CHAPTER 13

AREAS RELATED TO CIRCLES

13.1 INTRODUCTION

In earlier classes, we have studied methods of finding perimeters and areas of simple plane figures such as rectangles, squares, parallelograms, triangles and circles. In our daily life, we come across many objects which are related to circular shape in some form or the other. For example, cycle wheels, wheel arrow, drain cover, bangles, brooches, flower beds, circular paths etc. That is why the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall discuss problems on finding the areas of the two special parts of a circular region known as sector and segment of a circle. We shall also discuss problems on finding the areas of some combinations of plane figures involving circles or parts of circles. Let us first recall the concepts related to the perimeter and area of a circle.

13.2 REVIEW OF PERIMETER AND AREA OF A CIRCLE

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains same.

The fixed point is called the *centre* and the given constant distance is known as the *radius* of the circle.

CIRCUMFERENCE *The perimeter of a circle is generally known as its circumference.*

We know that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi').

Thus, if C denotes the circumference of a circle of radius r. Then,

$$\pi = \frac{\text{Circumference}}{\text{Diameter}} \Rightarrow \pi = \frac{C}{2r} \Rightarrow C = 2\pi r$$

Here, π stands for a particular irrational number whose approximate value upto two decimal place is 3.14 or $\frac{22}{7}$. The value of π upto four places of decimal is 3.1416 and up to eight decimal places its value is 3.14159265. For practical purposes, we generally take the value of π as $\frac{22}{7}$ or 3.14 approximately.

If r is the radius of a circle, then

(i) Circumference $= 2\pi r$

Also, Circumference $= \pi d$, where $d = 2r$ is the diameter of the circle.

(ii) Area $= \pi r^2$, Also Area $= \pi\left(\frac{d}{2}\right)^2 = \frac{1}{4}\pi d^2$

(iii) Area of semi-circle $= \frac{1}{2}\pi r^2$

(iv) Area of a quadrant of a circle $= \frac{1}{4}\pi r^2$

AREA ENCLOSED BY TWO CONCENTRIC CIRLES If R and r are radii of two concentric circles, then

Area enclosed by the two circles $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R+r)(R-r)$

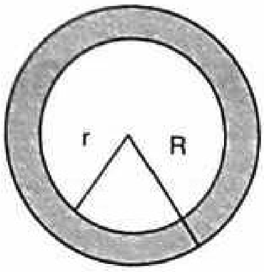

Fig. 13.1

Some useful results:

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in one minute

$$= \frac{\text{Distance moved in one minute}}{\text{Circumference}}$$

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Find the circumference and area of a circle of radius 8.4 cm.*

SOLUTION We know that the circumference C and area A of a circle of radius r are given by $C = 2\pi r$ and $A = \pi r^2$ respectively.

Here, $r = 8.4$ cm.

$$\therefore \quad C = \text{Circumference} = 2\pi r = 2 \times \frac{22}{7} \times 8.4 \text{ cm} = 52.8 \text{ cm}$$

$$A = \text{Area} = \pi r^2 = \frac{22}{7} \times 8.4 \times 8.4 \text{ cm}^2 = 221.76 \text{ cm}^2$$

EXAMPLE 2 *Find the area of a circle whose circumference is 22 cm.*

SOLUTION Let r be the radius of the circle. It is given that the circumfernce of the circle is 22 cm.

Now, Circumference = 22 cm

$$\Rightarrow \quad 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2} \text{ cm}$$

$$\therefore \quad \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2 = 38.5 \text{ cm}^2$$

EXAMPLE 3 *Find the area of a quadrant of a circle whose circumference is 22 cm.*

SOLUTION Let r be the radius of the circle. It is given that the circumference of the circle is 22 cm.

Now, Circumference = 22 cm

$\Rightarrow \quad 2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2}$ cm

$\therefore \quad$ Area of a quadrant $= \frac{1}{4}\pi r^2 = \left\{\frac{1}{4} \times \frac{22}{7}\left(\frac{7}{2}\right)^2\right\} \text{cm}^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\} \text{cm}^2 = \frac{77}{8} \text{cm}^2 = 9.625 \text{ cm}^2$

EXAMPLE 4 *If the perimeter of a semi-circular protractor is 108 cm, find the diameter of the protractor* (Take $\pi = 22/7$).

SOLUTION Let the radius of the protractor be r cm. It is given that its perimeter is 108 cm.

Now, Perimeter = 108 cm

$\Rightarrow \quad \frac{1}{2}(2\pi r) + 2r = 108 \qquad \left[\because \text{ Perimeter of a semi-circle} = \frac{1}{2}(2\pi r)\right]$

$\Rightarrow \quad \pi r + 2r = 108 \Rightarrow \frac{22}{7} \times r + 2r = 108 \Rightarrow 36r = 108 \times 7 \Rightarrow r = 3 \times 7 = 21$

$\therefore \quad$ Diameter of the protractor $= 2r = (2 \times 21)$ cm $= 42$ cm

EXAMPLE 5 *The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.*

SOLUTION Let the radius of the circle be r cm. Then,

Diameter = $2r$ cm and, Circumference = $2\pi r$ cm

It is given that the circumference exceeds the diameter by 16.8 cm. That is,

Circumference = Diameter + 16.8

$\Rightarrow \quad 2\pi r = 2r + 16.8$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 2r + 16.8 \qquad \left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow \quad 44r = 14r + 16.8 \times 7$

$\Rightarrow \quad 44r - 14r = 117.6 \Rightarrow 30r = 117.6 \Rightarrow r = \frac{117.6}{30} = 3.92$

Hence, radius of the circle is 3.92 cm.

EXAMPLE 6 *Find the diameter of the circle whose area is equal to the sum of the areas of two circles of diameters 20 cm and 48 cm.* [NCERT EXEMPLAR]

SOLUTION Let d be the diameter of the circle whose area is equal to the sum of the areas of two circles of diameters $d_1 = 20$ cm and $d_2 = 48$ cm. Then,

$$\pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{20}{2}\right)^2 + \pi\left(\frac{48}{2}\right)^2$$

$\Rightarrow \quad \frac{d^2}{4} = 10^2 + 24^2$

$\Rightarrow \quad \frac{d^2}{4} = 100 + 576 = 676$

$\Rightarrow \quad d^2 = 676 \times 4 = 26^2 \times 2^2$

$\Rightarrow \quad d = 26 \times 2 = 52$ cm.

EXAMPLE 7 *All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm²*. (Use $\pi = 3.14$) [NCERT EXEMPLAR]

SOLUTION Let $ABCD$ be a rhombus whose vertices A, B, C, D lie on a circle with centre O and radius r.

It is given that the area of the circle is 1256 cm^2.

$\therefore \quad \pi r^2 = 1256$

$\Rightarrow \quad 3.14 r^2 = 1256$

$\Rightarrow \quad r^2 = 400 \quad$ ...(i)

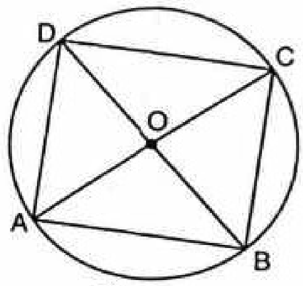

Fig. 13.2

We know that the diagonals of a rhombus intersect at right angle. Therefore, AC and BD are perpendicular and so these two are diameters of the circle.

$\therefore \quad AC = BD = 2r$

Area of rhombus $ABCD = \frac{1}{2}(AC \times BD) = \frac{1}{2}(2r \times 2r) = 2r^2 = 2 \times 400 \text{ cm}^2 = 800 \text{ cm}^2$

EXAMPLE 8 *A copper wire, when bent in the form of a square, encloses an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it. (Use $\pi = 22/7$).*

SOLUTION It is given that

Area of the square = 484 cm^2

$\therefore$ Side of the square $\sqrt{484}$ cm = 22cm $\quad \left[\because \text{Area} = (\text{Side})^2 \therefore \text{Side} = \sqrt{\text{Area}}\right]$

So, Perimeter of the square = 4 (Side) = (4 × 22) cm = 88 cm

Let r be the radius of the circle. Then,

Circumference of the circle = Perimeter of the square.

$\Rightarrow \quad 2\pi r = 88$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14 \text{ cm}$

$\therefore \quad$ Area of the circle $= \pi r^2 = \left\{\frac{22}{7} \times (14)^2\right\} \text{cm}^2 = 616 \text{ cm}^2$

EXAMPLE 9 *A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of the side of the square.*

SOLUTION Let the side of the square be x cm. The wire is in the form of a circle of radius 28 cm.

$\therefore$ Length of the wire = Circumference of the circle

$= \left\{2 \times \frac{22}{7} \times 28\right\} \text{cm} \quad$ [Using $C = 2\pi r$]

$= 176 \text{ cm} \quad$ ...(i)

The wire is re-bent in the form of a square of side x cm.

$\therefore$ Perimeter of the square = Length of the wire

$\Rightarrow \quad 4x = 176 \quad$ [Using (i)]

$\Rightarrow \quad x = 44 \text{ cm}$

Hence, the length of the side of the square is 44 cm.

EXAMPLE 10 *A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.*

SOLUTION Let the outer and inner radii of the ring be R metres and r metres respectively.

It is given that the outer and inner circumferences of the ring are 396 m and 352 m respectively.

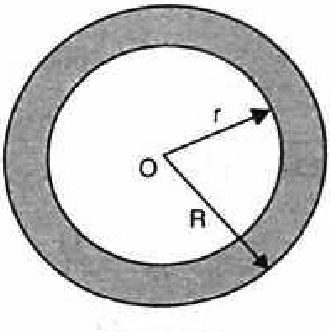

Fig. 13.3

$\therefore \quad 2\pi R = 396 \text{ and } 2\pi r = 352$

$\Rightarrow \quad 2 \times \frac{22}{7} \times R = 396 \text{ and } 2 \times \frac{22}{7} \times r = 352$

$\Rightarrow \quad R = 396 \times \frac{7}{22} \times \frac{1}{2} \text{ and } r = 352 \times \frac{7}{22} \times \frac{1}{2}$

$\Rightarrow \quad R = 63 \text{ m and } r = 56 \text{ m}$

Hence, width of the track $= (R - r)$ metres

$= (63 - 56)$ metres $= 7$ metres

EXAMPLE 11 *The inner circumference of a circular track is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹2 per metre.* (Use $\pi = 22/7$)

SOLUTION Let the inner and outer radii of the circular track be r metres and R metres respectively. It is given that

Inner circumference = 220 metres

$\Rightarrow \quad 2\pi r = 220 \Rightarrow 2 \times \frac{22}{7} \times r = 220 \Rightarrow r = 35 \text{ m}$

The track is 7 metre wide everywhere. Therefore, the outer radius R is given by

$R = r + 7 = (35 + 7) \text{ m} = 42 \text{ m}$

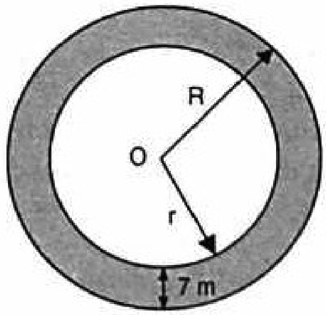

Fig. 13.4

$\therefore \quad$ Outer circumference $= 2\pi R = 2 \times \frac{22}{7} \times 42 \text{ m} = 264 \text{ m}$

Rate of fencing = ₹ 2 per metre

$\therefore \quad$ Total cost of fencing = (Circumference × Rate) = ₹ (264 × 2) = ₹ 528

EXAMPLE 12 *A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.*

SOLUTION Let the radius of the wheel be r cm. We observe that the distance covered by the wheel in one revolution is equal to the circumference of the wheel.

$\therefore \quad$ Distance covered by the wheel in one revolution $= 2\pi r$ cm

$\Rightarrow \quad$ Distance covered by the wheel in 5000 revolutions $= 5000 \times 2\pi r$ cm

$$= 10000 \times \frac{22}{7} \times r \text{ cm}$$

$$= \frac{10000 \times \frac{22}{7} \times 4}{100} \text{ m}$$

$$= \frac{10000 \times \frac{22}{7} \times r}{100 \times 1000} \text{ km} = \frac{11}{35} r \text{ km}$$

It is given that the bicycle wheel covers 11 km distance in 5000 revolutions.

$\therefore \quad \frac{11}{35} r = 11 \Rightarrow r = 35$

$\therefore \quad$ Diameter $= 2r$ cm $= (2 \times 35)$ cm $= 70$ cm

Hence, the diameter of the wheel is 70 cm.

EXAMPLE 13 *A wheel has diameter 84 cm. Find how many complete revolutions must it take to cover 792 meters.*

SOLUTION Suppose the wheel makes n complete revolutions in covering 792 meters.
Let r be the radius of the wheel. It is given that the diameter of the wheel is 84 cm.

$\therefore \quad 2r = 84 \Rightarrow r = 42$ cm

$\therefore \quad$ Circumference of the wheel $= 2\pi r$ cm $= 2 \times \frac{22}{7} \times 42$ cm $= 264$ cm $= 2.64$ m

Distance covered by the wheel in one revoluton = 2.64 cm
Distance covered by wheel in n revolutions = $(2.64)\, n$ metres
It is given that the wheel covers 792 metres in n revolutions.

$\therefore \quad (2.64)\, n = 792 \Rightarrow n = \frac{792}{2.64} = 300$

Hence, the wheel takes 300 revolutions in covering 792 meters.

EXAMPLE 14 *A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour with which the boy is cycling.*

SOLUTION The speed with which the boy is cycling per hour is the distance covered by the wheel in one hour. Clearly, distance covered by the wheel in one revolution is equal to its circumference. So, let us first compute the circumference.
We have,

Radius of the wheel $= r = \frac{60}{2}$ cm $= 30$ cm.

$\therefore \quad$ Circumference of the wheel $= 2\pi r = 2 \times \frac{22}{7} \times 30$ cm $= \frac{1320}{7}$ cm

So, $\quad$ Distance covered by the wheel in one revolution $= \frac{1320}{7}$ cm

$\Rightarrow \quad$ Distance covered by the wheel in 140 revolutions $= \frac{1320}{7} \times 140$ cm

$$= (1320 \times 20) \text{ cm} = 26400 \text{ cm}$$

$$= \frac{26400}{100} \text{ m} = 264 \text{ m} = \frac{264}{1000} \text{ km}$$

It is given that the wheels are making 140 revolutions per minute. So, distance covered by the wheels in one minute is equal to the distance covered by the wheels in 140 revolutions

$\Rightarrow$ Distance covered by the wheel in one minute $= \frac{264}{1000}$ km

$\Rightarrow$ Distance covered by the wheel in one hour i.e. in 60 minutes $= \frac{264}{1000} \times 60$ km $= 15.84$ km

Hence, the speed with which the boy is cycling is 15.84 km/hr.

EXAMPLE 15 *The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour?*

SOLUTION Suppose the driving wheel of the bus makes n revolutions per minute to keep a speed of 66 km/hr. Clearly, distance covered by the wheel in one revolution is equal to its circumference. It is given that r = Radius of the wheel = 70 cm.

$\therefore$ Circumference of the wheel $= 2\pi r = 2 \times \frac{22}{7} \times 70$ cm $= 440$ cm

So, distance covered by the wheel in one revolution = 440 cm

$\Rightarrow$ Distance covered by the wheel in n revolutions $= (440 \times n)$ cm

$\Rightarrow$ Distance covered by the wheel in one minute $= (440 \times n)$ cm

$\Rightarrow$ Distance covered by the wheel in one hour $= (440 \times n \times 60)$ cm

We are given that the speed of the bus is 66 km/hr. This means that the wheel covers 66 km in one hour.

So, the wheel covers $\frac{66 \times 1000 \times 100}{60} = 110000$ cm in one minute.

$\therefore \quad 440 \times n \times 60 = 110000 \Rightarrow n = \frac{110000}{440 \times 60} = 250$

Hence, the wheel makes 250 revolutions per minute.

EXAMPLE 16 *A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?* [NCERT]

SOLUTION Suppose each wheel of the car makes n complete revolutions in 10 minutes. This means that the distance covered by each wheel in n revolutions is same as the distance travelled by the car in 10 minutes.

It is given that :

Speed of the car = 66 km/hr

$\therefore$ Distance travelled by the car in 1 hour = 66 km

$\Rightarrow$ Distance travelled by the car in 10 min $= \left(\frac{66}{60} \times 10\right)$ km $= 11$ km $= 11 \times 1000 \times 100$ cm ...(i)

It is given that :

Radius of car wheels = 40 cm

$\therefore$ Circumference of the wheels $= 2 \times \frac{22}{7} \times 40$ cm

In a revolution each wheel covers the distance equal to its circumference.

$\therefore$ Distance covered by each wheel in one complete revolution $= 2\times\frac{22}{7}\times 40$ cm

$\Rightarrow$ Distance covered by each wheel in n revolutions $= \left(2\times\frac{22}{7}\times 40\times n\right)$ cm ...(ii)

But, distance covered by each wheel in completing n revoluitions is equal to the distance travelled by the car in 10 minutes.

$$\therefore \quad 2\times\frac{22}{7}\times 40\times n = 11\times 1000\times 100 \Rightarrow n = \frac{11\times 1000\times 100\times 7}{2\times 22\times 40} = 4375$$

Hence, each wheel makes 4375 revolutions in 10 minutes.

EXAMPLE 17 *Find the number of revolutions made by a circular wheel of area 1.54 m^2 in rolling a distance of 176 m.* [NCERT EXEMPLAR]

SOLUTION Let r be the radius of the circular wheel. It is given that its area is 1.54 m^2

$$\therefore \quad \pi r^2 = 1.54 \Rightarrow \frac{22}{7}r^2 = 1.54 \Rightarrow r^2 = 7\times 0.07 = 0.49 \Rightarrow r = 0.7$$

Suppose the wheel makes n revolutions in rolling a distance of 176 m.

$\therefore \quad n\times$ Distance rolled in one revolution $= 176$

$\Rightarrow \quad n\times 2\pi r = 176$ [$\because$ Distance rolled in one revolution = Circumference]

$$\Rightarrow \quad n\times 2\times\frac{22}{7}\times 0.7 = 176 \Rightarrow n = \frac{176\times 7}{2\times 22\times 0.7} = 40$$

Hence, the circular wheel makes 40 revolutions.

EXAMPLE 18 *The diameters of front and rear wheels of a tractor are 80 cm and 2 m respectively. Find the number of revolutions that rear wheel makes in covering a distance in which the front wheel makes 1400 revolutions.* [NCERT EXEMPLAR]

SOLUTION Let r_1 and r_2 be the radii of front and rear wheels of the tractor. It is given that $r_1 = 0.40$ m and $r_2 = 1$ m.

Distance covered by the front wheel in one revolution $= 2\pi r_1 = 2\pi\times 0.4$ m $= 0.8\,\pi$ m

$\therefore$ Distance covered by the front wheel in 1400 revolutions $= 1400\times 0.8\,\pi$ m $= 1120\,\pi$ m

Suppose the rear wheel makes n revolutions to cover this distance. Then,

(Distance covered by the rear wheel in one revolution) $\times n = 1120\,\pi$

$$\Rightarrow \quad 2\pi r_2\times n = 1120\pi \Rightarrow 2\pi\times 1\times n = 1120\,\pi \Rightarrow n = 560$$

Hence, the rear wheel makes 560 revolutions.

EXAMPLE 19 *The cost of fencing a circular field at the rate ₹ 24 per metre is ₹ 5280. The field is to be ploughed at the rate of ₹ 0.50 per m^2. Find the cost of ploughing the field.* (Take $\pi = 22/7$) [NCERT]

SOLUTION We have,

Rate of fencing = ₹ 24 per metre and, Total cost of fencing = ₹ 5280

$\therefore$ Length of the fence $= \frac{\text{Total cost}}{\text{Rate}} = \frac{5280}{24}$ metre $= 220$ metre

$\Rightarrow$ Circumference of the field $= 220$ metre

$\Rightarrow$ $2\pi r = 220$, where r is the radius of the field

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 220$

$\Rightarrow$ $r = \frac{220 \times 7}{22 \times 2} = 35$ metres

$\therefore$ Area of the field $= \pi r^2 = \frac{22}{7} \times 35 \times 35 \text{ m}^2 = 22 \times 5 \times 35 \text{ m}^2$

It is given that the field is ploughed at the rate of ₹ 0.50 per m^2

$\therefore$ Cost of ploughing the field $=$ ₹ $(22 \times 5 \times 35 \times 0.50) =$ ₹ 1925

ALITER Let the radius of the circular field be r metres. Then,

Length of its circular fence $= 2\pi r$ metres.

It is given that the cost of fencing the field at the rate of ₹ 24 per metre is ₹ 5280.

Length of the fence $\times 24 = 5280$

$\Rightarrow$ $2\pi r \times 24 = 5280$

$\Rightarrow$ $2 \times \frac{22}{7} \times r \times 24 = 5280$

$\Rightarrow$ $r = \frac{5280 \times 7}{2 \times 22 \times 24} = 35$ metre

$\therefore$ Area of the circular field $= \pi r^2 = \frac{22}{7} \times 35 \times 35 \text{ m}^2$

So, the cost of ploughing the field at the rate of ₹ 0.50 per square metre is $=$ ₹ $\left(\frac{22}{7} \times 35 \times 35 \times 0.50\right)$

$=$ ₹ 1925

EXAMPLE 20 *The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between the areas of the two circles is 1078 sq. cm. Find the radius of the smaller circle.* [CBSE 2017]

SOLUTION Let the lengths of the radii of the smaller and larger circles be r cm and R cm respectively.

It is given that

$$R - r = 7 \quad \text{...(i)}$$

It is also given that the difference between the areas of two circles is 1078 cm^2

$\therefore$ $\pi R^2 - \pi r^2 = 1078$

$\Rightarrow$ $\pi(R^2 - r^2) = 1078$

$\Rightarrow$ $\frac{22}{7}(R + r)(R - r) = 1078$

$\Rightarrow$ $\frac{22}{7}(R + r) \times 7 = 1078$ [Using (i)]

$\Rightarrow$ $R + r = 49$ ...(ii)

Subtracting (i) from (ii), we get

$2r = 42 \Rightarrow r = 21$

Hence, the radius of the smaller circle is of length 21 cm.

LEVEL-2

EXAMPLE 21 *Two circles touch externally. The sum of their areas is 130 π sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.*

SOLUTION If two circles touch externally, then the distance between their centres is equal to the sum of their radii. Let the radii of the two circles be r_1 cm and r_2 cm respectively. Let C_1 and C_2 be the centres of the given circles. Then,

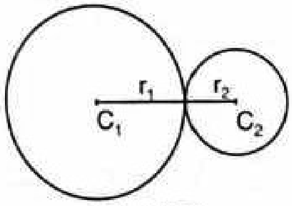

Fig. 13.5

$$C_1C_2 = r_1 + r_2$$

$\Rightarrow \quad 14 = r_1 + r_2 \qquad [\because C_1C_2 = 14 \text{ cm (given)}]$

$\Rightarrow \quad r_1 + r_2 = 14 \qquad \ldots(i)$

It is given that the sum of the areas of two circles is equal to 130 $\pi \text{ cm}^2$.

$\therefore \quad \pi r_1^2 + \pi r_2^2 = 130\pi$

$\Rightarrow \quad r_1^2 + r_2^2 = 130 \qquad \ldots(ii)$

Now, $\quad (r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \quad 14^2 = 130 + 2r_1r_2 \qquad$ [Using (i) and (ii)]

$\Rightarrow \quad 196 - 130 = 2r_1r_2$

$\Rightarrow \quad r_1r_2 = 33 \qquad \ldots(iii)$

Now,

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$

$\Rightarrow \quad (r_1 - r_2)^2 = 130 - 2 \times 33 \qquad$ [Using (ii) and (iii)]

$\Rightarrow \quad (r_1 - r_2)^2 = 64$

$\Rightarrow \quad r_1 - r_2 = 8 \qquad \ldots(iv)$

Solving (i) and (iv), we get $r_1 = 11$ cm and $r_2 = 3$ cm.

Hence, the radii of the two circles are 11 cm and 3 cm.

EXAMPLE 22 *Two circles touch internally. The sum of their areas is 116 π cm² and distance between their centres is 6 cm. Find the radii of the circles.*

SOLUTION Let R and r be the radii of the circles having centres at O and O' respectively. It is given that the sum of the areas is $116\pi \text{ cm}^2$ and the distance between the centres is 6 cm.

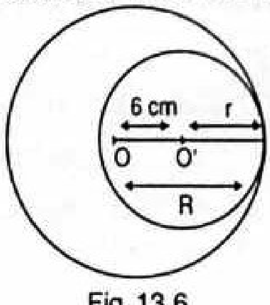

Fig. 13.6

Now, $\quad$ Sum of areas $= 116\pi \text{ cm}^2$

$\Rightarrow \quad \pi R^2 + \pi r^2 = 116\pi$

$\Rightarrow \quad R^2 + r^2 = 116 \qquad \ldots(i)$

Distance between the centres = 6 cm

$\Rightarrow$ $OO' = 6\text{cm}$

$\Rightarrow$ $R - r = 6$ ...(ii)

Now, $(R+r)^2 + (R-r)^2 = 2(R^2 + r^2)$

$\Rightarrow$ $(R+r)^2 + 36 = 2 \times 116$ [Using (i) and (ii)]

$\Rightarrow$ $(R+r)^2 = (2 \times 116 - 36) = 196$

$\Rightarrow$ $R + r = 14$ ...(iii)

Solving (ii) and (iii), we get: $R = 10$ and $r = 4$.

Hence, radii of the given circles are 10 cm and 4 cm respectively.

