanes when Q is vertically below P and $OP = 4000$ m. Let the angles of elevation of P and Q at a point A on the ground be 60° and 45° respectively.

In triangles AOP and AOQ, we have

$$\tan 60° = \frac{OP}{OA} \text{ and } \tan 45° = \frac{OQ}{OA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{4000}{OA} \text{ and } 1 = \frac{OQ}{OA}$$

$$\Rightarrow \quad OA = \frac{4000}{\sqrt{3}} \text{ and } OQ = OA$$

$$\Rightarrow \quad OQ = \frac{4000}{\sqrt{3}} \text{ m}$$

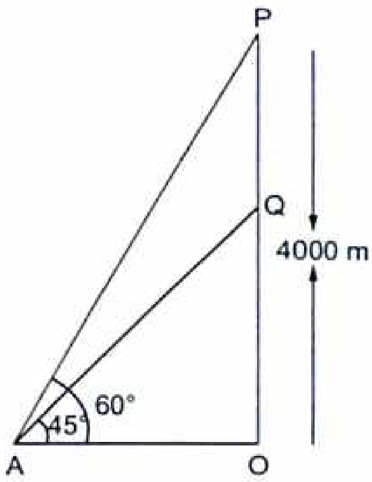

Fig. 12.32

∴ Vertical distance PQ between the aeroplanes is given by

$$PQ = OP - OQ$$

$$\Rightarrow \quad PQ = \left(4000 - \frac{4000}{\sqrt{3}}\right) \text{m} = 4000\frac{(\sqrt{3}-1)}{\sqrt{3}} \text{m} = 1690.53 \text{ m}$$

LEVEL-2

EXAMPLE 32 *A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are α and β respectively. Prove that the height of the tower is* $\frac{h \tan\alpha}{\tan\beta - \tan\alpha}$.

[NCERT EXEMPLAR]

SOLUTION Let AB be the tower and BC be the flag-staff. Let O be a point on the plane containing the foot of the tower such that the angles of elevation of the bottom B and top C of the flag-staff at O are α and β respectively. Let $OA = x$ metres, $AB = y$ metres and $BC = h$ metres.

In ΔOAB, we have

$$\tan\alpha = \frac{AB}{OA}$$

$$\Rightarrow \quad \tan\alpha = \frac{y}{x}$$

$$\Rightarrow \quad x = \frac{y}{\tan\alpha} \quad ...(i)$$

$$\Rightarrow \quad x = y\cot\alpha$$

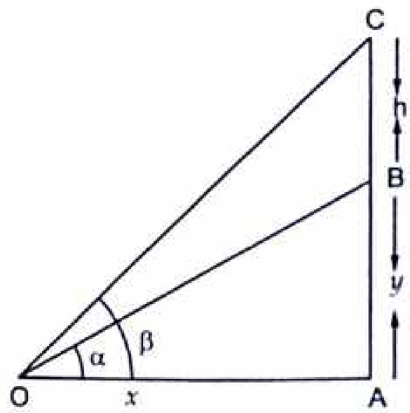

Fig. 12.33

In ΔOAC, we have

$$\tan\beta = \frac{y+h}{x}$$

$$\Rightarrow \quad x = \frac{y+h}{\tan\beta}$$

$$\Rightarrow \quad x = (y+h)\cot\beta \qquad \text{...(ii)}$$

On equating the values of x given in equations (i) and (ii), we get

$$\Rightarrow \quad y\cot\alpha = (y+h)\cot\beta$$

$$\Rightarrow \quad (y\cot\alpha - y\cot\beta) = h\cot\beta$$

$$\Rightarrow \quad y(\cot\alpha - \cot\beta) = h\cot\beta$$

$$\Rightarrow \quad y = \frac{h\cot\beta}{\cot\alpha - \cot\beta} = \frac{\frac{h}{\tan\beta}}{\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}} = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence, the height of the tower is $\frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

EXAMPLE 33 *The angles of elevation of the top of a tower from two points at distances a and b metres from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$ metres.* **[NCERT EXEMPLAR, CBSE 2002 C, 2004]**

SOLUTION Let AB be the tower. Let C and D be two points at distances a and b respectively from the base of the tower. Then, $AC = a$ and $AD = b$. Let $\angle ACB = \theta$ and $\angle ADB = 90° - \theta$. Let h be the height of the tower AB.

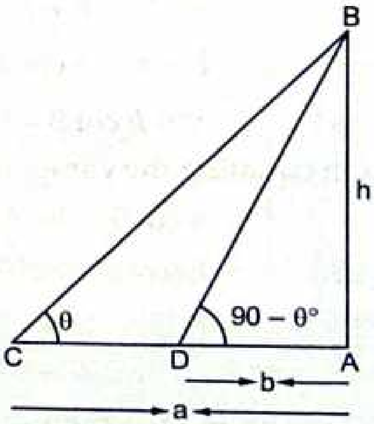

Fig. 12.34

In $\Delta\, CAB$, we have

$$\tan\theta = \frac{AB}{AC}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{a} \qquad \text{...(i)}$$

In $\Delta\, DAB$, we have

$$\tan(90° - \theta) = \frac{AB}{AD}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{b} \qquad \text{...(ii)}$$

From(i) and (ii), we have

$$\tan\theta \times \cot\theta = \frac{h^2}{ab}$$

$$\Rightarrow \quad 1 = \frac{h^2}{ab} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab} \text{ metres.}$$

Hence, the height of the tower is $\sqrt{ab}$ metres

EXAMPLE 34 *Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If α, β be the elevations of the [illegible] of the tower from these stations, prove that its inclination θ to the horizontal is given [illegible]*

$$\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$$

SOLUTION Let AB be the leaning tower and let C an[illegible] [illegible]en stations at distances a and b respectively from the foot A of the tower.

Let $AE = x$ and $BE = h$

In Δ *AEB*, we have

$$\tan\theta = \frac{BE}{AE}$$

$\Rightarrow \quad \tan\theta = \frac{h}{x}$

$\Rightarrow \quad x = h\cot\theta \quad$ ...(i)

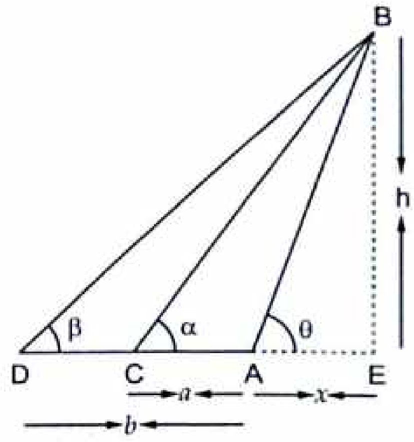

Fig. 12.35

In Δ *CEB*, we have

$$\tan\alpha = \frac{BE}{CE}$$

$\Rightarrow \quad \tan\alpha = \frac{h}{a+x}$

$\Rightarrow \quad a + x = h\cot\alpha$

$\Rightarrow \quad x = h\cot\alpha - a \quad$ ... (ii)

In Δ *DEB*, we have

$$\tan\beta = \frac{BE}{DE}$$

$\Rightarrow \quad \tan\beta = \frac{h}{b+x}$

$\Rightarrow \quad b + x = h\cot\beta$

$\Rightarrow \quad x = h\cot\beta - b \quad$ ... (iii)

On equating the values of x obtained from equations (i) and (ii), we have

$$h\cot\theta = h\cot\alpha - a$$

$\Rightarrow \quad h(\cot\alpha - \cot\theta) = a$

$\Rightarrow \quad h = \frac{a}{\cot\alpha - \cot\theta} \quad$ ... (iv)

On equating the values of x obtained from equations (i) and (iii), we get

$$h\cot\theta = h\cot\beta - b$$

$\Rightarrow \quad h(\cot\beta - \cot\theta) = b$

$\Rightarrow \quad h = \frac{b}{\cot\beta - \cot\theta} \quad$ ...(v)

Equating the values of h from equations (iv) and (v), we get

$$\frac{a}{\cot\alpha - \cot\theta} = \frac{b}{\cot\beta - \cot\theta}$$

$\Rightarrow \quad a(\cot\beta - a\cot\theta) = b(\cot\alpha - \cot\theta)$

$\Rightarrow \quad (b - a)\cot\theta = b\cot\alpha - a\cot\beta$

$\Rightarrow \quad \cot\theta = \frac{b\cot\alpha - a\cot\beta}{b - a}$

EXAMPLE 35 *If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is*

$$\frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$$

[NCERT EXEMPLAR]

SOLUTION Let AB be the surface of the lake and let P be a point of observation such that $AP = h$ metres. Let C be the position of the cloud and C' be its reflection in the lake. Then, $CB = C'B$. Let PM be perpendicular from P on CB. Then, $\angle CPM = \alpha$ and $\angle MPC' = \beta$. Let $CM = x$. Then, $CB = CM + MB = CM + PA = x + h$.