EXAMPLE 23 *Figure 13.7, depicts an archery target marked with its five scoring areas from the centre outwards as Gold, Red, Blue Black and white. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.* [NCERT]

SOLUTION We have,

r = Radius of the region representing Gold score = 10.5 cm

$\therefore$ r_1 = Radius of the region representing Gold and Red scoring areas
= (10.5 + 10.5) cm = 21 cm = $2r$ cm

r_2 = Radius of the region representing Gold, Red and Blue scoring areas
= (21 + 10.5) cm = 31.5 cm = $3r$ cm

r_3 = Radius of the region represing Gold, Red, Blue and Black scoring areas
= (31.5 + 10.5) cm = 42 cm = $4r$ cm

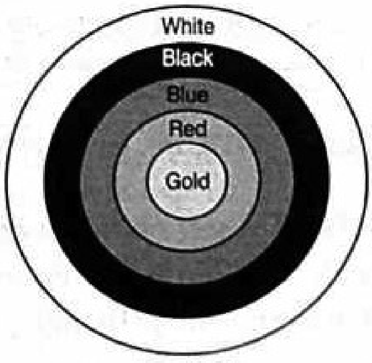

Fig. 13.7

r_4 = Radius of the region representing Gold, Red, Blue, Black and white scoring areas
= (42 + 10.5) cm = 52.5 cm = $5r$ cm

Now,

A_1 = Area of the region representing Gold scoring area $= \pi r^2 = \frac{22}{7} \times (10.5)^2 = \frac{22}{7} \times 10.5 \times 10.5$
$= 22 \times 1.5 \times 10.5 = 346.5 \text{ cm}^2$

A_2 = Area of the region representing Red scoring area $= \pi(2r)^2 - \pi r^2 = 3\pi r^2 = 3A_1$
$= 3 \times 346.5 \text{ cm}^2 = 1039.5 \text{ cm}^2$

A_3 = Area of the region representing Blue scoring area $= \pi(3r)^2 - \pi(2r)^2 = 9\pi r^2 - 4\pi r^2$
$= 5\pi r^2 = 5A_1 = 5 \times 346.5 \text{ cm}^2$
$= 1732.5 \text{ cm}^2$

A_4 = Area of the region representing Black scoring area $= \pi(4r)^2 - \pi(3r)^2 = 7\pi r^2 = 7A_1$
$= 7 \times 346.5 \text{ cm}^2 = 2425.5 \text{ cm}^2$

A_5 = Area of the region representing White scoring area = $\pi (5r)^2 - \pi (4r)^2 = 9\pi r^2 = 9A_1$

$= 9 \times 346.5 \text{ cm}^2 = 3118.5 \text{ cm}^2$

EXERCISE 13.1

LEVEL-1

1. Find the circumference and area of a circle of radius 4.2 cm.
2. Find the circumference of a circle whose area is 301.84 cm^2.
3. Find the area of a circle whose circumference is 44 cm.
4. The circumference of a circle exceeds the diameter by 16.8 cm. Find the circum-ference of the circle.
5. A horse is tied to a pole with 28 m long string. Find the area where the horse can graze. (Take $\pi = 22/7$).
6. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, find the area of the circle.
7. The circumference of two circles are in the ratio 2 : 3. Find the ratio of their areas.
8. The sum of the radii of two circles is 140 cm and the difference of their circumferences is 88 cm. Find the diameters of the circles.
9. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm. [NCERT EXEMPLAR]
10. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles. [NCERT]
11. The radii of two circles are 19 cm and 9 cm respectively. Find the radius and area of the circle which has its circumference equal to the sum of the circumferences of the two circles. [NCERT]
12. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹ 50 per metre. [NCERT EXEMPLAR]
13. The side of a square is 10 cm. Find the area of circumscribed and inscribed circles.
14. If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.
15. The area of a circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle. [Use $\pi = 22/7$ and $\sqrt{3} = 1.73$]
16. A field is in the form of a circle. A fence is to be erected around the field. The cost of fencing would be ₹ 2640 at the rate of ₹ 12 per metre. Then, the field is to be thoroughly ploughed at the cost of ₹ 0.50 per m^2. What is the amount required to plough the field? [Take $\pi = 22/7$].
17. A park is in the form of a rectangle 120 m × 100 m. At the centre of the park there is a circular lawn. The area of park excluding lawn is 8700 m^2. Find the radius of the circular lawn. (Use $\pi = 22/7$).
18. A car travels 1 kilometre distance in which each wheel makes 450 complete revolutions. Find the radius of the its wheels.
19. The area enclosed between the concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, find the radius of the inner circle.
20. An archery target has three regions formed by thre concentric circles as shown in Fig. 13.8. If the diameters of the concentric circles are in the ratio 1 : 2 : 3, then find the ratio of the areas of three regions. [NCERT EXEMPLAR]

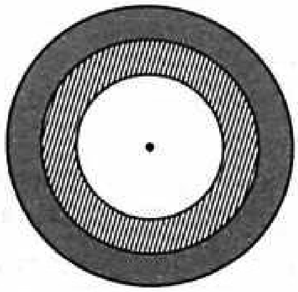

Fig. 13.8

21. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/hr? [NCERT EXEMPLAR]
22. A circular pond is 17.5 m in diameter. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ₹25 per m^2. [NCERT EXEMPLAR]
23. A circular park is surrounded by a rod 21 m wide. If the radius of the park is 105 m, find the area of the road. [NCERT EXEMPLAR]
24. A square of diagonal 8 cm is inscribed in a circle. Find the area of the region lying outside the circle and inside the square. [NCERT EXEMPLAR]
25. A path of 4 m width runs round a semi-circular grassy plot whose circumference is 163 $\frac{3}{7}$ m Find:
    (i) the area of the path
    (ii) the cost of gravelling the path at the rate of ₹1.50 per square metre
    (iii) the cost of turfing the plot at the rate of 45 paise per m^2.
26. Find the area enclosed between two concentric circles of radii 3.5 cm and 7 cm. A third concentric circle is drawn outside the 7 cm circle, such that the area enclosed between it and the 7 cm circle is same as that between the two inner circles. Find the radius of the third circle correct to one decimal place.
27. A path of width 3.5 m runs around a semi-circular grassy plot whose perimeter is 72 m. Find the area of the path. (Use $\pi = 22/7$) [CBSE 2015]
28. A circular pond is of diameter 17.5 m. It is surrounded by a 2 m wide path. Find the cost of constructing the path at the rate of ₹25 per square metre (Use $\pi = 3.14$) [CBSE 2014]
29. The outer circumference of a circular race-track is 528 m. The track is everywhere 14 m wide. Calculate the cost of levelling the track at the rate of 50 paise per square metre (Use $\pi = 22/7$).
30. A road which is 7 m wide surrounds a circular park whose circumference is 352 m. Find the area of the road.
31. Prove that the area of a circular path of uniform width h surrounding a circular region of radius r is $\pi h(2r + h)$.

ANSWERS

1. 26.4 cm, 55.44 cm^2
2. 61.6 cm
3. 154 cm^2
4. 24.64 cm
5. 2464 m^2
6. 154 cm^2
7. 4 : 9
8. 154 cm, 126 cm
9. 33 cm
10. 10 cm
11. 28 cm, 2464 cm^2
12. ₹ 26400
13. 157 cm^2, 78.5 cm^2
14. $\pi : 2$
15. 72.7 cm
16. ₹ 1925
17. 32.40 m
18. 35.35 cm
19. 14 cm
20. 1 : 3 : 5
21. 500
22. ₹ 3061.50
23. 15246 cm^2
24. $(16\pi - 32)$ cm^2
25. (i) 352 m^2 (ii) ₹ 528 (iii) ₹ 478

26. 115.5 cm^2, 9.26 cm 27. 173.25 m^2 28. ₹ 3061.50
29. ₹ 3388 30. 2618 m^2

HINT TO SELECTED PROBLEMS

5. Length of the string = 28 m. Area over which the horse can graze is the area of a circle of radius 28 m. Hence, required area $= \pi (28)^2 = 2464 \text{ m}^2$

6. Let r cm be the radius of the circle. Side of the square $= \sqrt{121}$ cm = 11 cm

   $\therefore$ Perimeter of the square = (4×11) cm = 44 cm

   So, length of the wire = 44 cm.

   Now, Circumference of the circle = Length of the wire $\Rightarrow$ $2\pi r = 44$ cm $\Rightarrow$ $r = 7$ cm

   Hence, Area of the circle $= \pi r^2 = \pi \times 7^2 \text{ cm}^2 = 154 \text{ cm}^2$

7. Let r_1 and r_2 be the radii of two given circles and C_1 and C_2 be their circumferences. Then,

$$C_1 : C_2 = 2 : 3$$

$$\Rightarrow \quad 2\pi r_1 : 2\pi r_2 = 2 : 3$$

$$\Rightarrow \quad r_1 : r_2 = 2 : 3 \Rightarrow r_1^2 : r_2^2 = 4 : 9 \Rightarrow \pi r_1^2 : \pi r_2^2 = 4\pi : 9\pi \Rightarrow \pi r_1^2 : \pi r_2^2 = 4 : 9$$

8. Let r_1 and r_2 be the radii of two given circles. Then,

$$r_1 + r_2 = 140 \text{ and } 2\pi r_1 - 2\pi r_2 = 88 \Rightarrow 2\pi(r_1 - r_2) = 88 \Rightarrow r_1 - r_2 = 14$$

10. Let r be the radius of the circle whose area is equal to the sum of the areas of circles of radii 8 cm and 6 cm. Then,

$$\pi r^2 = \pi \times 8^2 + \pi \times 6^2 \Rightarrow r^2 = 100 \Rightarrow r = 10 \text{ cm}$$

11. Let the radius of the circle be r cm. Then,

$$2\pi r = 2\pi \times 19 + 2\pi \times 9 \Rightarrow r = 28 \text{ cm}$$

$$\therefore \quad \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 2464 \text{ cm}^2$$

13. We have, Diameter of the circumscribed circle = Diagonal of the square $= \sqrt{10^2 + 10^2}$

$$= 10\sqrt{2} \text{ cm}$$

   Diameter of the inscribed circle = Length of the side of the square.

15. Let r be the radius of the inscribed circle. Then,

   Area = 154 cm^2 $\Rightarrow$ $\pi r^2 = 154$ $\Rightarrow$ $r = 7$ cm

   Let h be the height of the triangle. Then,

$$r = \frac{h}{3} \Rightarrow h = 3r = 21 \text{ cm}$$

   If a is the side of the triangle. Then,

$$h = \frac{\sqrt{3}}{2} a \Rightarrow a = \frac{2h}{\sqrt{3}} = \frac{42}{\sqrt{3}} = 14\sqrt{3} \text{ cm}$$

   Hence, perimeter $= 3a = 3 \times 14\sqrt{3}$ cm = 72.7 cm

16. Length of the fence $= \dfrac{\text{Total Cost}}{\text{Rate per metre}} = \dfrac{2640}{12} = 220$ m

$\therefore \quad 2\pi r = 220 \text{ m} \Rightarrow r = 35 \text{ m}$

$\therefore \quad \text{Area} = \pi r^2 = \pi (35)^2$

Cost of ploughing the whole field $= ₹\,[\pi (35)^2 \times 0.50]$

17. Area of the park $= (120 \times 100) \text{ m}^2 = 12000 \text{ m}^2$

Area of the park excluding the lawn $= 8700 \text{ m}^2$

$\therefore$ Area of the circular lawn $= (12000 - 8700) \text{ m}^2$

$$\Rightarrow \quad \pi r^2 = 3300 \Rightarrow r^2 = 3300 \times \frac{7}{22} = 150 \times 7 \Rightarrow r = \sqrt{150 \times 7} = 32.40 \text{ m}$$

18. Let the radius of each wheel be r metres. Then,

Circumference of each wheel $= 2\pi r = 2 \times \frac{22}{7} \times r$ metres

$\Rightarrow$ Distance covered by the wheels in one revolution $= 2 \times \frac{22}{7} \times r$ metres

$\Rightarrow$ Distance covered by the wheels in 450 revolutions $= 2 \times \frac{22}{7} \times r \times 450$ metres

It is given that the car travels 1 kilometre i.e. 1000 metres distance when its each wheel makes 450 revolutions.

$$2 \times \frac{22}{7} \times r \times 450 = 1000 \Rightarrow r = \frac{7 \times 1000}{2 \times 22 \times 450} = 0.3535 \text{ metres} = 35.35 \text{ cm}$$

19. Let the radius of the inner circle be r cm. Then,

$$\pi \times 21 \times 21 - \pi \times r^2 = 770 \Rightarrow \pi(441 - r^2) = 770 \Rightarrow \frac{22}{7} \times (441 - r^2) = 770 \Rightarrow 441 - r^2 = 245$$

$$\Rightarrow r^2 = 196 \Rightarrow r = 14 \text{ cm}$$

13.3 SECTOR OF A CIRCLE AND ITS AREA

Consider a circle of radius r having its centre at the point O. Let A, B, and C be three points on the circle as shown in Fig. 13.9. The area enclosed by the circle is divided into two regions, namely, OBA and $OBCA$. These regions are called sectors of the circle. Each of these two sectors has an arc of the circle as a part of its boundary. The sector OBA has arc AB as a part of its boundary whereas the sector $OBCA$ has arc ACB as a part of its boundary. These sectors are known as minor and major sectors of the circle as defined below.

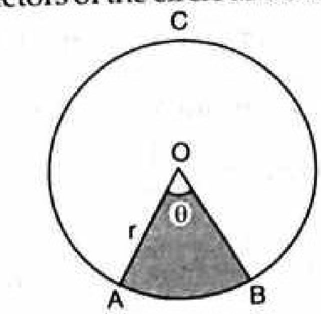

Fig. 13.9

MINOR SECTOR *A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary*

In Fig. 13.9, sector OAB is the minor sector.

[redacted]

MAJOR SECTOR *A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary.*

In Fig. 13.9, sector $OACB$ is the major sector.

Following are some important points to remember:

(i) *A minor sector has an angle* θ, *(say), subtended at the centre of the circle, whereas a major sector has no angle.*

(ii) *The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.*

(iii) *The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.*

(iv) *The boundary of a sector consists of an arc of the circle and the two radii.*

13.3.1 AREA OF A SECTOR

Consider a circle of radius r having its centre at O. Let AOB be a sector of the circle such that $\angle AOB = \theta$. If $\theta < 180°$, then the arc AB is a minor arc of the circle. Now, if θ increases the length of the arc AB also increases and if θ becomes 180°, then arc AB becomes the circumference of a semi-circle. Thus, if an arc subtends an angle of 180° at the centre, then its arc length is πr.

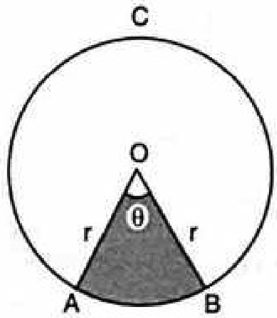

Fig. 13.10

$\therefore$ If the arc subtends an angle of θ at the centre, then its arc length $= \frac{\theta}{180} \times \pi r$

Hence, the arc length l of a sector of angle θ in a circle of radius r is given by

$$l = \frac{\theta}{180} \times \pi r \qquad ...(i)$$

$$\Rightarrow \quad l = \frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times \text{(Circumference of the circle)}$$

As discussed above, if the arc subtends an angle of 180° then the area of the corresponding sector is equal to the area of a semi-circle i.e $\frac{1}{2}\pi r^2$.

$\therefore$ If the arc subtends an angle θ, then area of the corresponding sector is $\frac{\theta}{180} \times \frac{1}{2}\pi r^2 = \frac{\pi r^2 \theta}{360}$

Thus, the area A of a sector of angle θ in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times \text{(Area of the circle)} \qquad ...(ii)$$

Also, $A = \frac{\theta}{360} \times \pi r^2 \Rightarrow A = \frac{1}{2}\left(\frac{\theta}{180} \times \pi r\right) r \Rightarrow A = \frac{1}{2} lr$ [Using (i)]

REMARK *Area of major sector* $= \pi r^2$ *– Area of minor segment*

Some useful results to remember:

(i) Angle described by minute hand in 60 minutes $= 360°$

$\therefore$ Angle described by minute hand in one minute $= \left(\frac{360}{60}\right)^\circ = 6°$

Thus, minute hand rotates through an angle of $6°$ in one minute.

(ii) Angle described by hour hand in 12 hours $= 360°$

$\therefore$ Angle described by hour hand in one hour $= \left(\frac{360}{12}\right)^\circ = 30°$

$\Rightarrow$ Angle described by hour hand in one minute $= \left(\frac{30}{60}\right)^\circ = \frac{1}{2}^\circ$

Thus, hour hand rotates through $\left(\frac{1}{2}\right)^\circ$ in one minute.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Find the area of a sector of a circle of radius 28 cm and central angle 45°.* [NCERT EXEMPLAR]

SOLUTION We know that the area A of a sector of a circle of radius r and central angle θ (in degrees) is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

Here, $r = 28$ cm and $\theta = 45$.

$$\therefore \quad A = \frac{45}{360} \times \pi \times (28)^2 = \frac{1}{8} \times \frac{22}{7} \times 28 \times 28 \text{ cm}^2 = 308 \text{ cm}^2$$

EXAMPLE 2 *Find the difference of the areas of a sector of angle 120° and its corresponding major sector of a circle of radius 21 cm.* [NCERT EXEMPLAR]

SOLUTION Let A_1 and A_2 be the areas of the given sector and the corresponding major sector respectively. We have, $\theta = 120$ and $r = 21$ cm.

$$\therefore \quad A_1 = \frac{\theta}{360} \times \pi r^2 = \frac{120}{360} \times \pi \times (21)^2 = 147\,\pi \text{ cm}^2$$

and, $A_2 =$ Area of the circle $- A_1$

$$\Rightarrow \quad A_2 = \{\pi \times (21)^2 - 147\pi\} \text{ cm}^2 = \pi(441 - 147) \text{ cm}^2 = 294\,\pi \text{ cm}^2$$

Required differences $= A_2 - A_1 = (294\pi - 147\pi)\text{ cm}^2 = 147\pi\text{ cm}^2 = \left(147 \times \frac{22}{7}\right)\text{cm}^2 = 462\text{ cm}^2$

EXAMPLE 3 *Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use $\pi = 3.14$).*

SOLUTION Here, $\theta = 30°$ and $r = 4$ cm.

$\therefore$ Area of sector $OAPB = \frac{\theta}{360} \times \pi r^2$

$$= \frac{30}{360} \times 3.14 \times 4 \times 4\text{ cm}^2 = \frac{3.14 \times 4}{3}\text{ cm}^2 = 4.153\text{ cm}^2$$

Let A be the area of corresponding major sector. Then,

A = Area of sector $OAQB$

$\Rightarrow$ A = Area of the circle – Area of the corresponding minor sector

$\Rightarrow$ $A = \pi r^2 - \frac{\theta}{360} \times \pi r^2$

$\Rightarrow$ $A = \pi r^2\left(1 - \frac{\theta}{360}\right)$

$\Rightarrow$ $A = 3.14 \times 4 \times 4\left(1 - \frac{30}{360}\right)\text{cm}^2$

$\Rightarrow$ $A = 3.14 \times 4 \times 4 \times \frac{11}{12}\text{ cm}^2 = \frac{3.14 \times 44}{3}\text{ cm}^2 = 46.05\text{ cm}^2$

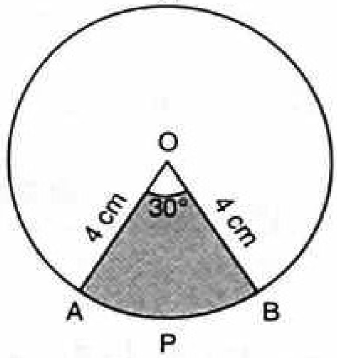

Fig. 13.11

EXAMPLE 4 *A sector is cut from a circle of radius 21 cm. The angle of the sector is 150°. Find the length of its arc and area.*

SOLUTION The arc length l and area A of a sector of angle θ in a circle of radius r are given by $l = \frac{\theta}{360} \times 2\pi r$ and $A = \frac{\theta}{360} \times \pi r^2$ respectively. Here, $r = 21$ cm and $\theta = 150$

$\therefore$ $l = \left\{\frac{150}{360} \times 2 \times \frac{22}{7} \times 21\right\}\text{cm} = 55\text{ cm}$

and, $A = \left\{\frac{150}{360} \times \frac{22}{7} \times (21)^2\right\}\text{cm}^2 = \frac{1155}{2}\text{ cm}^2 = 577.5\text{ cm}^2$

EXAMPLE 5 *The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the length of the corresponding arcs. What do you observe?* [NCERT EXEMPLAR]

SOLUTION

	Sector-I	Sector-II
Radius:	$r_1 = 7$ cm	$r_2 = 21$ cm
Sector angle:	$\theta_1 = 120°$	$\theta_2 = 40°$
Sector areas:	$A_1 = \frac{\theta_1}{360} \times \pi r_1^2$	$A_2 = \frac{\theta_2}{360} \times \pi r_2^2$

Sector arc: $l_1 = \frac{\theta_1}{360} \times 2\pi r_1$ $\qquad l_2 = \frac{\theta_2}{360} \times 2\pi r_2$

We find that

$$A_1 = \frac{\theta_1}{360} \times \pi r_1^2 = \frac{120}{360} \times \frac{22}{7} \times 7^2 \text{ cm}^2 = \frac{154}{3} \text{cm}^2$$

$$A_2 = \frac{\theta_2}{360} \times \pi r_2^2 = \frac{40}{360} \times \frac{22}{7} \times 21^2 \text{ cm}^2 = 154 \text{ cm}^2$$

$$l_1 = \frac{\theta_1}{360} \times 2\pi r_1 = \frac{120}{360} \times 2 \times \frac{22}{7} \times 7 \text{ cm}^2 = \frac{44}{3} \text{ cm}$$

$$l_2 = \frac{\theta_2}{360} \times 2\pi r_2 = \frac{40}{360} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = \frac{44}{3} \text{ cm}$$

We observe that the arc lengths of two circles of different radii may be same but areas need not be equal.

EXAMPLE 6 *A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.* [NCERT]

SOLUTION Clearly, each wiper sweeps a sector of a circle of radius 25 cm and sector angle 115°. Therefore, total area A cleaned at each sweep is given by

$$\therefore \quad A = 2 \times \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow \quad A = 2 \times \frac{115}{360} \times \frac{22}{7} \times 25 \times 25 \text{ cm}^2 = 1254.96 \text{ cm}^2$$

EXAMPLE 7 *To warm ships for underwater rocks, a light house throws a red coloured light over a sector of 80° angle to a distance of 16.5 km. Find the area of the sea over which the ships area warmed.* (*Use* $\pi = 3.14$) [NCERT]

SOLUTION We have, $r = 16.5$ km and $\theta = 80$.

Let A be the area of the sea over which the ships are warmed. Then,

$$A = \frac{\theta}{360} \times \pi r^2 = \frac{80}{360} \times 3.14 \times 16.5 \times 16.5 \text{ km}^2 = 189.97 \text{ km}^2$$

EXAMPLE 8 *In Fig. 13.12, there are shown sectors of two concentric circles of radii 7 cm and 3.5 cm. Find the area of the shaded region.* (*Use* $\pi = 22/7$).

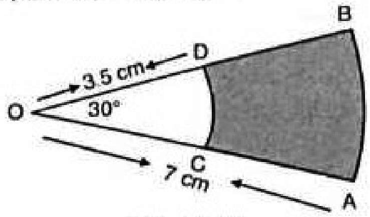

Fig. 13.12

SOLUTION Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then,

A_1 = Area of a sector of angle 30° in a circle of radius 7 cm

$$\Rightarrow \quad A_1 = \left\{\frac{30}{360} \times \frac{22}{7} \times 7^2\right\} \text{cm}^2 = \frac{77}{6} \text{cm}^2 \qquad \left[\text{Using: } A = \frac{\theta}{360} \times \pi r^2\right]$$

A_2 = Area of a sector of angle 30° in a circle of radius 3.5 cm.

$$\Rightarrow \quad A_2 = \left\{\frac{30}{360} \times \frac{22}{7} \times (3.5)^2\right\} \text{cm}^2 = \left\{\frac{1}{12} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right\} \text{cm}^2 = \frac{77}{24} \text{cm}^2$$

$\therefore$ Area of the shaded region $= A_1 - A_2$

$$= \left(\frac{77}{6} - \frac{77}{24}\right) \text{cm}^2$$

$$= \frac{77}{24} \times (4-1) \text{cm}^2 = \frac{77}{8} \text{cm}^2 = 9.625 \text{ cm}^2$$

EXAMPLE 9 *A pendulum swings through an angle of 30° and describes an arc 8.8 cm in length. Find the length of the pendulum.* [*Use* $\pi = 22/7$].