In ΔCPM, we have

$$\tan\alpha = \frac{CM}{PM}$$

$$\Rightarrow \quad \tan\alpha = \frac{x}{AB} \qquad [\because PM = AB]$$

$$\Rightarrow \quad AB = x\cot\alpha \qquad \text{...(i)}$$

In $\Delta PMC'$, we have

$$\tan\beta = \frac{C'M}{PM}$$

$$\Rightarrow \quad \tan\beta = \frac{x + 2h}{AB} \qquad [\because C'M = C'B + BM = x + h + h]$$

$$\Rightarrow \quad AB = (x + 2h)\cot\beta \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$x\cot\alpha = (x + 2h)\cot\beta$$

[On equating the values of AB]

$$\Rightarrow \quad x(\cot\alpha - \cot\beta) = 2h\cot\beta$$

$$\Rightarrow \quad x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow \quad x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

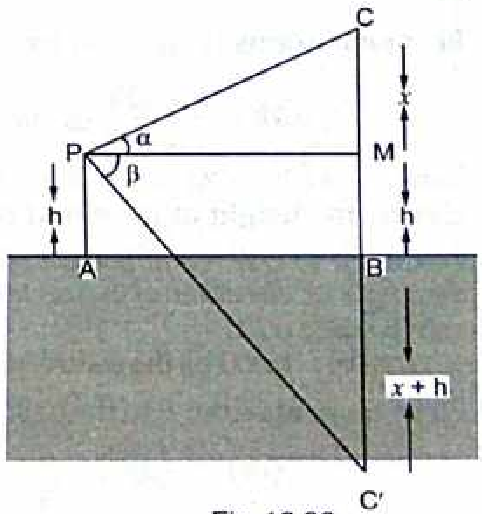

Fig. 12.36

Hence, the height CB of the cloud is given by

$$CB = x + h$$

$$\Rightarrow \quad CB = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$$

$$\Rightarrow \quad CB = \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha} = \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$

EXAMPLE 36 *The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud.*

[CBSE 2010, 2017]

SOLUTION Let AB be the surface of the lake and P be the point of observation such that $AP = 60$ metres. Let C be the position of the cloud and C' be its reflection in the lake. Then, $CB = C'B$. Let PM be perpendicular from P on CB. Then, $\angle CPM = 30°$ and $\angle C'PM = 60°$. Let $CM = h$. Then, $CB = h + 60$. Consequently, $C'B = h + 60$.

In ΔCMP, we have

$$\tan 30° = \frac{CM}{PM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{PM}$$

$$\Rightarrow \quad PM = \sqrt{3}\, h \qquad ...(i)$$

In $\Delta PMC'$, we have

$$\tan 60° = \frac{C'M}{PM}$$

$$\Rightarrow \quad \tan 60° = \frac{C'B + BM}{PM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 60 + 60}{PM}$$

$$\Rightarrow \quad PM = \frac{h + 120}{\sqrt{3}} \qquad ...(ii)$$

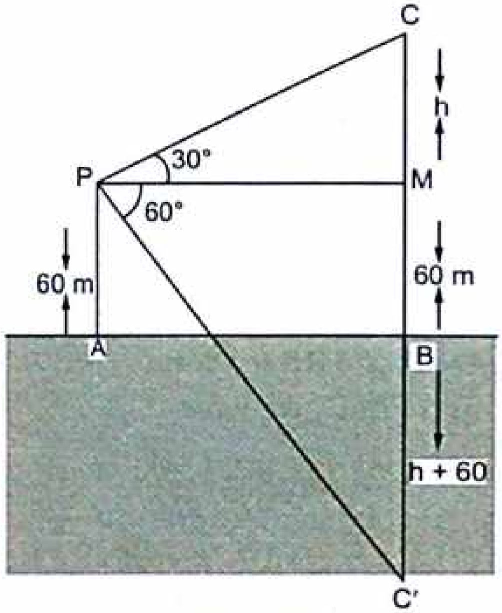

Fig. 12.37

From equations (i) and (ii), we get

$$\sqrt{3}\, h = \frac{h + 120}{\sqrt{3}} \Rightarrow 3h = h + 120 \Rightarrow 2h = 120 \Rightarrow h = 60$$

Now, $CB = CM + MB = h + 60 = 60 + 60 = 120$.

Hence, the height of the cloud from the surface of the lake is 120 metres.

EXAMPLE 37 *A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r \sin\beta \operatorname{cosec} \alpha/2$.* **[NCERT EXEMPLAR]**

SOLUTION Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA, PB be tangents from P to the balloon. Then, $\angle APB = \alpha$.

$$\therefore \quad \angle APO = \angle BPO = \frac{\alpha}{2}$$

Let OL be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the balloon is β i.e, $\angle OPL = \beta$.

In ΔOAP, we have

$$\sin\frac{\alpha}{2} = \frac{OA}{OP}$$

$$\Rightarrow \quad \sin\frac{\alpha}{2} = \frac{r}{OP}$$

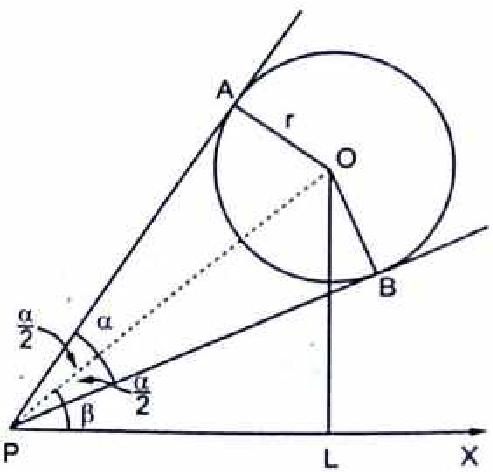

Fig. 12.38

$\Rightarrow \quad OP = r \operatorname{cosec} \frac{\alpha}{2}$ ...(i)

In $\Delta\, OPL$, we have

$$\sin\beta = \frac{OL}{OP}$$

$\Rightarrow \quad OL = OP\sin\beta = r \operatorname{cosec}\frac{\alpha}{2}\, \sin\beta$ [Using equation (i)]

Hence, the height of the centre of the balloon is $r \sin\beta \operatorname{cosec}\frac{\alpha}{2}$

EXAMPLE 38 *The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at an angle of φ, it is found that the angle of elevation is α. Show that the height of the cliff is*

$$\frac{k\,(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha} \textit{ metres}$$

SOLUTION Let AB be the cliff and O be the fixed point such that the angle of elevation of the cliff from O is θ i.e., $\angle AOB = \theta$. Let $\angle AOC = \phi$ and $OC = k$ metres. From C draw CD and CE perpendiculars on AB and OA respectively. Then, $\angle DCB = \alpha$. Let h be the height of the cliff AB.

In $\Delta\, OCE$, we have

$$\sin\phi = \frac{CE}{OC}$$

$\Rightarrow \quad \sin\phi = \frac{CE}{k}$

$\Rightarrow \quad CE = k\sin\phi$

$\Rightarrow \quad AD = k\sin\phi$ ...(i) $[\because CE = AD]$

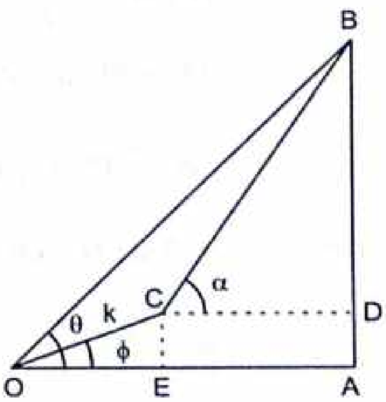

Fig. 12.39

and, $\cos\phi = \frac{OE}{OC}$

$\Rightarrow \quad \cos\phi = \frac{OE}{k}$

$\Rightarrow \quad OE = k\cos\phi$ ....(ii)

In ΔOAB, we have

$$\tan\theta = \frac{AB}{OA}$$

$\Rightarrow \quad \tan\theta = \frac{h}{OA}$

$\Rightarrow \quad OA = h\cot\theta$ ...(iii)

$\therefore \quad CD = EA = OA - OE = h\cot\theta - k\cos\phi$ ...(iv) [Using (ii) and (iii)]

and, $BD = AB - AD = AB - CE = h - k\sin\phi$ ...(v) [Using (i)]

In $\Delta\, BCD$, we have

$$\tan\alpha = \frac{BD}{CD}$$

$\Rightarrow \quad \tan\alpha = \dfrac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$ [Using (iv) and (v)]

$\Rightarrow \quad \dfrac{1}{\cot\alpha} = \dfrac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$

$\Rightarrow \quad h\cot\alpha - k\sin\phi\cot\alpha = h\cot\theta - k\cos\phi$

$\Rightarrow \quad h(\cot\theta - \cot\alpha) = k(\cos\phi - \sin\phi\cot\alpha)$

$\Rightarrow \quad h = \dfrac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$

EXAMPLE 39 *At the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.*

SOLUTION Let F be the foot and S be the summit of the mountain FOS. Then, $\angle OFS = 45°$ and therefore, $\angle OSF = 45°$. Consequently, $OF = OS = h$ km (say). Let $FP = 1000$ m $= 1$ km be the slope so that $\angle OFP = 30°$. Draw $PM \perp OS$ and $PL \perp OF$. Join PS. It is given that $\angle MPS = 60°$.