SOLUTION Here, $\theta = 30°$, l = arc = 8.8 cm

$$\therefore \quad l = \frac{\theta}{360} \times 2\pi r \Rightarrow 8.8 = \frac{30}{360} \times 2 \times \frac{22}{7} \times r \Rightarrow r = \frac{8.8 \times 6 \times 7}{22} \text{ cm} = 16.8 \text{ cm}$$

EXAMPLE 10 *The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.*

SOLUTION Let OAB be the given sector. It is given that

Perimeter of sector OAB = 16.4 cm

$\Rightarrow \quad OA + OB + \text{arc } AB = 16.4$ cm

$\Rightarrow \quad 5.2 + 5.2 + \text{arc } AB = 16.4$

$\Rightarrow \quad \text{arc } AB = 6 \text{ cm} \Rightarrow l = 6$ cm

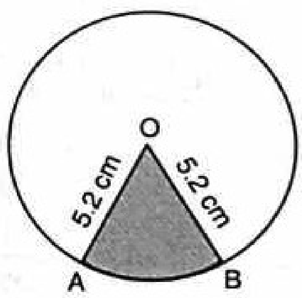

Fig. 13.13

$$\therefore \quad \text{Area of sector } OAB = \frac{1}{2} lr = \frac{1}{2} \times 6 \times 5.2 \text{ cm}^2 = 15.6 \text{ cm}^2$$

EXAMPLE 11 *An arc of a circle is of length 5π cm and the sector it bounds has an area of $20\pi \text{ cm}^2$. Find the radius of the circle.*

SOLUTION Let the radius of the circle be r cm and the arc AB of length 5π cm subtends angle θ at the centre O of the circle. Then,

Arc $AB = 5\pi$ cm and, Area of sector $OAB = 20\pi \text{ cm}^2$

$$\Rightarrow \quad \frac{\theta}{360} \times 2\pi r = 5\pi \text{ and, } \frac{\theta}{360} \times \pi r^2 = 20\pi$$

$$\Rightarrow \quad \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r} = \frac{20\pi}{5\pi}$$

$$\Rightarrow \quad \frac{r}{2} = 4$$

$$\Rightarrow \quad r = 8 \text{ cm}$$

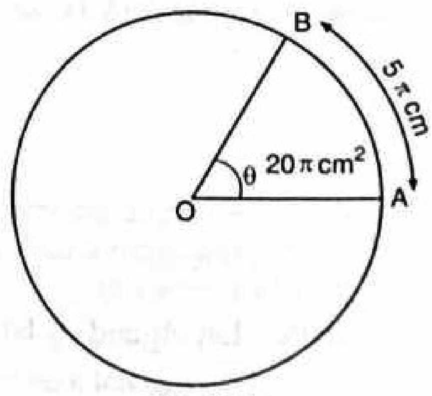

Fig. 13.14

ALITER We know that Area $= \frac{1}{2} lr \Rightarrow 20\pi = \frac{1}{2} \times 5\pi \times r \Rightarrow r = 8$ cm

EXAMPLE 12 *Area of a sector of a circle of radius 36 cm is $54\pi\ cm^2$. Find the length of the corresponding arc of the sector.*

SOLUTION Let A be the area of the sector of a circle of radius $r = 36$ cm and l be the length of the corresponding arc. Then,

$$A = \frac{1}{2} l r$$

$$\Rightarrow \quad 54\pi = \frac{1}{2} \times l \times 36 \qquad [\because A = 54\pi \text{ cm}^2 \text{ (given) and } r = 36 \text{ cm}]$$

$$\Rightarrow \quad l = 3\pi \text{ cm}$$

ALITER Let the central angle (in degrees) be θ. It is given that $r = 36$ cm and area of the sector is 54π cm^2.

$$\therefore \quad \frac{\theta}{360} \times \pi \times (36)^2 = 54\pi \qquad \left[\text{Using : Area} = \frac{\theta}{360} \times \pi r^2\right]$$

$$\Rightarrow \quad \theta = \frac{54\pi \times 360}{\pi (36)^2} = 15$$

Let l be the length of the corresponding arc. Then,

$$l = \frac{\theta}{360} \times 2\pi r \Rightarrow l = \frac{15}{360} \times 2\pi \times 36 \text{ cm} = 3\pi \text{ cm}$$

EXAMPLE 13 *A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.* **[NCERT EXEMPLAR]**

SOLUTION Let r be the radius of the circle. Here, $\theta = 60$ and $l = 20$ cm.

$$\therefore \quad l = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow \quad 20 = \frac{60}{360} \times 2\pi r$$

$$\Rightarrow \quad r = \frac{60}{\pi} \text{ cm}$$

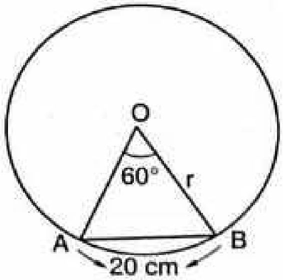

Fig. 13.15

Hence, the radius of the circle is $\frac{60}{\pi}$ cm.

EXAMPLE 14 *The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use $\pi = 22/7$)*

SOLUTION Clearly, minute hand of a clock describes a circle of radius equal to its length i.e. 14 cm. Since the minute hand rotates through 6° in one minute. Therefore, area swept by the minute hand in one minute is the area of a sector of angle 6° in a circle of radius 14 cm. Hence, required area A is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

$$\Rightarrow \quad A = \left\{\frac{6}{360} \times \frac{22}{7} \times (14)^2\right\} \text{cm}^2 = \left\{\frac{1}{60} \times \frac{22}{7} \times 14 \times 14\right\} \text{cm}^2 = \frac{154}{15} \text{cm}^2 = 10.26 \text{ cm}^2$$

EXAMPLE 15 *The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.*

SOLUTION We know that:

Angle described by the minute hand in one minute = 6°

$\therefore$ Angle described by the minute hand in 35 minutes $= (6 \times 35)^\circ = 210^\circ$

$\therefore$ Area swept by the minute hand in 35 minutes

= Area of a sector of angle 210° in a circle of radius 10 cm

$= \left\{\frac{210}{360} \times \frac{22}{7} \times (10)^2\right\} \text{cm}^2 = 183.3 \text{ cm}^2$ $\left[\text{Using: } A = \frac{\theta}{360^\circ} \times \pi r^2\right]$

EXAMPLE 16 *The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. (Take $\pi = 22/7$)*

SOLUTION In 2 days, the short hand will complete 4 rounds.

$\therefore$ Distance moved by its tip = 4 (Circumference of a circle of radius 4 cm)

$$= 4 \times \left(2 \times \frac{22}{7} \times 4\right) \text{cm} = \frac{704}{7} \text{ cm}$$

In 2 days, the long hand will complete 48 rounds.

$\therefore$ Distance moved by its tip = 48 (Circumference of a circle of radius 6 cm)

$$= 48 \times \left(2 \times \frac{22}{7} \times 6\right) \text{cm} = \frac{12672}{7} \text{ cm}$$

$\therefore$ Sum of the distances moved by the tips of two hands of the clock $= \left(\frac{704}{7} + \frac{12672}{7}\right) \text{cm}$

$= 1910.85 \text{ cm}$

LEVEL-2

EXAMPLE 17 *In a circle with centre O and radius 5 cm, AB is a chord of length $5\sqrt{3}$ cm. Find the area of sector AOB.*

SOLUTION It is given that $AB = 5\sqrt{3}$ cm.

$\Rightarrow \quad AL = BL = \frac{5\sqrt{3}}{2}$ cm

Let $\angle AOB = 2\theta$. Then, $\angle AOL = \angle BOL = \theta$.

In ΔOLA, we have

$$\sin\theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \theta = 60^\circ$

$\Rightarrow \quad \angle AOB = 120^\circ$

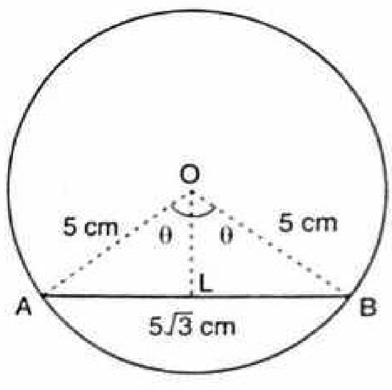

Fig. 13.16

$\therefore$ Area of sector $AOB = \frac{120}{360} \times \pi \times 5^2 \text{ cm}^2 = \frac{25\pi}{3} \text{ cm}^2$

EXAMPLE 18 *An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm. Find the area between the two consecutive ribs of the umbrella.* **[NCERT]**

SOLUTION We know that the ribs of an umbrella are equally spaced.

$\therefore$ Angle made by two consecutive ribs at the centre $= \frac{360^\circ}{8} = 45^\circ$

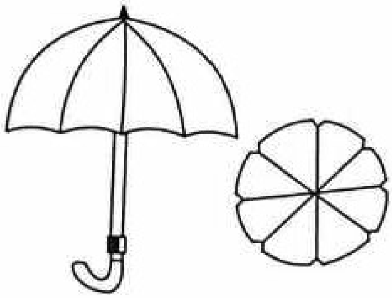

Fig. 13.17

Let A be the area between two consecutive ribs. Then,

A = Area of a sector of a circle of radius 45 cm and sector angle 45°

$$\Rightarrow \quad A = \left\{\frac{45}{360} \times \frac{22}{7} \times 45 \times 45\right\} \text{cm}^2 \qquad \left[\text{Using: Area} = \frac{\theta}{360} \times \pi r^2\right]$$

$$\Rightarrow \quad A = \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2 = 795.53 \text{ cm}^2$$

EXAMPLE 19 *A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 13.18. Find: (i) the total length of the silver wire required (ii) the area of each sector of the brooch.* **[NCERT]**

SOLUTION (i) Let l be the total length of the silver wire. Then,

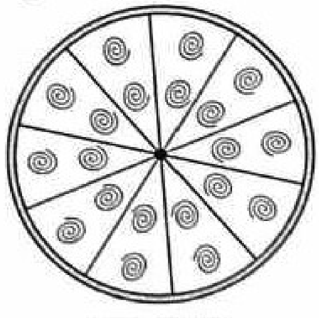

Fig. 13.18

l = Circumference of the circle of radius $\frac{35}{2}$ mm + Length of five diameters

$$\Rightarrow \quad l = 2\pi \times \frac{35}{2} + 5 \times 35 \text{ mm} = \left(2 \times \frac{22}{7} \times \frac{35}{2} + 175\right) \text{mm} = 285 \text{ mm}$$

(ii) The circle is divided into 10 equal sectors. Therefore, area A of each sector of the brooch is given by

$$A = \frac{1}{10} (\text{Area of the circle}) = \frac{1}{10} \times \pi \times \left(\frac{35}{2}\right)^2 \text{mm}^2 = \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{mm}^2 = \frac{385}{4} \text{mm}^2$$

EXAMPLE 20 *An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.* **[CBSE 2016]**

SOLUTION In Figure 13.19, let $\angle AOP = \angle BOP = \theta$. Clearly, portion AB of the belt is not in contact with the rim of the pulley in right triangle OAP, we have

$$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = 60° \Rightarrow \angle AOB = 2\theta = 120°$$

$\therefore$ Arc $AB = \frac{120° \times 2 \times \pi \times 5}{360}$ cm $= \frac{10\pi}{3}$ cm $\left[\text{Using: } l = \frac{\theta}{360} \times 2\pi r\right]$

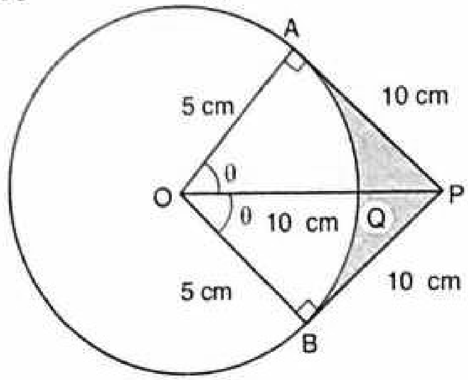

Fig. 13.19

Let l be the length of the belt that is in contact with the rim of the pulley. Then,

$$l = \text{Circumference of the rim} - \text{Length of arc } AB = 2\pi \times 5 \text{ cm} - \frac{10\pi}{3} \text{ cm} = \frac{20\pi}{3} \text{ cm}$$

Now,

$$\text{Area of sector } OAQB = \frac{120}{360} \times \pi \times 5^2 \text{ cm}^2 = \frac{25\pi}{3} \text{ cm}^2 \left[\text{Using: Area} = \frac{\theta}{360} \times \pi r^2\right]$$

Applying Pythagoras theorem in ΔOAP, we obtain

$$OP^2 = OA^2 + AP^2 \Rightarrow AP = \sqrt{OP^2 - OA^2} = \sqrt{100 - 25} = 5\sqrt{3} \text{ cm}.$$

$\therefore$ Area of quadrilateral $OAPB = 2(\text{Area of } \Delta OAP)$

$$= 2 \times \left(\frac{1}{2} \times OA \times AP\right) = 5 \times 5\sqrt{3} \text{ cm}^2 = 25\sqrt{3} \text{ cm}^2$$

Hence, Shaded area = Area of quadrilateral $OAPB$ − Area of sector $OAQB$.

$$= \left(25\sqrt{3} - \frac{25\pi}{3}\right) \text{cm}^2 = \frac{25}{3}(3\sqrt{3} - \pi) \text{ cm}^2$$

EXERCISE 13.2

LEVEL-1

1. Find, in terms of π, the length of the arc that subtends an angle of 30° at the centre of a circle of radius 4 cm.
2. Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length $(5\pi/3)$ cm.
3. An arc of length 20π cm subtends an angle of 144° at the centre of a circle. Find the radius of the circle.
4. An arc of length 15 cm subtends an angle of 45° at the centre of a circle. Find in terms of π, the radius of the circle.
5. Find the angle subtended at the centre of a circle of radius 'a' by an arc of length $(a\pi/4)$ cm.
6. A sector of a circle of radius 4 cm contains an angle of 30°. Find the area of the sector.
7. A sector of a circle of radius 8 cm contains an angle of 135°. Find the area of the sector.

8. The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector.
9. The area of a sector of a circle of radius 5 cm is 5 π cm^2. Find the angle contained by the sector.
10. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm. [NCERT EXEMPLAR]
11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and area of the sector.
12. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.
13. The perimeter of a certain sector of a circle of radius 5.6 m is 27.2 m. Find the area of the sector.
14. A sector is cut-off from a circle of radius 21 cm. The angle of the sector is 120°. Find the length of its arc and the area.
15. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.00 AM and 7.05 AM.
16. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 8 AM and 8.25 AM.
17. A sector of 56° cut out from a circle contains area 4.4 cm^2. Find the radius of the circle.
18. Area of a sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector. [NCERT EXEMPLAR]
19. The length of minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 amd and 6: 40 am. [NCERT EXEMPLAR]
20. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes. [CBSE 2013]
21. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc (ii) area of the sector formed by the arc. (Use π = 22/7) [CBSE 2013, 2017]
22. From a circular piece of carboard of radius 3 cm two sectors of 90° have been cut off. Find the perimeter of the remaining portion nearest hundredth centimeters (Take $\pi = 22/7$).
23. The area of a sector is one-twelfth that of the complete circle. Find the angle of the sector.

LEVEL-2

24. AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm. Find the area of the sector of the circle formed by chord AB.
25. In a circle of radius 6 cm, a chord of length 10 cm makes an angle of 110° at the centre of the circle. Find:
    (i) the circumference of the circle, (ii) the area of the circle,
    (iii) the length of the arc AB, (iv) the area of the sector OAB.
26. Figure 13.20, shows a sector of a circle, centre O, containing an angle $\theta°$. Prove that:

    (i) Perimeter of the shaded region is $r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$

    (ii) Area of the shaded region is $\frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$

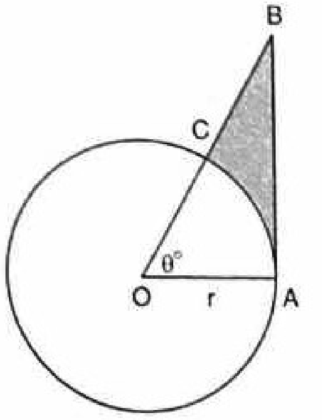

Fig. 13.20

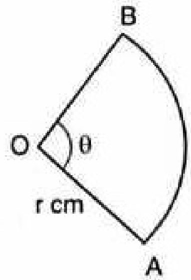

Fig. 13.21

27. Figure 13.21 shows a sector of a circle of radius r cm containing an angle $\theta°$. The area of the sector is A cm^2 and perimeter of the sector is 50 cm. Prove that

(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$ (ii) $A = 25r - r^2$

ANSWERS

1. $\frac{2\pi}{3}$ cm	2. 60°	3. 25 cm	4. $\frac{60}{\pi}$ cm
5. 45°	6. $\frac{4\pi}{3}$ cm^2	7. 24π cm^2	8. 90°
9. 72°	10. 8.7 cm^2	11. 44 cm, 770 cm^2	12. 45.03 m^2
13. 44.8 m^2	14. 44 cm, 462 cm^2		15. 5.5 cm^2
16. 130.95 cm^2	17. 3 cm	18. $\frac{220}{3}$ cm	19. $45\frac{5}{6}$ cm^2
20. 51.30 cm^2	21. (i) 22 cm, (ii) 231 cm^2		22. 9.428 cm
23. 30°	24. $\frac{8\pi}{3}$ cm^2		
25. (i) 37.68 cm	(ii) 113.1 cm^2	(iii) 11.51 cm	(iv) 34.5 cm^2

13.4 SEGMENT OF A CIRCLE AND ITS AREA

Consider a circle of radius r having centre at point O. Let PQ be a chord of the circle and let R and S be two points on it as shown in Fig. 13.22. The area enclosed by the circle is divided by the chord PQ into two segments, viz. PRQ and PSQ. Each of these two segments has an arc of the circle as a part of its boundary. Arc PRQ is the minor one and the arc PSQ is the major one.

SEGMENT OF A CIRCLE *The region enclosed by an arc and a chord is called the segment of the circle.*

In Fig. 13.22, the shaded region PRQ is a segment of the circle. The boundary of a segment consists of an arc of the circle and the chord determining the segment.

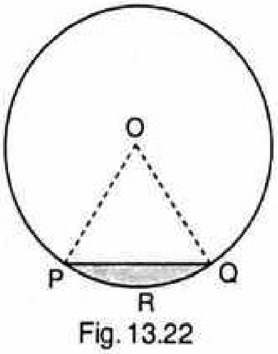

Fig. 13.22

MINOR SEGMENT *If the boundary of a segment is a minor arc of a circle, then the corresponding segment is called a minor segment.*

In Fig. 13.22, segment PQR is a minor segment.

MAJOR SEGMENT *A segment corresponding a major arc of a circle is known as the major segment.*

In Fig. 13.22, segment PQS is a major segment.

13.4.1 AREA OF A SEGMENT OF A CIRCLE

Draw a circle of radius r. Let O be the centre of the circle and PQ be a chord dividing the circle into two segments PRQ and PSQ as shown in Fig. 13.23. Suppose we wish to find the area of the minor segment PRQ (shaded region in Fig. 13.23). Let $\angle POQ = \theta$.

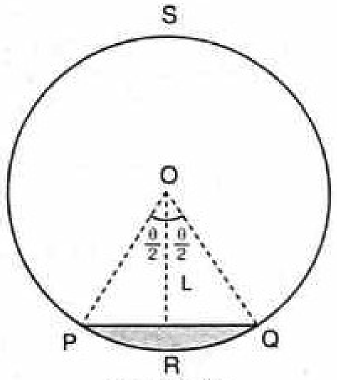

Fig. 13.23

It is evident from Fig. 13.23 that

Area of the sector $OPRQ$ = Area of the segment PRQ + Area of ΔOPQ

$\Rightarrow$ Area of the segment PRQ = Area of the sector $OPRQ$ − Area of ΔOPQ

Clearly, Area of the sector $OPRQ = \frac{\theta}{360} \times \pi r^2$

In ΔOLP, we have

$$\cos\frac{\theta}{2} = \frac{OL}{OP} \text{ and, } \sin\frac{\theta}{2} = \frac{PL}{OP}$$

$$\Rightarrow \quad OL = OP\cos\frac{\theta}{2} = r\cos\frac{\theta}{2} \text{ and, } PL = OP\sin\frac{\theta}{2} = r\sin\frac{\theta}{2}$$

$$\Rightarrow \quad OL = r\cos\frac{\theta}{2} \text{ and, } PQ = 2PL = 2r\sin\frac{\theta}{2}$$

$$\therefore \quad \Delta OPQ = \frac{1}{2}(PQ \times OL) = \frac{1}{2}\left(2r\sin\frac{\theta}{2} \times r\cos\frac{\theta}{2}\right) = r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

Hence,

$$\text{Area of segment } PRQ = \frac{\theta}{360} \times \pi r^2 - r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

<u>REMARK</u> *Area of the major segment PSQ* = πr^2 − *Area of minor segment PQR.*

<u>NOTE</u> *It should be noted that the area of the minor segment of a circle is always less than the area of its corresponding sector but the area of the major segment of a circle is greater than the area of its corresponding sector.*

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *Find the area of the segment of a circle, given that the angle of the sector is 120° and the radius of the circle is 21 cm. (Take $\pi = 22/7$)*

SOLUTION The area A of a minor segment of a circle of radius r and the corresponding sector angle θ (in degrees) is given by

$$A = \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

Here, $r = 21$ cm and $\theta = 120°$.

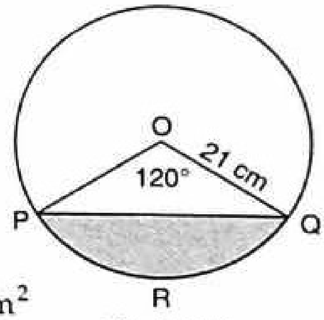

Fig. 13.24

$$\therefore \quad \text{Area of the segment} = \left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

$$= \left\{\frac{22}{7} \times \frac{120}{360} - \sin 60° \cos 60°\right\} (21)^2 \text{ cm}^2$$

$$= \left\{\frac{22}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right\} (21)^2 \text{ cm}^2$$

$$= \left\{\frac{22}{21} \times (21)^2 - (21)^2 \times \frac{\sqrt{3}}{4}\right\} \text{cm}^2$$

$$= \left(462 - \frac{441}{4}\sqrt{3}\right) \text{cm}^2 = \frac{21}{4}\left(88 - 21\sqrt{3}\right) \text{cm}^2$$

EXAMPLE 2 *A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the major and minor segments (Take $\pi = 3.14$)* **[CBSE 2016]**

SOLUTION We know that the area of a minor segment of angle θ (in degrees) in a circle of radius r is given by

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

Here, $r = 10$ and $\theta = 90°$

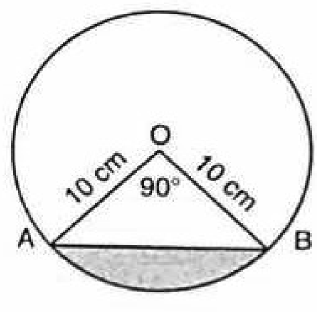

Fig. 13.25

$$\therefore \quad A = \left\{\frac{3.14 \times 90}{360} - \sin 45° \cos 45°\right\} (10)^2 \text{ cm}^2$$

$$\Rightarrow \quad A = \left\{\frac{3.14}{4} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right\} (10)^2 \text{ cm}^2$$

$$\Rightarrow \quad A = \{3.14 \times 25 - 50\} \text{ cm}^2 = (78.5 - 50) \text{ cm}^2 = 28.5 \text{ cm}^2$$

Area of the major segment = Area of the circle – Area of the minor segment

$$= \left(3.14 \times 10^2 - 28.5\right) \text{cm}^2 = (314 - 28.5) \text{ cm}^2 = 285.5 \text{ cm}^2$$

EXAMPLE 3 *A chord AB of a circle of radius 15 cm makes an angle of 60° at the centre of the circle. Find the area of the major and minor segment. (Take $\pi = 3.14$, $\sqrt{3} = 1.73$)* **[CBSE 2017]**

SOLUTION We know that the area of a minor segment of angle θ (in degrees) in a circle of radius r is given by

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

$\Rightarrow \quad A = \left\{\frac{3.14 \times 60}{360} - \sin 30^\circ \cos 30^\circ\right\} (15)^2 \text{ cm}^2$

$\Rightarrow \quad A = \left\{\frac{3.14}{6} - \frac{\sqrt{3}}{4}\right\} \times 225 \text{ cm}^2$

$\Rightarrow \quad A = (0.5233 - 0.4330)\, 225 \text{ cm}^2 = 225 \times 0.902 \text{ cm}^2 = 20.295 \text{ cm}^2$

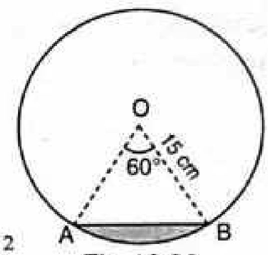

Fig. 13.26

Area of the major segment = Area of the circle – Area of the minor segment

$= \{3.14 \times (15)^2 - 20.295\} \text{ cm}^2 = \{706.5 - 20.295\} \text{ cm}^2 = 686.205 \text{ cm}^2$

EXAMPLE 4 *In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:*
(i) length of the arc *(ii) area of the sector formed by the arc* [NCERT]
(iii) area of the segment formed by the corresponding chord of the arc.

SOLUTION Let O be the centre of the circle of radius 21 cm such that an arc APB subtends 60° angle at the centre O.

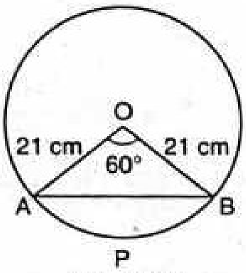

Fig. 13.27

(i) Length of the arc $APB = \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 \text{ cm} = 22 \text{ cm}$

(ii) Area of sector $OAPB = \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 231 \text{ cm}^2$

(iii) Area of the segment $APB = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$

$= \left\{\frac{22}{7} \times \frac{60}{360} - \sin 30^\circ \cos 30^\circ\right\} \times 21 \times 21 \text{ cm}^2$

$= \left\{\frac{11}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right\} \times 21 \times 21 \text{ cm}^2$

$= \left\{11 \times 21 - \frac{\sqrt{3}}{4} \times 21 \times 21\right\} \text{ cm}^2$

$= \left\{231 - \frac{441\sqrt{3}}{4}\right\} \text{ cm}^2 = (231 - 190.95) \text{ cm}^2 = 40.05 \text{ cm}^2$

EXAMPLE 5 *A chord of a circle of radius 10 cm subtends a right angle at the centre. Find:*
(i) area of the minor sector (ii) area of the minor segment
(iii) area of the major sector (iv) area of the major segment (Use $\pi = 3.14$) [CBSE 2016]

SOLUTION The area of the minor sector is a circle of radius r and sector angle θ (in degrees) is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

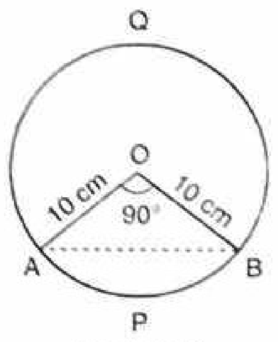

Fig. 13.28

The area of the corresponding minor segment is given by $\left\{\frac{\pi}{360} \times \theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$.