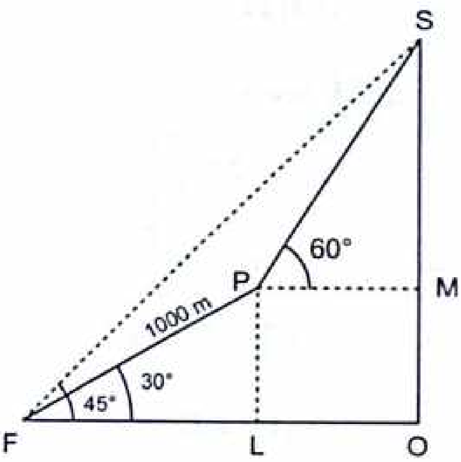

Fig. 12.40

In $\Delta\, FPL$, we have

$$\sin 30° = \frac{PL}{PF}$$

$\Rightarrow \quad PL = PF\sin 30° = \left(1 + \dfrac{1}{2}\right)\text{km} = \dfrac{1}{2}\text{km}$

$\therefore \quad OM = PL = \dfrac{1}{2}\text{km}$

$\Rightarrow \quad MS = OS - OM = \left(h - \dfrac{1}{2}\right)\text{km}$ ...(i)

Also, $\cos 30° = \dfrac{FL}{PF}$

$\Rightarrow \quad FL = PF\cos 30° = \left(1 \times \dfrac{\sqrt{3}}{2}\right)\text{km} = \dfrac{\sqrt{3}}{2}\text{km}$

Now, $h = OS = OF = OL + LF$

$\Rightarrow \quad h = OL + \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad OL = \left(h - \dfrac{\sqrt{3}}{2}\right)\text{km}$

$\Rightarrow \quad PM = \left(h - \dfrac{\sqrt{3}}{2}\right)\text{km}$ ...(ii)

In $\Delta\, SPM$, we have

$$\tan 60° = \frac{SM}{PM}$$

$\Rightarrow \quad SM = PM \cdot \tan 60°$

$\Rightarrow \quad \left(h - \frac{1}{2}\right) = \left(h - \frac{\sqrt{3}}{2}\right)\sqrt{3}$ [Using (i) and (ii)]

$\Rightarrow \quad h - \frac{1}{2} = h\sqrt{3} - \frac{3}{2}$

$\Rightarrow \quad \sqrt{3}h - h = \frac{3}{2} - \frac{1}{2}$

$\Rightarrow \quad h(\sqrt{3} - 1) = 1$

$\Rightarrow \quad h = \frac{1}{\sqrt{3} - 1} = \frac{\sqrt{3} + 1}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{\sqrt{3} + 1}{2} = \frac{2.732}{2} = 1.366 \text{ km}$

Hence, the height of the mountain is 1.366 km.

EXAMPLE 40 *The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β. If AB = d, show that the height of the tower is* $\frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$.

SOLUTION Let OP be the tower and let A and B be two points due south and east respectively of the tower such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let $OP = h$.

In ΔOAP, we have

$\tan \alpha = \frac{h}{OA}$

$\Rightarrow \quad OA = h \cot \alpha$ ...(i)

In ΔOBP, we have

$\tan \beta = \frac{h}{OB}$

$\Rightarrow \quad OB = h \cot \beta$ ...(ii)

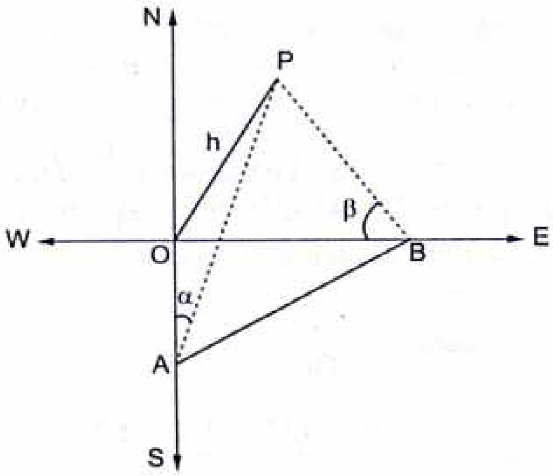

Fig. 12.41

Since OAB is a right-angled triangle.

$\therefore \quad AB^2 = OA^2 + OB^2$

$\Rightarrow \quad d^2 = h^2 \cot^2 \alpha + h^2 \cot^2 \beta$

$\Rightarrow \quad h = \frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$ [Using (i) and (ii)]

EXAMPLE 41 *The elevation of a tower at a station A due north of it is α and at a station B due west of A is β. Prove that the height of the tower is* $\frac{AB \sin \alpha \sin \beta}{\sqrt{\sin^2 \alpha - \sin^2 \beta}}$.

SOLUTION Let OP be the tower and let A be a point due north of the tower OP and let B be the point due west of A. Such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let h be the height of the tower. In right-angled triangle OAP and OBP, we have

$\tan \alpha = \frac{h}{OA}$ and $\tan \beta = \frac{h}{OB}$

$\Rightarrow \quad OA = h\cot\alpha$ and $OB = h\cot\beta$

In ΔOAB, we have

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad AB^2 = OB^2 - OA^2$$

$$\Rightarrow \quad AB^2 = h^2\cot^2\beta - h^2\cot^2\alpha$$

$$\Rightarrow \quad AB^2 = h^2\left[\cot^2\beta - \cot^2\alpha\right]$$

$$\Rightarrow \quad AB^2 = h^2\left[(\text{cosec}^2\beta - 1) - (\text{cosec}^2\alpha - 1)\right]$$

$$\Rightarrow \quad AB^2 = h^2(\text{cosec}^2\beta - \text{cosec}^2\alpha)$$

$$\Rightarrow \quad AB^2 = h^2\left(\frac{\sin^2\alpha - \sin^2\beta}{\sin^2\alpha\sin^2\beta}\right)$$

$$\Rightarrow \quad h = \frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$$

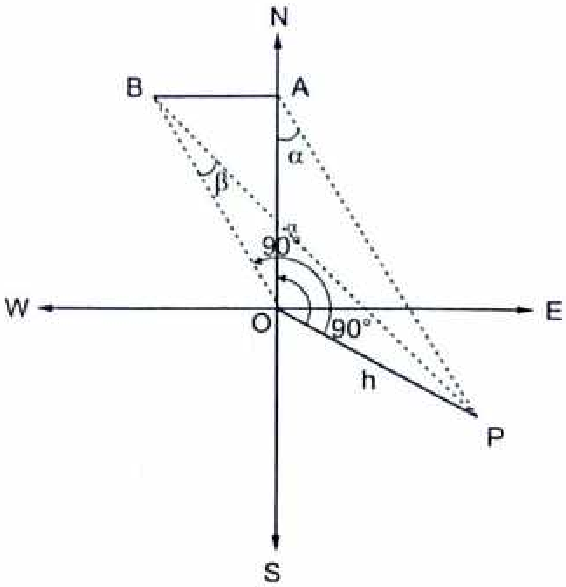

Fig. 12.42

EXAMPLE 42 *A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.* **[NCERT]**

SOLUTION Let P be the position of the balloon when its angle of elevation from the eyes of the girl is 60° and Q be the position when angle of elevation is 30°.

In ΔOLP, we have

$$\tan 60° = \frac{PL}{OL}$$

$$\Rightarrow \quad \sqrt{3} = \frac{PL' - LL'}{OL} = \frac{88.2 - 1.2}{OL}$$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{OL}$$

$$\Rightarrow \quad OL = \frac{87}{\sqrt{3}}$$

In ΔOMQ, we have

$$\tan 30° = \frac{QM}{OM} = \frac{QM' - MM'}{OM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{88.2 - 1.2}{OM}$$

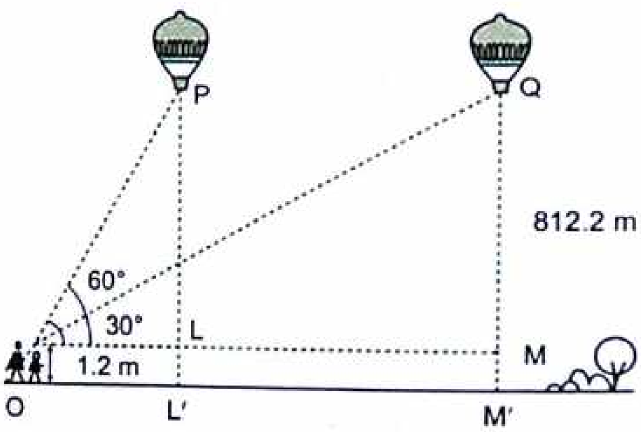

Fig. 12.43

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{OM}$$

$$\Rightarrow \quad OM = 87 \times \sqrt{3}$$

$\therefore$ Distance travelled by the balloon $= PQ = LM = OM - OL$

$$= \left(87 \times \sqrt{3} - \frac{87}{\sqrt{3}}\right) \text{m}$$

$$= 87 \times \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) \text{m} = \frac{87 \times 2}{\sqrt{3}} \text{m} = \frac{174}{\sqrt{3}} \text{m}$$

$$= \frac{174}{3}\sqrt{3} \text{ m} = 58\sqrt{3} \text{ m}.$$

EXAMPLE 43 *A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at angle of depression of 30°, which is approaching to the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the further time taken by the car to reach the foot of the tower.* **[NCERT, CBSE 2008, 2009, 2017]**

SOLUTION Let P be the foot of the vertical tower PQ of height h metres. Let the speed of the car be v m/sec. At A the angle of depression of the car is 30° and six seconds later it reaches to B where the angle of depression is 60°.

Clearly, car travels distance AB in 6 seconds with speed v m/sec.

$\therefore \quad AB = 6v$ metres

Suppose car takes t seconds to reach to P from point B. Then, $BP = vt$ metres.

$\therefore \quad AP = AB + BP = 6v + vt$

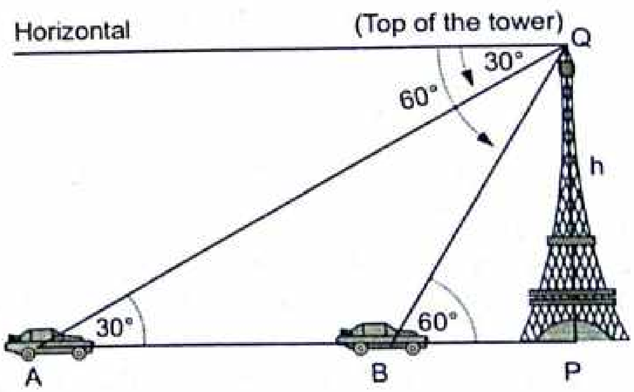

Fig. 12.44

In ΔAPQ, we have

$$\tan 30° = \frac{PQ}{AP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{6v + vt}$$

$$\Rightarrow \quad \sqrt{3}h = 6v + vt \quad \ldots(\text{i})$$

In ΔBPQ, we have

$$\tan 60° = \frac{PQ}{BP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{vt}$$

$$\Rightarrow \quad \sqrt{3}\, vt = h \quad \ldots(\text{ii})$$

From (i) and (ii), we have

$$\sqrt{3} \times \sqrt{3}\, vt = 6v + vt \Rightarrow 3vt = 6v + vt \Rightarrow 2vt = 6v \Rightarrow t = \frac{6v}{2v} = 3 \text{ seconds}$$

Hence, further time taken by the car to reach the foot of the tower is 3 seconds.

EXAMPLE 44 *A man on a cliff observes a boat at an angle of depression of 30° which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.*

SOLUTION Let OA be the cliff and P be the initial position of the boat when the angle of depression is 30°. After 6 minutes the boat reaches to Q such that the angle of depression at Q is 60°. Let $PQ = x$ metres.

In Δ's POA and QOA, we have

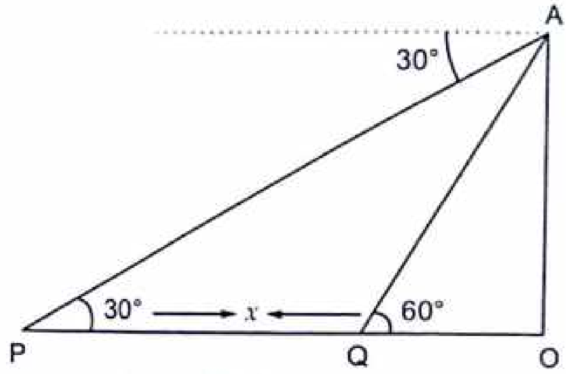

Fig. 12.45

$$\tan 30° = \frac{OA}{OP} \text{ and } \tan 60° = \frac{OA}{OQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{OA}{OP} \text{ and } \sqrt{3} = \frac{OA}{OQ}$$

$$\Rightarrow \quad OA = \frac{OP}{\sqrt{3}} \text{ and } OA = \sqrt{3}\, OQ$$

$$\Rightarrow \quad \frac{OP}{\sqrt{3}} = \sqrt{3}\, OQ$$

$$\Rightarrow \quad OP = 3\, OQ$$

$$\Rightarrow \quad PQ = OP - OQ = OP - \frac{OP}{3} = \frac{2}{3} OP \qquad \left[\because OQ = \frac{1}{3} OP\right]$$

Let the speed of the boat be v metre/minute. Then,

PQ = Distance travelled by the boat is 6 minutes

$$\Rightarrow \quad PQ = 6v$$

$$\Rightarrow \quad \frac{2}{3}(OP) = 6v \qquad \left[\because PQ = \frac{2}{3} OP\right]$$

$$\Rightarrow \quad OP = 9v$$

∴ Time taken by the boat to reach at the shore is given by

$$T = \frac{OP}{v} \qquad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad T = \frac{9v}{v} \text{ minutes} = 9 \text{ minutes}.$$

EXAMPLE 45 *A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the tower? Give your answer to the nearest second.*

[CBSE 2006C]

SOLUTION Let AB be the tower of height h metres. Let C be the initial position of the car and let after 12 minutes the car be at D. It is given that the angles of depression at C and D are 30° and 45° respectively.

Let the speed of the car be v metre per minute. Then,

CD = Distance travelled by the car in 12 minutes.

$$\Rightarrow \quad CD = 12\, v \text{ metres} \qquad [\because \text{Distance} = \text{speed} \times \text{time}]$$

Suppose the car takes t minutes to reach the tower AB from D. Then, $DA = vt$ metres.

In Δ DAB, we have

$$\tan 45° = \frac{AB}{AD}$$

$\Rightarrow \quad 1 = \frac{h}{vt}$

$\Rightarrow \quad h = vt \qquad \text{...(i)}$

In ΔCAB, we have

$$\tan 30° = \frac{AB}{AC}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{vt + 12v}$

$$\sqrt{3}h = vt + 12v \qquad \text{...(ii)}$$

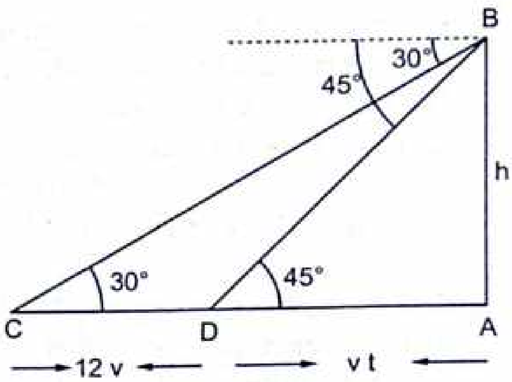

Fig. 12.46

Substituting the value of h from equation (i) in equation (ii), we get

$$\sqrt{3}\, vt = vt + 12v$$

$\Rightarrow \quad \sqrt{3}\, t = t + 12$

$\Rightarrow \quad t(\sqrt{3} - 1) = 12$

$\Rightarrow \quad t = \frac{12}{\sqrt{3} - 1} = \frac{12(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$\Rightarrow \quad t = 6(\sqrt{3} + 1) = 16.39$ minutes

$\Rightarrow \quad t = 16$ minutes 23 seconds $\qquad [\because 0.39 \text{ minutes} = 0.39 \times 60 \text{ seconds}]$

Thus, the car will reach the tower from D in 16 minutes and 23 seconds.

EXERCISE 12.1

LEVEL-1

1. A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of the tower is 60°. What is the height of the tower?
2. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.
3. A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60° with the level of the ground. Determine the height of the wall.
4. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.
5. A kite is flying at a height of 75 metres from the ground level, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest metre.
6. A ladder 15 metres long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall. **[NCERT EXEMPLAR]**
7. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of elevation of the top and the bottom of the flag-staff are respectively 60° and 45°. Find the height of the flag-staff and that of the tower. **[CBSE 2014]**

8. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 5 metres. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30° and 60°. Find the height of the tower. **[CBSE 2015, 2016]**

10. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

11. The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 m. longer than when it was 60°. Find the height of the tower.

12. A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

13. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

14. The angle of elevation of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 metres towards the foot of the tower, the angle of elevation of the tower becomes 60°. Show that the height of the tower is 129.9 metres (Use $\sqrt{3} = 1.732$). **[CBSE 2006]**

15. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626] **[CBSE 2001C]**

16. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B the angle of elevation increases to 60°. Find the height of the tower and the distance of the tower from the point A. **[CBSE 2002, 2015, 2017]**

17. From the top of a building 15 m high the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and building. **[CBSE 2002]**

18. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it. **[CBSE 2005]**

19. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

20. From a point P on the ground the angle of elevation of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from P is 45°. Find the length of the flag-staff and the distance of the building from the point P. (Take $\sqrt{3} = 1.732$).

21. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

22. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building. **[NCERT, CBSE 2014]**

23. The shadow of a tower standing on a level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Find the height of the tower. **[NCERT EXEMPLAR]**

24. From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transimission tower. **[NCERT]**

25. The angles of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of the multistoried building and the distance between the two buildings. **[NCERT, CBSE 2009]**

26. A statue 1.6 m tall stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal. **[NCERT, CBSE 2008,2014]**

27. A T.V. Tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the river. **[NCERT]**

28. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. **[NCERT, CBSE 2014, 2017]**

29. As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. **[NCERT]**

30. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building. **[NCERT, CBSE 2012, 2015, 2017]**

31. From a point on a bridge across a river the angles of depression of the banks on opposite side of the river are 30° and 45° respectively. If bridge is at the height of 30 m from the banks, find the width of the river. **[NCERT]**

32. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. **[NCERT]**

33. A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

34. A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower. **[CBSE 2016]**

35. The length of the shadow of a tower standing on level plane is found to be $2x$ metres longer when the sun's altitude is $30°$ than when it was $45°$. Prove that the height of tower is $x(\sqrt{3}+1)$ metres.

36. A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of $30°$ with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 metres. Find the height of the tree.

37. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at $60°$ to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

38. Two men on either side of the cliff 80 m high observes the angles of elevation of the top of the cliff to be $30°$ and $60°$ respectively. Find the distance between the two men. **[CBSE 2016]**

39. Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height.

40. An aeroplane is flying at a height of 210 m. Flying at this height at some instant the angles of depression of two points in a line in opposite directions on both the banks of the river are $45°$ and $60°$. Find the width of the river. (Use $\sqrt{3} = 1.73$) **[CBSE 2015]**

41. The angle of elevation of the top of a chimney from the top of a tower is $60°$ and the angle of depression of the foot of the chimney from the top of the tower is $30°$. If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the polution norms. What value is discussed in this question? **[CBSE 2014]**

42. Two ships are there in the sea on either side of a light house in such away that the ships and the light house are in the same straight line. The angles of depression of two ships are observed from the top of the light house are $60°$ and $45°$ respectively. If the height of the light house is 200 m, find the distance between the two ships. (Use $\sqrt{3} = 1.73$)

43. The horizontal distance between two poles is 15 m. The angle of depression of the top of the first pole as seen from the top of the second pole is $30°$. If the height of the second pole is 24 m, find the height of the first pole. ($\sqrt{3} = 1.732$) **[CBSE 2013]**

44. The angles of depression of two ships from the top of a light house and on the same side of it are found to be $45°$ and $30°$ respectively. If the ships are 200 m apart, find the height of the light house. **[CBSE 2012]**

45. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m. **[NCERT, CBSE 2016]**

46. From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be $45°$ and $60°$ respectively. Find the height of the pole. **[CBSE 2016]**

47. The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree when seen from the top of the second tree is $45°$. If the height of the second tree is 80 m, find the height of the first tree.

48. A flag-staff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is $60°$ and from the same point, the angle of elevation of the top of the tower is $45°$. Find the height of the flag-staff. **[CBSE 2013]**

49. The angle of elevation of the top of a vertical tower PQ from a point X on the ground is $60°$. At a point Y, 40 m vertically above X, the angle of elevation of the top is $45°$. Calculate the height of the tower.

50. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are $30°$ and $45°$. If one ship is directly behind the other, find the distance between the two ships. **[CBSE 2013]**

51. The angles of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively $30°$ and $45°$. Find the height of the rock.

52. A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are $30°$ and $60°$ respectively. What is the distance between the two cars and how far is each car from the tower?

53. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be $30°$ and $60°$ respectively. Find
    (i) the horizontal distance between AB and CD.
    (ii) the height of the lamp post.
    (iii) the difference between the heights of the building and the lamp post.
    [CBSE 2009]

54. Two boats approach a light house in mid-sea from opposite directions. The angles of elevation of the top of the light house from two boats are $30°$ and $45°$ respectively. If the distance between two boats is 100 m, find the height of the light house. **[CBSE 2014]**

55. The angle of elevation of the top of a hill at the foot of a tower is $60°$ and the angle of elevation of the top of the tower from the foot of the hill is $30°$. If the tower is 50 m high, what is the height of the hill? **[CBSE 2006C, 2013]**

56. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from $60°$ to $45°$ in 2 minutes. Find the speed of the boat in m/h. **[CBSE 2017]**

57. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as $60°$ and $45°$. Find the distance between the cars. (Take $\sqrt{3} = 1.732$) **[CBSE 2017]**

58. Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are $60°$ and $45°$ respectively. If the height of the tower is 15 m, then find the distance between these points. **[CBSE 2017]**

LEVEL-2

59. A fire in a building B is reported on telephone to two fire stations P and Q, 20 km apart from each other on a straight road. P observes that the fire is at an angle of $60°$ to the road and Q observes that it is at an angle of $45°$ to the road. Which station should send its team and how much will this team have to travel?

60. A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is $45°$ and the angle of depression of the base is $30°$. Calculate the distance of the cliff from the ship and the height of the cliff.

61. A man standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as $60°$ and the angle of depression of the base of the hill as $30°$. Calculate the distance of the hill from the ship and the height of the hill.

62. There are two temples, one on each bank of a river, just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

63. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

64. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/ hr.

65. A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β. Prove that the height of the top from the ground is

$$\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}.$$

66. The angle of elevation of a stationery cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level? (Use $\tan 15° = 0.268$)

67. If the angle of elevation of a cloud from a point h metres above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is

$$\frac{2h\sec\alpha}{\tan\beta-\tan\alpha}$$ [CBSE 2004]

68. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by

$$\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$$ [CBSE 2004]

69. PQ is a post of given height a, and AB is a tower at some distance. If α and β are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

70. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that

$$\frac{a}{b}=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$$ [NCERT EXEMPLAR]

71. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b metres just above A is β. Prove that the height of the tower is $b\tan\alpha\cot\beta$.

72. An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the top of the tower from his eye. [NCERT EXEMPLAR]

73. A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also, each leg is inclined at an angle of 60° to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distances.

74. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of $30°$. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of $45°$. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

75. From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h metres and the line joining the ships passes through the foot of the light house, show that the distance between the ship is $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha\tan\beta}$ metres.

76. From the top of a tower h metre high, the angles of depression of two objects, which are in the line with the foot of the tower are α and β $(\beta > \alpha)$. Find the distance between the two objects. [NCERT EXEMPLAR]

77. A window of a house is h metre above the ground. From the window, the angles of elevation and depression of the top and bottom of another house situated on the opposite side of the lane are found to be α and β respectively. Prove that the height of the house is $h(1 + \tan\alpha\tan\beta)$ metres. [NCERT EXEMPLAR]

78. The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be $60°$ and $30°$ respectively. Find the height of the balloon above the ground. [NCERT EXEMPLAR]

ANSWERS

1. $20\sqrt{3}$ m
2. 19 m
3. $2\sqrt{3}$ m
4. 14.1 m
5. 87 m
6. 7.5 m
7. 51.24 m, 70 m
8. 6.9 m
9. 2.5 m
10. 43.25 m
11. 23.66 m
12. 236.6 m, 136.6 m
13. 63.4 m
15. 91.65 m, 146.7 m
16. Height = 17.3 m, Distance = 30 m
17. Height = 22.5 m, Distance = 12.975 m
18. $3\sqrt{3}$ m, $6\sqrt{3}$ m
19. $8\sqrt{3}$ m
20. 7.32 m, 17.32 m
21. $\frac{8}{3}$ m
22. $19\sqrt{3}$ m
23. $20\sqrt{3}$ m
24. $20(\sqrt{3} - 1)$ m
25. $4(3 + \sqrt{3})$ m, $4(3 + \sqrt{3})$ m
26. $\frac{4(\sqrt{3} + 1)}{5}$ m
27. $10\sqrt{3}$ m, 10 m
28. $7(\sqrt{3} + 1)$ m
29. $75(\sqrt{3} - 1)$ m
30. $\frac{50}{3}$ m
31. $30(\sqrt{3} + 1)$ m
32. $20\sqrt{3}$ m, 20 m, 60 m
33. $\frac{80}{\sqrt{3}}$ m
34. 9.56 m
36. 17.3 m
37. 186 m
38. 184.8 m
39. $45°$
40. 331.38 m
41. 160 m, Yes polution control
42. 315.6 m
43. 15.34 m
44. 273.2 m
46. 21.13 m
47. 20 m
48. 3.65 m
49. 94.64 m
50. 109.5 m
51. 236.5 m
52. 57.67 m, 86.5 m, 28.83 m
53. (i) 34.64 m (ii) 40 m. (iii) 20 m.
54. $50(\sqrt{3} - 1)$ m
55. 150 m
56. 1902 m/hr
57. 189.28 m
58. 6.340 m
59. Station P, 14.64 km
60. Distance = $10\sqrt{3}$ m, Height = 27.32 m
61. Distance = $8\sqrt{3}$ m, Height = 32 m
62. 28.83 m, 33.33 m
63. 527.04 km/hr