Here, $r = 10$ cm and $\theta = 90°$

(i) Area of the minor sector $OAPB = \frac{\theta}{360} \times \pi r^2 = \frac{90}{360} \times 3.14 \times 10 \times 10 \text{ cm}^2 = 78.5 \text{ cm}^2$

(ii) Let A be the area of the minor segment APB. Then,

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$

$$\Rightarrow \quad A = \left\{3.14 \times \frac{90}{360} - \sin 45° \cos 45°\right\} \times 10 \times 10 \text{ cm}^2$$

$$\Rightarrow \quad A = \left\{\frac{3.14}{4} - \frac{1}{2}\right\} \times 100 \text{ cm}^2 = \left(\frac{314}{4} - 50\right) \text{cm}^2 = (78.5 - 50) \text{ cm}^2 = 28.5 \text{ cm}^2$$

(iii) Let A_1 be the area of the major sector $OAQB$. Then,

$A_1 =$ Area of the circle – Area of the minor sector $OA\ PB$.

$$\Rightarrow \quad A_1 = (3.14 \times 10 \times 10 - 78.5) \text{ cm}^2 = (314 - 78.5) \text{ cm}^2 = 235.5 \text{ cm}^2$$

(iv) Let A_2 be the area of the major segment AQB. Then,

$A_2 =$ Area of the circle – Area of the minor segment APB

$$\Rightarrow \quad A_2 = (3.14 \times 10 \times 10 - 28.5) \text{ cm}^2 = 285.5 \text{ cm}^2$$

LEVEL-2

EXAMPLE 6 *Figure 13.29 shows two arcs, A and B. Arc A is part of the circle with centre O and radius OP. Arc B is part of the circle with centre M and radius PM, where M is the mid-point of PQ. Show that the area enclosed by the two arcs is equal to* $25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2$. [CBSE 2016]

SOLUTION Let A_1 be the area enclosed by arc B and chord PQ. Then,

$$A_1 = \text{Area of semi-circle of radius 5 cm} = \frac{1}{2} \times \pi \times 5^2 \text{ cm}^2 = \frac{25\pi}{2} \text{cm}^2$$

Let $\angle MOQ = \angle MOP = \theta$

In ΔOMP, we have

$$\sin\theta = \frac{PM}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30^\circ$$

$$\Rightarrow \quad \angle POQ = 2\theta = 60^\circ$$

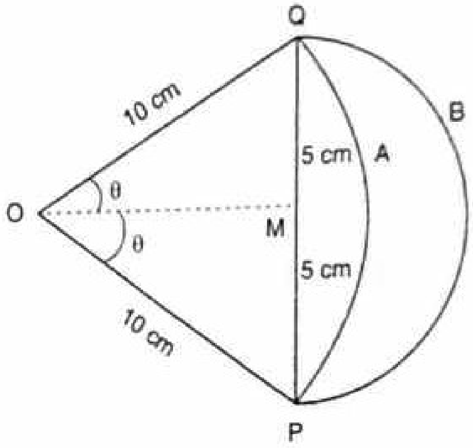

Fig. 13.29

Let A_2 be the area enclosed by arc A and chord PQ. Then,

A_2 = Area of segment of circle of radius 10 cm and sector containing angle 60°

$$\Rightarrow \quad A_2 = \left\{\frac{\pi \times 60}{360} - \sin 30^\circ \times \cos 30^\circ\right\} \times 10^2 \text{ cm}^2 \qquad \left[\because A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$$

$$\Rightarrow \quad A_2 = \left\{\frac{50\pi}{3} - 25\sqrt{3}\right\} \text{cm}^2$$

Clearly, Required area $= A_1 - A_2 = \left\{\frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)\right\} \text{cm}^2$

$$= \left\{25\sqrt{3} - \frac{25\pi}{6}\right\} \text{cm}^2 = 25\left\{\sqrt{3} - \frac{\pi}{6}\right\} \text{cm}^2$$

EXAMPLE 7 *Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre.* [NCERT EXEMPLAR]

SOLUTION Let r be the radius of the circle. Using Pythagoras theorem in ΔAOB, we obtain

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad 5^2 = r^2 + r^2$$

$$\Rightarrow \quad 2r^2 = 25 \Rightarrow r^2 = \frac{25}{2} \Rightarrow r = \frac{5}{\sqrt{2}}$$

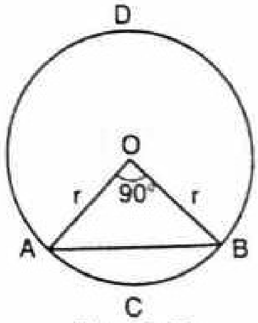

Fig. 13.30

Let A_1 and A_2 be the areas of minor segment ACB and major segment ADB respectively. Then,

$$A_1 = \left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) r^2$$

$$\Rightarrow \quad A_1 = \left(\frac{\pi}{360} \times 90 - \sin 45^\circ \cos 45^\circ\right) \times \left(\frac{5}{\sqrt{2}}\right) \qquad \left[\because \theta = 90^\circ \text{ and } r = \frac{5}{\sqrt{2}}\right]$$

$$\Rightarrow \quad A_1 = \left(\frac{\pi}{4} - \frac{1}{2}\right) \times \frac{25}{2} \text{ cm}^2 = \left(\frac{25\pi}{8} - \frac{25}{4}\right) \text{cm}^2$$

and,

$$A_2 = \text{Area of the circle} - A_1$$

$$\Rightarrow \quad A_2 = \left\{\pi \times \left(\frac{5}{\sqrt{2}}\right)^2 - \left(\frac{25\pi}{8} - \frac{25}{4}\right)\right\} \text{cm}^2 = \left(\frac{25\pi}{2} - \frac{25\pi}{8} + \frac{25}{4}\right) \text{cm}^2 = \left(\frac{75\pi}{8} + \frac{25}{4}\right) \text{cm}^2$$

$$\therefore \quad \text{Required difference} = A_2 - A_1 = \left\{\left(\frac{75\pi}{8} + \frac{25}{4}\right) - \left(\frac{25\pi}{8} - \frac{25}{4}\right)\right\} \text{cm}^2$$

$$= \left(\frac{25\pi}{4} + \frac{25}{2}\right) \text{cm}^2 = \frac{25}{4}(\pi + 2) \text{ cm}^2$$

EXERCISE 13.3

LEVEL-1

1. AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.
2. A chord PQ of length 12 cm subtends an angle of 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ. [NCERT]
3. A chord of a circle of radius 14 cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.
4. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find area of both the segments. (Take $\pi = 3.14$).
5. A chord AB of a circle, of radius 14 cm makes an angle of 60° at the centre of the circle. Find the area of the minor segment of the circle. (Use $\pi = 22/7$)
6. Find the area of the minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°. [NCERT EXEMPLAR]
7. A chord of a circle of radius 20 cm subtends an angle of 90° at the centre. Find the area of the corresponding major segment of the circle. (Use $\pi = 3.14$) [NCERT EXEMPLAR]
8. The radius of a circle with centre O is 5 cm (Fig. 13.31). Two radii OA and OB are drawn at right angles to each other. Find the areas of the segments made by the chord AB (Take $\pi = 3.14$).

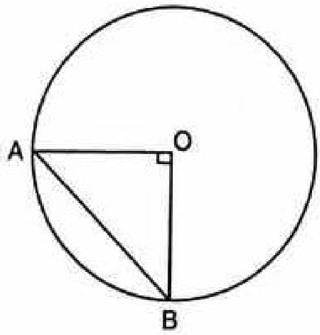

Fig. 13.31

LEVEL-2

9. AB is the diameter of a circle, centre O. C is a point on the circumference such that $\angle COB = \theta$. The area of the minor segment cut off by AC is equal to twice the area of the sector BOC. Prove that $\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120}\right)$.

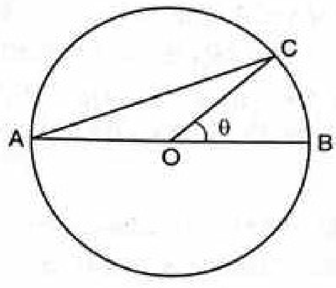

Fig. 13.32

10. A chord of a circle subtends an angle of θ at the centre of the circle. The area of the minor segment cut off by the chord is one eighth of the area of the circle. Prove that $8\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$.

ANSWERS

1. $\left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ cm^2
2. $4(4\pi - 3\sqrt{3})$ cm^2
3. 56 cm^2, 560 cm^2
4. 14.25 cm^2, 142.75 cm^2
5. 17.80 cm^2
6. $\left(\frac{308}{3} - 49\sqrt{3}\right)$ cm^2
7. 285.5 cm^2
8. 7.135 cm^2, 71.425 cm^2

13.5 AREAS OF COMBINATIONS OF PLANE FIGURES

In our daily life we come across various plane figures which are combinations of two or more plane figures. For example, window designs, flower beds, drain covers, circular paths etc. In this section, we shall discuss problems on calculating areas of such figures by using the knowledge of computing areas of different plane figures studied in earlier classes.

Following examples will illustrate the process of computing areas of plane figures which are combinations of two or more plane figures.

ILLUSTRATIVE EXAMPLES

LEVEL-1

EXAMPLE 1 *In figure 13.33, find the area of the shaded region* [Use $\pi = 3.14$]

SOLUTION Let r be the radius of the circle. Clearly, Diameter of the circle = Diagonal BD of rectangle $ABCD$.

Applying Pythagoras theorem in ΔBCD, we obtain

$BD = \sqrt{BC^2 + CD^2} = \sqrt{6^2 + 8^2}$ cm $= 10$ cm

$\therefore\ 2r = BD \Rightarrow 2r = 10 \Rightarrow r = 5$

Area of rectangle $ABCD = AB \times BC = (8 \times 6)\ \text{cm}^2 = 48\ \text{cm}^2$

Area of the circle $= \pi r^2 = 3.14 \times (5)^2\ \text{cm}^2 = 78.50\ \text{cm}^2$

$\therefore$ Area of the shaded region = Area of the circle – Area of rectangle $ABCD$

$= (78.50 - 48)\ \text{cm}^2 = 30.50\ \text{cm}^2$

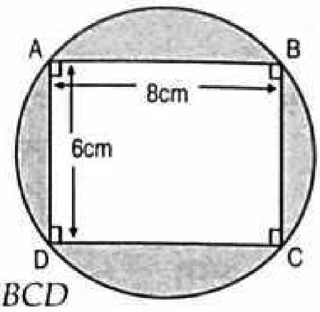

Fig. 13.33

EXAMPLE 2 *A paper is in the form of a rectangle ABCD in which AB = 20 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of a remaining part.*

SOLUTION We have,

Length of the rectangle $ABCD = AB = 20$ cm

Breadth of the rectangle $ABCD = BC = 14$ cm

$\therefore$ Area of rectangle $ABCD = (20 \times 14)\ \text{cm}^2 = 280\ \text{cm}^2$

Diameter of the semi-circle $= BC = 14$ cm

$\therefore$ Radius of the semi-circle = 7 cm

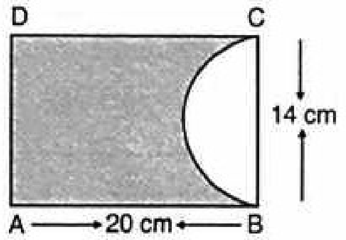

Fig. 13.34

Let A_1 be the area of the semi-circular portion cut off from the rectangle $ABCD$. Then,

$$A_1 = \frac{1}{2}(\pi r^2) = \left(\frac{1}{2} \times \frac{22}{7} \times 7^2\right) \text{cm}^2 = 77\ \text{cm}^2$$

$\therefore$ Area of the remaining part = Area of rectangle $ABCD$ – Area of semi-circle

$= (280 - 77)\ \text{cm}^2 = 203\ \text{cm}^2$

EXAMPLE 3 *Find the area of the shaded region in Fig. 13.35, if ABCD is a square of side 14 cm and APD and BPC are semi-circles.* **[NCERT]**

SOLUTION Let A be the area of the shaded region. Then,

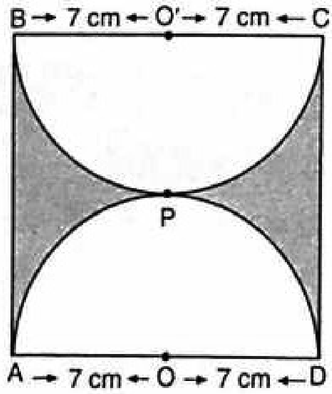

Fig. 13.35

A = Area of square $ABCD$ – Area of two semi-circles

$\Rightarrow \quad A = 14 \times 14 \text{ cm}^2 - 2\left(\frac{1}{2} \times \frac{22}{7} \times 7^2\right) \text{cm}^2 = 196 \text{ cm}^2 - 154 \text{ cm}^2 = 42 \text{ cm}^2$

EXAMPLE 4 *A horse is placed for grazing inside a rectangular field 70 m by 52 m and is tethered to one corner by a rope 21 m long. On how much area can it graze?*

SOLUTION Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius $r = 21$ m.

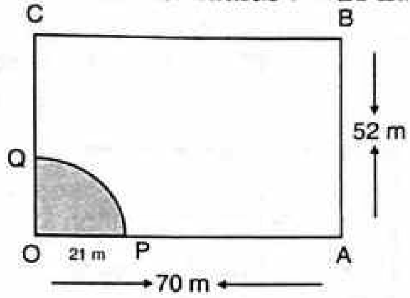

Fig. 13.36

$\therefore \quad \text{Required area} = \frac{1}{4}\pi r^2$

$\Rightarrow \quad \text{Required area} = \left\{\frac{1}{4} \times \frac{22}{7} \times (21)^2\right\} \text{cm}^2 = \frac{693}{2} \text{cm}^2 = 346.5 \text{ cm}^2$

EXAMPLE 5 *A square park has each side of 100 m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14 m as shown in Fig. 13.37. Find the area of the remaining part of the park (Use $\pi = 22/7$).*

SOLUTION Let A be the area of each quadrant of a circle of radius 14 m. Then,

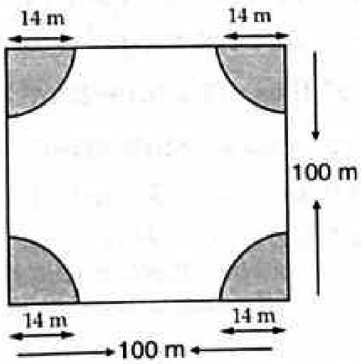

Fig. 13.37

$$A = \frac{1}{4}(\pi r^2) = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ m}^2$$

$\therefore \quad$ Area of 4 quadrants $= 4A = (4 \times 154) \text{ m}^2 = 616 \text{ m}^2$

Area of square park having side 100 m long $= (100 \times 100) \text{ m}^2 = 10{,}000 \text{ m}^2$

Hence,

Area of the remaining part of the park $= 10{,}000 - 616 = 9384 \text{ m}^2$

EXAMPLE 6 *A drain cover is made from a square metal plate of side 40 cm having 441 holes of diameter 1 cm each drilled in it. Find the area of the remaining square plate.*

SOLUTION Area of square metal plate $= 40 \times 40 \text{ cm}^2 = 1600 \text{ cm}^2$

Area of each hole $= \pi r^2 = \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \text{ cm}^2 = \frac{11}{14} \text{ cm}^2$

$\therefore$ Area of 441 holes $= 441 \times \frac{11}{14} \text{ cm}^2 = 346.5 \text{ cm}^2$

Hence, Area of the remaining square plate $= (1600 - 346.5) \text{ cm}^2 = 1253.5 \text{ cm}^2$

EXAMPLE 7 *Floor of a room is a dimensions 5m × 4m and it is covered with circular tiles of diameter 50 cm each as shown in Fig. 13. 38. Find the area of the floor that remains uncovered with tiles (Use π 3.14).* **[NCERT EXEMPLAR]**

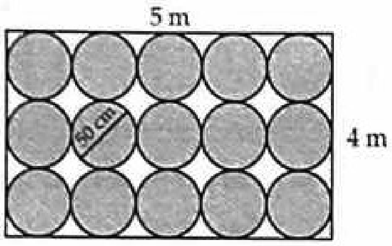

Fig. 13.38

SOLUTION Length of the room = 5 m, Breadth of the room = 4 m,

Diameter of each tile = 50 cm = 0.5 m

$\therefore$ Number of tiles along the length in a row $= \frac{5}{0.5} = 10$

Number of tiles along the breadth in a column $= \frac{4}{0.5} = 8$

$\therefore$ Total number of tiles used to cover the floor $= 10 \times 8 = 80$

Area of each tile $= \pi r^2 = 3.14 \times (0.25)^2 \text{ m}^2 = 0.19625 \text{ m}^2$

Total area covered by 80 tiles $= 80 \times 0.19625 \text{ m}^2 = 15.7 \text{ m}^2$

Area of the floor of the room $= 5 \times 4 \text{ m}^2 = 20 \text{ m}^2$

$\therefore$ Area of the uncovered floor $= (20 - 15.7) \text{ m}^2 = 4.3 \text{ m}^2$

EXAMPLE 8 *On a square cardboard sheet of area 784 cm², four circular plates of maximum size are placed such that each circular plate touches the other two plates and each side of the square sheet is tangent to circular plates. Find the area of the square sheet not covered by the circular plates.*

[NCERT EXEMPLAR]

SOLUTION Let the radius of each circular plate be r cm. Then,

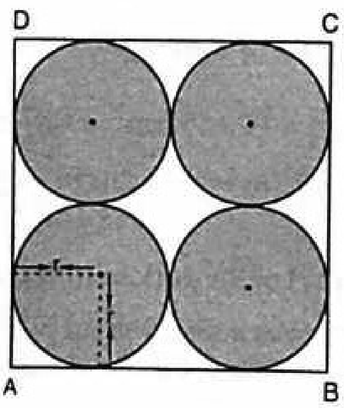

Fig. 13.39

Length of each side of the square sheet = $4r$ cm.

$\therefore$ Area of the square cardboard sheet = $(4r \times 4r)\ cm^2 = 16r^2\ cm^2$

But, the area of the cardboard sheet is given to be $784\ cm^2$

$\therefore$ $16r^2 = 784 \Rightarrow r^2 = 49 \Rightarrow r = 7$

Area of one circular plate $= \pi r^2 = \frac{22}{7} \times 7^2\ cm^2 = 154\ cm^2$

$\therefore$ Area of four circular plates $= 4 \times 154\ cm^2 = 616\ cm^2$

$\therefore$ Uncovered area of the square sheet $= (784 - 616)\ cm^2 = 168\ cm^2$

EXAMPLE 9 *On a square handkerchief, nine circular designs each of radius 7 cm are made. Find the area of the remaining portion of the handkerchief See (Fig. 13.40).* **[NCERT]**

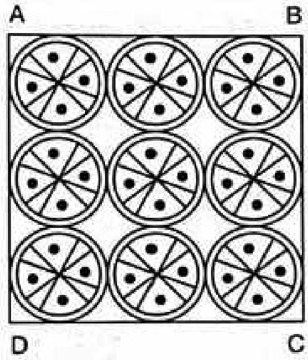

Fig. 13.40

SOLUTION Radius of each circle = 7 cm

$\therefore$ Diameter of each circle = 14 cm.

Length of each side of the square = 14 cm + 14 cm + 14 cm = 42 cm

So, area of the handkerchief $= 42 \times 42\ cm^2 = 1764\ cm^2$

Area of 9 circles each of 7 cm radius $= (9 \times \pi \times 7^2)\ cm^2 = \left(9 \times \frac{22}{7} \times 7^2\right) cm^2$

$= 1386\ cm^2$

Hence, Area of the remaining portion of handkerchief $= 1764\ cm^2 - 1386\ cm^2 = 378\ cm^2$

EXAMPLE 10 *Four equal circles are described about the four corners of a square so that each touches two of the others as shown in Fig. 13.41. Find the area of the shaded region, each side of the square measuring 14 cm.* **[NCERT]**

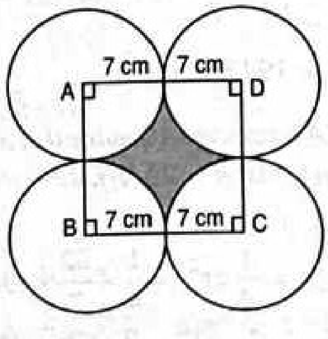

Fig. 13.41

SOLUTION Let $ABCD$ be the given square each side of which is 14 cm long. Clearly, the radius of each circle is 7 cm.

Area of the square of side 14 cm long $= (14 \times 14)\ cm^2 = 196\ cm^2$

Area of each quadrant of a circle of radius 7 cm $= \frac{1}{4}(\pi r^2)$

$$= \left\{\frac{1}{4} \times \frac{22}{7} \times (7)^2\right\} cm^2 = 38.5\ cm^2$$

$\therefore$ Area of 4 quadrants $= 4 \times 38.5\ cm^2 = 154\ cm^2$

Hence,

Area of the shaded region = Area of the square $ABCD$ − Area of 4 quadrants

$$= (196 - 154)\ cm^2 = 42\ cm^2$$

EXAMPLE 11 *ABCD is a flower bed. If OA = 21 m and OC = 14 m, find the area of the bed. (Take π = 22/7).* [NCERT]

SOLUTION We have, $OA = R = 21$ m and $OC = r = 14$ m

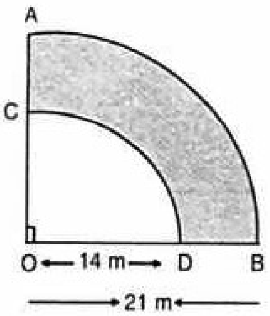

Fig. 13.42

$\therefore$ Area of the flower bed = Area of a quadrant of a circle of radius R − Area of a quadrant of a circle of radius r

$$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$$

$$= \frac{\pi}{4}(R^2 - r^2)$$

$$= \frac{1}{4} \times \frac{22}{7}(21^2 - 14^2)\ cm^2 \quad [\because R = 21 \text{ m and } r = 14 \text{ m}]$$

$$= \left\{\frac{1}{4} \times \frac{22}{7} \times (21+14)(21-14)\right\} m^2 = \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\} m^2$$

$$= 192.5\ m^2$$

EXAMPLE 12 *In Fig. 13.43, AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion (Take π = 22/7).* [CBSE 2014, 2017, NCERT]

SOLUTION We find that:

Area of quadrant $AOBCA = \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{27}{8}\ cm^2 = 9.625\ cm^2$$

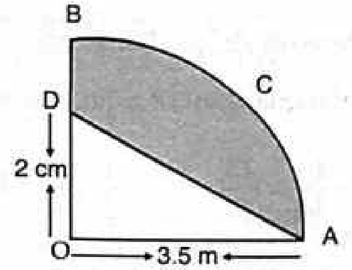

Fig. 13.43

Area of $\Delta AOD = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}(OA \times OB) = \frac{1}{2}(3.5 \times 2) \text{ cm}^2 = 3.5 \text{ cm}^2$

Hence,

Area of the shaded portion = Area of quadrant − Area of ΔAOD

$= (9.625 - 3.5) \text{ cm}^2 = 6.125 \text{ cm}^2$

EXAMPLE 13 *A circular grassy plot of land, 42 m in diameter, has a path 3.5 m wide running round it on the outside. Find the cost of gravelling the path at ₹ 4 per square metre.*

SOLUTION Radius of the plot = 21 m.

Radius of the plot including the path = (21 + 3.5) m = 24.5 m

$\therefore$ Area of the path $= \{\pi(24.5)^2 - \pi(21)^2\} \text{ m}^2$

$= \pi\{(24.5)^2 - (21)^2\} \text{ m}^2$

$= \pi\{(24.5 + 21)(24.5 - 21)\} \text{ m}^2$

$= \{\pi(45.5) \times (3.5)\} \text{ m}^2$

$= \frac{22}{7} \times 45.5 \times 3.5 \text{ m}^2 = 500.5 \text{ m}^2$

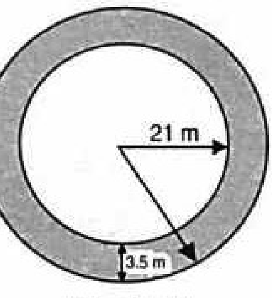

Fig. 13.44

Hence, cost of gravelling the path = ₹ (500.5 × 4) = ₹ 2002.