64. 415.68 km/hr 65. 20.87 m, 33.33 m 66. $2500\sqrt{3}$ m

69. Distance $= \dfrac{a}{\tan\alpha - \tan\beta}$, Height $= \dfrac{a\tan\alpha}{\tan\alpha - \tan\beta}$ 72. 45°

73. 1.732 m, 1.1077 m, 1.654 m 74. $40\sqrt{2}$ m 76. $h(\cot\alpha - \cot\beta)$ 78. 8 m

HINT TO SELECTED PROBLEMS

22. In ΔACB, we have

$$\tan 30^\circ = \frac{QC}{AC}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{30 - 1.5}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{AC}$$

$$\Rightarrow \quad AC = 28.5 \times \sqrt{3}\text{ m}$$

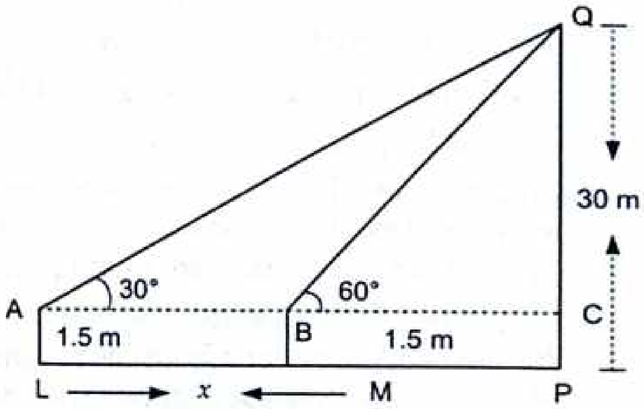

Fig. 12.47

In ΔBCQ, we have

$$\tan 60^\circ = \frac{QC}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{30 - 1.5}{BC}$$

$$\Rightarrow \quad BC = \frac{28.5}{\sqrt{3}}\text{ m}$$

$$\therefore \quad AB = AC - BC = 28.5 \times \sqrt{3} - \frac{28.5}{\sqrt{3}} = \frac{28.5 \times 2}{\sqrt{3}} = 19\sqrt{3}\text{ m}.$$

24. Let PQ be the building of height 20 metre and QR be the transmission tower of height h metre.

Let the angles of elevation of the bottom and top of the tower at point O be 45° and 60° respectively.

Then, in triangles OPQ and OPR, we have

$$\tan 45^\circ = \frac{PQ}{OP} \text{ and } \tan 60^\circ = \frac{PR}{OP}$$

$$\Rightarrow \quad 1 = \frac{20}{OP} \text{ and } \sqrt{3} = \frac{20 + h}{OP}$$

$$\Rightarrow \quad OP = 20\text{ m and } \sqrt{3} \times OP = 20 + h$$

$$\Rightarrow \quad 20\sqrt{3} = 20 + h$$

$$\Rightarrow \quad h = (20\sqrt{3} - 20)\text{ m} = 20(\sqrt{3} - 1)\text{ m}.$$

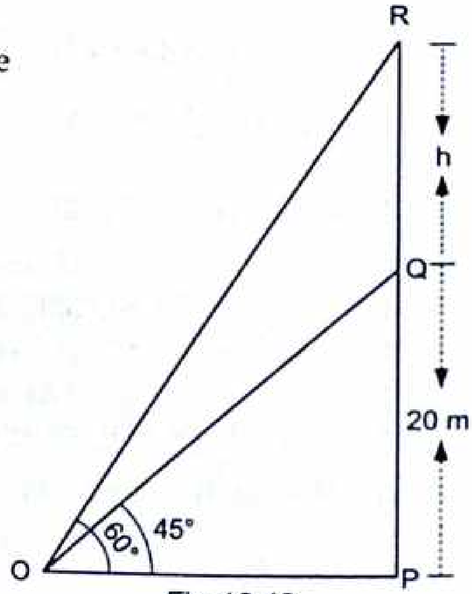

Fig. 12.48

26. Let OP be the pedestal and PQ be the statue of height 1.6 m

In Δ's AOP and AOQ, we have

$$\tan 45° = \frac{OP}{OA} \text{ and } \tan 60° = \frac{OQ}{OA}$$

$$\Rightarrow \quad 1 = \frac{OP}{OA} \text{ and } \sqrt{3} = \frac{OP + 1.6}{OA}$$

$$\Rightarrow \quad OA = OP \text{ and } \sqrt{3}\, OA = OP + 1.6$$

$$\Rightarrow \quad \sqrt{3}\, OP = OP + 1.6$$

$$\Rightarrow \quad (\sqrt{3} - 1)\, OP = 1.6$$

$$\Rightarrow \quad OP = \frac{1.6}{\sqrt{3} - 1} = 0.8\,(\sqrt{3} + 1)\text{ m}$$

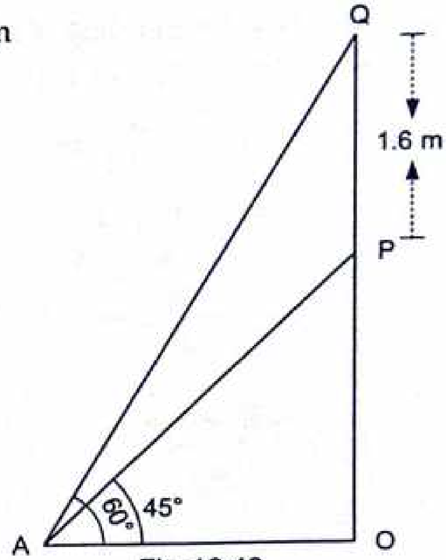

Fig. 12.49

27. Let AB be the tower of height h metre on a bank of the river and D be a point on the opposite bank or the river.

In Δ's DBA and CBA, we have

$$\Rightarrow \quad \tan 30° = \frac{AB}{DB} \text{ and } \tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20 + BC} \text{ and } \sqrt{3} = \frac{h}{BC}$$

$$\Rightarrow \quad \sqrt{3}\, h = 20 + BC \text{ and } BC = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = 20 + \frac{h}{\sqrt{3}} \qquad \text{[On eliminating } BC\text{]}$$

$$\Rightarrow \quad \frac{2h}{\sqrt{3}} = 20$$

$$\Rightarrow \quad h = 10\sqrt{3}\text{ m}$$

$$\therefore \quad BC = \frac{h}{\sqrt{3}} = 10\text{ m}$$

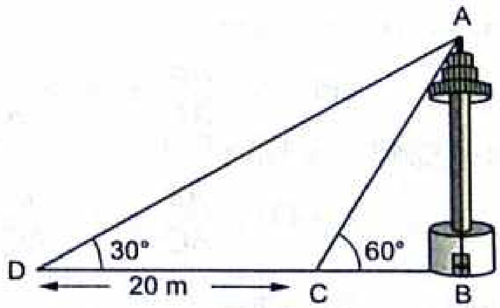

Fig. 12.50

29. Let OA be the light house of height 75 m and P and Q be the positions of two ships.

In Δ's AOQ and AOP, we have

$$\tan 45° = \frac{OA}{OQ} \text{ and } \tan 30° = \frac{OA}{OP}$$

$$\Rightarrow \quad 1 = \frac{75}{OQ} \text{ and } \frac{1}{\sqrt{3}} = \frac{75}{OP}$$

$$\Rightarrow \quad OQ = 75 \text{ and } OP = 75\sqrt{3}$$

$$\therefore \quad PQ = (75\sqrt{3} - 75)\text{ m} = 75\,(\sqrt{3} - 1)\text{ m}$$

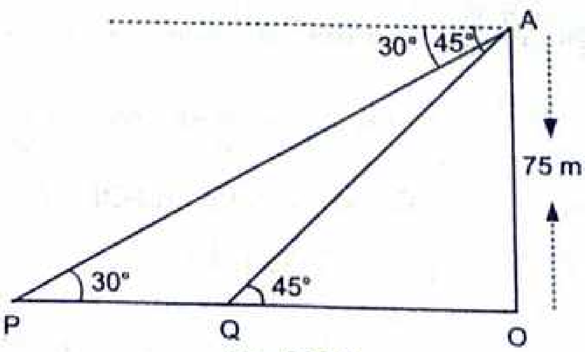

Fig. 12.51

30. Let AD be the building of height h metre.

In Δ's ABC and ABD, we have

$$\tan 60^\circ = \frac{BC}{AB} \text{ and } \tan 30^\circ = \frac{AD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50}{AB} \text{ and } \frac{1}{\sqrt{3}} = \frac{h}{AB}$$

$$\Rightarrow \quad AB = \frac{50}{\sqrt{3}} \text{ and } AB = \sqrt{3}h$$

$$\Rightarrow \quad \sqrt{3}h = \frac{50}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{50}{3}$$

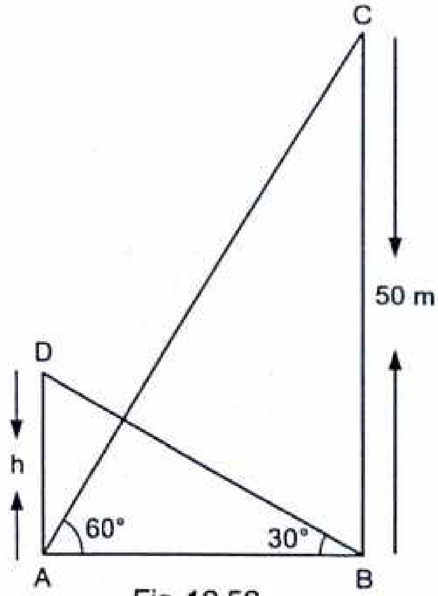

Fig. 12.52

59. Let AB be the building of height h.

Clearly, $\angle APB > \angle AQB$.