EXAMPLE 14 *ABCP is a quadrant of a circle of radius 14 cm. With AC as diameter, a semi-circle is drawn. Find the area of the shaded portion.* **[NCERT, CBSE 2008, 2014]**

SOLUTION Applying Pythagoras theorem in the right-angled triangle *ABC*, we obtain

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 14^2 + 14^2 = 2 \times 14^2$

$\Rightarrow AC = \sqrt{2 \times 14^2} = 14\sqrt{2} \text{ cm}$

$\Rightarrow \frac{1}{2}AC = \frac{14\sqrt{2}}{2} \text{ cm} = 7\sqrt{2} \text{ cm}$

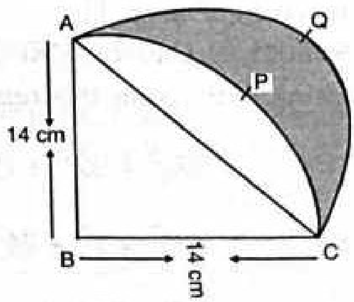

Fig. 13.45

Let *A* be the area of the shaded portion. Then,

$A = \text{Area } APCQA$

$\Rightarrow A = \text{Area } ACQA - \text{Area } ACPA$

$\Rightarrow A = \text{Area } ACQA - (\text{Area } ABCPA - \text{Area of } \Delta ABC)$

$\Rightarrow \quad A = $ (Area of sem-circle with AC as diameter)

–[Area of a quadrant of a circle with AB as radius– Area of $\Delta\ ABC$]

$$\Rightarrow \quad A = \left[\frac{1}{2}\left\{\frac{22}{7}\times(7\sqrt{2})^2\right\} - \left\{\frac{1}{4}\times\frac{22}{7}\times 14^2 - \frac{1}{2}\times 14\times 14\right\}\right]$$

$$\Rightarrow \quad A = \left\{\frac{1}{2}\times\frac{22}{7}\times 49\times 2 - \frac{1}{4}\times\frac{22}{7}\times 14\times 14 + \frac{1}{2}\times 14\times 14\right\}\text{cm}^2$$

$$\Rightarrow \quad A = (154 - 154 + 98)\ \text{cm}^2 = 98\ \text{cm}^2$$

EXAMPLE 15 *The inner and outer diameters of ring I of a dartboard are 32 cm and 34 cm respectively and those of rings II are 19 cm and 21 cm respectively. What is the total area of these two rings?*

SOLUTION We find that:

$$A_1 = \text{Area of ring I} = (\pi\times 17^2 - \pi\times 16^2)\ \text{cm}^2$$

$$= \frac{22}{7}\times(17^2 - 16^2)\ \text{cm}^2$$

$$= \frac{22}{7}\times(17+16)(17-16)\ \text{cm}^2$$

$$= \frac{22}{7}\times 33\ \text{cm}^2$$

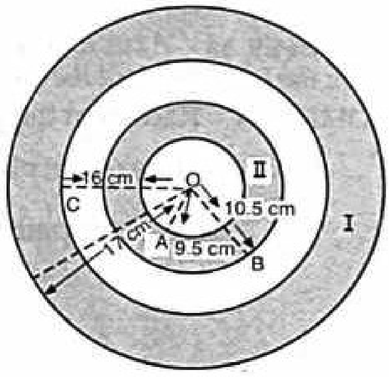

Fig. 13.46

$$A_2 = \text{Area of ring II} = (\pi\times 10.5^2 - \pi\times 9.5^2)\ \text{cm}^2$$

$$= \pi(10.5^2 - 9.5^2)\ \text{cm}^2 = \frac{22}{7}\times(10.5+9.5)(10.5-9.5)\ \text{cm}^2 = \frac{22}{7}\times 20\ \text{cm}^2$$

Hence,

$$\text{Total area of two rings} = A_1 + A_2 = \frac{22}{7}\times 33 + \frac{22}{7}\times 20\ \text{cm}^2$$

$$= \frac{22}{7}\times(33+20)\ \text{cm}^2 = 166.57\ \text{cm}^2$$

EXAMPLE 16 *Find the area of the shaded region in Fig. 13.47, if $PQ = 24$ cm $PR = 7$ cm and O is the centre of the circle.* **[NCERT, CBSE 2009]**

SOLUTION Clearly, $\angle RPQ$ is the angle in a semi-circle. Therefore, it is a right angle. Using Pythagoras theorem in $\Delta\ RPQ$, we obtain

$$RQ^2 = RP^2 + PQ^2$$

$$\Rightarrow \quad RQ^2 = 7^2 + 24^2 = 625$$

$$\Rightarrow \quad RQ = 25\ \text{cm}$$

$$\therefore \quad \text{Radius of the circle} = \frac{1}{2}RQ = \frac{25}{2}\ \text{cm}$$

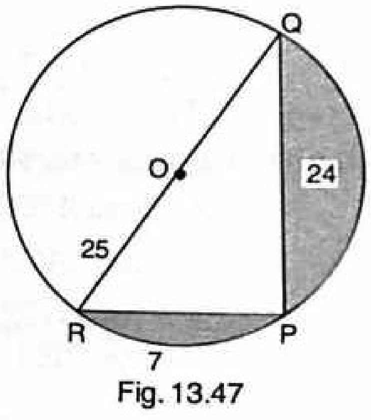

Fig. 13.47

Let A be the area of the shaded region. Then,

$$A = \text{Area of the semi-circle} - \text{Area of } \Delta RPQ.$$

$$\Rightarrow \quad A = \frac{1}{2}\pi r^2 - \frac{1}{2} \times PR \times PQ$$

$$\Rightarrow \quad A = \left\{\frac{1}{2}\times\frac{22}{7}\times\left(\frac{25}{2}\right)^2 - \frac{1}{2}\times 7 \times 24\right\}\text{cm}^2 = \left\{\frac{6875}{28} - 84\right\}\text{cm}^2 = \frac{4523}{28}\text{cm}^2$$

EXAMPLE 17 *Find the area of the shaded region in Fig. 13.48, where radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.* **[CBSE 2014, NCERT]**

SOLUTION We have,

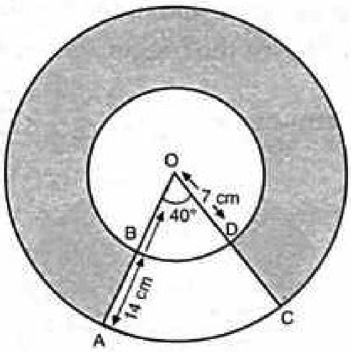

Fig. 13.48

$$\text{Area of the region } ABDC = \text{Area of sector } AOC - \text{Area of sector } BOD$$

$$= \left(\frac{40}{360}\times\frac{22}{7}\times 14\times 14 - \frac{40}{360}\times\frac{22}{7}\times 7\times 7\right)\text{cm}^2$$

$$= \left(\frac{1}{9}\times 22\times 14\times 2 - \frac{1}{9}\times 22\times 7\times 1\right)\text{cm}^2$$

$$= \frac{22}{9}\times(28-7)\text{ cm}^2 = \frac{154}{3}\text{ cm}^2$$

$$\text{Area of the circular ring} = \left(\frac{22}{7}\times 14\times 14 - \frac{22}{7}\times 7\times 7\right)\text{cm}^2$$

$$= (22\times14\times2 - 22\times7\times1)\text{ cm}^2 = 22\times 21\text{ cm}^2 = 462\text{ cm}^2$$

Hence, Required shaded area $= \left(462 - \frac{154}{3}\right)\text{cm}^2 = \frac{1232}{3}\text{ cm}^2 = 410.67\text{ cm}^2$

EXAMPLE 18 *AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30°$, find the area of the shaded region.* **[NCERT, CBSE 2012]**

SOLUTION Let A be the area oif the shaded region. Then,

$$A = \text{Area of sector } OAB - \text{Area of sector } OCD$$

$$\Rightarrow \quad A = \left(\frac{30}{360}\times\frac{22}{7}\times 21\times 21 - \frac{30}{360}\times\frac{22}{7}\times 7\times 7\right)\text{cm}^2$$

$$\Rightarrow \quad A = \frac{30}{360}\times\frac{22}{7}\times(21\times 21 - 7\times 7)\text{ cm}^2$$

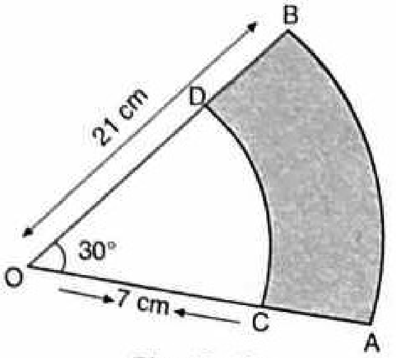

Fig. 13.49

$$\Rightarrow \quad A = \frac{11}{42} \times (21+7) \times (21-7) \text{ cm}^2 = \frac{11}{42} \times 28 \times 14 \text{ cm}^2 = 102.67 \text{ cm}^2$$

EXAMPLE 19 *Find the areas of the shaded region in the Fig. 13.50.*

SOLUTION It is given that the radius of the bigger semi-circle is $r = 14$ cm

$$\therefore \quad A_1 = \text{area of the bigger semi-circle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times (14)^2 \text{ cm}^2 = 308 \text{ cm}^2$$

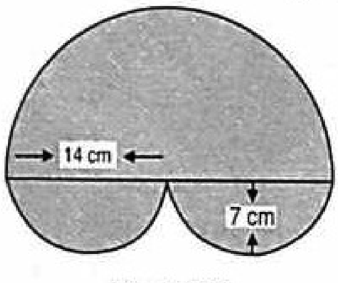

Fig. 13.50

Radius of each of the smaller circle is $r_1 = 7$ cm

$$\therefore \quad A_2 = \text{Area of 2 smaller semi-circles} = 2\left(\frac{1}{2}\pi r_1^2\right) = 2\left(\frac{1}{2} \times \frac{22}{7} \times 7^2\right) \text{cm}^2 = 154 \text{ cm}^2$$

Hence, required area $= A_1 + A_2 = (308 + 154) \text{ cm}^2 = 462 \text{ cm}^2$

EXAMPLE 20 *In Fig. 13.51, ABCD is a square of side 10 cm. Semi-circles are drawn with each side of square as diameter. Find the area of (i) the unshaded region (ii) the shaded region*

[NCERT, CBSE 2016]

SOLUTION Let us mark the four unshaded regions as R_1, R_2, R_3 and R_4.

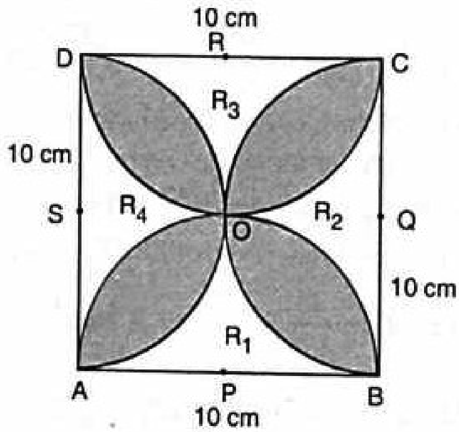

Fig. 13.51

Clealry,

Area of R_1 + Area of R_3

= Area of square $ABCD$ – Area of two semi-circles having centres at Q and S

$= \left(10 \times 10 - 2 \times \frac{1}{2} \times 3.14 \times 5^2\right) \text{cm}^2$ [∵ Radius = AP = 5 cm]

$= (100 - 3.14 \times 25) \text{ cm}^2 = (100 - 78.5) \text{ cm}^2 = 21.5 \text{ cm}^2$

Similarly, we have

Area of R_2 + Area of R_4 = 21.5 cm²

(i) Area of the unshaded region = Area R_1 + Area R_2 + Area R_3 + Area R_4

= (Area R_1 + Area R_3) + (Area R_2 + Area R_4)

$= 2(21.5) \text{ cm}^2 = 43 \text{ cm}^2$

(ii) Area of the shaded region

= Area of square $ABCD$ – (Area of R_1 + Area of R_2 + Area of R_3 + Area of R_4)

$= (100 - 2 \times 21.5) \text{ cm}^2 = 57 \text{ cm}^2$

EXAMPLE 21 *In Fig. 13.52, ABC is an equilateral triangle inscribed in a circle of radius 4 cm with centre O. Find the area of the shaded region.*

SOLUTION Let h be the height of ΔABC and R be the radius of the circumcircle. Then,

$$OA = \frac{2}{3} AD \text{ and } h = \frac{\sqrt{3}}{2} a$$

$$\Rightarrow \quad R = \frac{2}{3} h \text{ and } h = \frac{\sqrt{3}a}{2} \Rightarrow R = \frac{a}{\sqrt{3}} \Rightarrow a = R\sqrt{3} \Rightarrow a = 4\sqrt{3}$$

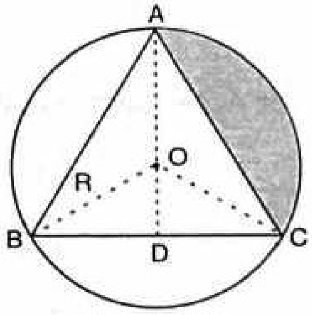

Fig. 13.52

Let the length of each side of equilateral ΔABC be a.

We find that : $\angle AOC = 2\angle ABC = 2 \times 60° = 120°$

∴ Required area $= \frac{1}{3}$(Area of the circle – Area of ΔABC)

$$= \frac{1}{3}\left\{\pi R^2 - \frac{\sqrt{3}}{4} \times (\text{Side})^2\right\}$$

$$= \frac{1}{3}\left\{16\pi - \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2\right\} \text{cm}^2$$

$$= \frac{1}{3}(16\pi - 12\sqrt{3}) \text{ cm}^2 = \frac{4}{3}(4\pi - 3\sqrt{3}) \text{ cm}^2$$

EXAMPLE 22 *PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. 13.53. Find the perimeter and area of the shaded region.*

SOLUTION PS = Diameter of a circle of radius 6 cm = 12 cm

$$\therefore \quad PQ = QR = RS = \frac{12}{3} = 4 \text{ cm}, \quad QS = QR + RS = (4 + 4) \text{ cm} = 8 \text{ cm}$$

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm

+ Arc of semi-circle of radius 2 cm

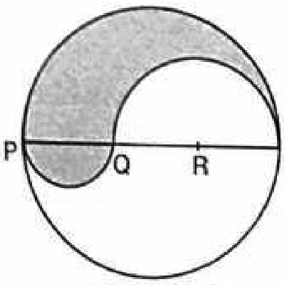

Fig. 13.53

$\Rightarrow \quad P = (\pi \times 6 + \pi \times 4 + \pi \times 2)\text{ cm} = 12\pi \text{ cm}$

and,

A = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter – Area of semi-circle with QS as diameter.

$$\Rightarrow \quad A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$$

$$\Rightarrow \quad A = \frac{1}{2} \times \frac{22}{7}(6^2 + 2^2 - 4^2) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7}\text{ cm}^2 = 37.71\text{ cm}^2$$

EXAMPLE 23 *Find to the three places of decimals the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use $\pi = 22/7$).*

SOLUTION For the first triangle, we have $a = 35$, $b = 53$ and $c = 66$.

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{ cm}$$

Let Δ_1 be the area of the first triangle. Then,

$$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$$

$$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} = \sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924\text{ cm}^2 \quad \ldots\text{(i)}$$

For the second triangle, we have $a = 33$, $b = 56$, $c = 65$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{ cm}$$

Let Δ_2 be the area of the second triangle. Then,

$$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$$

$$\Rightarrow \quad \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12} = \sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4} = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$$

$$\Rightarrow \quad \Delta_2 = 7 \times 11 \times 4 \times 3 = 924\text{ cm}^2 \quad \ldots\text{(ii)}$$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$ [Using: (i) & (ii)]

$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}$ cm

EXAMPLE 24 *In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. Find the area of the remaining portion of the triangle* (*Take* $\sqrt{3} = 1.732$).

SOLUTION Let ABC be an equilateral triangle of side 24 cm, and let AD be perpendicular from A on BC. Since the triangle is equilateral, so D bisects BC.

$\therefore \quad BD = CD = 12$ cm

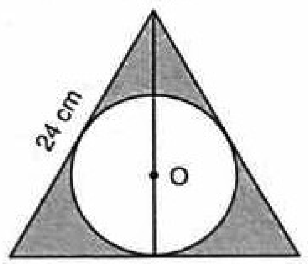

Fig. 13.54

The centre of the inscribed circle will coincide with the centroid of ΔABC.

$\therefore \quad OD = \frac{1}{3} AD$

In ΔABD, we have

$AB^2 = AD^2 + BD^2$ [Using Pythagoras Theorem]

$\Rightarrow \quad 24^2 = AD^2 + 12^2$

$\Rightarrow \quad AD = \sqrt{24^2 - 12^2} = \sqrt{(24-12)(24+12)} = \sqrt{36 \times 12} = 12\sqrt{3}$ cm

$\therefore \quad OD = \frac{1}{3} AD = \left(\frac{1}{3} \times 12\sqrt{3}\right)$ cm $= 4\sqrt{3}$ cm

Now, Area of the incircle $= \pi (OD)^2 = \left\{\frac{22}{7} \times (4\sqrt{3})^2\right\}$ cm^2 $= \left\{\frac{22}{7} \times 48\right\}$ cm^2 $= 150.85$ cm^2

and, Area of the triangle $ABC = \frac{\sqrt{3}}{4}(\text{Side})^2 = \frac{\sqrt{3}}{4}(24)^2 = 249.4$ cm^2

$\therefore$ Area of the remaining portion of the triangle $= (249.4 - 150.85)$ cm^2 $= 98.55$ cm^2

EXAMPLE 25 *Find the area of the shaded region in Fig. 13.55, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.* **[NCERT]**

SOLUTION Let A be the area of the shaded region. Then,

A = Area of ΔOAB + Area of the circle − Area of a sector of a circle of radius 6 cm and of angle 60°

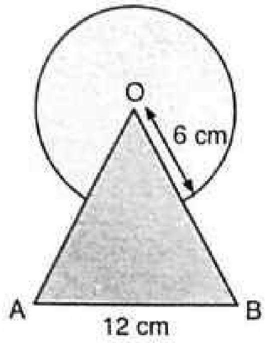

Fig. 13.55

$$\Rightarrow \quad A = \left\{ \frac{\sqrt{3}}{4} \times 12^2 + \pi \times 6^2 - \frac{60}{360} \times \pi \times 6^2 \right\} \text{cm}^2$$

$$\Rightarrow \quad A = \left(36\sqrt{3} + 36\pi - 6\pi \right) \text{cm}^2 = \left(36\sqrt{3} + 30 \times \frac{22}{7} \right) \text{cm}^2 = \left(\frac{660}{7} + 36\sqrt{3} \right) \text{cm}^2$$

EXAMPLE 26 *The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking eqch angular point as centre, a circle is described with radius equal to half the length of the side of the triangle as shown in Fig. 13.56. Find the area of the triangle not included in the circle.* **[CBSE 2009]**

SOLUTION Let each side of the triangle be a cm. Then,

Area of $\Delta\, ABC = 49\sqrt{3}$ cm^2

$$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 = 49\sqrt{3} \qquad \left[\because \text{ Area} = \frac{\sqrt{3}}{4} (\text{Side})^2 \right]$$

$$\Rightarrow \quad a^2 = 49 \times 4 \Rightarrow a = 14 \text{ cm}$$

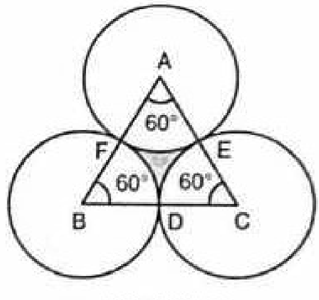

Fig. 13.56

Thus, the radius of each circle is $r = 7$ cm

Let A be the required area. Then,

A = Area of $\Delta\, ABC - 3 \times$ (Area of a sector of angle 60° in a circle of radius 7 cm)

$$\Rightarrow \quad A = \left\{ 49\sqrt{3} - 3\left(\frac{60}{360} \times \frac{22}{7} \times 7^2 \right) \right\} \text{cm}^2 = \left(49\sqrt{3} - 77 \right) \text{cm}^2 = \left(49 \times 1.73 - 77 \right) \text{cm}^2 = 7.77 \text{ cm}^2$$

EXAMPLE 27 *The area of an equilateral triangle is 1732.05 cm^2. About each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles. (Use $\pi = 3.14$).* **[NCERT]**

SOLUTION Let each side of the equilateral triangle be a cm. It is given that its area is 1732.05 cm^2

$$\therefore \quad \frac{\sqrt{3}}{4} a^2 = 1732.05$$

$$\Rightarrow \quad \frac{a^2}{4} = \frac{1732.05}{\sqrt{3}} \Rightarrow \left(\frac{a}{2} \right)^2 = \frac{1732.05}{\sqrt{3}} \qquad \ldots \text{(i)}$$

Clearly, radius of each circle is $\frac{a}{2}$ cm.

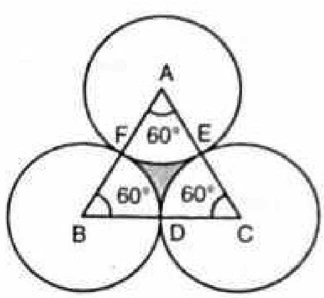

Fig. 13.57

Let A be the area of three sectors each of angle 60° in a circle of radius $\frac{a}{2}$ cm. Then,

$$A = 3\left\{\frac{60}{360}\times 3.14\times\left(\frac{a}{2}\right)^2\right\}\text{cm}^2 = \frac{1}{2}\times 3.14\times\left(\frac{a}{2}\right)^2 = \frac{1}{2}\times 3.14\times\frac{1732.05}{\sqrt{3}} = 1570.05\ \text{cm}^2$$

Let A_1 be the required area. Then,

$$A = \text{Area of } \Delta ABC - A_1$$

$\Rightarrow$ $$A = (1732.05 - 1570.04)\ \text{cm}^2 = 162.01\ \text{cm}^2$$

EXAMPLE 28 *An athletic track 14 m wide consists of two straight sections 120 m long joining semi-circular ends whose inner radius is 35 m. Calculate the area of the shaded region.*

SOLUTION Let A be the area of the shaded region.

We have, $OB = O'C = 35$ m and $AB = CD = 14$ m

$\therefore$ $$OA = O'D = (35 + 14)\ \text{m} = 49\ \text{m}$$

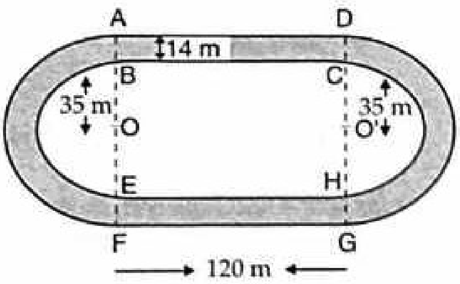

Fig. 13.58

The area A of the shaded region is given by

A = Area of rectangle $ABCD$ + Area of rectangle $EFGH$ + 2 {Area of the semi-circle with radius 49 m} − {Area of the semi-circle with radius 35 m}

$$A = (14\times 120) + (14\times 120) + 2\left\{\frac{1}{2}\times\frac{22}{7}\times(49)^2\right\} - 2\left\{\frac{1}{2}\times\frac{22}{7}\times(35)^2\right\}$$

$\Rightarrow$ $$A = \left\{1680 + 1680 + \frac{22}{7}(49^2 - 35^2)\right\}\text{m}^2 = \left\{3360 + \frac{22}{7}(49+35)(49-35)\right\}\text{m}^2$$

$\Rightarrow$ $$A = \left\{3360 + \frac{22}{7}\times 84\times 14\right\}\text{m}^2 = \{3360 + 44\times 84\}\ \text{m}^2 = 7056\ \text{m}^2$$

Hence, the area of the shaded region is 7056 m^2

EXAMPLE 29 *It is proposed to add to a square lawn measuring 58 cm on a side, two circular ends. The centre of each circle being the point of intersection of the diagonals of the square. Find the area of the whole lawn.*

SOLUTION The length of each side of a square lawn is 58 cm.

$\therefore$ Length of the diagonal of the square $= \sqrt{58^2 + 58^2} = 58\sqrt{2}$ cm

So, radius of the circle having centre at the point of intersection of diagonals is $29\sqrt{2}$ cm.

Let A be the area of one of the circular ends. Then,

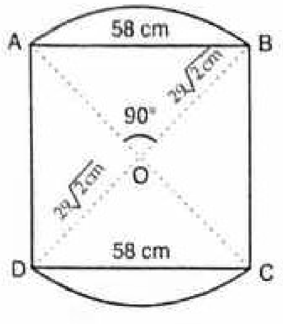

Fig. 13.59

A = Area of a segment of angle 90° in a circle of radius $29\sqrt{2}$ cm

$\Rightarrow \quad A = \left\{\frac{22}{7} \times \frac{90}{360} - \sin 45° \cos 45°\right\} \times (29\sqrt{2})^2 \text{ cm}^2 \quad \left[\because \text{Area} = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2\right]$

$\Rightarrow \quad A = \left(\frac{11}{14} - \frac{1}{2}\right) \times 29 \times 29 \times 2 \text{ cm}^2 = 29 \times 29 \times 2 \times \frac{4}{14} \text{ cm}^2 = \frac{3364}{7} \text{ cm}^2$

$\therefore$ Area of the whole lawn = Area of the square + 2 (Area of a circular end)

$$= \left\{58 \times 58 + 2 \times \frac{3364}{7}\right\} \text{cm}^2 = \left\{3364 + 2 \times \frac{3364}{7}\right\} \text{cm}^2$$

$$= 3364\left(1 + \frac{2}{7}\right) \text{cm}^2 = 3364 \times \frac{9}{7} \text{cm}^2 = 4325.14 \text{ cm}^2$$

EXAMPLE 30 *In Fig. 13.60, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection of the diagonals of the square lawn, find the sum of the areas of the lawns and the flower beds.* **[CBSE 2014]**

SOLUTION Using Pythagoras theorem in ΔABD, we obtain

$$BD^2 = AB^2 + AD^2 = 56^2 + 56^2 = 2 \times (56)^2$$

$\Rightarrow \quad BD = 56\sqrt{2}$

$\therefore \quad AC = BD = 56\sqrt{2}$ m

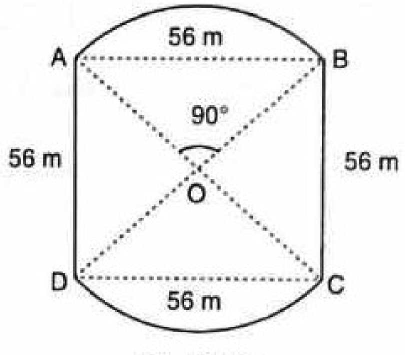

Fig. 13.60

$\therefore \quad OA = OB = \frac{1}{2} AC = 28\sqrt{2}$ m

So, the radius of the circle having centre at the point of intersection of diagonals is $28\sqrt{2}$ m. Let A be the area of one of the circular ends. Then,

A = Area of a segment of angle 90° in a circle of radius $28\sqrt{2}$ m.

$\Rightarrow \quad A = \left\{\frac{22}{7} \times \frac{90}{360} - \sin 45^\circ \cos 45^\circ\right\} \times (28\sqrt{2})^2 \text{ m}^2 \quad \left[\begin{array}{l}\text{Substituting } r = 28\sqrt{2} \text{ and } \theta = 90^\circ \\ \text{in } A = \left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)r^2\end{array}\right]$

$\Rightarrow \quad A = \left\{\frac{11}{14} - \frac{1}{2}\right\} \times 28 \times 28 \times 2 \text{ m}^2 = 28 \times 28 \times 2 \times \frac{4}{14} \text{ cm}^2 = 448 \text{ m}^2$

$\therefore$ Area of two flower beds $2A = 2 \times 448 \text{ m}^2 = 896 \text{ m}^2$

Area of the square lawn $= 56 \times 56 \text{ m}^2 = 3136 \text{ m}^2$

Hence, Total area $= (3136 + 896) \text{ m}^2 = 4032 \text{ m}^2$

EXAMPLE 31 *A round table cover has six equal designs as shown in Fig. 13.61. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹3.50 per cm². (Use $\sqrt{3} = 1.7$)* **[NCERT]**

SOLUTION We observe that the designs form six segments of a circle of radius $r = 28$ cm and each of angle $\theta = 60^\circ$.

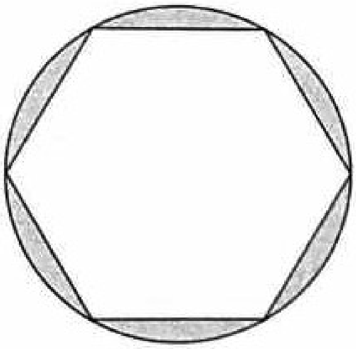

Fig. 13.61

Let A be the area of the six designs. Then,

$$A = 6\left\{\frac{\theta}{360} \times \pi r^2 - \sin\frac{\theta}{2}\cos\frac{\theta}{2} r^2\right\} \text{cm}^2$$

$\Rightarrow \quad A = 6\left\{\frac{60}{360} \times \frac{22}{7} \times (28)^2 - \sin 30^\circ \cos 30^\circ \times (28)^2\right\} \text{cm}^2 \; [\because r = 28 \text{ cm and } \theta = 60^\circ]$

$\Rightarrow \quad A = 6\left\{\frac{1}{6} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 28 \times 28\right\} \text{cm}^2$

$\Rightarrow \quad A = (88 \times 28 - 6 \times \sqrt{3} \times 7 \times 28) \text{ cm}^2 = (2464 - 1999.2) \text{ cm}^2 = 464.8 \text{ cm}^2$

Hence, Cost of making the designs at the rate of ₹ 3.50 per cm² = ₹ 464.8 × 3.50 = ₹ 1626.80

EXAMPLE 32 *In Figure 13.62, ABC is a right angled triangle at A. Find the area of the shaded region, if AB = 6 cm, BC = 10 cm and I is the centre of incircle of ΔABC.* **[CBSE 2009]**

SOLUTION Applying Pythagoras theorem in ΔABC, we obtain

$$BC^2 = AB^2 + AC^2$$

$\Rightarrow \quad AC^2 = BC^2 - AB^2$

$\Rightarrow \quad AC^2 = 100 - 36 = 64$

$\Rightarrow \quad AC = 8 \text{ cm}$

$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 6 \times 8 \text{ cm}^2 = 24 \text{cm}^2$

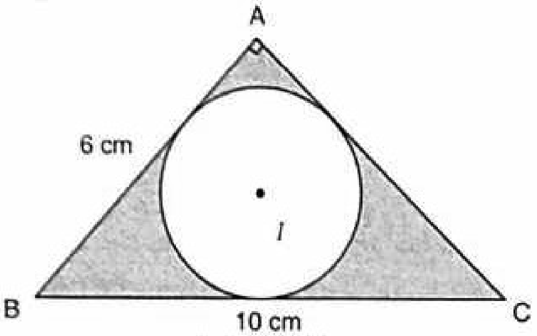

Fig. 13.62

Let r cm be the radius of the incircle. (circle inscribed in ΔABC). We observe that :

$$\text{Area of } \Delta ABC = \text{Area of } \Delta IBC + \text{Area of } \Delta ICA + \text{Area of } \Delta IAB$$

$\Rightarrow \quad 24 = \frac{1}{2}(BC \times r) + \frac{1}{2}(CA \times r) + \frac{1}{2}(AB \times r)$

$\Rightarrow \quad 24 = \frac{1}{2} r (BC + CA + AB)$

$\Rightarrow \quad 24 = \frac{1}{2} \times r \times (10 + 8 + 6)$

$\Rightarrow \quad 24 = 12r$

$\Rightarrow \quad r = 2$

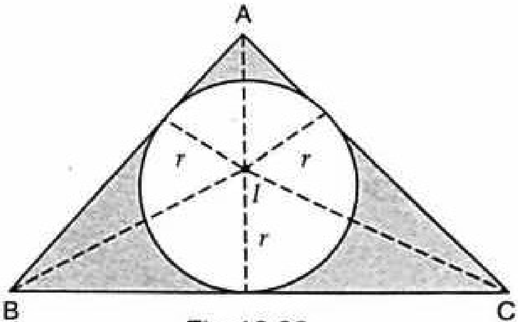

Fig. 13.63

Let A be the area of the shaded region. Then,

$$A = \text{Area of } \Delta ABC - \text{Area of the incircle}$$

$\Rightarrow \quad A = 24 - \pi r^2 = \left(24 - \frac{22}{7} \times 4\right) \text{cm}^2 = \frac{80}{7} \text{cm}^2$

LEVEL-2

EXAMPLE 33 *ABCD is a field in the shape of a trapezium. $AB \parallel DC$ and $\angle ABC = 90°$, $\angle DAB = 60°$. Four sectors are formed with centres A, B, C and D (See Fig. 13.64). The radius of each sector is 17.5 m. Find the*

(i) total area of the four sectors.

(ii) area of remaining portion given that $AB = 75$ m and $CD = 50$ m.

SOLUTION Since $AB \parallel CD$ and $\angle ABC = 90°$. Therefore $\angle BCD = 90°$. Also, $\angle BAD = 60°$.

$\therefore \quad \angle CDA = 180° - 60° = 120°$ [Co-interior angles]

(i) Let A be the total area of the four sectors. Then,

$$A = \text{Area of sector at } A + \text{Area of sector at } B + \text{Area of sector at } C + \text{Area of sector at } D.$$

$\Rightarrow \quad A = \frac{60}{360} \times \pi \times (17.5)^2 + \frac{90}{360} \times \pi \times (17.5)^2 + \frac{90}{360} \times \pi \times (17.5)^2 + \frac{120}{360} \times \pi \times (17.5)^2$

$\Rightarrow \quad A = \left\{\left(\frac{1}{6}+\frac{1}{4}+\frac{1}{4}+\frac{1}{3}\right)\times\pi\times(17.5)^2\right\} m^2$

$\Rightarrow \quad A = \pi\times\left(\frac{35}{2}\right)^2 m^2 = \frac{22}{7}\times\frac{35}{2}\times\frac{35}{2} m^2 = 962.5 m^2$

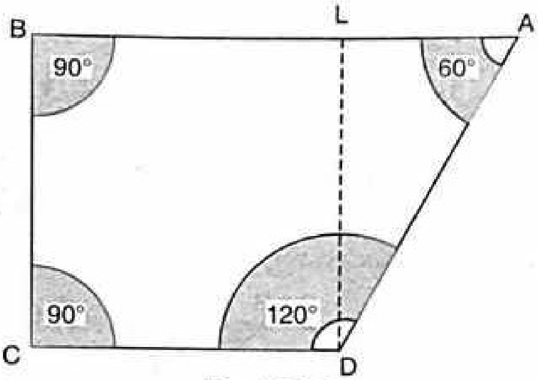

Fig. 13.64

(ii) Let DL be perpendicular drawn from D on AB. Then,

$$AL = AB - BL = AB - CD = (75 - 50) \text{ m} = 25 \text{ m}$$

In ΔALD, we have

$$\tan 60° = \frac{DL}{AL} \Rightarrow \sqrt{3} = \frac{DL}{25} \Rightarrow DL = 25\sqrt{3} \text{ m}$$

$\therefore$ Area of trapezium $ABCD = \frac{1}{2}(AB + CD)\times DL$

$$= \frac{1}{2}(75+50)\times 25\sqrt{3} \text{ m}^2 = 1562.5\times 1.732 \text{ m}^2 = 2706.25 \text{ m}^2$$

Let A be the area of the remaining portion. Then,

A = Area of trapezium $ABCD$ – Area of 4 sectors

$\Rightarrow \quad A = 2706.25 \text{ m}^2 - 962.5 \text{ m}^2 = 1743.75 \text{ m}^2$

EXAMPLE 34 *On a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 13.65. Find the area of the design (shaded region).* **[NCERT]**

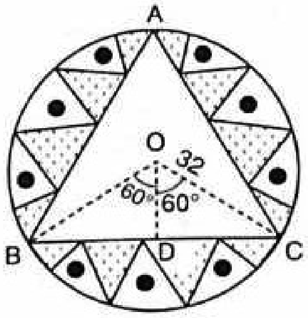

Fig. 13.65

SOLUTION In ΔOBD, we have

$$\cos 60° = \frac{OD}{OB} \text{ and } \sin 60° = \frac{BD}{OB}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{OD}{32} \text{ and } \frac{\sqrt{3}}{2} = \frac{BD}{32}$$

$\Rightarrow \quad OD = 16$ and $BD = 16\sqrt{3}$

$\Rightarrow \quad BC = 2\,BD = 32\sqrt{3}$

Let A be the area of the shaded region. Then,

$$A = \text{Area of the circle} - \text{Area of } \Delta\, ABC = \left\{\pi \times 32^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2\right\} \text{cm}^2$$

$$\Rightarrow \quad A = \left\{\frac{22}{7} \times 32 \times 32 - 768\sqrt{3}\right\} \text{cm}^2 = \left\{\frac{22528}{7} - 768\sqrt{3}\right\} \text{cm}^2$$

EXAMPLE 35 *In Fig. 13.66, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.* **[NCERT, CBSE 2010, 2013]**

SOLUTION Let A be the area of the shaded region. Then,

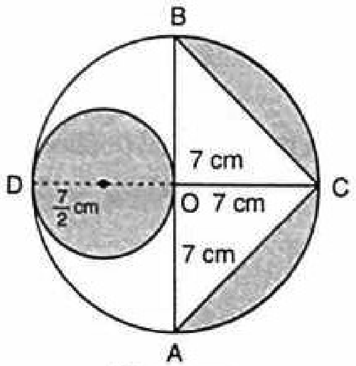

Fig. 13.66

A = (Area of circle with OD (= 7 cm) as diameter)

+ Area of semi-circle with AB as diameter − Area of $\Delta\, ABC$

$$\Rightarrow \quad A = \pi \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \pi \times (7)^2 - \frac{1}{2} \times AB \times OC = \left\{\frac{\pi}{4} \times 49 + \frac{\pi}{2} \times 49 - \frac{1}{2} \times 14 \times 7\right\} \text{cm}^2$$

$$\Rightarrow \quad A = \left(\frac{3\pi}{4} \times 49 - 49\right) \text{cm}^2 = \left(\frac{3}{4} \times \frac{22}{7} \times 49 - 49\right) \text{cm}^2 = \frac{231 - 98}{2} \text{cm}^2 = 66.5 \text{ cm}^2$$

EXAMPLE 36 *Calculate the area of the designed region in Fig. 13.67 common between two quadrants of circles of radius 8 cm each.* **[NCERT]**

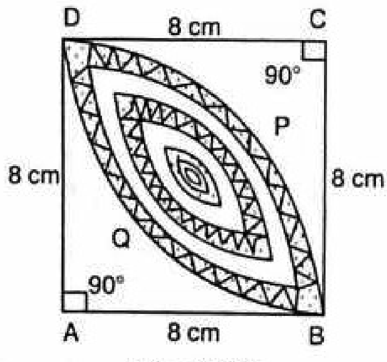

Fig. 13.67

SOLUTION Let A be the area of the shaded region. Then,

$$A = 2\,(\text{Area of quadrant } ABPD - \text{Area of } \Delta\, ABD)$$

$$\Rightarrow \quad A = 2\left\{\frac{\pi}{4} \times (8)^2 - \frac{1}{2} \times 8 \times 8\right\} \text{cm}^2$$

$\Rightarrow \quad A = 2\left\{\frac{22}{7} \times \frac{1}{4} \times 64 - 32\right\} \text{cm}^2$

$\Rightarrow \quad A = 2\left\{\frac{22 \times 16}{7} - 32\right\} \text{cm}^2 = 2\left(\frac{352 - 224}{7}\right) \text{cm}^2 = \frac{256}{7} \text{cm}^2$

EXAMPLE 37 *In Fig. 13.68, a crescent is formed by two circles which touch at A. C is the centre of the larger circle. The width of the crescent at BD is 9 cm and at EF it is 5 cm. Find (i) the radii of two circles (ii) the area of the shaded region.*

SOLUTION (i) Let the radii of the larger and smaller circles be R and r respectively. Then,

$BD = 9 \text{ cm} \Rightarrow 2R - 2r = 9 \Rightarrow R - r = 4.5 \quad \ldots(i)$

Join AE and DE. Let $\angle CAE = \theta$ Then, $\angle AEC = 90° - \theta$.

Now, $\quad \angle AED = 90° \Rightarrow \angle AEC + \angle DEC = 90° \Rightarrow \angle DEC = 90° - (90° - \theta) = \theta$.

Thus, in Δ's ACE and DCE, we have

$\angle CAE = \angle CED = \theta$ and $\angle ACE = \angle ECD = 90°$

So, by AA similarity criterion, we obtain

$\Delta ACE \sim \Delta ECD$

$\Rightarrow \quad \frac{AC}{EC} = \frac{CE}{CD}$

$\Rightarrow \quad \frac{AC}{CF - EF} = \frac{CF - EF}{BC - BD}$

$\Rightarrow \quad \frac{R}{R-5} = \frac{R-5}{R-9}$

$\Rightarrow \quad R(R-9) = (R-5)^2 \Rightarrow 0 = -R + 25 \Rightarrow R = 25 \text{ cm}$

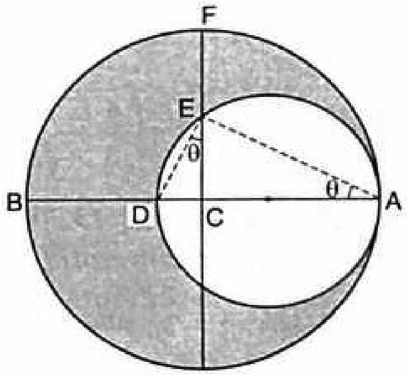

Fig. 13.68

Substituting the value of R in (i), we get

$25 - r = 4.5 \Rightarrow r = 20.5 \text{ cm}$

Hence, the radii of two circles are $r = 20.5$ cm and $R = 25$ cm.

(ii) Let A be the area of the shaded region. Then,

$$A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$$

$$= 3.14(25 + 20.5)(25 - 20.5) \text{ cm}^2 = 3.14 \times 45.5 \times 4.5 \text{ cm}^2 = 642.915 \text{ cm}^2$$

EXAMPLE 38 *In Fig. 13.69, three circles of radius 2 cm touch one another externally. These circle are circumscribed by a circle of radius R cm. Find the value of R and the area of the shaded region in terms of π and $\sqrt{3}$.*

SOLUTION Clearly, ΔABC is an equilateral triangle of side 4 cm.

In ΔBDO, we have

$\cos \angle OBD = \frac{BD}{OB}$

$\Rightarrow \quad \cos 30° = \frac{2}{OB} \quad [\because \angle OBD = 30°]$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{2}{OB}$

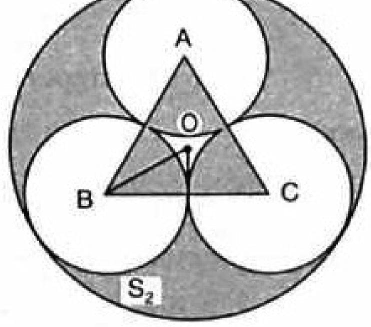

Fig. 13.69

$\Rightarrow \quad OB = \frac{4}{\sqrt{3}}$

$\therefore \quad OP = OB + BP \Rightarrow R = \left(\frac{4}{\sqrt{3}} + 2\right) \text{cm}$

Let A be the area of the shaded region. Then,

A = Area of the larger circle of radius $R - 3 \times$ Area of a smaller circle of radius 2 cm
+ 3 (Area of a sector of angle 60° in a circle of radius 2 cm)
− {Area of ΔABC − 3 (Area of sector of angle 60° in a circle of radius 2 cm)}

$\Rightarrow$ A = Area of the larger circle of radius $R - 3 \times$ Area of a smaller circle of radius 2 cm
+ 6 × Area of a sector of angle 60° in a circle of radius 2 cm − Area of $\Delta\, ABC$

$$\Rightarrow \quad A = \left\{\pi\left(\frac{4}{\sqrt{3}} + 2\right)^2 - 3 \times \pi \times 2^2 + 6 \times \left(\frac{60}{360} \times \pi \times 2^2\right) - \frac{\sqrt{3}}{4} \times 4^2\right\} \text{cm}^2$$

$$\Rightarrow \quad A = \left\{\pi\left(\frac{16}{3} + 4 + \frac{16}{\sqrt{3}}\right) - 12\pi + 4\pi - 4\sqrt{3}\right\} \text{cm}^2$$

$$\Rightarrow \quad A = \left\{\pi\left(\frac{4}{3} + \frac{16}{\sqrt{3}}\right) - 4\sqrt{3}\right\} \text{cm}^2 = \left\{\frac{4\pi}{3}(4\sqrt{3} + 1) - 4\sqrt{3}\right\} \text{cm}^2$$

EXAMPLE 39 *In Fig. 13.70, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 60°$. If BFEC is a sector of a circle with centre C and AB = BC = 7 cm and DE = 4 cm, then find the area of the shaded region (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$).* **[CBSE 2010]**

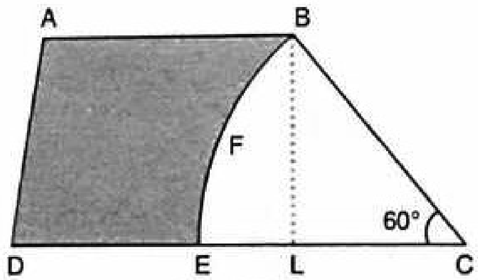

Fig. 13.70

SOLUTION Clearly, $CE = CB = 7$ cm.

$\therefore \quad CD = CE + ED = (7 + 4)$ cm $= 11$ cm

In ΔCLB, we have

$$\sin 60° = \frac{BL}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BL}{7} \Rightarrow BL = \frac{7\sqrt{3}}{2} \text{ cm}$$

$$\therefore \quad \text{Area of trapezium} = \frac{1}{2}(AB + CD) \times BL = \frac{1}{2}(7 + 11) \times \frac{7\sqrt{3}}{2} \text{cm}^2 = \frac{63\sqrt{3}}{2} \text{cm}^2$$

and,

$$\text{Area of sector } BFEC = \frac{60°}{360°} \times \frac{22}{7} \times 7^2 \text{ cm}^2 = \frac{77}{3} \text{cm}^2$$

Let A be the area of the shaded region. Then,

$$A = \left(\frac{63\sqrt{3}}{2} - \frac{77}{3}\right) \text{cm}^2 = (54.558 - 25.666) \text{ cm}^2 = 28.89 \text{ cm}^2$$

EXAMPLE 40 *With vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in Fig. 13.71. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region. (Use $\pi = 3.14$).*

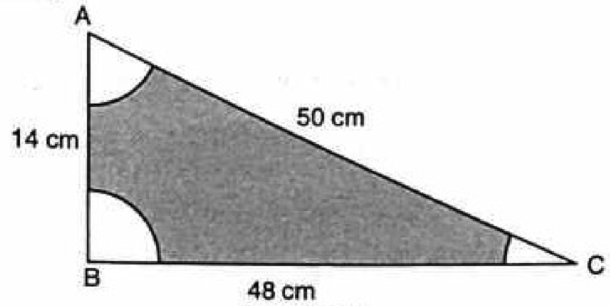

Fig. 13.71

SOLUTION In ΔABC, we have

$$a = BC = 48 \text{ cm}, b = CA = 50 \text{ cm and } c = AB = 14 \text{ cm}$$

Let s be the semi-perimeter of ΔABC. Then,

$$s = \frac{a+b+c}{2} = \frac{48+50+14}{2} = 56 \text{ cm}$$

Let Δ be the area of ΔABC. Then, by Heron's formula

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{56 \times 8 \times 6 \times 42} \text{ cm}^2 = 336 \text{ cm}^2$$

Let A_1, A_2 and A_3 be the areas of sectors with sector angles A, B and C respectively and sector radius $r = 5$ cm. Then,

$$A_1 = \frac{A}{360} \times \pi r^2 = \frac{A}{360} \times \pi \times 5^2 \text{ cm}^2 = \frac{A}{360} \times 25\pi \text{ cm}^2$$

$$A_2 = \frac{B}{360} \times \pi r^2 = \frac{B}{360} \times \pi \times 5^2 \text{ cm}^2 = \frac{B}{360} \times 25\pi \text{ cm}^2$$

$$A_3 = \frac{C}{360} \times \pi r^2 = \frac{C}{360} \times \pi \times 5^2 \text{ cm}^2 = \frac{C}{360} \times 25\pi \text{ cm}^2$$

$$\therefore \quad A_1 + A_2 + A_3 = \left(\frac{A}{360} \times 25\pi + \frac{B}{360} \times 25\pi + \frac{C}{360} \times 25\pi\right) \text{cm}^2$$

$$= (A+B+C) \times \frac{25\pi}{360} \text{cm}^2$$

$$= \frac{180}{360} \times 25\pi \text{ cm}^2 = \frac{25\pi}{2} \text{cm}^2 = \frac{25 \times 3.14}{2} \text{cm}^2 = 39.25 \text{ cm}^2$$

Let A be the area of the shaded region. Then,

$$A = \text{Area of } \Delta ABC - (A_1 + A_2 + A_3) = (336 - 39.25) \text{ cm}^2 = 296.75 \text{ cm}^2$$

REMARK *The above solution is the general solution. In this case, ΔABC is a right triangle right angled at B. So, its area can also be computed as follows:*

$$\Delta = \frac{1}{2}(BC \times AB) = \frac{1}{2} \times 48 \times 14 \text{ cm}^2 = 336 \text{ cm}^2$$

EXERCISE 13.4

LEVEL-1

1. A plot is in the form of a rectangle $ABCD$ having semi-circle on BC as shown in Fig. 13.72. If $AB = 60$ m and $BC = 28$ m, find the area of the plot.

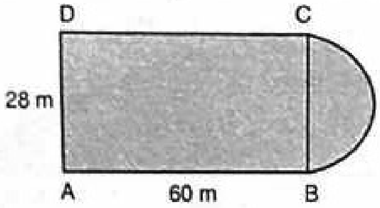

Fig. 13.72

2. A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36 m and 24.5 m, find the area of the playground. (Take $\pi = 22/7$).
3. Find the area of the circle in which a square of area 64 cm^2 is inscribed. [Use $\pi = 3.14$]
4. A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.
5. In Fig. 13.73, $PQRS$ is a square of side 4 cm. Find the area of the shaded square. [NCERT]

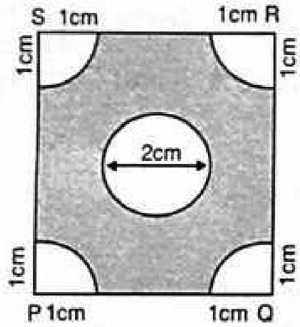

Fig. 13.73

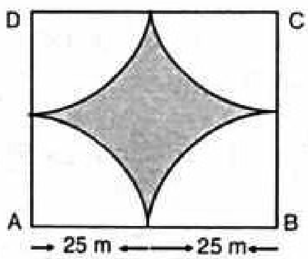

Fig. 13.74

6. Four cows are tethered at four corners of a square plot of side 50 m, so that they just cannot reach one another. What area will be left ungrazed? (Fig. 13.74) [CBSE 2018]
7. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20 m × 16 m, find the area of the field in which the cow can graze. [NCERT EXEMPLAR]
8. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze. [NCERT EXEMPLAR]
9. A square water tank has its side equal to 40 m. There are four semi-circular grassy plots all round it. Find the cost of turfing the plot at ₹ 1.25 per square metre (Take $\pi = 3.14$).
10. A rectangular park is 100 m by 50 m. It is surrounded by semi-circular flower beds all round. Find the cost of levelling the semi-circular flower beds at 60 paise per square metre (Use $\pi = 3.14$).
11. The inside perimeter of a running track (shown in Fig. 13.75) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If the track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.