$\Rightarrow \quad \angle ABP < \angle ABQ$

$\Rightarrow \quad AP < AQ$

$\Rightarrow$ Station P is nearer to the building.

So, station P must send its team.

In ΔPAB, we have

$$\tan 60^\circ = \frac{AB}{AP} \Rightarrow \sqrt{3} = \frac{h}{AP} \Rightarrow AP = \frac{h}{\sqrt{3}}$$

In ΔQAB, we have

$$\tan 45^\circ = \frac{AB}{AQ} \Rightarrow 1 = \frac{h}{AQ} \Rightarrow AQ = h$$

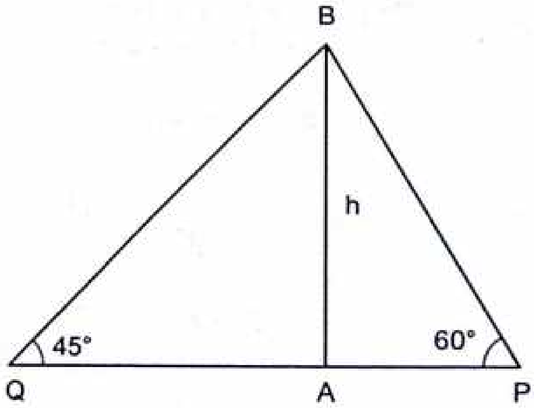

Fig. 12.53

Now, $\quad PQ = 20$ km

$\Rightarrow \quad AP + AQ = 20$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + h = 20 \Rightarrow h = \frac{20\sqrt{3}}{\sqrt{3}+1} = 10(3-\sqrt{3}) = 17.32 \text{ km}$$

65. Let OP be the tree and A, B be two points such that $OA = a$ and $OB = b$.

In Δ's ALP and BLP, we have

$$\tan \alpha = \frac{h}{OL + a} \text{ and } \tan \beta = \frac{h}{OL + b}$$

$$\Rightarrow \quad OL + a = h \cot \alpha \text{ and } OL + b = h \cot \beta$$

$$\Rightarrow \quad b - a = h \cot \beta - h \cot \alpha$$

$$\Rightarrow \quad h = \frac{(b-a)}{\cot \beta - \cot \alpha} = \frac{(b-a)\tan \alpha \tan \beta}{\tan \alpha - \tan \beta}$$

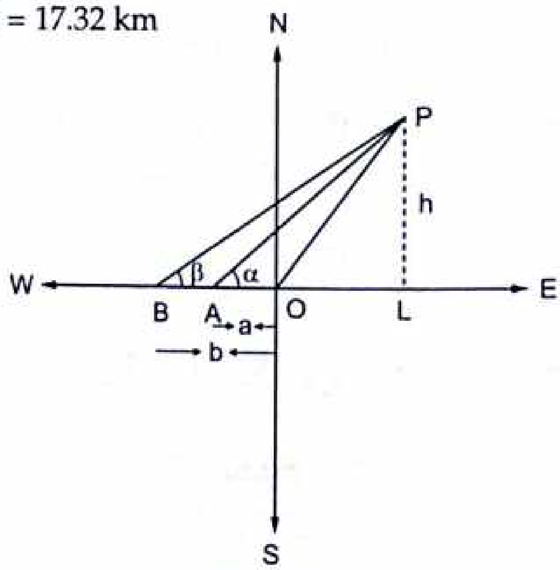

Fig. 12.54

67. Let C' be the image of clould C in the lake.

In Δ's PQC and PQC', we have

$$\tan\alpha = \frac{x}{PQ} \text{ and } \tan\beta = \frac{QC'}{PQ}$$

$$\Rightarrow \quad \tan\alpha = \frac{x}{PQ} \text{ and } \tan\beta = \frac{x+2h}{PQ}$$

$$\Rightarrow \quad \tan\beta - \tan\alpha = \frac{x+2h}{PQ} - \frac{x}{PQ}$$

$$\Rightarrow \quad \tan\beta - \tan\alpha = \frac{2h}{PQ}$$

$$\Rightarrow \quad PQ = \frac{2h}{\tan\beta - \tan\alpha}$$

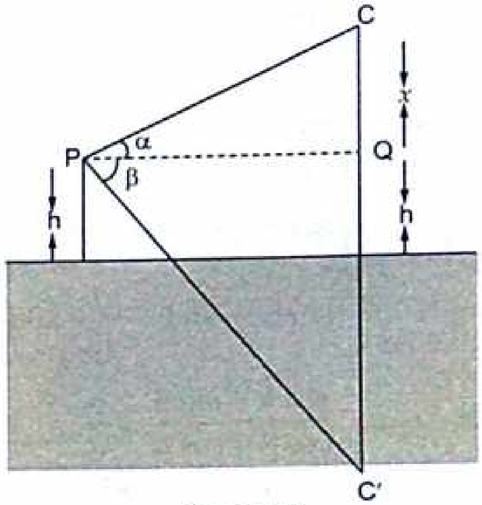

Fig. 12.55

Again, in ΔPQC, we have

$$\cos\alpha = \frac{PQ}{CP}$$

$$\Rightarrow \quad CP = PQ \sec\alpha$$

$$\Rightarrow \quad CP = \frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$$

68. Let h be the height of aeroplane P above the road and A and B be two consecutive milestones. Then,

$$AB = 1 \text{ mile}$$

In $\Delta^{'s} AQP$ and BQP, we have

$$\tan\alpha = \frac{h}{AQ} \text{ and } \tan\beta = \frac{h}{BQ}$$

$$\Rightarrow \quad AQ = h\cot\alpha \text{ and } BQ = h\cot\beta$$

$$\Rightarrow \quad AQ + BQ = h(\cot\alpha + \cot\beta)$$

$$\Rightarrow \quad AB = h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right)$$

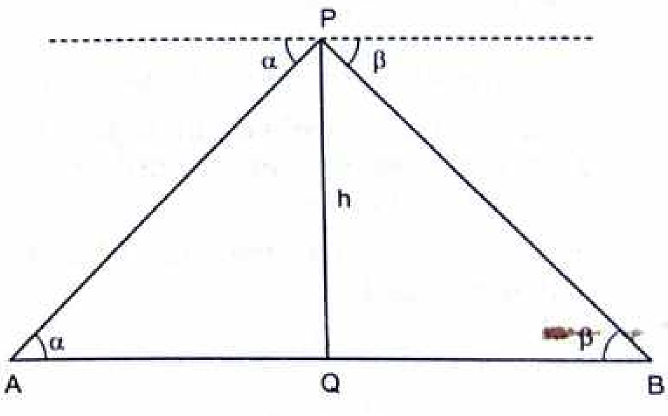

Fig. 12.56

$$\Rightarrow \quad h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta} \qquad [\because AB = 1]$$

69. Let PQ be the ladder such that its top Q is on the wall OQ and bottom P is on the ground. The ladder is pulled away from the wall through a distance a, so that its top Q slides and takes position Q'.

Clearly, $PQ = P'Q'$.

In $\Delta^{'s}$ POQ and $P'OQ'$, we have

$$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}, \sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$$

$$\Rightarrow \quad \sin\alpha = \frac{b+y}{PQ}, \cos\alpha = \frac{x}{PQ}, \sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$$

$$\Rightarrow \quad \sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ} \text{ and } \cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$$

$$\Rightarrow \quad \sin\alpha - \sin\beta = \frac{b}{PQ} \text{ and } \cos\beta - \cos\alpha = \frac{a}{PQ}$$

$$\Rightarrow \quad \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{b}{a}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

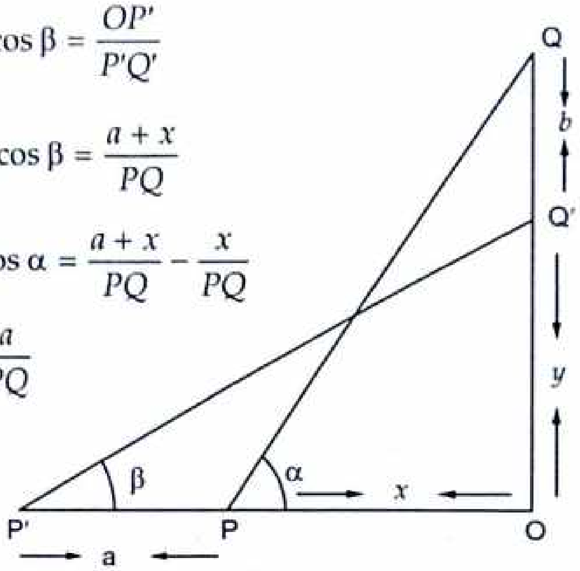

Fig. 12.57

VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

Answer each of the following questions either in one word or one sentence or as per requirement of the questions:

1. The height of a tower is 10 m. What is the length of its shadow when Sun's altitude is 45°?
2. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$, what is the angle of elevation of the Sun? [CBSE 2017]
3. What is the angle of elevation of the Sun when the length of the shadow of a vertical pole is equal to its height?
4. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°, what is the height of the tower?
5. If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower.
6. In Fig. 12.58, what are the angles of depression from the observing positions O_1 and O_2 of the object at A?