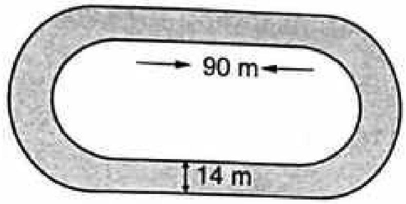

Fig. 13.75

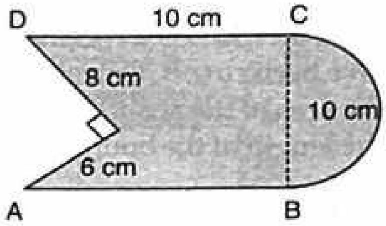

Fig. 13.76

12. Find the area of Fig. 13.76, in square cm, correct to one place of decimal. (Take $\pi = 22/7$).
13. In Fig. 13.77, from a rectangular region $ABCD$ with $AB = 20$ cm, a right triangle AED with $AE = 9$ cm and $DE = 12$ cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. (Use $\pi = 22/7$). **[CBSE 2014]**

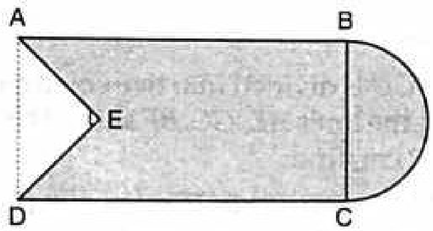

Fig. 13.77

14. From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the centre as shown in Fig. 13.78. Find the area of the remaining (shaded) portion of the square. (Use $\pi = 22/7$). **[CBSE 2010]**

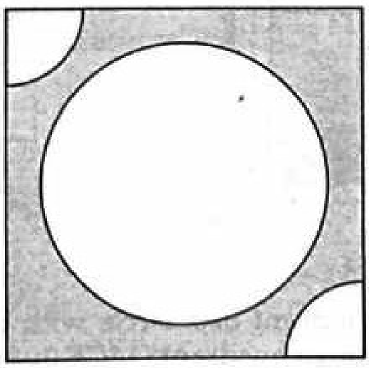

Fig. 13.78

15. In Fig. 13.79, $ABCD$ is a rectangle with $AB = 14$ cm and $BC = 7$ cm. Taking DC, BC and AD as diameters, three semi-circles are drawn as shown in the figure. Find the area of the shaded region.

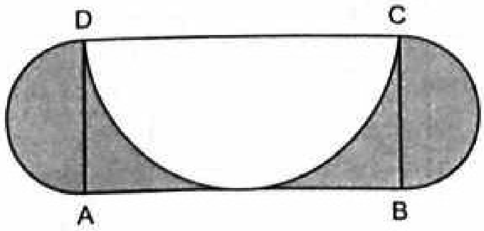

Fig. 13.79

16. In Fig. 13.80, *ABCD* is a rectangle, having $AB = 20$ cm and $BC = 14$ cm. Two sectors of $180°$ have been cut off. Calculate:
   (i) the area of the shadded region.
   (ii) the length of the boundary of the shaded region.

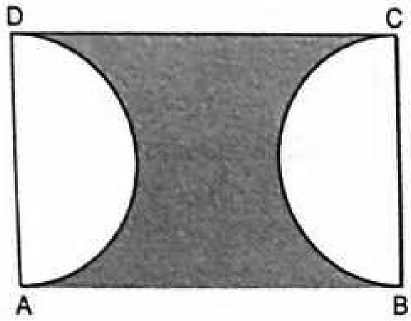

Fig. 13.80

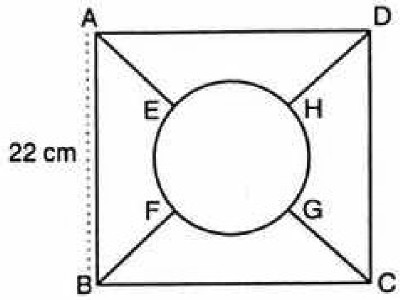

Fig. 13.81

17. In Fig. 13.81, the square *ABCD* is divided into five equal parts, all having same area. The central part is circular and the lines *AE, GC, BF* and *HD* lie along the diagonals *AC* and *BD* of the square. If $AB = 22$ cm, find:
   (i) the circumference of the central part. (ii) the perimeter of the part *ABEF*.

18. In Fig. 13.82, find the area of the shaded region. (Use $\pi = 3.14$). **[CBSE 2015]**

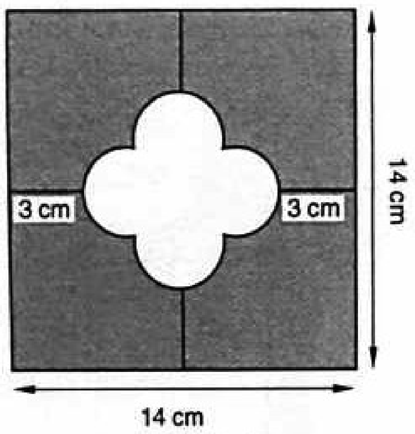

Fig. 13.82

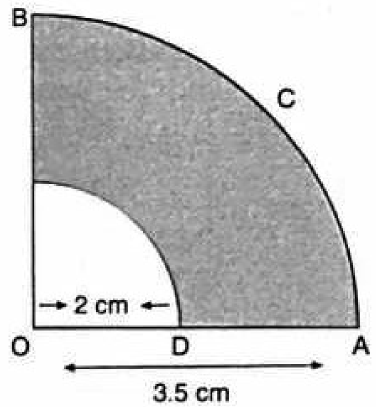

Fig. 13.83

19. In Fig. 13.83, *OACB* is a quadrant of a circle with centre *O* and radius 3.5 cm. If $OD = 2$ cm, find the area of the (i) quadrant *OACB* (ii) shaded region.

20. In Fig. 13.84, a square *OABC* is inscribed in a quadrant *OPBQ* of a circle. If $OA = 21$ cm, find the area of the shaded region. **[CBSE 2013, 2014]**

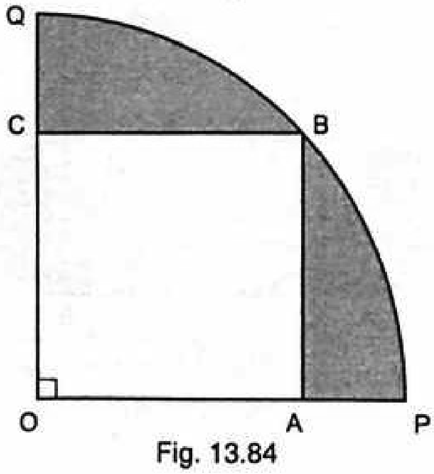

Fig. 13.84

21. In Fig. 13.85, $OABC$ is a square of side 7 cm. If $OAPC$ is a quadrant of a circle with centre O, then find the area of the shaded region. (Use $\pi = 22/7$) **[CBSE 2012]**

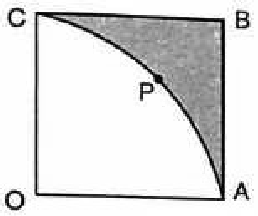

Fig. 13.85

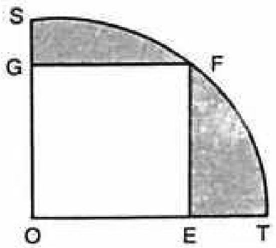

Fig. 13.86

22. In Fig. 13.86, $OE = 20$ cm. In sector $OSFT$, square $OEFG$ is inscribed. Find the area of the shaded region. **[CBSE 2013, 2014]**

23. Find the area of the shaded region in Fig. 13.87, if $AC = 24$ cm, $BC = 10$ cm and O is the centre of the circle. (Use $\pi = 3.14$) **[CBSE 2010]**

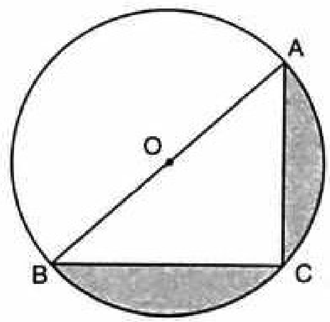

Fig. 13.87

24. A circle is inscribed in an equilateral triangle ABC is side 12 cm, touching its sides (Fig. 13.88). Find the radius of the inscribed circle and the area of the shaded part. **[CBSE 2014]**

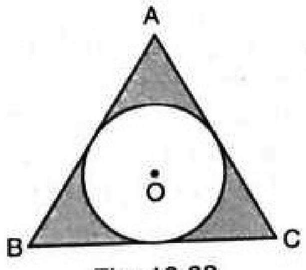

Fig. 13.88

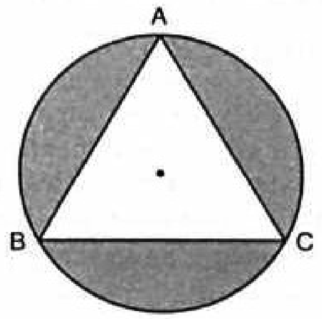

Fig. 13.89

25. In Fig. 13.89, an equilateral triangle ABC of side 6 cm has been inscribed in a circle. Find the area of the shaded region. (Take $\pi = 3.14$).

26. A circular field has a perimeter of 650 m. A square plot having its vertices on the circumference of the field is marked in the field. Calculate the area of the square plot.

27. Find the area of a shaded region in the Fig. 13.90, where a circular arc of radius 7 cm has been drawn with vertex A of an equilateral triangle ABC of side 14 cm as centre. (Use $\pi = 22/7$ and $\sqrt{3} = 1.73$) **[CBSE 2015, 2016]**

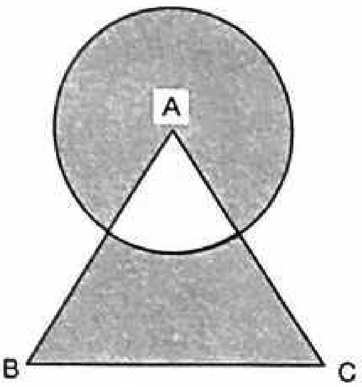

Fig. 13.90

28. A regular hexagon is inscribed in a circle. If the area of hexagon is $24\sqrt{3}$ cm^2, find the area of the circle. (Use $\pi = 3.14$) [CBSE 2015]

29. *ABCDEF* is a regular hexagon with centre *O* (Fig. 13.91). If the area of triangle *OAB* is 9 cm^2, find the area of: (i) the hexagon and (ii) the circle in which the haxagon is incribed.

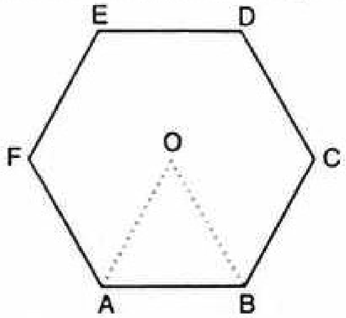

Fig. 13.91

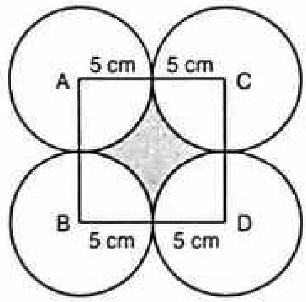

Fig. 13.92

30. Four equal circles, each of radius 5 cm, touch each other as shown in Fig. 13.92. Find the area included between them (Take $\pi = 3.14$).

31. Four equal circles, each of radius *a*, touch each other. Show that the area between them is $\frac{6}{7}a^2$. (Take $\pi = 22/7$).

32. A child makes a poster on a chart paper drawing a square *ABCD* of side 14 cm. She draws four circles with centre *A, B, C* and *D* in which she suggests different ways to save energy. The circles are drawn in such a way that each circle touches externally two of the three remaining circles (Fig. 13.93). In the shaded region she write a message 'Save Energy'. Find the perimeter and area of the shaded region. (Use $\pi = 22/7$) [CBSE 2015]

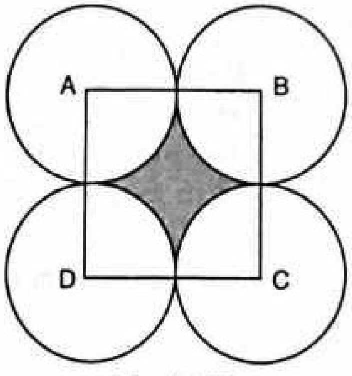

Fig. 13.93

33. The diameter of a coin is 1 cm (Fig. 13.94). If four such coins be placed on a table so that the rim of each touches that of the other two, find the area of the shaded region (Take $\pi = 3.1416$).

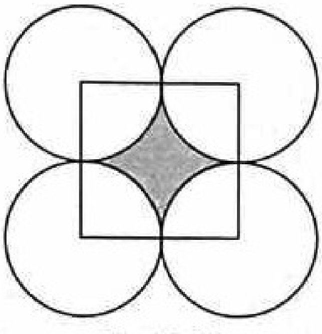

Fig. 13.94

34. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm × 7 cm. Find the area of the remaining card board. (Use $\pi = 22/7$) [CBSE 2013]

35. In Fig. 13.95, AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region.

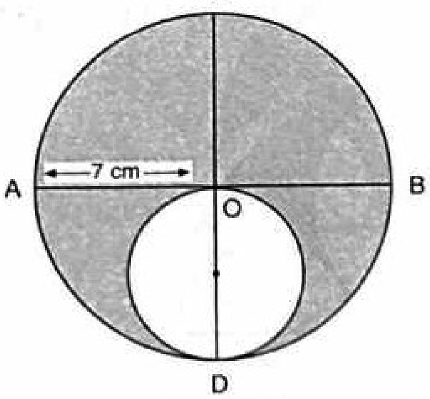

Fig. 13.95

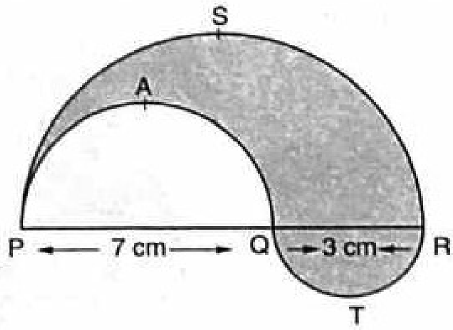

Fig. 13.96

36. In Fig. 13.96, PSR, RTQ and PAQ are three semi-circles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. [CBSE 2014]

37. In Fig. 13.97, two circles with centres A and B touch each other at the point C. If $AC = 8$ cm and $AB = 3$ cm, find the area of the shaded region.

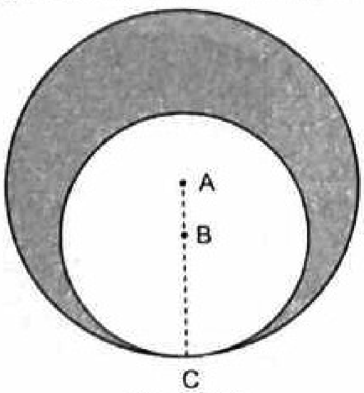

Fig. 13.97

38. In Fig. 13.98, $ABCD$ is a square of side $2a$. Find the ratio between
    (i) the circumferences
    (ii) the areas of the incircle and the circum-circle of the square.

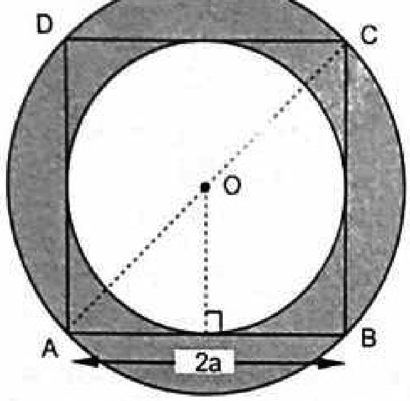

Fig. 13.98

39. In Fig. 13.99, there are three semicircles, A, B and C having diameter 3 cm each, and another semicircle E having a circle D with diameter 4.5 cm are shown. Calculate:
    (i) the area of the shaded region
    (ii) the cost of painting the shaded region at the rate of 25 paise per cm^2, to the nearest rupee.

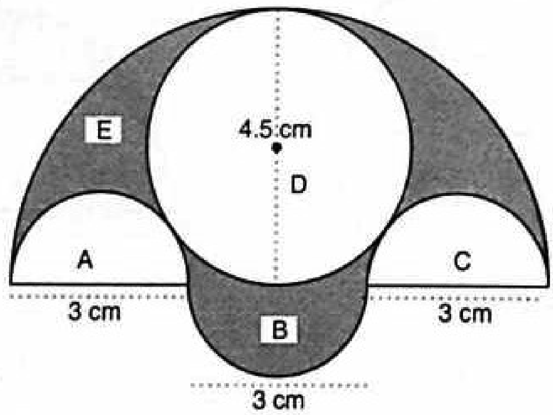

Fig. 13.99

40. In Fig. 13.100, ABC is a right-angled triangle, $\angle B = 90°$, $AB = 28$ cm and $BC = 21$ cm. With AC as diameter a semicircle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region correct to two decimal places.

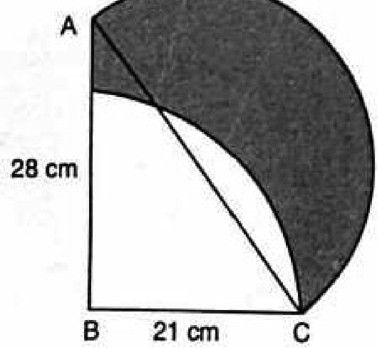

Fig. 13.100

41. In Fig. 13.101, O is the centre of a circular arc and AOB is a straight line. Find the perimeter and the area of the shaded region correct to one decimal place. (Take $\pi = 3.142$)

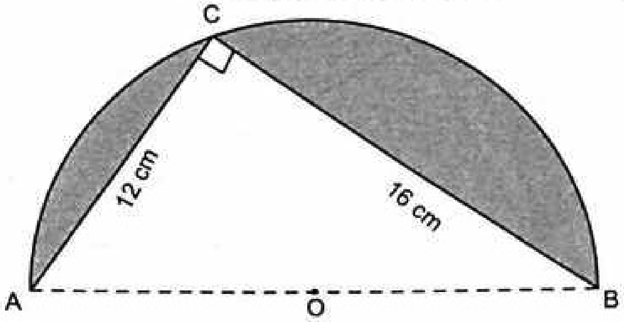

Fig. 13.101

42. In Fig. 13.102, the boundary of the shaded region consists of four semi-circular arcs, the smallest two being equal. If the diameter of the largest is 14 cm and of the smallest is 3.5 cm, find
   (i) the length of the boundary. (ii) the area of the shaded region.
   [CBSE 2010, 2016]

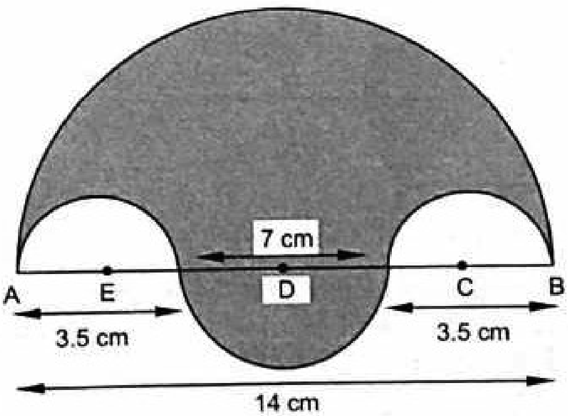

Fig. 13.102

43. In Fig. 13.103, $AB = 36$ cm and M is mid-point of AB. Semi-circles are drawn on AB, AM and MB as diameters. A circle with centre C touches all the three circles. Find the area of the shaded region.

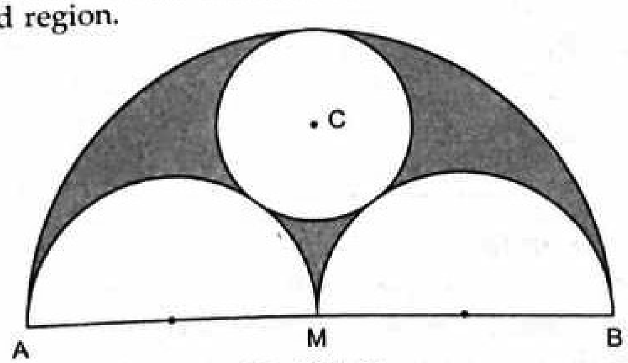

Fig. 13.103

44. In Fig. 13.104, ABC is a right angled triangle in which $\angle A = 90°$, $AB = 21$ cm and $AC = 28$ cm. Semi-circles are described on AB, BC and AC as diameters. Find the area of the shaded region.

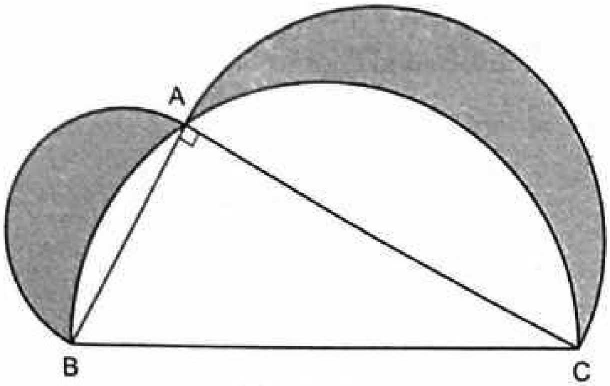

Fig. 13.104

45. Figure 13.105, shows the cross-section of railway tunnel. The radius OA of the circular part is 2 m. If $\angle AOB = 90°$, calculate:
 (i) the height of the tunnel
 (ii) the perimeter of the cross-section
 (iii) the area of the cross-section.

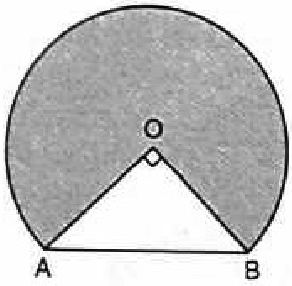

Fig. 13.105

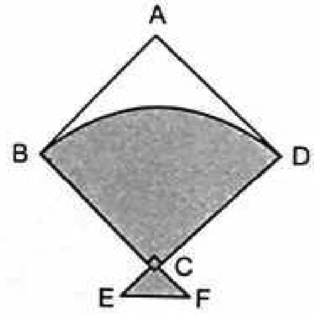

Fig. 13.106

46. Figure 13.106., shows a kite in which BCD is the shape of a quadrant of a circle of radius 42 cm. $ABCD$ is a square and $\Delta\, CEF$ is an isosceles right angled triangle whose equal sides are 6 cm long. Find the area of the shaded region.

47. In Fig. 13.107, $ABCD$ is a trapezium of area 24.5 cm^2. In it, $AD \parallel BC$, $\angle DAB = 90°$, $AD = 10$ cm and $BC = 4$ cm. If ABE is a quadrant of a circle, find the area of the shaded region. (Take $\pi = 22/7$). **[CBSE 2014]**

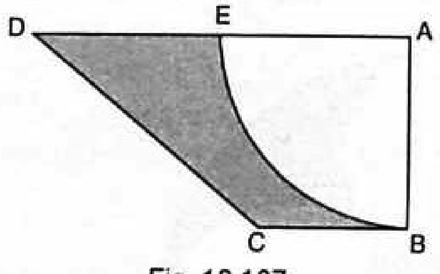

Fig. 13.107

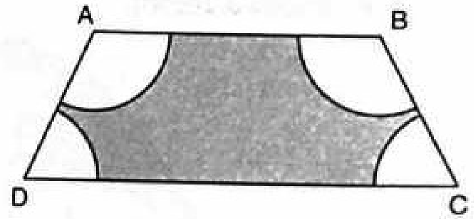

Fig. 13.108

48. In Fig. 13.108, $ABCD$ is a trapezium with $AB \parallel DC$, $AB = 18$ cm, $DC = 32$ cm and the distance between AB and DC is 14 cm. Circles of equal radii 7 cm with centres A, B, C and D have been drawn. Then, find the area of the shaded region of the figure. (Use $\pi = 22/7$). **[CBSE 2014]**

49. From a thin metallic piece, in the shape of a trapezium $ABCD$, in which $AB \parallel CD$ and $\angle BCD = 90°$, a quarter circle $BEFC$ is removed (see Fig. 13.109). Given $AB = BC = 3.5$ cm and $DE = 2$ cm, calculate the area of the remaining piece of the metal sheet.

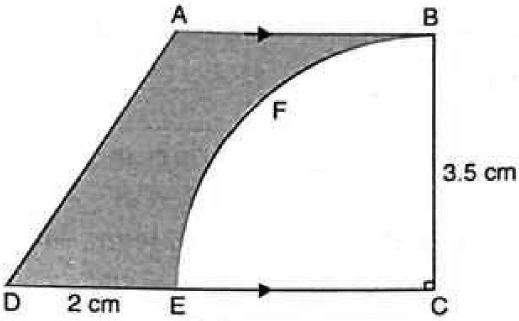

Fig. 13.109

50. In Fig. 13.110, ABC is an equilateral triangle of side 8 cm. A, B and C are the centres of circular arcs of radius 4 cm. Find the area of the shaded region correct upto 2 decimal places. (Take $\pi = 3.142$ and $\sqrt{3} = 1.732$).

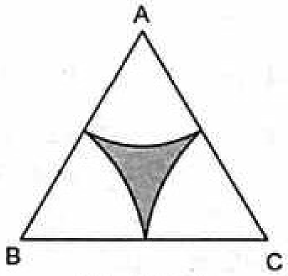

Fig. 13.110

51. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7 m each to graze in the field. Find the area of the field which cannot be grazed by three animals.