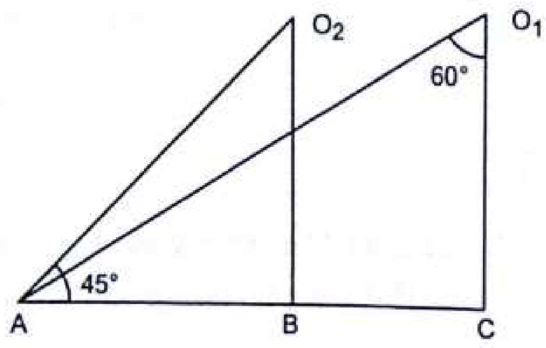

Fig. 12.58

7. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$. [CBSE 2015]

8. The angle of elevation of the top of a tower at a point on the ground is 30°. What will be the angle of elevation, if the height of the tower is tripled? [CBSE 2015]

9. AB is a pole of height 6 m standing at a point B and CD is a ladder inclined at angle of 60° to the horizontal and reaches upto a point D of pole. If $AD = 2.54$ m, find the length of the ladder. (Use $\sqrt{3} = 1.73$) [CBSE 2016]

10. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of an observer to the top of tower is 30°. Find the height of the tower. [CBSE 2016]

11. An observer, 1.5 m tall, is 28.5 m away from a 30 m high tower. Determine the angle of elevation of the top of the tower from the eye of the observer. [CBSE 2017]

ANSWERS

1. 10 m
2. 60°
3. 45°
4. $20\sqrt{3}$ m
5. 6 m
6. 30°, 45°
7. 1 : 3
8. 60°
9. 4 m
10. 21.7 m
11. 45°

MULTIPLE CHOICE QUESTIONS (MCQs)

Mark the correct alternative in each of the following:

1. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sum is
   (a) 30° (b) 45° (c) 60° (d) 90°

2. If the angle of elevation of a tower from a distance of 100 metres from its foot is 60°, then the height of the tower is
   (a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m (c) $50\sqrt{3}$ m (d) $\frac{200}{\sqrt{3}}$ m

3. If the altitude of the sum is at 60°, then the height of the vertical tower that will cast a shadow of length 30 m is
   (a) $30\sqrt{3}$ m (b) 15 m (c) $\frac{30}{\sqrt{3}}$ m (d) $15\sqrt{2}$ m

4. If the angles of elevation of a tower from two points distant a and b $(a > b)$ from its foot and in the same straight line from it are 30° and 60°, then the height of the tower is
   (a) $\sqrt{a+b}$ (b) $\sqrt{ab}$ (c) $\sqrt{a-b}$ (d) $\sqrt{\frac{a}{b}}$

5. If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary, then the height of the tower is
   (a) ab (b) $\sqrt{ab}$ (c) $\frac{a}{b}$ (d) $\sqrt{\frac{a}{b}}$

6. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30° and 45°. If the height of the light house is h metres, the distance between the ships is

(a) $(\sqrt{3}+1)\,h$ metres (b) $(\sqrt{3}-1)\,h$ metres

(c) $\sqrt{3}\,h$ metres (d) $1+\left(1+\frac{1}{\sqrt{3}}\right)h$ metres

7. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. The height of the tower is

(a) $\frac{d}{\cot\alpha + \cot\beta}$ (b) $\frac{d}{\cot\alpha - \cot\beta}$ (c) $\frac{d}{\tan\beta - \tan\alpha}$ (d) $\frac{d}{\tan\beta + \tan\alpha}$

8. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with horizontal, then the length of the wire is

(a) 12 m (b) 10 m (c) 8 m (d) 6 m

9. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is

(a) 25 m (b) 50 m (c) 75 m (d) 100 m

10. The angles of depression of two ships from the top of a light house are 45° and 30° towards east. If the ships are 100 m apart, the height of the light house is

(a) $\frac{50}{\sqrt{3}+1}$ m (b) $\frac{50}{\sqrt{3}-1}$ m (c) $50(\sqrt{3}-1)$ m (d) $50(\sqrt{3}+1)$ m

11. If the angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°, then the height of the cloud above the lake, is

(a) 200 m (b) 500 m (c) 30 m (d) 400 m

12. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is

(a) 100 m (b) $100\sqrt{3}$ m (c) $100(\sqrt{3}-1)$m (d) $\frac{100}{\sqrt{3}}$ m

13. Two persons are a metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then the height of the shorter post is

(a) $\frac{a}{4}$ (b) $\frac{a}{\sqrt{2}}$ (c) $a\sqrt{2}$ (d) $\frac{a}{2\sqrt{2}}$

14. The angle of elevation of a cloud from a point h metre above a lake is θ. The angle of depression of its reflection in the lake is 45°. The height of the cloud is

(a) $h\tan(45° + \theta)$ (b) $h\cot(45° - \theta)$ (c) $h\tan(45° - \theta)$ (d) $h\cot(45° + \theta)$

15. A tower subtends an angle of 30° at a point on the same level as its foot. At a second point h metres above the first, the depression of the foot of the tower is 60°. The height of the tower is

(a) $\frac{h}{2}$ m (b) $\sqrt{3}h$ m (c) $\frac{h}{3}$ m (d) $\frac{h}{\sqrt{3}}$ m

16. It is found that on walking x meters towards a chimney in a horizontal line through its base, the elevation of its top changes from 30° to 60°. The height of the chimney is

(a) $3\sqrt{2}x$ (b) $2\sqrt{3}x$ (c) $\frac{\sqrt{3}}{2}x$ (d) $\frac{2}{\sqrt{3}}x$

17. The length of the shadow of a tower standing on level ground is found to be $2x$ metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower in metres is

(a) $(\sqrt{3}+1)\,x$ (b) $(\sqrt{3}-1)\,x$ (c) $2\sqrt{3}x$ (d) $3\sqrt{2}x$

18. Two poles are 'a' metres apart and the height of one is double of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the smaller is

(a) $\sqrt{2}a$ metres (b) $\frac{a}{2\sqrt{2}}$ metres (c) $\frac{a}{\sqrt{2}}$ metres (d) $2a$ metres

19. The tops of two poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then l =

(a) 26 (b) 16 (c) 12 (d) 10

20. If a 1.5 m tall girl stands at a distance of 3 m from a lamp-post and casts a shadow of length 4.5 m on the ground, then the height of the lamp-post is

(a) 1.5 m (b) 2 m (c) 2.5 m (d) 2.8 m

21. The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is

(a) 45° (b) 30° (c) 60° (d) 90° [CBSE 2012]

22. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is

(a) $25\sqrt{3}$ (b) $50\sqrt{3}$ (c) $75\sqrt{3}$ (d) 150 [CBSE 2013]

23. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is

(a) $15\sqrt{3}$ m (b) $\frac{15\sqrt{3}}{2}$ m (c) $\frac{15}{2}$ m (d) 15 m [CBSE 2013]

24. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is

(a) $50\sqrt{3}$ (b) $150\sqrt{3}$ (c) $150\sqrt{2}$ (d) 75 [CBSE 2014]

25. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the sun at that time is

(a) 30° (b) 60° (c) 45° (d) 75° [CBSE 2014]

26. The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the tower is 45°. Then the height of the tower (in metres) is

(a) $50\sqrt{3}$ (b) 50 (c) $\frac{50}{\sqrt{2}}$ (d) $\frac{50}{\sqrt{3}}$ [CBSE 2014]

27. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is

(a) $\frac{4}{\sqrt{3}}$ (b) $4\sqrt{3}$ (c) $2\sqrt{2}$ (d) 4 [CBSE 2014]

ANSWERS

1. (a)	2. (b)	3. (a)	4. (b)	5. (b)	6. (a)
7. (b)	8. (a)	9. (b)	10. (d)	11. (d)	12. (c)
13. (d)	14. (a)	15. (c)	16. (c)	17. (a)	18. (b)
19. (c)	20. (c)	21. (b)	22. (a)	23. (c)	24. (a)
25. (b)	26. (b)	27. (d)			

SUMMARY

1. The line drawn from the eye of an observer to a point in the object where the person is viewing is called the line of sight.
2. The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.
3. The angle formed by the line of sight with the horizontal when the object is below the horizontal level is called the angle of depression.
4. The height of an object or the distance between distant objects can be determined with the help of trigonometric ratios.