[NCERT EXEMPLAR]

52. In the given Fig. 13.111, the side of square is 28 cm, and radius of each circle is half of the length of the side of the square where O and O' are centres of the circles. Find the area of shaded region. **[CBSE 2017]**

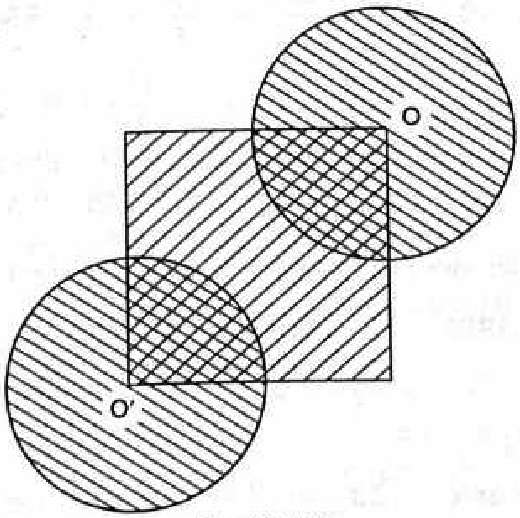

Fig. 13.111

53. In a hospital used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and

breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park? [CBSE 2017]

ANSWERS

1. 1988 m^2	2. 1353.625 m^2	3. 100.48 cm^2	4. 261.5 m^2
5. $(16 - 2\pi)$ cm^2	6. 535.71 m^2	7. 154 m^2	8. 75.625 m^2
9. ₹ 3140	10. ₹ 5887.50	11. 6216 m^2, 488 m	
12. 115.3 cm^2	13. 334.3125 cm^2	14. 5.48 cm^2	15. 59.5 cm^2
16. (i) 126 cm^2 (ii) 84 cm		17. (i) 34.88 cm (ii) 50.64 cm	
18. 154.88 cm^2	19. (i) 9.625 cm^2	(ii) 6.482 cm^2	20. 252 cm^2
21. 10.5 cm^2	22. 228 cm^2	23. 145.33 cm^2	24. $2\sqrt{3}$ cm, 24.638 cm^2
25. 22.126 cm^2	26. 21387 m^2	27. 187.44 cm^2	28. 50.24 cm^2
29. (i) 54 cm (ii) 65.23 cm^2		30. 21.5 cm^2	31. $\frac{6}{7}a^2$
32. Area = 42 cm^2, Perimeter = 44 cm		33. 0.2146 cm^2	34. 21 cm^2
35. 115.5 cm^2	36. 31.4 cm	37. 122.57 cm^2	38. (i) $1 : \sqrt{2}$ (ii) $1 : 2$
39. (i) 12.375 cm^2 (ii) ₹ 3		40. 428.75 cm^2	41. 59.4 cm, 61.1 cm^2
42. (i) 44 cm (ii) 86.625 cm^2		43. 45π cm^2	44. 294 cm^2
45. (i) $(2+\sqrt{2})$ m	(ii) $(3\pi + 2\sqrt{2})$ m	(iii) $(3\pi + 2)$ m^2	46. 1404 cm^2
47. 14.875 cm^2	48. 196 cm^2	49. 6.125 cm^2	50. 2.576 cm^2
51. $(24\sqrt{21} - 77)$ m^2	52. 3688 cm^2	53. π cm	

HINTS TO SELECTED PROBLEMS

17. Let the radius of the central part be r cm. Then,

Area of the central part $= \frac{1}{5} \times$ Area of the square

$$\Rightarrow \frac{22}{7} \times r^2 = \frac{1}{5} \times 22 \times 22 \Rightarrow r^2 = \frac{22 \times 7}{5} = \frac{154}{5} \Rightarrow r = 5.549 \simeq 5.55 \text{ cm}$$

(i) Circumference of central part $= 2\pi r = 2 \times \frac{22}{7} \times 5.55 = 34.88$ cm

(ii) Let O be the centre of the central part. Clearly, O is also the centre of the square.

$AE = BF = OA - OE = 11\sqrt{2} - 5.55 = 15.51 - 5.55 = 9.96$ cm

$EF = \frac{1}{4}$(Circumference of the circle) $= \frac{1}{4}(2\pi r) = \frac{1}{2}(\pi r) = \frac{1}{2} \times \frac{22}{7} \times 5.55 = 8.72$ cm

$\therefore$ Perimeter of part $ABEF = AB + AE + EF + BF = 22 + 2 \times 9.96 + 8.72$ cm $= 50.64$ cm

37. Required area $= (\pi \times 8^2 - \pi \times 5^2) = 39\pi$ $cm^2 = 122.57$ cm^2

38. $AC = \sqrt{2} \times 2a = 2\sqrt{2}\,a$

$\therefore$ Radius of larger circle $= \sqrt{2}\,a$ and, Radius of smaller circle $= a$

(i) Ratio of circumferences $= 2\pi a : 2\pi\sqrt{2}\,a = 1 : \sqrt{2}$

(ii) Ratio of area's $= \pi a^2 : \pi(\sqrt{2}a)^2 = 1 : 2$

41. Area of the shaded region = Area of semi-cricle with AB as diameter – Area of $\Delta\ ABC$

$$= \left(\frac{1}{2} \times \pi \times 10^2 - \frac{1}{2} \times 12 \times 16\right) \text{cm}^2 = 61.1 \text{ cm}^2$$

Perimeter of the shaded region $= (\pi \times 10 + 12 + 16)$ cm $= 59.4$ cm

42. (i) Length of the boundary $= \left\{\pi \times 7 + \pi \times \frac{7}{2} + \pi\left(\frac{7}{4}\right) + \pi\left(\frac{7}{4}\right)\right\}$ cm $= 14\pi$ cm $= 44$ cm

(ii) Area of the shaded region $= \frac{\pi}{2} \times 7^2 + \frac{\pi}{2} \times \left(\frac{7}{2}\right)^2 - \frac{\pi}{2} \times \left(\frac{7}{4}\right)^2 - \frac{\pi}{2} \times \left(\frac{7}{4}\right)^2$

$$= \frac{\pi}{2} \times 7^2\left(1 + \frac{1}{4} - \frac{1}{16} - \frac{1}{16}\right) = 86.625 \text{ cm}^2$$

43. Radius of circle with C as centre $= \frac{1}{6}AB = 6$ cm

$\therefore$ Area of the shaded region $= \frac{1}{2}\pi \times 18^2 - 2\left(\frac{1}{2} \times \pi \times 9^2\right) - \pi \times 6^2 = 45\pi \text{ cm}^2$

44. Area of the shaded region = Area of semi-circle with AB as diamater

+ Area of semi-circle with AC as diameter + Area of $\Delta\ ABC$

– Area of semi-circle with BC as diameter

$$= \frac{\pi}{2}\left\{\left(\frac{21}{2}\right)^2 + \left(\frac{28}{2}\right)^2\right\} + \frac{1}{2} \times 21 \times 28 - \frac{\pi}{2} \times \left(\frac{35}{2}\right)^2 = 294 \text{ cm}^2$$

46. Area of shaded region = Area of quadrant BCD + Area of $\Delta\ EFC$

$$= \frac{1}{4} \times \frac{22}{7} \times 42^2 + \frac{1}{2} \times 6 \times 6 \text{ cm}^2 = 1404 \text{ cm}^2$$

50. Area of shaded region = Area of $\Delta\ ABC$ – Area of 3 sectors of sector angle 60°

= Area of $\Delta\ ABC$ – Area of semi-circle of radius 4 cm

$$= \left(\frac{\sqrt{3}}{4} \times 8^2 - \frac{1}{2} \times 3.142 \times 4^2\right) \text{cm}^2 = (27.712 - 25.136) \text{ cm}^2 = 2.576 \text{ cm}^2$$

VERY SHORT ANSWER TYPE QUESTIONS (VSAQs)

Answer each of the following questions either in one word or one sentence or as per requirement of the questions:

1. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal?
2. If the circumference of two circles are in the ratio 2 : 3, what is the ratio of their areas?
3. Write the area of the sector of a circle whose radius is r and length of the arc is l.
4. What is the length (in terms of π) of the arc that subtends an angle of 36° at the centre of a circle of radius 5 cm?
5. What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3 π cm?

6. What is the area of a sector of a circle of radius 5 cm formed by an arc of length 3.5 cm?
7. In a circle of radius 10 cm, an arc subtends an angle of 108° at the centre. What is the area of the sector in terms of π?
8. If a square is inscribed in a circle, what is the ratio of the areas of the circle and the square?
9. Write the formula for the area of a sector of angle θ (in degrees) of a circle of radius r.
10. Write the formula for the area of a segment in a circle a circle of radius r given that the sector angle is θ (in degrees).
11. If the adjoining figure is a sector of a circle of radius 10.5 cm, what is the perimeter of the sector? (Take $\pi = 22/7$)

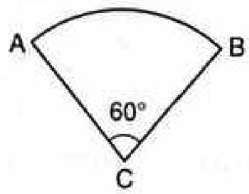

Fig. 13.112

12. If the diameter of a semi-circular protractor is 14 cm, then find its perimeter. **[CBSE 2009]**
13. An arc subtends an angle of 90° at the centre of the circle of radius 14 cm. Write the area of minor sector thus formed in terms of π.
14. Find the area of the largest triangle that can be inscribed in a semi-circle of radius r units. **[CBSE 2015]**
15. Find the area of a sector of circle of radius 21 cm and central angle 120°.
16. What is the area of a square inscribed in a circle of diameter p cm?
17. Is it true to say that area of a segment of a circle is less than the area of its corresponding sector? Why?
18. If the numerical value of the area of a circle is equal to the numerical value of its circumference, find its radius.
19. How many revolutions a circular wheel of radius r metres makes in covering a distance of s metres?
20. Find the ratio of the area of the circle circumscribing a square to the area of the circle inscribed in the square.

ANSWERS

1. $4\pi : \sqrt{3}$ 2. $4:9$ 3. $\frac{1}{2}lr$ 4. π cm 5. 90° 6. 8.75 cm^2 7. 30π cm^2

8. $\pi:2$ 9. $\frac{\theta}{360} \times \pi r^2$ 10. $\left(\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)r^2$ 11. 32 cm 12. 36 cm

13. 49π cm^2 14. r^2 15. 462 cm^2 16. $\frac{p^2}{2}$ cm^2

17. No; it is only true for minor segment.
18. 2 units
19. $\frac{s}{2\pi r}$
20. 2 : 1

MULTIPLE CHOICE QUESTIONS (MCQs)

1. If the circumference and the area of a circle are numerically equal, then diameter of the circle is
   (a) $\frac{\pi}{2}$ (b) 2π (c) 2 (d) 4
2. If the difference between the circumference and radius of a circle is 37 cm., then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is
   (a) 154 (b) 44 (c) 14 (d) 7 [CBSE 2013]
3. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be
   (a) 3520 cm^2 (b) 6400 cm^2 (c) 7744 cm^2 (d) 8800 cm^2
4. If a wire is bent into the shape of a square, then the area of the square is 81 cm^2. When wire is bent into a semi-circular shape, then the area of the semi-circle will be
   (a) 22 cm^2 (b) 44 cm^2 (c) 77 cm^2 (d) 154 cm^2
5. A circular park has a path of uniform width around it. The difference between the outer and inner circumferences of the circular path is 132 m. Its width is
   (a) 20 m (b) 21 m (c) 22 m (d) 24 m
6. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km will be
   (a) 2800 (b) 4000 (c) 5500 (d) 7000
7. The ratio of the outer and inner perimeters of a circular path is 23 : 22. If the path is 5 metres wide, the diameter of the inner circle is
   (a) 55 m (b) 110 m (c) 220 m (d) 230 m
8. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is
   (a) $50\sqrt{2}$ cm (b) $\frac{100}{\pi}$ cm (c) $\frac{50\sqrt{2}}{\pi}$ cm (d) $\frac{100\sqrt{2}}{\pi}$ cm
9. The area of the incircle of an equilateral triangle of side 42 cm is
   (a) $22\sqrt{3}$ cm^2 (b) 231 cm^2 (c) 462 cm^2 (d) 924 cm^2
10. The area of incircle of an equilateral triangle is 154 cm^2. The perimeter of the triangle is
    (a) 71.5 cm (b) 71.7 cm (c) 72.3 cm (d) 72.7 cm
11. The area of the largest triangle that can be inscribed in a semi-circle of radius r, is
    (a) r^2 (b) $2r^2$ (c) r^3 (d) $2r^3$
12. The perimeter of a triangle is 30 cm and the circumference of its incircle is 88 cm. The area of the triangle is
    (a) 70 cm^2 (b) 140 cm^2 (c) 210 cm^2 (d) 420 cm^2
13. The area of a circle is 220 cm^2. The area of a square inscribed in it is
    (a) 49 cm^2 (b) 70 cm^2 (c) 140 cm^2 (d) 150 cm^2

14. If the circumference of a circle increases from 4π to 8π, then its area is
(a) halved (b) doubled (c) tripled (d) quadrupled

15. If the radius of a circle is diminished by 10%, then its area is diminished by
(a) 10% (b) 19% (c) 20% (d) 36%

16. If the area of a square is same as the area of a circle, then the ratio of their perimeters, in terms of π, is
(a) $\pi : \sqrt{3}$ (b) $2 : \sqrt{\pi}$ (c) $3 : \pi$ (d) $\pi : \sqrt{2}$

17. The area of the largest triangle that can be inscribed in a semi-circle of radius r is
(a) $2r$ (b) r^2 (c) r (d) $\sqrt{r}$

18. The ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal, is
(a) $\pi : \sqrt{2}$ (b) $\pi : \sqrt{3}$ (c) $\sqrt{3} : \pi$ (d) $\sqrt{2} : \pi$

19. If the sum of the areas of two circles with radii r_1 and r_2 is equal to the area of a circle of radius r, then $r_1^2 + r_2^2$
(a) $> r^2$ (b) $= r^2$ (c) $< r^2$ (d) None of these

20. If the perimeter of a semi-circular protractor is 36 cm, then its diameter is
(a) 10 cm (b) 12 cm (c) 14 cm (d) 16 cm

21. The perimeter of the sector *OAB* shown in Fig. 13.113, is
(a) $\frac{64}{3}$ cm (b) 26 cm (c) $\frac{64}{5}$ cm (d) 19 cm

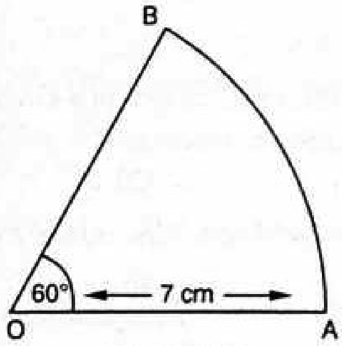

Fig. 13.113

22. If the perimeter of a sector of a circle of radius 6.5 cm is 29 cm, then its area is
(a) 58 cm^2 (b) 52 cm^2 (c) 25 cm^2 (d) 56 cm^2

23. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π cm^2, then its radius is
(a) 12 cm (b) 16 cm (c) 8 cm (d) 10 cm

24. The area of the circle that can be inscribed in a square of side 10 cm is
(a) 40π cm^2 (b) 30π cm^2 (c) 100π cm^2 (d) 25π cm^2

25. If the difference between the circumference and radius of a circle is 37 cm, then its area is
(a) 154 cm^2 (b) 160 cm^2 (c) 200 cm^2 (d) 150 cm^2

26. The area of a circular path of uniform width h surrounding a circular region of radius r is

(a) $\pi(2r+h)r$ (b) $\pi(2r+h)h$ (c) $\pi(h+r)r$ (d) $\pi(h+r)h$

27. If AB is a chord of length $5\sqrt{3}$ cm of a circle with centre O and radius 5 cm, then area of sector OAB is

(a) $\frac{3\pi}{8}$ cm^2 (b) $\frac{8\pi}{3}$ cm^2 (c) 25π cm^2 (d) $\frac{25\pi}{3}$ cm^2

28. The area of a circle whose area and circumference are numerically equal, is

(a) 2π sq. units (b) 4π sq. units (c) 6π sq. units (d) 8π sq. units

29. If diameter of a circle is increased by 40%, then its area increases by

(a) 96% (b) 40% (c) 80% (d) 48%

30. In Fig. 13.114, the shaded area is

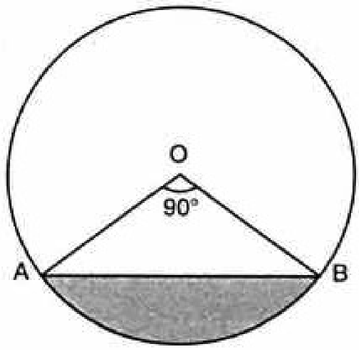

Fig. 13.114

(a) $50(\pi-2)$ cm^2 (b) $25(\pi-2)$ cm^2 (c) $25(\pi+2)$ cm^2 (d) $5(\pi-2)$ cm^2

31. In Fig. 13.115, the area of the segment PAQ is

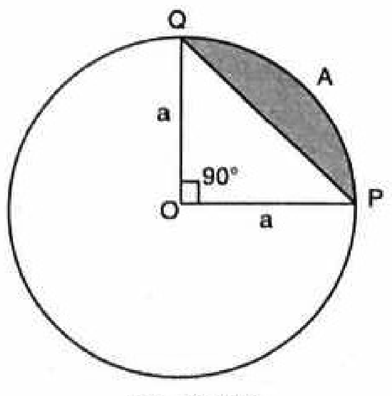

Fig. 13.115

(a) $\frac{a^2}{4}(\pi+2)$ (b) $\frac{a^2}{4}(\pi-2)$ (c) $\frac{a^2}{4}(\pi-1)$ (d) $\frac{a^2}{4}(\pi+1)$

32. In Fig. 13.116, the area of segment ACB is

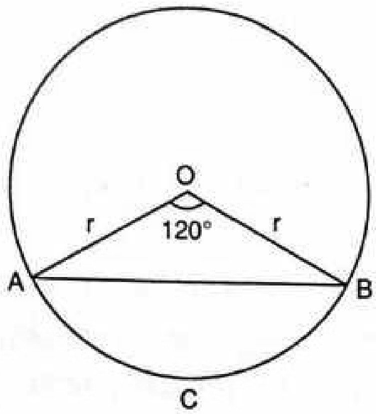

Fig. 13.116

(a) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)r^2$ (b) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)r^2$ (c) $\left(\frac{\pi}{3}-\frac{2}{\sqrt{3}}\right)r^2$ (d) None of these

33. If the area of a sector of a circle bounded by an arc of length 5π cm is equal to 20π cm^2, then the radius of the circle is
(a) 12 cm (b) 16 cm (c) 8 cm (d) 10 cm

34. In Fig. 13.117, the ratio of the areas of two sectors S_1 and S_2 is

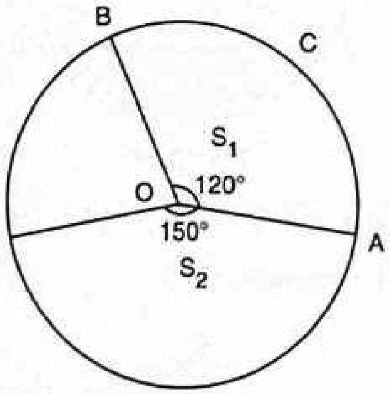

Fig. 13.117

(a) 5 : 2 (b) 3 : 5 (c) 5 : 3 (d) 4 : 5

35. If the area of a sector of a circle is $\frac{5}{18}$ of the area of the circle, then the sector angle is equal to
(a) 60° (b) 90° (c) 100° (d) 120°

36. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the sector angle is equal to
(a) 110° (b) 130° (c) 100° (d) 126°

37. In Fig. 13.118, if *ABC* is an equilateral triangle, then shaded area is equal to

(a) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)r^2$ (b) $\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)r^2$ (c) $\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)r^2$ (d) $\left(\frac{\pi}{3}+\sqrt{3}\right)r^2$

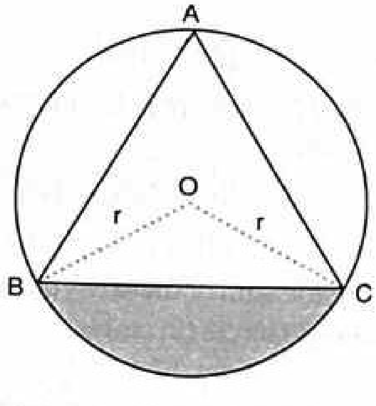

Fig. 13.118

38. In Fig. 13.119, the area of the shaded region is

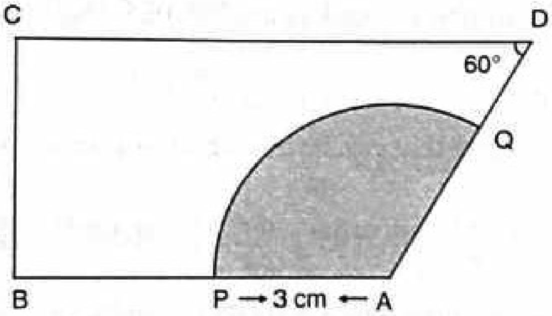

Fig. 13.119

(a) 3π cm^2 (b) 6π cm^2 (c) 9π cm^2 (d) 7π cm^2

39. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(a) 13 : 22 (b) 14 : 11 (c) 22 : 13 (d) 11: 14

40. The radius of a circle is 20 cm. It is divided into four parts of equal area by drawing three concentric circles inside it. Then, the radius of the largest of three concentric circles drawn is
(a) $10\sqrt{5}$ cm (b) $10\sqrt{3}$ cm (c) 10 cm (d) $10\sqrt{2}$ cm

41. The area of a sector whose perimeter is four times its radius r units, is
(a) $\frac{r^2}{4}$ sq. units (b) $2r^2$ sq. units (c) r^2 sq. units (d) $\frac{r^2}{2}$ sq. units

42. If a chord of a circle of radius 28 cm makes an angle of 90° at the centre, then the area of the major segment is
(a) 392 cm^2 (b) 1456 cm^2 (c) 1848 cm^2 (d) 2240 cm^2

43. If area of a circle inscribed in an equilateral triangle is 48π square units, then perimeter of the triangle is
(a) $17\sqrt{3}$ units (b) 36 units (c) 72 units (d) $48\sqrt{3}$ units

44. The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is
(a) 2.75 cm^2 (b) 5.5 cm^2 (c) 11 cm^2 (d) 10 cm^2

45. $ABCD$ is a square of side 4 cm. If E is a point in the interior of the square such that ΔCED is equilateral, then area of ΔACE is
(a) $2(\sqrt{3}-1)\ cm^2$ (b) $4(\sqrt{3}-1)\ cm^2$
(c) $6(\sqrt{3}-1)\ cm^2$ (d) $8(\sqrt{3}-1)\ cm^2$

46. If the area of a circle is equal to the sum of the areas of two circles of diameters 10 cm and 24 cm, then diameter of the larger circle (in cm) is
(a) 34 (b) 26 (c) 17 (d) 14 **[CBSE 2012]**

47. If π is taken as 22/7, the distance (in metres) covered by a wheel of diameter 35 cm, in one revolution, is
(a) 2.2 (b) 1.1 (c) 9.625 (d) 96.25 **[CBSE 2013]**

48. $ABCD$ is a rectangle whose three vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$. The length of one of its diagonals is
(a) 5 (b) 4 (c) 3 (d) 25 **[CBSE 2014]**

49. Area of the largest triangle that can be inscribed in a semi-circle of radius r units is
(a) r^2 sq. units (b) $\frac{1}{2}r^2$ sq. units (c) $2r^2$ sq. units (d) $\sqrt{2}r^2$ sq. units

50. If the sum of the areas of two circles with radii r_1 and r_2 is equal to the area of a circle of radius r, then
(a) $r = r_1 + r_2$ (b) $r_1^2 + r_2^2 = r^2$ (c) $r_1 + r_2 < r$ (d) $r_1^2 + r_2^2 < r^2$

51. If the sum of the circumferences of two circles with radii r_1 and r_2 is equal to the circumference of a circle of radius r, then
(a) $r = r_1 + r_2$ (b) $r_1 + r_2 > r$ (c) $r_1 + r_2 < r$ (d) None of these

52. If the circumference of a circle and the perimeter of a square are equal, then
(a) Area of the circle = Area of the square (b) Area of the circle < Area of the square
(c) Area of the circle > Area of the square (d) Nothing definite can be said

53. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is
(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11 : 14

ANSWERS

1. (d)	2. (b)	3. (c)	4. (c)	5. (b)
6. (d)	7. (c)	8. (c)	9. (c)	10. (d)
11. (a)	12. (c)	13. (c)	14. (d)	15. (b)
16. (d)	17. (b)	18. (c)	19. (b)	20. (c)
21. (a)	22. (b)	23. (c)	24. (d)	25. (a)
26. (b)	27. (d)	28. (b)	29. (a)	30. (b)

31. (b)	32. (b)	33. (c)	34. (d)	35. (c)
36. (d)	37. (a)	38. (a)	39. (d)	40. (b)
41. (c)	42. (a)	43. (d)	44. (b)	45. (b)
46. (b)	47. (b)	48. (a)	49. (a)	50. (b)
51. (a)	52. (c)	53. (b)		

SUMMARY

1. For a circle of a radius r, we have
   (i) Circumference $= 2\pi r$ (ii) Area $= \pi r^2$
   (iii) Area of semi-circle $= \frac{\pi r^2}{2}$ (iv) Area of a quadrant $= \frac{\pi r^2}{4}$
2. If R and r are the radii of two concentric circles such that $R > r$ then.
   Area enclosed by the two circles $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$
3. If a sector of a circle of radius r contains an angle of $\theta°$. Then,
   (i) Length of the arc of the sector $= \frac{\theta}{360} \times 2\pi r$
   $= \frac{\theta}{360} \times$ (Circumference of the circle)
   (ii) Perimeter of the sector $= 2r + \frac{\theta}{360} \times 2\pi r$
   (iii) Area of the sector $= \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times$ (Area of the circle)
   (iv) Area of the segment
   = Area of the corresponding sector – Area of the corresponding triangle
   $$= \frac{\theta}{360} \times \pi r^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$$